Econ 702, Spring 2006
Problem set 9
Due Tuesday March 28th

Problem 1 *Consider the following growth problem*

$$\sum_{t=0}^{\infty} \beta^t N_t u\left[\frac{C_t}{N_t}\right]$$

subject to

$$C_t + K_{t+1} = F\left(K_t, N_t\right)$$

The number of individuals is growing at rate γ. Set $N_0 = 1$ and define $\hat{x} = \gamma^{-t} x_0$. Rewrite the growth problem with hats and look for the steady state. At the SS, what is the growth rate of all variables?

Problem 2 *The FOC of the usual growth problem can be expressed like this*

$$\frac{u_c\left(c_t\right)}{u_c\left(c_{t+1}\right)} = \beta\left(1 + r\right)$$

We saw in class that with CRRA preferences, we could find an expression for the stead state of the form $1 = \beta\gamma^{-\sigma}\left(1 + r\right)$. Show that if we use the same CRRA preferences but with a minimum level of consumption needed (i.e., $\hat{u}\left(c\right) = \frac{(c-b)^{1-\sigma}-1}{1-\sigma}$ with $b > 0$) the result doesn't go through.

Problem 3 *Show that CRRA preferences have constant elasticity of intertemporal substitution equal to σ, which is equal to the coefficient of relative risk aversion.*

Problem 4 *Imagine an economy where there is a total factor productivity growth, i.e., the production function is*

$$Y_t = A_t F\left(K_t, N_t\right)$$

where $A_t = \gamma^t A_0$. Show that this economy exhibits a balanced growth path and calculate the rate at which all variables grow. Is it equal to γ?

Problem 5 *Imagine a Lucas-tree model with CRRA utility and where the fruit is constant every period ($d_t = 1\ \forall t$). Price this tree. Now, assume that the fruit starts growing at rate γ. Calculate the price of the new tree. Is it bigger/smaller than before?*