Dating as Learning

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Abstract

This paper extends the Becker (1973) and Burdett-Coles (1997) models of marriage markets by assuming that, on initial contact, people do not know the payoff to marriage. Instead, they receive a signal of this payoff, and can choose between marrying, separating, or entering a temporary relationship called dating. Within this context, people learn characteristics about the match over time, and one person may know his or her true value of the match while the other does not know. I characterize the resulting equilibrium.

1 Introduction

In this study, I investigate the situation where singles meet, but they do not learn everything about the payoff to marrying the other. Instead, they simply receive a noisy signal. Given two singles meet and learn the noisy signal about the payoff to marriage, they can choose one of three options: 1) they can separate, 2) they can choose to marry immediately, or 3) they can choose to date. If they date or marry, each accrues information about the other over time, which may change the expected payoff to marriage to at least one person. Marriage yields a greater utility flow, but subsequent separation is more costly, relative to dating. As time continues, one person may discover his or her true value of the match while the other does not yet know.

There is currently a large literature on marriage markets with and without frictions (see, for example, Becker (1973), Burdett-Coles (1997), Shimer-Smith (2000)). In these studies it is typically assumed that when two singles meet, they instantly learn all relevant information to make the marriage decision there and then. In some of the existing literature, matches are temporary, and in other studies, matches are

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permanent. While the nature of the matching environment can dictate the permanence of a match, there are times when agents can control the ease of its termination. This paper studies this choice given the uncertain payoff to a permanent match.

In Jovanovic (1979), workers and firms match based on a noisy signal of their marginal productivity. As information about the match was revealed, they could choose to terminate this match if marginal productivity fell below a specific reservation level. This is, therefore, a temporary match. A temporary match can be terminated when new information demonstrates that it is no longer worthwhile to maintain that match. These matches require no testing to determine the accuracy of the match quality signal. This type of match is a fairly accurate depiction of what happens in many real life scenarios. A job distributing flyers in a public place is one that a firm can terminate quickly if it is not done well. This does not require a testing state, and if work is not adequate, it is often simple to terminate the match. Jovanovic's model may describe this labor market quite well. Under different circumstances, a model of permanent matches would be more appropriate, due to the fact that termination could be quite costly or impossible. In some European labor markets, the firm's cost of terminating the work is so high that matching is effectively permanent. However, matches often fail to fall into either extreme category, and many times there is a testing stage (or temporary matching stage) that precedes a permanent match. For example, a college or university will often hire an assistant professor and evaluate his or her performance over a number of years before deciding whether to offer tenure and make this match permanent. In that case, testing is an essential stage before forming a permanent match and the level of testing is a decision that the college or university must decide. Alternatively, there are times when both agents get little to no benefit from the testing state. Other times, this process can provide benefit to both agents while allowing them to acquire information about the match over time.

Although this paper utilizes a marriage market framework, the results obtained can easily be applied to a labor market framework. It is reasonable to argue that a typical firm does not know all about a potential employee when they first meet. Likewise, a worker does not have full information about the benefits that be acquired if employment is accepted. Both know, however, that they will learn over time the true payoff to the match if they form an employment relationship. Further, it is costly at least to the firm, if it wants to separate after the worker begins to work there. In such a situation, the firm and worker may choose to form a temporary relationship and test rigorously before forming a permanent relationship. Whether this type of model describes the market depends on whether a state can be formed in which separating is costless and information can be acquired. In those European labor markets in which separating an employment relationship is very costly, firms test carefully before hiring a worker in order to decrease the likelihood that they will regret this decision, as this worker will likely remain with them for a long time. This results in an increased duration of vacancies in Europe, as compared to the United States where terminating a match is easier.

Many similar environments allow agents to choose how permanent a match can be. As mentioned earlier, a worker and a firm may choose to learn about each other, either through an interviewing process or a trial hiring period without a long-term contract, and would form a permanent match only after doing so. Naturally, different types of business partnerships can vary in their degree of commitment as well. Two internet websites can agree to advertise for each other on a given day or can sign a contract to permanently advertise for each other. In each of these decisions, the permanence of the match is the choice of the agents.

Consider now a man and woman who are attracted to each other as potential permanent partners. People date for two primary reasons—because they enjoy it, and because they want to learn about a potential permanent partner. The decision to date is to form a less committed partnership, and is often chosen to learn about a potential permanent partner as well. The decision to marry is to form a permanent partnership, or at least one which is difficult to terminate.

Permanent partnerships often provide more benefit than less committed ones in the short term as well as the long term. A married couple can spend more time together than a couple who is dating, enjoy legal benefits unavailable to unmarried couples, benefit from sharing costs on housing and utilities, and enjoy social advantages of their status. With a long term contract, a firm can provide training for its worker to increase future productivity for the firm and provide higher wages for the worker as a result. Without a long term contract to increase the likelihood of the firm benefitting from this investment, the firm may not be willing to invest in that worker and the match may be less productive for them both.

There are trade-offs that agents must balance in deciding between making temporary or permanent matches. With a permanent match, agents benefit from a larger match surplus. With a temporary match, agents benefit from the ability to terminate a match that is no longer desirable. When a couple decides whether to date or marry, each person considers the current benefits of making their match permanent and any potential regret that they may face if the marriage ceases to be beneficial to them. They must also consider the impact of their partner's decision making process. For example, the other partner may wish to separate if they do not marry. Thus, the first partner may choose to make the partnership permanent to prevent this possibility.

Economists have studied marriage for decades. Indeed, the decision theoretic aspect of maximizing utility subject to resource and informational constraints lends itself quite well to economics. The usefulness of modeling marriage is obvious not only when we look at its effect on economic issues such as labor force participation, population growth, and housing, but also when applying these marriage models to the study of matching. These matching models have the potential for generating insight into labor markets, goods markets, and business partnerships. Becker (1973) first established an application of economic analysis to marriage. His model did not include frictions. Later work often focused on search frictions (e.g. Burdett-Coles (1997), Shimer-Smith (2000)), as a person deciding to marry must consider who else they may have the opportunity to marry in the future. Informational frictions also play a large role in marriage, since when initially meeting a potential partner, it is difficult to accurately assess the quality of the match. Learning about the match quality over time through the process of dating can help in making informed decisions. This paper focuses on the informational frictions and search frictions in marriage markets. The trade-off that is highlighted is whether to sacrifice utility now in exchange for easier separation later.

I assume nontransferable utility. While this is a more reasonable assumption in some matching frameworks than in others, it is quite appropriate in the context of marriage. One cannot pay his or her spouse a sum of money to remain in love. While bargaining may take place over some aspects of a marriage, it is difficult to enforce. In the labor market, nontransferable utility may or may not be reasonable. The wage of a job itself is often set by an external mechanism such as a union contract. Even if there is not an official wage set in place, firms play a repeated game with each other when hiring similar workers and may explicitly or implicitly collude to fix a certain wage. Alternatively, firms may pay the same wage to an entire set of workers, and offering a higher wage than this to one worker may require raising the wage of too many other workers to make doing so worthwhile. Therefore, as wages are not perfectly negotiable and without externalities, it is important that much of the benefit from a job comes from its nonmonetary qualities since that generates nontransferable utility. Workers value the culture of the firm they select, their coworkers, the location, the prestige associated with working at the firm, and experience that they may receive that allows them to acquire higher paying jobs in the future.

However, the assumption of nontransferable utility does not remove the importance one places on his or her partner's value for being matched. When a couple decides to date and maintain a temporary match, each partner forgoes some utility that he or she would have received while married in order to maintain the flexibility to terminate the match if he or she no longer deems it desirable. However, in doing so, he or she is also providing the same flexibility to his or her partner. In that sense, understanding the probability that one's partner will be willing to marry him or her in the future is important. At times, the couple will prefer different relationship states, and I resolve this in a game theoretic framework. This paper discusses when a couple will agree to date.

The set of signals for which couples date is not quite intuitive. One might expect that the best signals lead to marriages, the worst signals lead to separation, and those signals in the middle lead couples to date to learn the true value of the match. However, an individual must consider his or her partner's willingness to marry in the future. The lower one's partner's signal is, the less one values dating. Acquiring information about the match for oneself is not necessarily worth the risk of losing one's partner altogether. As a result, an individual may propose marriage when his or her partner has a lower match value, but will agree to date if his or her partner has a higher match value. Therefore, inferior matches may actually marry earlier. The fear of one's partner terminating a match actually increases the probability of marriage. This has implications in many labor markets and other matching environments if there is some form of nontransferable utility. A firm may offer a worker a contract early in the hiring process, because it fears the worker may actually be able to match better with another firm. The worker may choose the security of accepting that offer even if he or she would prefer to wait longer to learn about his or her personal value of the match. Firms may choose to merge earlier if one firm fears the other may end up rejecting the merger with more information. Even if more efficient mergers may be possible, and even if the firm who is proposing the merger might want to learn more information before committing to the match, the firm may choose to secure the merger rather than risk the result of whatever potential information the other may learn (even if there is no private information). It is typical in matching models to assume that all information is known at the time agents initially meet, but this result highlights that the timing of the revelation of information has a large effect on the resulting equilibrium.

Furthermore, if one person knows that his or her partner is aware that he or she would be willing to marry in the future, dating becomes less attractive to that person. The value of dating is that one can learn information about the potential match. However, if one is less likely to be able to learn this information, as his or her partner may propose marriage earlier, dating itself becomes less appealing now. A couple may actually separate that would have dated if one partner had a lower signal of match quality, as that partner would have believed that dating would give them more time to learn about the match (if both partners knew he or she was unwilling to marry). Similarly, a worker may not be willing to agree to a temporary match if they believe the firm will soon offer them an exploding contract. However, if the worker and firm both know that the worker would reject that contract, the worker will be more inclined to take a temporary position, believing an ultimatum is less likely to occur. Once again, the timing of the revelation of information affects which matches are made, and the resulting equilibrium is less efficient than if all information could be learned immediately.

In Section 2, I present the model and establish the properties of marriage, dating, and singleness, as well as the variables that affect those decisions. In Section 3, I present the equilibrium strategies and outcomes when people have full information about their potential partners, followed by the equilibrium strategies and outcomes when only one person in a couple has full information about his or her potential partner. In Section 4, I discuss what happens when neither person has received full information about his or her partner. Section 5 concludes, and discusses future applications of this model.

2 Model

Suppose Jack is looking to get married, as is Jill. To achieve their goals, both search for a partner. An individual can be in one of three states: single, dating, or married. All agents live forever and discount the future at rate r. Let V_s denote the expected return to any single individual looking for a spouse, which I assume is exogenous and the same for both men and women. Singles date in order to accumulate information about potential life partners. Dating is exclusive, so the only way for a person to find another partner is to leave his or her current partner. An individual will date both to learn about his or her potential life partner and because he or she enjoys dating that person.

When Jack and Jill contact each other, each receives a noisy signal of his or her value for marrying the other, and knows both his or her own signal and his or her partner's signal. Jack's signal, s, and Jill's signal, s', are positive real numbers which are unbiased estimators of Jack and Jill's flow utilities of marrying each other. One can think of this signal as summarizing one's "first impression" of how he or she feels about marrying a potential partner. Upon learning this signal, potential partners decide between three choices: dating, getting married, or separating.

Suppose that Jack and Jill choose to date. For a given signal, s, Jack only receives flow utility $\rho s dt$ in a period of length dt, where $0 < \rho < 1$. If they choose to marry, Jack receives flow utility sdt in a period of length dt.

When Jack and Jill are dating, each learns his or her true feeling about the other with probability αdt in a given period of length dt. Jack's true feeling takes the form $f = s + e$, and is a positive real number. The distribution function of f is $G(f|s)$, with associated density function $g(f|s)$, where e is independent of s, $E\{e\} = 0$, and $g(.|.)$ is log concave. As time is continuous, Jack and Jill learn their true feelings at different times with probability 1. After Jack or Jill learns his or her true feeling, each can then reassess the situation and decide again if they want to marry, date, or separate. If they choose to date after one of them is fully informed about his or her true feeling about the other, but the other knows only his or her signal, they continue to do so until they are both fully informed, at which point they then choose again between dating, marriage, and separating. I establish the details of their payoffs to each later, but their preferences over relationship states assume that they are able to obtain that state (e.g., If Jack prefers dating Jill to marrying her or separating, this explicitly means that Jack prefers dating Jill when she will date him to marrying her when she will marry him and to separating when he is able to do this.)

There are a number of ways to construct a game in which the relationship state is determined. Without specifying exact rules of the game, the game rules should ensure that the result has the following properties:

1. Anyone who would like to become single can do so at any time, unless he or she is married.

2. No relationship state should ever be selected if a different relationship state is preferred by both partners.

3. When one partner prefers to get married, but prefers dating to being single, and when the other partner prefers to date, but prefers marriage to being single, then the couple will marry.

The third rule is debatable, but I have selected it for mathematical simplicity. The results and ideas are similar for the alternative approach, assuming that those couples date.

A game with these three rules generates the following outcomes:

1st/2nd/3rdchoice	**M/D/S**	**M/S/D**	**D/M/S**	**D/S/M**	**S/M/D** or **S/D/M.**
M/D/S	M	M	$\mathcal{M}$	$\mathcal{D}$	S
M/S/D	M	M	$\mathcal{M}$	$\mathcal{S}$	S
D/M/S	$\mathcal{M}$	$\mathcal{M}$	D	D	S
D/S/M	$\mathcal{D}$	$\mathcal{S}$	D	D	S
S/M/D or **S/D/M**	S	S	S	S	S

If Jack and Jill decide to marry, they will not be allowed to divorce. For that reason, they do not care what each other feels for them if they are married. There are four possible states of information in which Jack and Jill can be when married: (a) Jack and Jill may both know their true feelings, (b) Jack and Jill may both know only their signals, (c) Jack may know his true feeling while Jill only knows her signal, or (d) Jack may only know his signal while Jill knows her true feeling.

Let the man's value of dating be denoted by $V_d(j, k')$ and the women's value of dating be denoted by $V_d(k', j)$ where $(j, k') \in \{f, s\} X \{f', s'\}$. As agents live forever and never get divorced, it must be that Jack's (Jill's) value of marriage is f/r (f'/r) or s/r (s'/r). What remain to be analyzed are the value functions and choices made when a couple chooses to date. This is discussed in the next two sections.

3 Analysis of dating when at least one partner knows true feeling

Suppose Jack and Jill know their true feelings about the other. Jack's utility per period if they marry is f, whereas Jill obtains f'. As they both know their true feelings about the other, there is no point in dating, since there is nothing new to learn and the utility flow is less than in marriage. Hence, they both agree to marry if and only if $f, f' \geq f^*$, where $f^*/r = V_s$. If one of them does not want to marry, they separate and continue to search for a partner.

Suppose now that Jack knows his true feelings about Jill, but Jill has not learned her true feelings about Jack yet. Again, each obtains payoff V_s, if they separate. If they choose to marry, then Jack's expected return is f/r, whereas Jill's expected return is f'/r. They can, of course, choose to date, in which case Jack obtains utility flow $\rho f dt$ in dt, and Jack's return can be written as

$$(1+rdt)V_d(f,s') = \rho f dt + \alpha dt[(1-G(f^*|s'))\max\{\frac{f}{r}, V_s\} + G(f^*|s')V_s] + (1-\alpha dt)V_d(f,s')$$

$$rV_d(f,s') = \rho f + \alpha[1-G(f^*|s')]\max\{\frac{f}{r}, V_s\} + \alpha G(f^*|s')V_s - \alpha V_d(f,s')$$

Jack obtains utility flow ρf per unit of time until Jill learns her true feelings about Jack. When she does, with probability $(1-G(f^*|s'))$ she will then want to marry Jack, and with probability $G(f^*|s')$ she will choose to separate. Hence, the only situation in which Jack can make a choice after Jill learns her true feeling arises when Jill wants to marry him. Of course, Jack's answer is immediately obvious, and therefore Jack's return to dating can be written as

$$rV_d(f,s') = \begin{cases} \rho f + \alpha[V_s - V_d(f,s')], \text{ if } f < f^* \\ \rho f + \alpha[(1-G(f^*|s'))\frac{f}{r} + G(f^*|s')V_s - V_d(f,s')], \text{ if } f \geq f^* \end{cases}$$

and therefore

$$(\alpha + r)V_d(f,s') = \begin{cases} \rho f + \alpha \frac{f^*}{r}, \text{ if } f < f^* \\ \rho f + \alpha[1-G(f^*|s')][\frac{f}{r} - V_s] + \alpha V_s, \text{ if } f \geq f^* \end{cases}$$

At f^*, $(\alpha + r)V_d(f^*,s') = [\rho r + \alpha](f^*/r)$. Hence, $V_s = f^*/r > V_d(f^*,s')$, i.e., at f^*, Jack prefers to be single rather than date. Clearly, $V_d(.,s')$ is strictly increasing in f. Define $f^d(s')$ by $V_d(f^d(s'),s') = V_s$. It follows immediately that

$$f^d(s') = \frac{[r + \alpha(1-G(f^*|s'))]}{[r\rho + \alpha(1-G(f^*|s'))]}f^*$$

Since $\frac{[r+\alpha(1-G(f^*|s'))]}{[r\rho+\alpha(1-G(f^*|s'))]} > 1$, then $f^d(s') > f^*$ and $f^d(s') > 0$.

Given (f,s'), Jack's behavior can be described as follows:

- If $f \geq f^d(s')$, then Jack's preferences can be represented by $M \succ D \succ S$ (henceforth denoted "Jack has preferences MDS").
- If $f^* \leq f < f^d(s')$, then Jack has preferences MSD.
- If $f < f^*$, then Jack has preferences SMD or SDM.

Jack's expected payoffs to marriage, dating, and separating are represented by the following diagram.

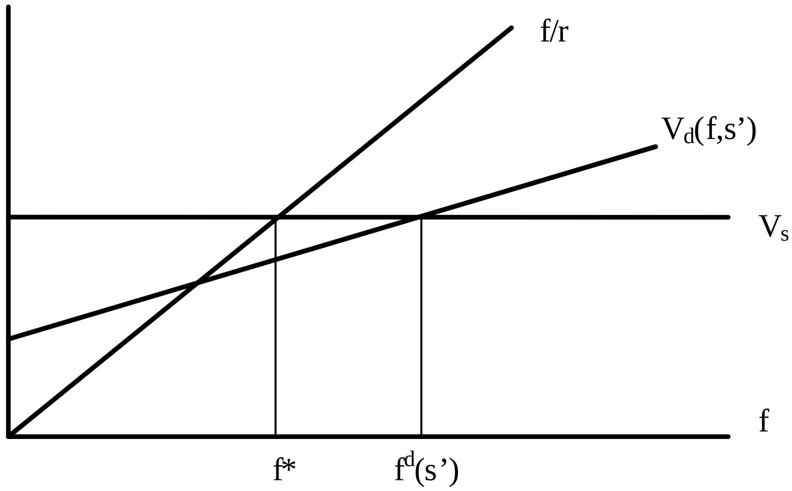

Fig. 1: Payoffs to marriage, dating, and separating

Consider now Jill's problem when (f, s') describes the information. As shown earlier, if Jack is willing to date Jill, he prefers to marry her. Hence, given $f \geq f^*$, Jill's expected return to dating Jack can be written as

$$\begin{aligned}(1 + rdt)V_d(s', f) &= \rho s'dt + \alpha dt[\int_{f^*}^{f'+\bar{e}} (f'/r)dG(f'|s') + G(f^*|s')V_s] + (1 - \alpha dt)V_d(s', f) \\ rV_d(s', f) &= \rho s' + \frac{\alpha}{r} \int_{f^*}^{f'+\bar{e}} f'dG(f'|s') + \alpha G(f^*|s')V_s - \alpha V_d(s', f)\end{aligned}$$

Manipulation and then integration by parts implies

$$V_d(s', f) = \frac{r\rho s' + \alpha\mu(f^*|s') + \alpha s^*}{r(r + \alpha)}$$

where

$$\mu(f^*|s') = \int_{f^*-s'}^{\bar{e}} [1 - G(x|0)]dx$$

Note, $\partial\mu(f^*|s')/\partial s' = 1 - G(f^*|s') \in [0, 1]$. This implies $\partial V_d(s', f)/\partial s' \in (0, 1/r)$.

Using these results it is simple to establish that there exists an s^m such that $s'/r \lesseqgtr V_d(s', f)$ as $s' \lesseqgtr s^m$, where

$$s^m = \frac{\alpha\mu(f^*|s^m) + \alpha s^*}{\alpha + r(1 - \rho)}$$

In a similar fashion we can defines s^d by $V_d(s', f) \lesseqgtr V_s$ as $s' \lesseqgtr s^d$, where

$$\rho s^d + \frac{\alpha\mu(f^*|s^d)}{r} = s^*$$

$$s^* - s^d = \frac{\alpha\mu(f^*|s^d)}{r} - (1-\rho)s^d$$

Since the right-hand side could be positive or negative, we have two possibilities depicted in Figures 2a and 2b.

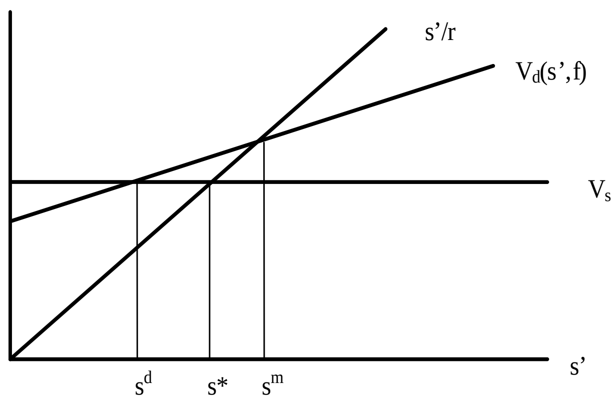

Fig. 2a: $s^d < s^* < s^m$

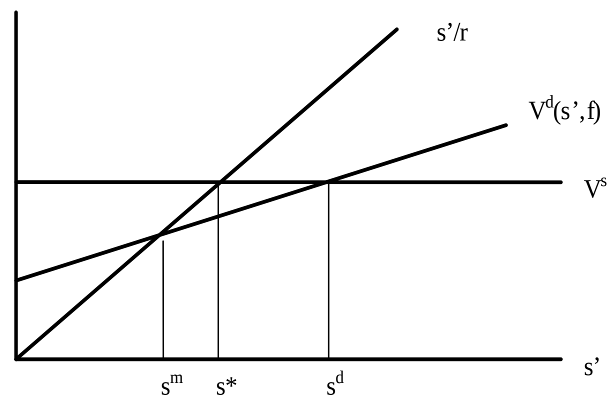

Fig. 2b: $s^d > s^* > s^m$

It may be that $s^d < s^* < s^m$. In this case, given (f, s'), Jill's behavior can be described as follows:

- If $s' < s^d$, then Jill has preferences SDM.
- If $s^d \leq s' < s^*$, then Jill has preferences DSM.
- If $s^* \leq s' < s^m$, then Jill has preferences DMS.
- If $s' \geq s^m$, then Jill has preferences MDS.

The other case is when $s^d \geq s^* \geq s^m$. This is possible depending on the sign of the equation above. In this case, Jill's behavior can described as follows.

- If $s' < s^m$, then Jill has preferences SDM.

- If $s^m \leq s' < s^*$, then Jill has preferences SMD.
- If $s^* \leq s' < s^d$, then Jill has preferences MSD.
- If $s' \geq s^d$, then Jill has preferences MDS.

The first case is described in Figure 2a and the second case is described in Figure 2b. Manipulation establishes that $V_d(s^*, f) \gtrless (s^*/r)$ as $\alpha\mu(0|0) \gtrless r^2(1-\rho)V_s$. When $V_d(s^*, f) > s^*/r$, Figure 2a describes the value functions accurately; otherwise Figure 2b holds.

The equilibrium outcomes in Figures 2a and 2b above can depicted by the following figure, below. The color green represents marriage, yellow represents dating, and red represents singleness.

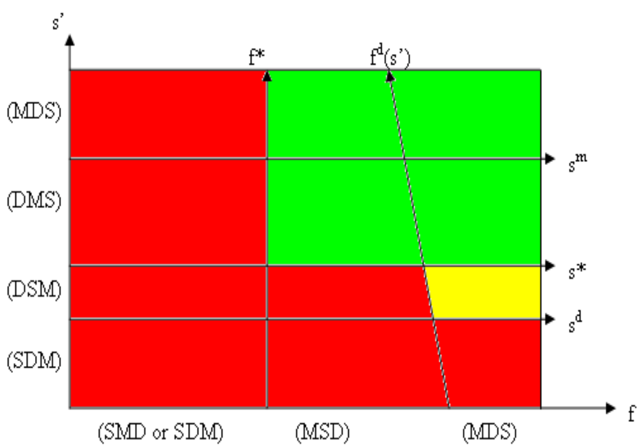

Fig. 3a: Equilibrium when $s^d < s^* < s^m$

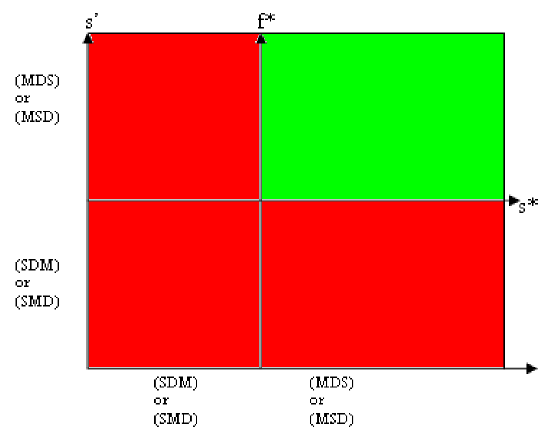

Fig. 3b: Equilibrium when $s^d > s^* > s^m$

Note that the only reason that a couple will date in this state is if one person is unwilling to marry and when $\alpha\mu(0|0) > r^2(1-\rho)V_s$, since a person who knows his or her true feeling will never prefer dating as his or her first choice.

4 Analysis of dating when both partners only know signals

Suppose that Jack and Jill meet. The signals they obtain, (s, s'), are revealed to both of them immediately. Again, if they choose to separate, each obtains payoff

V_s. If they choose to marry, Jack expects payoff s/r, and Jill expects payoff s'/r. Alternatively, if they choose to date, Jack's expected payoff will be a function of (a) his flow utility while dating (ρs), (b) his expected return if Jill learns her true feeling first, and (c) his expected return if he learns his own true feeling first.

Denote Jack's expected payoff if Jill learns her true feeling first as $\varphi_w(s,s')$, and his expected payoff when he learns his own true feeling first as $\varphi_m(s,s')$. If they date, Jack's expected return can be written as

$$\begin{aligned}(1+rdt)V_d(s,s') &= \rho s dt + \alpha dt \varphi_m(s,s') + \alpha dt \varphi_w(s,s') + (1-2\alpha dt)V_d(s,s') \\ V_d(s,s') &= \frac{\rho s + \alpha\varphi_m(s,s') + \alpha\varphi_w(s,s')}{r+2\alpha}\end{aligned}$$

Recall in the previous section, where Jack knows his true feeling but Jill only knows her signal, we found two general conditions.

- **Condition D** (for dating): If $\alpha\mu(0|0) > r^2(1-\rho)V_s$, there was the possibility that Jack and Jill would date.
- **Condition ND** (for no dating): If $\alpha\mu(0|0) \le r^2(1-\rho)V_s$, Jack and Jill would never date.

where $\mu(x|y) = \int_{x-y}^{\overline{e}}(1-G(e|0))de$.

As we will discover, the same conditions that allowed for or negated the possibility of dating are the same when Jack and Jill first meet. In the following section, we consider Condition D.

4.1 Condition D

Assuming Condition D holds, we observed in the previous section that if Jack knows his true feeling but Jill only knows her signal, they will date for some values of (f,s'). In this subsection, we find that when Jack and Jill first meet, they date for some values of (s,s') as well. Assuming Condition D, Figure 4a illustrates the outcomes when Jack knows his true feeling f and Jill only knows her signal s'. The figure highlights again the cutoffs in terms of true feelings/signals that determine whether Jack and Jill marry (green), date (yellow), or become single (red). Figure 4b is the symmetric figure when Jack knows his signal s and Jill knows her true feeling f'. These will be important in the following derivations.

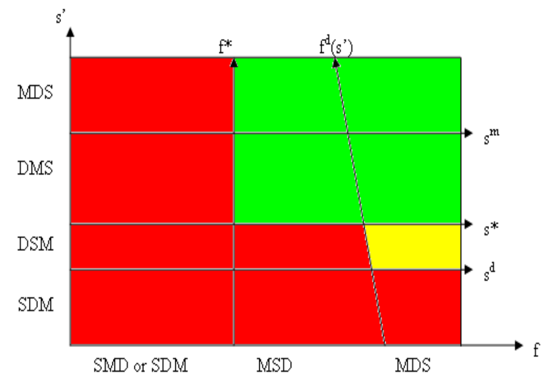

Fig. 4a:
Jack knows f, Jill knows s'

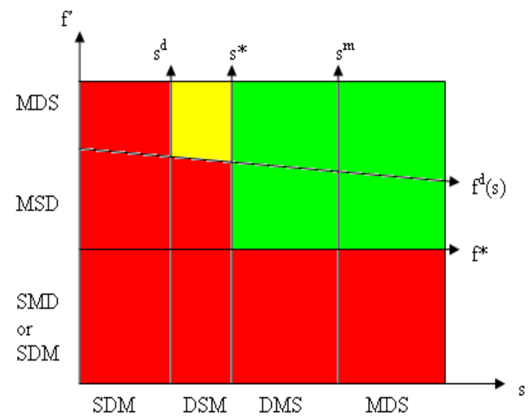

Fig. 4b:
Jack knows s, Jill knows f'

We use these figures to solve the expected payoff to Jack if Jill learns her true feeling first ($\varphi_w(s, s')$) and the expected payoff to Jack if he learns his own true feeling first ($\varphi_m(s, s')$).

Consider first $\varphi_w(s, s')$. In this case, Jill learns first and Figure 4b depicts the potential outcomes. Jack knows his signal s and therefore knows the horizontal coordinate of the point on Figure 4b at which they would arrive.

- If $s \geq s^*$, Jack knows that he and Jill will marry if $f' \geq f^*$ or separate otherwise.
- If $s \in [s^d, s^*)$, Jack knows that he and Jill will date if $f' \geq f^d(s)$ or separate otherwise.
- If $s < s^*$, Jack knows that he and Jill will separate for any f'.

It follows that

$$\varphi_w(s, s') = \begin{cases} G(f^*|s')V_s + [1 - G(f^*|s')]s/r, \text{ if } s \geq s^* \\ G(f^d(s)|s')V_s + [1 - G(f^d(s)|s')]V^d(s, f'), \text{ if } s \in [s^d, s^*) \\ V_s, \text{ if } s < s^d \end{cases}$$

Now consider $\varphi_m(s, s')$. In this case, Jack learns first and Figure 4a depicts the potential outcomes. Jack knows Jill's signal s' and therefore knows the vertical coordinate of the point on Figure 4a at which they would arrive.

- If $s' \geq s^*$, Jack knows that he and Jill will marry if $f \geq f^*$ or separate otherwise.

- If $s' \in [s^d, s^*)$, Jack knows that he and Jill will date if $f \geq f^d(s')$ or separate otherwise.
- If $s' < s^*$, Jack knows that he and Jill will separate for any f.

It follows that

$$\varphi_m(s,s') = \begin{cases} G(f^*|s)V_s + \int_{f^*}^{s+\overline{e}} \frac{f}{r} dG(f|s), \text{ if } s' \geq s^* \\ G(f^d(s')|s)V_s + \int_{f^d(s')}^{s+\overline{e}} V_d(f,s')dG(f|s), \text{ if } s' \in [s^d, s^*) \\ V_s, \text{ if } s' < s^d \end{cases}$$

We now show that an increase in Jack's signal s increases Jack's expected payoff to marriage more than his expected payoff to dating, which in turn increases more than his expected payoff to singleness (when Jack's payoff to dating is continuous and differentiable).

Claim 1 *$\partial V_d(s,s')/\partial s \geq 0$ for $s \neq s^*, s^d$, and if $g(.|.)$ is log-concave, $\partial V_d(s,s')/\partial s \leq 1/r = \partial(s/r)/\partial s$ for $s \neq s^*, s^d$.*

Proof. See Appendix 6.1.1. ■

Furthermore, Jack's expected payoff to dating increases with respect to Jill's signal. This is because 1) if Jack learns his own true feeling first, Jill is more likely to date him for high signals, and if she is in fact willing to marry him, he is able to marry if he prefers this most, and 2) because if Jill learns her true feeling first, Jack knows that Jill is more likely to be willing to marry or date him if she has a higher signal before learning her true feeling.

Claim 2 *$V_d(s,s')$ is increasing in s'.*

Proof. See Appendix 6.1.2. ■

Assuming Condition D, if Jill learns first, Jack will either marry her or separate if $s \geq s^*$, but will either date her or separate if $s < s^*$. As Jack's payoffs for dating and marriage are not equal at s^*, we can see that if Jill learns first, Jack's expected payoff for dating Jill is discontinuous at $s = s^*$. (Note that Jack's payoff to dating is continuous at $s = s^d$, because Jack is indifferent between dating and singleness at this point, so the payoff if Jack learns first at $s = s^d$ is equal to the payoff to singleness.)

Given Claim 1, above, as well as the discontinuity of the payoff to dating at $s = s^*$, one of the following three figures depicts Jack's payoffs to marriage, dating, and separating and the subsequent reservation values. We can define a signal $s^m(s')$ such that dating and marriage have equal expected payoffs, and another signal $s^d(s')$

such that dating and separating have equal expected payoffs. If there are two values of $s^d(s')$ (as in Figure 5c), then they will be called $s_1^d(s')$ and $s_2^d(s')$ (and $s_1^d(s') < s_2^d(s')$).

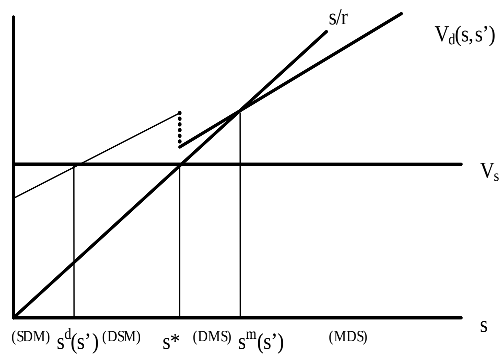

Fig. 5a: $s^d(s') < s^* < s^m(s')$

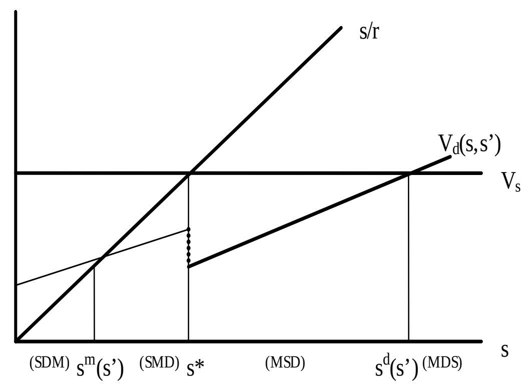

Fig. 5b: $s^m(s') < s^* < s^d(s')$

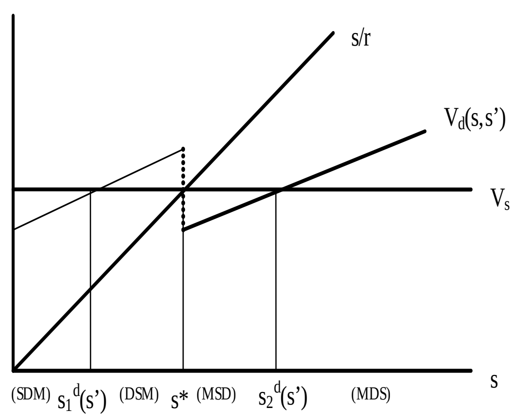

Fig. 5c: $s_1^d(s') < s^* < s_2^d(s')$

Corollary 3 *Given Jill's signal, s', Jack has either one or two values of $s^d(s')$, and, assuming $g(.|.)$ is log-concave, at most one value of $s^m(s')$.*

Corollary 4 *When it exists, $s^m(s')$ is increasing with respect to s'; where $s^d(s')$ is unique, it is decreasing with respect to s'; and where $s^d(s')$ is not unique, $s_1^d(s')$ and $s_2^d(s')$ are each decreasing with respect to s'.*

Next, we study the cases and signals for which Jack and Jill date, marry, or separate.

4.1.1 Outcome when Jack and Jill are both indifferent between marriage and separating in Condition D

Consider Condition D when $s = s' = s^*$. If Jack were to learn his true feeling first and prefers to marry Jill, Figure 6a would depict Jill's payoffs for marriage, dating, and separating, with respect to her signal s', and Figure 6b would depict Jack's payoffs for marriage, dating, and singleness with respect to his true feeling f (given Jill's signal s').

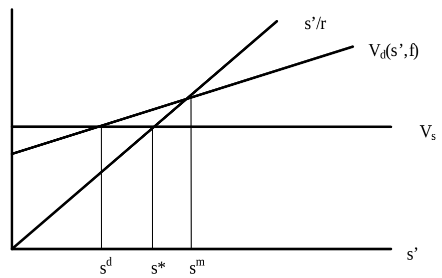

Fig. 6a

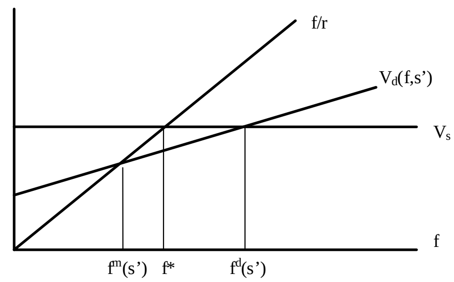

Fig. 6b

Since $s' = s^*$, Jill would have preferences DMS if Jack were to learn first (we assume Jill would select marriage over singleness if she were indifferent). After learning his true feeling, Jack would have preferences MDS, MSD, SMD, or SDM. Therefore, Jack and Jill would marry if $f \geq f^*$, or separate otherwise. Likewise, if Jill were to learn first, Jack and Jill would marry if $f' \geq f^*$, or separate otherwise. Thus, Jack's expected payoffs if he learns first or if Jill learns first are, respectively:

$$\begin{aligned} \varphi_m(s^*, s^*) &= \mu(0|0)/r + V_s \\ \varphi_w(s^*, s^*) &= (1 - G(f^*|s^*))(s^*/r) + G(f^*|s^*)V_s = V_s \end{aligned}$$

Therefore, Jack's expected payoff to dating when he and Jill only know their signals is

$$V_d(s^*, s^*) = \frac{r\rho s^* + \alpha\mu(0|0) + 2\alpha s^*}{r(r + 2\alpha)}$$

As Condition D assumes that $\alpha\mu(0|0) > r(1 - \rho)s^*$, Jack strictly prefers dating to separating when both he and Jill have signals of s^* since

$$V_d(s^*, s^*) > \frac{r\rho s^* + r(1 - \rho)s^* + 2\alpha s^*}{r(r + 2\alpha)} = V_s.$$

Hence, Figure 7, below, depicts Jack's payoffs to marriage, dating, and separating, with respect to his signal (given that Jill has a signal $s' = s^*$):

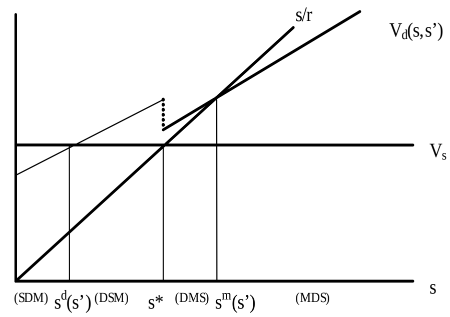

Fig. 7

Jill's payoffs are identical, given Jack's signal $s = s^*$. Therefore, when Condition D holds, and $s = s' = s^*$, both Jack and Jill have preferences DMS, so they will date. As Jack's and Jill's expected returns to dating are continuous for $s \geq s^*$ and $s' \geq s^*$, they will also both have preferences DMS near the point at which $(s, s') = (s^*, s^*)$ (with $s, s' > s^*$ such that $s^m(s'), s^m(s) > s^m(s^*)$). Both partners have an incentive to date here since they will not end up much worse off if their partner leaves, and if they learn first, they can restrict marriage only to when they have good true feelings, and choose to separate for bad true feelings.

Conclusion 5 *If $s \in [s^*, s^m(s'))$ and $s' \in [s^*, s^m(s))$, and if Condition D holds, Jack and Jill both have preferences DMS and date.*

Note that if Jack's and Jill's signals are both larger than s^* and they do not date, they marry as both prefer this to singleness.

4.1.2 Outcome when Jill prefers marriage to singleness but Jack prefers singleness to marriage

Consider Condition D, when Jack and Jill date for $s = s^d$ and $s' \geq s^*$. If Jill were to learn first, Figure 8a would depict Jack's potential payoffs, and Figure 8b, Jill's potential payoffs, respectively.

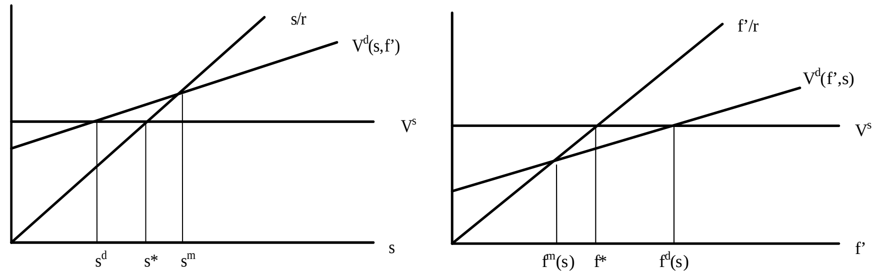

Fig. 8a Fig. 8b

Jack would be indifferent between dating and separating if Jill were to learn her true feeling first and was willing to date him (since $s = s^d$). Therefore, if Jill were to learn first, Jack would expect a payoff equal to V_s whether she would date him or not. If Jack were to learn first, Jill would be willing to marry Jack since $s' \geq s^*$. Thus, Jack has the same flow utility and continuation payoff while dating when $s = s^d$ and $s' \geq s^*$ as he does when $s = s^d$ and Jill knows $f' \geq f^d(s)$. Therefore, his payoff to dating is equivalent as well, and we know this is equal to his payoff of singleness. As Jack's payoff to dating increases with respect to his signal s, Jack's reservation value for dating when $s' \geq s^*$ is the same in information state (s, s') as it is in information state (s, f').[1]

Therefore, when $s \in [s^d, s^*)$, Jack has preferences DSM, and the following diagram, Figure 9, depicts his payoffs to marriage, dating, and singleness (given Jill has signal $s' = s^*$) under Condition D:

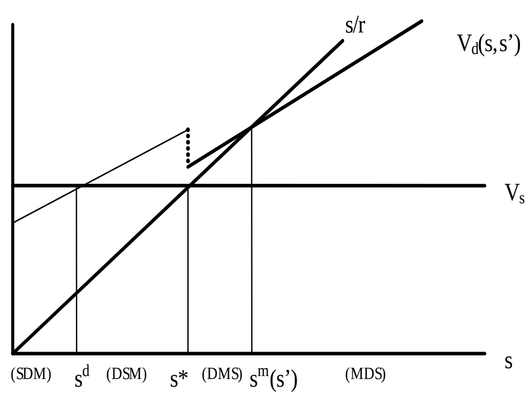

Fig. 9

(Note that for $s' > s^*$, $s^d(s') = s^d(s^*) = s^d$ and $s^m(s') \geq s^m(s^*)$.)

[1] The mathematical details are left to Appendix 6.2.1.

Depending on Jack's signal s, one of Figures 5a-5c will depict Jill's payoffs if Jack learns first, and Jill's preferences are either MDS, MSD or DMS since $s' \geq s^*$, and they will date if $s' \geq s^d(s)$ as well, or separate otherwise. Jack has an incentive to date here, since he can restrict marriage only to when he has a good true feeling and can separate otherwise. Jill may prefer marriage, but is willing to date because she has a high flow utility from dating, or because she thinks that she will have a good true feeling if they do get married, and she finds the probability of marriage high enough.

Conclusion 6 *If Jack has signal $s \in [s^d, s^*)$ and Jill has signal $s' \geq \max\{s^d(s), s^*\}$, and if Condition D holds, then Jack and Jill will date.*

Conclusion 7 *If Jill has signal $s' \in [s^d, s^*)$ and Jack has signal $s \geq \max\{s^d(s'), s^*\}$, and if Condition D holds, then Jack and Jill will date.*

Note that if either Jack's or Jill's signal is less than s^* and they do not date, they will separate since if either has a signal less than s^*, then he or she prefers singleness to marriage.

4.1.3 Outcome when Jack and Jill both prefer singleness to marriage

In the previous two subsections, we studied examples when either Jack's or Jill's signal is larger than s^* under Condition D. In this subsection, we study examples when both Jack and Jill have signals smaller than s^* under Condition D. In the previous two subsections, marriage often occurred after one person learned his or her true feeling. In this section, Jack and Jill either date or separate when the first of them learns his or her true feeling, and if marriage does occur, it will only occur after both have learned their true feelings.

If Jack has a signal less than s^d, then Jack and Jill do not date– Jack would not date when Jill would marry him if he learned a good true feeling first, so he certainly does not date when Jill would not marry him if he learned a good true feeling first. Similarly, Jill will not date when she has a signal less than s^d either. Therefore, the only case to study is one in which both of their signals are in the interval $[s^d, s^*)$.

As $s < s^*$, if Jill were to learn first, then Jack and Jill would continue dating if $f' \geq f^d(s)$, or separate otherwise. Likewise, if Jack were to learn first, Jack and Jill would continue dating if $f \geq f^d(s')$, or separate otherwise. If $s = s^d$, then if Jill learns first and is willing to date, Jack's payoff is the same as with singleness, and his payoff if he learns first is lower now than if Jill were willing to marry him, since she will only date him in this state. As Jack was only indifferent between dating and singleness when Jill would marry him if he learned true feeling first, Jack now strictly prefers singleness to dating when $s = s^d$ and $s' < s^*$. Similarly, Jill does not date for $s' = s^d$ and $s < s^*$. Depending on parameters, Jack and Jill may not date for any pair of signals less than s^*. If they do date, they certainly do as $s = s' \to s^{*-}$, since both of their payoffs are increasing with respect to their own signals and with respect to each other's signal for all (s, s') not equal to s^*.[2]

Conclusion 8 *If* $\lim_{s \to s^{*-}} V_d(s,s) < V_s$*, and if Condition D holds, and Jack and Jill will never date for any pair of signals which are less than* s^**. However, if* $s \geq s^d(s')$ *and* $s' \geq s^d(s)$*, and if Condition D and* $\lim_{s \to s^{*-}} V_d(s,s) > V_s$ *both hold, then Jack and Jill date.*

Note that if Jack and Jill both have signals less than s^* and they do not date, they will separate, as they both prefer singleness to marriage.

4.1.4 Possible outcomes when Jack and Jill meet in Condition D

In summary, parameter values determine whether there are either three or four regions of (s, s') in which Jack and Jill choose to date for Condition D.

1. Jack and Jill choose to date for $s \in [s^*, s^m(s')), s' \in [s^*, s^m(s))$
2. Jack and Jill choose to date for $s \in [s^d, s^*), s' \geq \max\{s^d(s), s^*\}$
3. Jack and Jill choose to date for $s \geq \max\{s^d(s'), s^*\}, s' \in [s^d, s^*)$
4. If $\lim_{s \to s^{*-}} V_d(s, s') > V_s$, Jack and Jill choose to date for $s \in [s^d(s'), s^*), s' \in [s^d(s), s^*)$.

Therefore, there are two general possibilities of equilibrium outcomes when Jack and Jill know their signals in Condition D. They are depicted in the following figures.

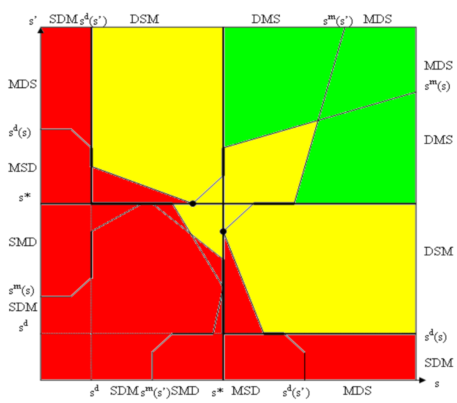

Fig. 10a: Outcomes under Condition D, when dating occurs for s, s' less than s^*

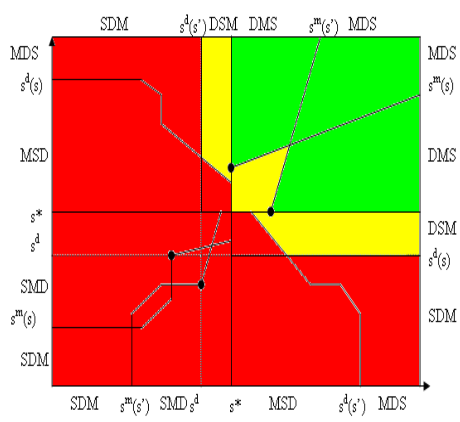

Fig. 10b: Outcomes under Condition D, when dating does not occur for s, s' less than s^*

[2]The details of this analysis are left to Appendix 6.2.2.

Both of these figures depict dating in situations where one person has a sufficiently high signal and the other person has a signal nearly as large as expected flow payoff of singleness. Both of these also depict dating when both people have signals slightly larger than the expected flow payoff of singleness. Under conditions specified above, Jack and Jill may date when both people have signals slightly less than the expected flow payoff of singleness as depicted in Figure 10a.

The set of signals for which couples date is not quite intuitive. Specifically, there are two properties of the outcomes depicted above that may seem counter-intuitive, but are logical upon further analysis. First, people may not prefer to date when they fear their partners are likely to leave them; and second, dating is not necessarily as valuable if one expects to be confronted with an ultimatum in the near future. In the first instance, there are some couples who marry who would have only chosen to date with higher signals. Consider on either figure $s' = s^*$, $s = s^m(s^*)$. In this case, Jack and Jill marry, since Jack prefers marriage. However, $s^m(s')$ is increasing with respect to Jill's signal s', so if $s' > s^*$, then Jack will prefer dating ($s < s^m(s')$) and Jack and Jill date. The fear that Jill would leave Jack makes dating less attractive to him when $s' = s^*$, so he prefers marriage, but when $s' > s^*$, Jack knows Jill is more likely to be willing to marry later, so he prefers dating. In this sense, the outcome does not imply that the couples with the highest signals will marry. In the second instance, there are couples who separate who would have dated with slightly lower signals. Consider on Figure 13a, when $s_1^d(s') < s = s^* < s_2^d(s')$ and $s' > \lim_{s \to s^{*-}} s^d(s)$. In this case, Jack is unwilling to date Jill because he knows that if Jill were to learn first and want to marry him, they would marry, and therefore his expected payoff if Jill learns first is equal to the expected payoff of singleness (since $s^*/r = V_s$). Jack is unwilling to date when he and Jill only know signals, so they separate. However, if $s < s^*$, Jack knows that if Jill were to learn first and were willing to date, they would date, and therefore his expected payoff if Jill learns first is now greater than the expected payoff of singleness (since $V_d(s^*, f') > V_s$). Therefore, dating when he and Jill only know signals has a higher expected payoff due to this higher future payoff. In other words, Jack and Jill separate with higher signals, but if Jack had a slightly lower signal, he would be willing to date Jill, and they would date.

4.2 Condition ND

Recall that Condition ND did not result in dating when one person knew his or her true feeling. As we will see in this section, Jack and Jill also will never date when neither person knows his or her own true feeling. Recall that in (f, s') and (s, f'), the equilibrium outcomes were as follows.

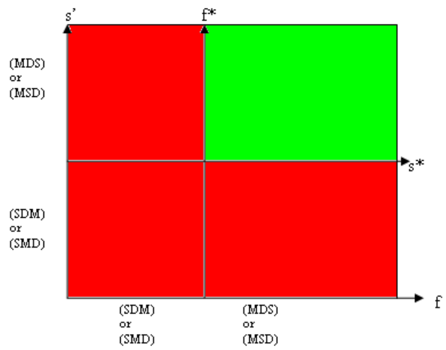

Fig. 11a:
Jack knows f, Jill knows s'

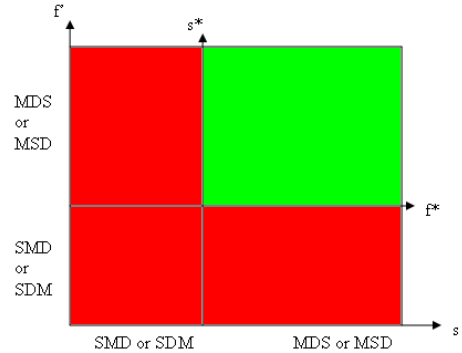

Fig. 11b:
Jack knows s, Jill knows f'

Jack and Jill both know that there will be no dating after the first of them learns his or her signal. As a result, dating is only chosen in this state if Jack would prefer to date Jill if he knew that she would marry him if he learned first and proposed marriage. As before, this would need to entail Jack preferring to date when $s = s^*$. However, for the same reasons as in information state (s, f'), Jack does not prefer dating in this state either, and hence, Jack and Jill will not date upon learning their signals and will marry if and only if $\min(s, s') \geq s^*$.[3]

Conclusion 9 *Jack and Jill will never date for any signals if Condition ND holds.*

The following figures depict the value functions and equilibrium outcomes for Condition ND.

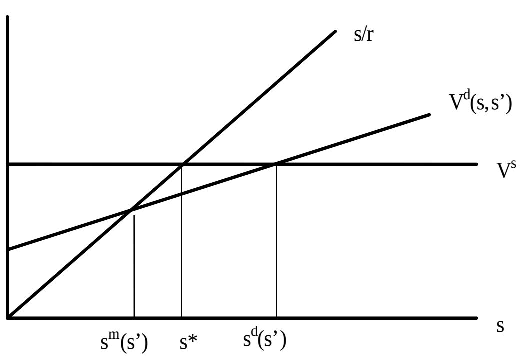

Fig. 12a: Value functions with respect to s in Condition ND

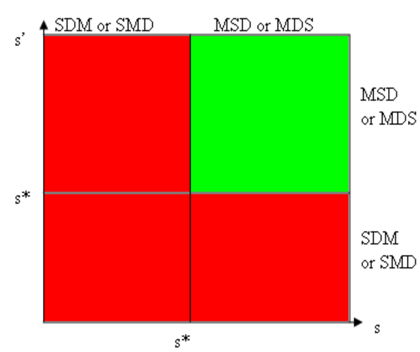

Fig. 12b: Outcomes

[3]The mathematical details of this analysis are left to Appendix 6.2.3.

Thus, Jack and Jill marry when they meet if the expected flow payoff to marriage exceeds the expected flow payoff to singleness for both of them, or separate otherwise.

5 Conclusion

This paper used the example of dating and marriage to address when a pair of agents will choose a temporary match over a permanent match, given that match quality is unknown upon initial contact. People may choose to date, a temporary match, in order to maintain the flexibility to terminate a match if it ceases to be desirable. When choosing this option, one loses some flow utility that he or she would have received in a permanent match (marriage).

I described the set of signals for which dating and acquiring information was preferred to both marrying and separating upon meeting, as well as sets of signals for which marriage and separating were preferred. I also described the conditions for which a couple married, dated, and separated when only one person knew true match quality as well. Couples in which both people know true match quality did not prefer to date, as flow utility was lower and there was no new information to be acquired.

In general, temporary matching upon meeting was more likely to occur in environments where one's initial signal of match quality was more blurry, when information could be received relatively quickly, and when one did not lose a lot of flow utility while waiting for that information. A temporary match was more beneficial to a person who believed that his or her partner was willing to stay temporarily matched upon learning more information, because choosing a temporary match was less likely to limit one's options in the future in that case. Conversely, a temporary match was less beneficial when one was concerned that his or her partner was likely to terminate the match in the future, because if one's partner terminated the match, he or she would lose the possibility of maintaining this match if it were beneficial.

Where there existed signals for which couples dated, people chose to date when both partners were nearly indifferent between marriage and singleness, but each preferred marriage if dating were unavailable. This occurred because, upon learning one's true feeling, marriage would be possible. Similarly, dating occurred in asymmetric situations in which one person slightly preferred singleness to marriage and the other strongly preferred marriage to singleness. The person who preferred dating could continue to date to learn full information, with the knowledge that his or her partner would not be likely to prematurely terminate the match. The other would be willing to date because his or her match quality was high. In the setting in which both partners preferred singleness to marriage, a dating relationship sometimes formed. However, this match was less valuable to each partner, because both partners wait until both know their true feeling before deciding whether to marry.

In situations where at least one person preferred separating to marriage, couples often continued to date until both learned their true feelings. Dating when one person knew his or her true feeling also occurred when he or she had a very high match quality and his or her partner was reasonably likely to want to marry in the future.

Threat points played an important role in the determination of a couple's relationship state, causing the set of signals for which couples dated to be less intuitive than expected. If a person knew that if he or she later removed the option of dating, his or her partner would choose marriage over separation, dating was more valuable as a result. Similarly, dating was less valuable if one knew that he or she might later be forced to choose between marriage and singleness, as his or her partner might have removed the option of dating. As a result, inefficiencies arose in which pairs with superior match quality were not always more likely to marry than date, nor were they always more likely to date than separate. This highlights the importance of specifying when information is known in matching models. As information is revealed over time, agents make decisions with incomplete information, and ignoring this fact would inaccurately specify which matches are made.

It is important to note that these principles apply to other types of partnerships as well. When utility is not fully transferable, as in many labor markets and business partnerships, the result of a potential future ultimatum can greatly affect whether agents will match. The timing of the revelation of information is important, and it can lead to a different equilibrium than if all information were known when agents initially met.

A number of extensions to this research are worth exploring. Allowing for divorce is an obvious extension. Depending on the model used, if divorce were permissible at some cost, it is likely that the results would not qualitatively change. Analyzing cohabitation with this model is also possible, as the model could allow the flow utility to be the same or nearly the same while cohabiting as while married. Alternatively, the model could allow information to arrive more quickly while cohabiting. Furthermore, studying the impact of bargaining and transferrable utility with this model of temporary matching may provide significant insight into many environments.

6 APPENDIX

6.1 Proofs

6.1.1 Jack's return to dating with respect to his signal

Claim 1 $\partial V_d(s, s')/\partial s \geq 0$ for $s \neq s^*, s^d$, and if $g(.|.)$ is log-concave, $\partial V_d(s, s')/\partial s \leq 1/r = \partial(s/r)/\partial s$ for $s \neq s^*, s^d$.

Proof. As the derivative of V_s with respect to s is 0, it is sufficient to show that $\partial[V_d(s, s')]/\partial s > 0$ to prove that there are either one or two values of $s^d(s')$.

$$\partial[\rho s/r(r + 2\alpha)]/\partial s = \rho/(r + 2\alpha) > 0$$

$$\partial[\alpha\varphi_m(s,s')]/\partial s = \left.\begin{cases} \alpha(1-G(f^*|s))/[r(r+2\alpha)], \text{ if } s' \geq s^* \\ \alpha([r\rho+\alpha(1-G(f^*|s'))]/[r+\alpha])(1-G(f^d(s')|s))/[r(r+2\alpha)], \text{ if } s' \in [s^d, s^*) \\ 0, \text{ if } s' < s^d \end{cases}\right| \geq 0$$

$$\partial[\alpha\varphi_w(s,s')]/\partial s = \left.\begin{cases} \alpha(1-G(f^*|s'))/[r(r+2\alpha)], \text{ if } s \geq s^* \\ [\alpha[1-G(f^d(s)|s')](\partial V_d(s,f')/\partial s)+ \\ \alpha[1-G(f^d(s)|s')](rV_d(s,f')-s^*)[-\partial f^d(s)/\partial s]/[r(r+2\alpha)], \text{ if } s \in [s^d, s^*) \\ 0, \text{ if } s < s^d \end{cases}\right| \geq 0$$

Thus, $V_d(s,s')$ is increasing for $s \neq s^*, s^d$.

Next, we see that the derivative is less than $1/r$, if $g(.|.)$ is log-concave for $s \neq s^*, s^d$.

As the derivative of s/r with respect to s is $1/r$, it is sufficient to show that $\partial[V_d(s,s')]/\partial s < 1/r$ to prove that there can be at most one $s^m(s')$.

$\partial[\rho s/r(r+2\alpha)]/\partial s = \rho/(r+2\alpha) < r/r(r+2\alpha)$

$\partial[\alpha\varphi_m(s,s')]/\partial s \leq \alpha(1-G(f^*|s))/r(r+2\alpha) < \alpha/r(r+2\alpha)$

Therefore, it is sufficient to show that $\partial[\alpha\varphi_w(s,s')]/\partial s \leq \alpha/r(r+2\alpha)$

When $s \geq s^*$, $\partial[\alpha\varphi_w(s,s')]/\partial s = \alpha(1-G(f^*|s'))/r(r+2\alpha) \leq \alpha/r(r+2\alpha)$

When $s < s^d$, $\partial[\alpha\varphi_w(s,s')]/\partial s = 0 \leq \alpha/r(r+2\alpha)$.

Therefore, what remains to be shown is that $\partial[\alpha\varphi_w(s,s')]/\partial s \leq \alpha/r(r+2\alpha)$ when $s \in [s^d, s^*)$.

$$\begin{aligned}\partial[\alpha\varphi_w(s,s')]/\partial s &= \frac{\alpha[1-G(f^d(s)|s')]}{r(r+2\alpha)}(\frac{\partial V_d(s,f')}{\partial s}) + \frac{\alpha g(f^d(s)|s'))}{r(r+2\alpha)}(rV_d(s,f')-s^*)\frac{-\partial f^d(s)}{\partial s} \\ &= \frac{\alpha[1-G(f^d(s)|s')]}{r(r+2\alpha)}(\frac{r\rho+\alpha(1-G(f^*|s))}{r+\alpha}) \\ &\quad + \frac{\alpha g(f^d(s)|s')}{r(r+2\alpha)}(\frac{\alpha\mu(f^*|s)+r\rho s-rs^*}{r+\alpha})(\frac{r(1-\rho)f^*\alpha g(f^*|s)}{[r\rho+\alpha(1-G(f^*|s))]^2})\end{aligned}$$

Therefore, it is sufficient to show that

$$g(f^d(s)|s')\mu(f^*|s)(\frac{r(1-\rho)f^*\alpha g(f^*|s)}{[r\rho+\alpha(1-G(f^*|s))]^2}) < G(f^*|s)$$

As $r(1-\rho)f^* < \alpha\mu(f^*|s)$ by assumption, and since log-concavity implies that $\mu(f^*-s)g(f^*|s) < (1-G(f^*|s))^2$, it is sufficient to show that:

$$g(f^d(s)|s')\mu(f^*|s) \leq G(f^*|s) \qquad (*)$$

Note that $\mu(f^*|s) < \int_{-\bar{e}}^{f^*-s} G(e)de$ for $s < f^*$.

Consider three possible assumptions
(1) $f^d(s) - s' \geq f^* - s$
(2) $f^d(s) - s' < f^* - s$ and $G(f^*|s) \leq 0.5$
(3) $f^d(s) - s' < f^* - s$ and $G(f^*|s) > 0.5$
For assumption (1), $g(f^d(s)|s') \leq g(f^*|s)G(f^d(s)|s')/G(f^*|s) \leq g(f^*|s)/G(f^*|s)$
Therefore, showing (*) is true requires showing that

$$[g(f^*|s)/G(f^*|s)] \int_{-\bar{e}}^{f^*-s} G(e)de \leq G(f^*|s)$$

which is true by log-concavity.
For assumptions (2) and (3):$g(f^d(s)|s') < g(f^*|s)[1 - G(f^d(s)|s')]/[1 - G(f^*|s)] < g(f^*|s)/[1 - G(f^*|s)]$
For assumption (2), showing (*) is true requires showing that

$$\{g(f^*|s)/[1 - G(f^*|s)]\}\mu(f^*|s) < G(f^*|s)$$

Due to log-concavity and because $\mu(f^*|s) < \int_{-\bar{e}}^{f^*-s} G(e)de$ for $s < f^*$, it is sufficient to show that

$$G(f^*|s)^2/[1 - G(f^*|s)] < G(f^*|s)$$

which is trivially true for assumption (2).
For assumption (3), showing (*) is true requires showing that

$$g(f^*|s)/[1 - G(f^*|s)]\}\mu(f^*|s) < G(f^*|s)$$

Due to log-concavity, it is sufficient to show that

$$[1 - G(f^*|s)] < G(f^*|s)$$

which is trivially true for assumption (3). ■

6.1.2 Jack's return to dating with respect to Jill's signal

Claim 2 $V_d(s, s')$ is increasing in s'.

Proof.

$$\partial[\rho s/r(r + 2\alpha)]/\partial s' = 0$$

$$\partial[\varphi_m(s, s')]/\partial s'$$
$$= \left\{ \begin{array}{l} 0, \text{ if } s' \geq s^* \\ (r\rho + \alpha(1 - G(f^*|s'))/[r + \alpha](1 - G(f^d(s')|s))(-\partial f^d(s')/\partial s')), \text{ if } s' \in [s^d, s^*) \\ 0, \text{ if } s' < s^d \end{array} \right| \geq 0$$

$$\partial[\varphi_w(s,s')]/\partial s'$$

$$= \left\{\begin{array}{l} \alpha(g(f^*|s')(s-s^*),\ \text{if } s \geq s^* \\ \alpha(g(f^*|s')(s-s^*)+s^*)+\alpha(\frac{\alpha}{r+\alpha}g(f^*|s')\mu(f^d(s')-s),\ \text{if } s \in [s^d,s^*) \\ 0,\ \text{if } s < s^d \end{array}\right| \geq 0$$

Furthermore, $\lim_{s'\to s^*}\varphi_w(s,s') < \varphi_w(s,s^*)$. ■

6.2 Details of analysis of dating when Jack and Jill only know signals

6.2.1 Details of analysis when Jack prefers singleness to marriage but Jill prefers marriage to singleness

In Section 4.1.2, we considered Condition D when Jack and Jill date for $s < s^* \leq s'$. This appendix details the mathematics of this case.

Since $s < s^*$, Jack would have preferences SDM, DSM or SMD if Jill learned first. Therefore, Jack and Jill would date if $f' \geq f^d(s)$ and $s \geq s^d$, or separate otherwise. Thus, Jack's expected payoff if Jill learns first is

$$\varphi_w(s,s') = (1-G(f^d(s)|s'))(\max\{V_d(s,f')-V_s,0\})+V_s$$

If Jack were to learn first, Jill would have preferences DMS or MDS as $s' \geq s^*$, and Jack would have preferences MDS, MSD, SDM, or SMD, and therefore, they would either marry if $f \geq f^*$, or separate otherwise. Thus, Jack's expected payoff if he learns first is

$$\varphi_m(s,s') = \mu(f^*|s)/r+V_s$$

Therefore, Jack's expected payoff to dating when he and Jill only know their signals is

$$V_d(s,s') = \frac{\rho s+\alpha[(1-G(f^d(s)|s'))(\max\{V_d(s,f')-V_s,0\})+V_s]+\alpha[\mu(f^*|s)/r+V_s]}{r+2\alpha}$$

and manipulation establishes that

$$V_d(s,s')-V_s = \frac{r(\rho s-s^*)+\alpha\mu(f^*|s)+\frac{\alpha(1-G(f^d(s)|s'))}{r+\alpha}\max\{r(\rho s-s^*)+\alpha\mu(f^*|s),0\}}{r(r+2\alpha)}$$

Thus, Jack prefers dating to singleness if and only if $r(\rho s-s^*)+\alpha\mu(f^*|s) \geq 0$ (i.e. if and only if $s \geq s^d$, where $V_d(s^d,f') = V_s$).

Therefore, when $s \in [s^d,s^*)$, Jack has preferences DSM, and they date if Jill's signal $s' \geq s^d(s)$ as well, or separate otherwise.

6.2.2 Details of analysis when Jack and Jill both prefer singleness to marriage

In Section 4.1.3, we considered Condition D when Jack and Jill both had signals smaller than s^*. This appendix details the mathematics of this case.

Suppose $s < s^d$. In this case, $\varphi_w(s, s') = V_s$, since Jack does not date Jill if she learns first, and or $\varphi_m(s, s') = V_s$.

$$\varphi_m(s, s') = \begin{cases} G(f^d(s')|s)V_s + \int_{f^d(s')}^{s+\bar{e}} V_d(f, s')dG(f|s), \text{ if } s' \in [s^d, s^*) \\ V_s, \text{ if } s' < s^* \end{cases}$$

If $s' < s^*$, then

$$V_d(s, s') = \frac{[r\rho s + 2\alpha s^*]}{r[r + 2\alpha]}$$

and trivially this is inferior to being single.

If $s' \in [s^d, s^*)$, then

$$\begin{aligned} \varphi_m(s, s') &= V_s + \int_{f^d(s')}^{s+\bar{e}} (V_d(f, s') - V_s)dG(f|s) \\ &< V_s + \int_{f^*}^{s+\bar{e}} ((s + e)/r - V_s)dG(f|s) \\ &= V_s + \mu(f^*|s)/r \\ &< V_s + \mu(f^*|s^d)/r \end{aligned}$$

since we have assumed $s < s^d$, and so

$$V_d(s, s') < \frac{1}{r(r + 2\alpha)}[r\rho s + 2\alpha s^* + \alpha\mu(f^*|s^d)] = V_s$$

Thus, Jack prefers singleness to marriage when $s < s^d$. Similarly, Jill prefers singleness to marriage when $s' < s^d$.

Now suppose $s = s^d$. In this case, Jack's payoff when Jill learns first, $\varphi_w(s, s') = V_s$, since he gets the same payoff to dating if she is willing as when she is not. Similarly to above, $\varphi_m(s, s') = V_s$ for $s' < s^*$, and if $s' \in [s^d, s^*)$, $\varphi_m(s, s') < V_s + \mu(f^*|s^d)/r$. Therefore

$$V_d(s^d, s') < \frac{r\rho s^d + 2\alpha s^* + \alpha\mu(f^*|s^d)}{r(r + 2\alpha)} = V_s$$

when $s' < s^*$ as well.

Now suppose $s \in (s^d, s^*)$. If Jill were to learn first, then Jack and Jill would continue dating if $f' \geq f^d(s)$, or separate otherwise. Thus, Jack's expected payoff to dating if Jill learns first is

$$\varphi_w(s, s') = (1 - G(f^d(s)|s'))[V_d(s, f') - V_s] + V_s$$

Likewise, if Jack were to learn first, Jack and Jill would continue dating if $f \geq f^d(s')$, or separate otherwise. Thus, Jack's expected payoff to dating if he learns his own true feeling first is

$$\varphi_m(s,s') = \frac{r\rho + \alpha(1 - G(f^*|s'))}{r(r+\alpha)}\mu(f^d(s')|s) + V_s$$

Therefore, Jack's payoff to dating is

$$V_d(s,s') = \frac{r\rho s + 2\alpha s^* + \alpha(1 - G(f^d(s)|s))[rV_d(s,f') - s^*] + \alpha\frac{r\rho+\alpha(1-G(f^*|s))}{(r+\alpha)}\mu(f^d(s)|s)}{r(r+2\alpha)}$$

This may or may not be larger than the payoff to singleness. To see if Jack and Jill ever date for $s, s' < s^*$, consider $s = s' \to s^{*-}$. Since Jack's payoff to dating is increasing in both his and her signals, if Jack ever prefers dating for any pair of signals $s, s' < s^*$, this payoff must be larger than V_s. The limit of the difference between the payoff to dating and the payoff to singleness is

$$\lim_{s\to s^{*-}} V_d(s,s) - V_s$$
$$= \frac{\alpha\frac{r\rho+\alpha(1-G(f^*|s^*))}{(r+\alpha)}\mu(f^d(s^*)|s^*) - r(1-\rho)s^* + \frac{\alpha(1-G(f^d(s^*)|s^*))}{r+\alpha}[\alpha\mu(0|0) - r(1-\rho)s^*]}{r(r+2\alpha)}$$

Again,

$$\frac{r\rho + \alpha(1 - G(f^*|s^*))}{(r+\alpha)}\mu(f^d(s^*)|s^*) < \mu(0|0)$$

so this term can be positive or negative depending on parameter values, and therefore Jack and Jill do not necessarily date for any pair of signals both less than s^*.

6.2.3 Details of analysis under Condition ND

In Section 4.2, we considered Condition ND and found that Jack and Jill never dated

in this case. This section details the mathematics of this analysis.

As before,

$$V_d(s,s') = [\rho s + \alpha\varphi_w(s,s') + \alpha\varphi_m(s,s')]/[r+2\alpha]$$

Under the assumptions in Condition ND, consider $\varphi_w(s,s')$ and $\varphi_m(s,s')$. In this case, if Jill learns her true feeling first, and Figure 11b depicts the potential outcomes, and if Jack learns his true feeling first, Figure 11a does. Jack knows his signal s and therefore knows the horizontal coordinate of the point on Figure 11b at which they will arrive if Jill learns first. If $s \geq s^*$, Jack knows that he and Jill will marry if $f' \geq f^*$ or separate otherwise. If $s < s^*$, Jack knows that they will separate for any

f'. Similarly, Jack knows Jill's signal s', and therefore knows that if he learns first, they will marry if $f \geq f^*$ or separate otherwise. It follows that

$$\varphi_w(s,s') = \begin{cases} G(f^*|s')V_s + [1 - G(f^*|s')]s/r, \text{ if } s \geq s^* \\ V_s, \text{ if } s < s^* \end{cases}$$
$$\varphi_m(s,s') = \begin{cases} G(f^*|s)V_s + \int_{f^*}^{s+\bar{e}} \frac{f}{r} dG(f|s), \text{ if } s' \geq s^* \\ V_s, \text{ if } s' < s^* \end{cases}$$

It can be shown that an increase in Jack's signal s will increase Jack's expected payoff to marriage more than his expected payoff to dating, which in turn increases more than his expected payoff to singleness. This requires showing that the payoff to dating is increasing by less than $1/r$ with respect to s, which can be done by checking the derivative at all differentiable points ($s \neq s^*$) and noticing that the function is continuous with respect to s. We can now define two reservation values for Jack at each value of s': $s^d(s')$ such that dating and singleness have the same payoff, and $s^m(s')$ such that dating and marriage have the same payoff, which are each unique given s'.

Since $\varphi_w(s,s')$ increases with respect to s' due to the increased possibility that Jill would marry Jack, and $\varphi_m(s,s')$ and ρs are nondecreasing with respect to s', it can also be shown that $V_d(s,s')$ is increasing with respect to s'. Thus, $s^d(s')$ decreases and $s^m(s')$ increases with respect to s'.

Now, we see that $s^m(s') < s^* < s^d(s')$ for all s'. Consider Jack's payoff to dating when his signal $s = s^*$ and Jill's signal s' is such that she would definitely marry him if he learns his true feeling first. If this payoff is less than the payoff to singleness at s^*, Jack never prefers dating to both marriage and singleness, and hence neither does Jill. Consider Jack's payoff if Jill learns her true feeling first, and his payoff if he learns his own true feeling first, respectively:

$$\varphi_w(s^*,s') = G(f^*|s')V_s + [1 - G(f^*|s')]s^*/r = V_s$$
$$\varphi_m(s^*,s') = \mu(f^*|s^*)/r + V_s = \mu(0|0)/r + V_s$$

Therefore, Jack's payoff to dating Jill is

$$V_d(s^*,s') = \frac{r\rho s^* + \alpha\mu(0|0) + 2\alpha s^*}{r(r+2\alpha)}$$

In Condition ND, $\alpha\mu(0|0) \leq r(1-\rho)s^*$, and therefore for all s',

$$V_d(s^*,s') \leq \frac{r\rho s^* + r(1-\rho)s^* + 2\alpha s^*}{r(r+2\alpha)} = V_s$$

Therefore, Jack never prefers dating to both marriage and singleness, and neither does Jill.

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