

# Pro-cyclical Unemployment Benefits? Optimal Policy in an Equilibrium Business Cycle Model

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## Abstract

In this paper, we characterize the optimal cyclical behavior of unemployment insurance. We integrate risk-averse workers, endogenous job search effort, and unemployment benefit expiration into the standard Diamond-Mortensen-Pissarides search and matching model. We solve the optimal policy problem of the government within this framework, allowing both the benefit level and benefit duration to depend on the history of past aggregate shocks. Contrary to the current US policy, we find that the path of optimal unemployment benefits is pro-cyclical - positively correlated with productivity and employment. However, this overall pro-cyclical masks richer short-run dynamics of optimal benefits in response to productivity shocks. Specifically, in response to a recessionary shock, optimal benefits rise on impact and then fall significantly below their pre-recession level. Optimal benefit levels and optimal benefit duration co-move positively in response to productivity shocks, thus operating as complementary policy instruments over the business cycle. As compared to the current US unemployment insurance policy, the optimal benefits smooth cyclical fluctuations in unemployment and deliver substantial welfare gains.

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# 1 Introduction

How should unemployment insurance respond to fluctuations in productivity and unemployment? This question has gained importance in light of the high and persistent unemployment rates following the 2007-2009 recession. In the United States, existing legislation automatically extends unemployment benefit duration in times of high unemployment. Nationwide benefit extensions have been enacted in every major recession since 1958, including the most recent one, in which the maximum duration of unemployment benefits reached an unprecedented 99 weeks. The desirability of such extensions is the subject of an active policy debate, which has only recently begun to receive attention in economic research. In this paper, we use an equilibrium search and matching model to determine how unemployment insurance should optimally respond to business cycle conditions.

To study optimal state-contingent unemployment insurance (UI), we integrate risk-averse workers, endogenous search effort, and UI benefit expiration into the workhorse Diamond-Mortensen-Pissarides model. Business cycles in the model are driven by shocks to aggregate labor productivity. Unemployment benefits provide insurance to workers but create distortions on both worker and firm margins. More generous unemployment benefits discourage unemployed workers from searching. They also raise the worker outside option in wage bargaining, thereby discouraging firms from posting vacancies. We are the first to study optimal UI over the business cycle within this classic equilibrium search framework. Moreover, we jointly characterize the optimal behavior of benefit levels and benefit duration. This allows us to disentangle the role of these two dimensions of UI policy, and to directly address the current policy debate.

Since the focus of our analysis is optimal policy over the business cycle, it is crucial to use a model consistent with the cyclical behavior of the US labor market, in particular the unemployment experience in the recent recession. We calibrate the model to match the mean and standard deviation of the vacancy-unemployment ratio and micro evidence on the responsiveness of unemployment duration to benefit generosity. We find that the model matches post-war US unemployment volatility. Moreover, using the realized time series of productivity and UI benefit extensions enacted during the 2007-2011 period, the model generates persistently high unemployment consistent with the data.

We characterize the optimal state-contingent UI policy by solving the Ramsey problem of the government, taking the equilibrium conditions of the model as constraints. Specifically, we allow the government to choose both benefit levels and benefit duration optimally over

the business cycle, and to condition its policy choices on the entire past history of aggregate productivity shocks. We find that, contrary to the current US policy, the optimal benefit schedule is *pro-cyclical* over long time horizons: when the model is simulated under the optimal policy, optimal benefit levels and optimal benefit duration are positively correlated with labor productivity and negatively correlated with the unemployment rate. This overall procyclicality of benefits, however, masks richer dynamics of the optimal policy. In particular, the optimal policy response to a one-time productivity drop is different in the short run and in the long run: optimal benefit levels and duration initially rise in response to a negative shock, but both subsequently fall below their pre-recession level. Thus, behavior of optimal benefits in response to productivity is non-monotonic, and the fall in benefit generosity lags the fall in productivity.

The intuition for the behavior of the optimal policy is that, when productivity falls, so do the social gains from creating additional jobs, hence the opportunity cost of raising the generosity of UI benefits is low. On the other hand, a negative shock to productivity leads to a rise in the unemployment rate. Higher unemployment rates tighten incentive constraints on both firms and workers, because periods of high unemployment raise the social gains to posting vacancies and searching for jobs, but do not raise the private incentives to do so. As a consequence, the government optimally lowers unemployment benefit generosity in response to the rise in unemployment. Furthermore, the dynamics of the optimal policy reflect the fact that workers and firms in the model are forward-looking and their expectations about future policies matter for their behavior. Future benefit generosity thus affects job creation incentives in the current period.

A novel feature of our analysis is the joint characterization of optimal benefit level and optimal benefit duration. The key mechanism that enables us to disentangle these two policy dimensions is that, while benefit levels trade off insurance and incentives for workers who are eligible for benefits, the expiration rate of benefits regulates the fraction of eligible workers in the unemployed population. We find that optimal benefit levels and optimal benefit duration co-move positively, thus operating as complementary policy instruments over the business cycle.

## 1.1 Relationship to the literature

A large literature has taken the principal-agent approach to optimal UI, starting with Baily (1978), Shavell and Weiss (1979), Hopenhayn and Nicolini (1997), and Shimer and Werning (2008). This strand of the literature focuses on the tradeoff between insurance and incentive

provision for an individual unemployed worker. Kroft and Notowidigdo (2010) extend the principal-agent model of optimal UI to the design of a state-contingent policy and argue that optimal UI should be countercyclical if unemployment benefits distort job search behavior less in a recession than they do in a boom. In common with this line of research, our model incorporates worker search effort decisions, which are unobservable to the government. However, our paper is innovative in its consideration of general equilibrium effects of UI through firm vacancy creation decisions, which are abstracted from in the aforementioned papers. For the analysis of large-scale policy changes, such as the benefit extensions proposed and implemented in the recent recession, general equilibrium effects through firm behavior are an important concern. This concern is corroborated by available empirical evidence. For example, micro studies estimating the disincentive effects of UI on search effort, which have typically considered changes in benefit generosity between small groups of workers, usually find small effects of UI as compared to cross-country or cross-regional estimates based on large policy differences.<sup>1</sup> This difference between micro and macro estimates suggests that large policy changes have general equilibrium effects that amplify the response of unemployment to benefit generosity. The Diamond-Mortensen-Pissarides model we use is ideal for studying the combined effect of policy on worker and firm behavior. Our paper thus builds on the analysis of Fredriksson and Holmlund (2001), Cahuc and Lehmann (2000), Coles and Masters (2006), and Lehmann and van der Linden (2007), who study optimal UI design in equilibrium models with endogenous job creation and wage bargaining, emphasizing the tradeoff between insurance and firm vacancy creation. In contrast with these papers, which all focus on steady state optimal policy analysis, our paper characterizes the business cycle dynamics of optimal policy.

Landais, Michailat, and Saez (2010) also examine optimal UI policy over the business cycle but use a general equilibrium model very different from ours. Unlike our paper, they find that optimal UI benefits should be countercyclical. However, a key implication of the model in Landais, Michailat, and Saez (2010) is that general equilibrium effects *dampen* the responsiveness of unemployment to UI policy. The reason for this is that in their model wages are assumed to be exogenous function of labor productivity. Jobs are therefore rationed, and an increase in search intensity by all workers results in a crowding-out effect that partially mitigates the effect on unemployment. Thus, Landais, Michailat, and Saez (2010) predicts that the sensitivity of unemployment to economy-wide changes in benefit policy should

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<sup>1</sup>See e.g. Moffitt (1985) and Meyer (1990) for classic micro studies and Krueger and Meyer (2002) for a survey of available micro estimates. See Fredriksson and Soderstrom (2008) for a cross-regional study of Sweden and Hagedorn and Stetsenko (2010) for estimates based on cross-country regressions.

be smaller, in percentage terms, than its sensitivity to policy changes for a small group of workers. In contrast, our model predicts that general equilibrium effects *amplify* the responsiveness of unemployment to UI policy, by discouraging firms from posting vacancies. As a result, our model implies, consistent with the available evidence, that the sensitivity of unemployment to large-scale policy changes should be greater than what would be measured in small-scale experiments.

Our results that optimal benefits respond non-monotonically to a productivity shock, and that the optimal path of benefits is pro-cyclical, are new to the above literature on optimal unemployment insurance. Furthermore, introducing the joint analysis of optimal benefit level and optimal benefit duration is particularly important, since the current debate on the optimality of UI benefit extensions has focused almost entirely on the duration of benefits. To our knowledge, our paper is the first to incorporate both policy dimensions in the context of optimal UI provision over the business cycle.

The paper is organized as follows. We present the model in section 2. In section 3, we describe our calibration strategy and show that the model is consistent with the rise in unemployment in the 2007-2009 recession. Section 4 defines the optimal policy and contains our main optimal policy results. In section 5, we discuss our results and conduct sensitivity analysis. Finally, we conclude in section 6. All tables and figures are in section 7.

## 2 Model Description

### 2.1 Economic Environment

We consider a Diamond-Mortensen-Pissarides model with aggregate productivity shocks. Time is discrete and the time horizon is infinite. The economy is populated by a unit measure of workers and a larger continuum of firms.

**Agents.** In any given period, a worker can be either employed (matched with a firm) or unemployed. Workers are risk-averse expected utility maximizers and have expected lifetime utility

$$U = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [u(x_t) - c(s_t)],$$

where  $\mathbb{E}_0$  is the period-0 expectation operator,  $\beta \in (0, 1)$  is the discount factor,  $x_t$  denotes consumption in period  $t$ , and  $s_t$  denotes search effort exerted in period  $t$  if unemployed. Only unemployed workers can supply search effort: there is no on-the-job search. The

within-period utility of consumption  $u : \mathbb{R}_+ \rightarrow \mathbb{R}$  is twice differentiable, strictly increasing, strictly concave, and satisfies  $u'(0) = \infty$ . The cost of search effort for unemployed workers  $c : [0, 1] \rightarrow \mathbb{R}$  is twice differentiable, strictly increasing, strictly convex, and satisfies  $c'(0) = 0$ ,  $c'(1) = \infty$ . An unemployed worker produces  $h$ , which stands for the combined value of leisure and home production. There do not exist private insurance markets and workers cannot save or borrow.

Firms are risk-neutral and maximize profits. Workers and firms have the same discount factor  $\beta$ . A firm can be either matched to a worker or vacant. A firm posting a vacancy incurs a flow cost  $k$ .

**Matching.** Unemployed workers and vacancies match in pairs to produce output. The number of new matches in period  $t$  equals

$$M(S_t(1 - L_{t-1}), v_t),$$

where  $1 - L_{t-1}$  is the unemployment level in period  $t-1$ ,  $S_t$  is the average search effort exerted by unemployed workers in period  $t$ , and  $v_t$  is the measure of vacancies posted in period  $t$ . The quantity  $\mathcal{N}_t = S_t(1 - L_{t-1})$  represents the measure of efficiency units of worker search.

The matching function  $M$  exhibits constant returns to scale, is strictly increasing and strictly concave in both arguments, and has the property that the number of new matches cannot exceed the number of potential matches:  $M(\mathcal{N}, v) \leq \min\{\mathcal{N}, v\} \forall \mathcal{N}, v$ . We define

$$\theta_t = \frac{v_t}{\mathcal{N}_t}$$

to be the market tightness in period  $t$ . We define the functions

$$\begin{aligned} f(\theta) &= \frac{M(\mathcal{N}, v)}{\mathcal{N}} = M(1, \theta) \quad \text{and} \\ q(\theta) &= \frac{M(\mathcal{N}, v)}{v} = M\left(\frac{1}{\theta}, 1\right) \end{aligned}$$

where  $f(\theta)$  is the job-finding probability per efficiency unit of search and  $q(\theta)$  is the probability of filling a vacancy. By the assumptions on  $M$  made above, the function  $f(\theta)$  is increasing in  $\theta$  and  $q(\theta)$  is decreasing in  $\theta$ . For an individual worker exerting search effort  $s$ , the probability of finding a job is  $sf(\theta)$ . When workers choose the amount of search effort  $s$ , they take as given the aggregate job-finding probability  $f(\theta)$ .

Existing matches are exogenously destroyed with a constant job separation probability

$\delta$ . Thus, any of the  $L_{t-1}$  workers employed in period  $t - 1$  has a probability  $\delta$  of becoming unemployed in period  $t$ .

**Production.** All worker-firm matches are identical: the only shocks to labor productivity are aggregate shocks. Specifically, a matched worker-firm pair produces output  $z_t$  in period  $t$ , where  $z_t$  is aggregate labor productivity. We assume that  $\ln z_t$  follows an AR(1) process

$$\ln z_t = \rho \ln z_{t-1} + \sigma_\varepsilon \varepsilon_t,$$

where  $0 \leq \rho < 1$ ,  $\sigma_\varepsilon > 0$ , and  $\varepsilon_t$  are independent and identically distributed standard normal random variables. We will write  $z^t = \{z_0, z_1, \dots, z_t\}$  to denote the history of shocks up to period  $t$ .

## 2.2 Government Policy

The US UI system is financed by payroll taxes on firms and is administered at the state level. However, under the provisions of the Social Security Act, each state can borrow from a federal unemployment insurance trust fund, provided it meets certain federal requirements. Motivated by these features of the UI system, we assume that the government in the model economy can insure against aggregate shocks by buying and selling claims contingent on the aggregate state and is required to balance its budget only in expectation. Further, we assume that the price of a claim to one unit of consumption in state  $z_{t+1}$  after a history  $z^t$  is equal to the probability of  $z_{t+1}$  conditional on  $z^t$ ; this would be the case, e.g., in the presence of a large number of out-of state risk-neutral investors with the same discount factor.

Government policies are restricted to take the following form. The government levies a constant lump sum tax  $\tau$  on firm profits and uses its tax revenues to finance unemployment benefits. The government is allowed to choose both the level of benefits and the rate at which they expire. We assume stochastic benefit expiration. This assumption is similar to Fredriksson and Holmlund (2001) and will ensure the stationarity of the worker's optimization problem.

A benefit policy at time  $t$  thus consists of a pair  $(b_t, e_t)$ , where  $b_t \geq 0$  is the level of benefits provided to those workers who are eligible for benefits at time  $t$ , and  $e_t \in [0, 1]$  is the probability that an unemployed worker eligible for benefits becomes ineligible the following period. The eligibility status of a worker evolves as follows. A worker employed in period  $t$  is automatically eligible for benefits in case of job separation. An unemployed worker eligible for benefits in period  $t$  becomes ineligible the following period with probability  $e_t$ , and an

ineligible worker does not regain eligibility until he finds a job. All eligible workers receive the same benefits  $b_t$ ; ineligible workers receive no unemployment benefits, but instead receive an exogenously given welfare payment  $p$ .

We allow the benefit policy to depend on the entire history of past aggregate shocks; thus the policy  $b_t = b_t(z^t), e_t = e_t(z^t)$  must be measurable with respect to  $z^t$ . Benefits are constrained to be non-negative: the government cannot tax  $h$ .

## 2.3 Timing

The government commits to a policy  $(\tau, b_t(\cdot), e_t(\cdot))$  once and for all before the period-0 shock realizes. Within each period  $t$ , the timing is as follows.

1. The economy enters period  $t$  with a level of employment  $L_{t-1}$ . Of the  $1 - L_{t-1}$  unemployed workers, a measure  $D_{t-1} \leq 1 - L_{t-1}$  are eligible for benefits, i.e. will receive benefits in period  $t$  if they do not find a job.
2. The aggregate shock  $z_t$  then realizes. Firms observe the aggregate shock and decide how many vacancies to post, at cost  $k$  per vacancy. At the same time, workers choose their search effort  $s_t$  at the cost of  $c(s_t)$ . Letting  $S_t^E$  and  $S_t^I$  be the search effort exerted by an eligible unemployed worker and an ineligible unemployed worker, respectively, the aggregate search effort is then equal to  $S_t^E D_{t-1} + S_t^I (1 - L_{t-1} - D_{t-1})$ , and the market tightness is therefore equal to

$$\theta_t = \frac{v_t}{S_t^E D_{t-1} + S_t^I (1 - L_{t-1} - D_{t-1})} \quad (1)$$

3.  $f(\theta) (S_t^E D_{t-1} + S_t^I (1 - L_{t-1} - D_{t-1}))$  unemployed workers find jobs. At the same time, a fraction  $\delta$  of the existing  $L_{t-1}$  matches are exogenously destroyed.
4. All the workers who are now employed produce  $z_t$  and receive a bargained wage  $w_t$  (below we describe wage determination in detail). Workers who (i) were employed and lost a job, or (ii) were eligible unemployed workers and did not find a job, consume  $h$  plus unemployment benefits,  $h + b_t$  and lose their eligibility for the next period with probability  $e_t$ . Ineligible unemployed workers who have not found a job consume  $h$  plus public assistance,  $h + p$ , and remain ineligible for the following period.

This determines the law of motion for employment

$$L_t(z^t) = (1 - \delta) L_{t-1}(z^{t-1}) + f(\theta_t(z^t)) [S_t^E(z^t) D_{t-1}(z^{t-1}) + S_t^I(z^t) (1 - L_{t-1}(z^{t-1}) - D_{t-1}(z^{t-1}))] \quad (2)$$

and the law of motion for the measure of eligible unemployed workers:

$$D_t(z^t) = (1 - e_t(z^t)) [\delta L_{t-1}(z^{t-1}) + (1 - S_t^E(z^t) f(\theta_t(z^t))) D_{t-1}(z^{t-1})] \quad (3)$$

Thus, the measure of workers receiving benefits in period  $t$  is

$$\delta L_{t-1} + (1 - S_t^E f(\theta_t)) D_{t-1} = \frac{D_t}{1 - e_t}$$

Since we assume that the government has access to financial markets in which a full set of state-contingent claims is traded, its budget constraint is a present-value budget constraint

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ L_t(z^t) \tau - \left( \frac{D_t(z^t)}{1 - e_t(z^t)} \right) b_t(z^t) \right\} \geq 0 \quad (4)$$

## 2.4 Worker Value Functions

A worker entering period  $t$  employed retains his job with probability  $1 - \delta$  and loses it with probability  $\delta$ . If he retains his job, he consumes his wage  $w_t(z^t)$  and proceeds as employed to period  $t + 1$ . If he loses his job, he consumes  $h + b_t(z^t)$  and proceeds as unemployed to period  $t + 1$ . With probability  $1 - e_t(z^t)$  he then retains his eligibility for benefits in period  $t + 1$ , and with probability  $e_t(z^t)$  he loses his eligibility. Denote by  $W_t(z^t)$  the value after a history  $z^t$  for a worker who enters period  $t$  employed.

A worker entering period  $t$  unemployed and eligible for benefits chooses search effort  $s_t^E$  and suffers the disutility  $c(s_t^E)$ . He finds a job with probability  $s_t^E f(\theta_t(z^t))$  and remains unemployed with the complementary probability. If he finds a job, he earns the wage  $w_t(z^t)$  and proceeds as employed to period  $t + 1$ . If he remains unemployed, he consumes  $h + b_t(z^t)$ , and proceeds as unemployed to the next period. With probability  $1 - e_t(z^t)$  he retains his eligibility for benefits in period  $t + 1$ , and with probability  $e_t(z^t)$  he loses his eligibility. Denote by  $U_t^E(z^t)$  the value after a history  $z^t$  for a worker who enters period  $t$  as eligible unemployed.

Finally, a worker entering period  $t$  unemployed and ineligible for benefits chooses search effort  $s_t^I$  and suffers the disutility  $c(s_t^I)$ . He finds a job with probability  $s_t^I f(\theta_t(z^t))$  and

remains unemployed with the complementary probability. If he finds a job, he earns the wage  $w_t(z^t)$  and proceeds as employed to period  $t + 1$ . If he remains unemployed, he consumes  $h + p$  and proceeds as ineligible unemployed to the next period. Denote by  $U_t^I(z^t)$  the value after a history  $z^t$  for a worker who enters period  $t$  as ineligible unemployed.

The Bellman equations for the three types of workers are then:

$$W_t(z^t) = (1 - \delta) [u(w_t(z^t)) + \beta \mathbb{E}W_{t+1}(z^{t+1})] + \delta [u(h + b_t(z^t)) + \beta(1 - e_t) \mathbb{E}U_{t+1}^E(z^{t+1}) + \beta e_t \mathbb{E}U_{t+1}^I(z^{t+1})] \quad (5)$$

$$U_t^E(z^t) = \max_{s_t^E} -c(s_t^E) + s_t^E f(\theta_t(z^t)) [u(w_t(z^t)) + \beta \mathbb{E}W_{t+1}(z^{t+1})] + (1 - s_t^E f(\theta_t(z^t))) [u(h + b_t(z^t)) + \beta(1 - e_t(z^t)) \mathbb{E}U_{t+1}^E(z^{t+1}) + \beta e_t \mathbb{E}U_{t+1}^I(z^{t+1})] \quad (6)$$

$$U_t^I(z^t) = \max_{s_t^I} -c(s_t^I) + s_t^I f(\theta_t(z^t)) [u(w_t(z^t)) + \beta \mathbb{E}W_{t+1}(z^{t+1})] + (1 - s_t^I f(\theta_t(z^t))) [u(h + p) + \beta \mathbb{E}U_{t+1}^I(z^{t+1})] \quad (7)$$

It will be useful to define the worker's surplus from being employed. The surplus utility from being employed, as compared to eligible unemployed, in period  $t$  is

$$\Delta_t(z^t) = [u(w_t(z^t)) + \beta \mathbb{E}_t W_{t+1}(z^{t+1})] - [u(h + b_t(z^t)) + \beta(1 - e_t) \mathbb{E}U_{t+1}^E(z^{t+1}) + \beta e_t \mathbb{E}U_{t+1}^I(z^{t+1})] \quad (8)$$

Similarly, we define the surplus utility from being employed as compared to being unemployed and ineligible for benefits:

$$\Xi_t(z^t) = [u(w_t(z^t)) + \beta \mathbb{E}_t W_{t+1}(z^{t+1})] - [u(h + p) + \beta \mathbb{E}U_{t+1}^I(z^{t+1})] \quad (9)$$

## 2.5 Firm Value Functions

A matched firm retains its worker with probability  $1 - \delta$ . In this case, the firm receives the output net of wages and taxes,  $z_t - w_t(z^t) - \tau$ , and then proceeds into the next period as a matched firm. If the firm loses its worker, it gains nothing in the current period and proceeds into the next period unmatched. A firm that posts a vacancy incurs a flow cost  $k$  and finds a worker with probability  $q(\theta_t(z^t))$ . If the firm finds a worker, it gets flow profits  $z_t - w_t(z^t) - \tau$  and proceeds into the next period as a matched firm. Otherwise, it proceeds unmatched into the next period.

Denote by  $J_t(z^t)$  the value of a firm that enters period  $t$  matched to a worker, and denote

by  $V_t(z^t)$  the value of an unmatched firm posting a vacancy. These value functions satisfy the following Bellman equations:

$$J_t(z^t) = (1 - \delta) [z_t - w_t(z^t) - \tau + \beta \mathbb{E}_t J_{t+1}(z^{t+1})] + \delta \beta \mathbb{E}_t V_{t+1}(z^{t+1}) \quad (10)$$

$$V_t(z^t) = -k + q(\theta_t(z^t)) [z_t - w_t(z^t) - \tau + \beta \mathbb{E}_t J_{t+1}(z^{t+1})] + (1 - q(\theta_t(z^t))) \beta \mathbb{E}_t V_{t+1}(z^{t+1}) \quad (11)$$

The firm's surplus from employing a worker in period  $t$  is denoted

$$\Gamma_t(z^t) = z_t - w_t(z^t) - \tau + \beta \mathbb{E}_t J_{t+1}(z^{t+1}) - \beta \mathbb{E}_t V_{t+1}(z^{t+1}) \quad (12)$$

## 2.6 Wage Bargaining

We make the assumption, standard in the literature, that wages are determined according to Nash bargaining: the wage is chosen to maximize a weighted product of the worker's surplus and the firm's surplus. Further, the worker's outside option is being unemployed and eligible for benefits, since he becomes eligible upon locating an employer and retains eligibility if negotiations with the employer break down. The worker-firm pair therefore chooses the wage  $w_t(z^t)$  to maximize

$$\Delta_t(z^t)^\xi \Gamma_t(z^t)^{1-\xi}, \quad (13)$$

where  $\xi \in (0, 1)$  is the worker's bargaining weight.

## 2.7 Equilibrium Given Policy

In this section, we define the equilibrium of the model, taking as given a government policy  $(\tau, b_t(\cdot), e_t(\cdot))$  and characterize it.

### 2.7.1 Equilibrium Definition

Taking as given an initial condition  $(z_{-1}, L_{-1})$ , we define an equilibrium given policy:

**Definition 1** *Given a policy  $(\tau, b_t(\cdot), e_t(\cdot))$  and an initial condition  $(z_{-1}, L_{-1})$  an equilibrium is a sequence of  $z^t$ -measurable functions for wages  $w_t(z^t)$ , search effort  $S_t^E(z^t)$ ,  $S_t^I(z^t)$ , market tightness  $\theta_t(z^t)$ , employment  $L_t(z^t)$ , measures of eligible workers  $D_t(z^t)$ , and value functions*

$$\{W_t(z^t), U_t^E(z^t), U_t^I(z^t), J_t(z^t), V_t(z^t), \Delta_t(z^t), \Xi_t(z^t), \Gamma_t(z^t)\}$$

such that:

1. The value functions satisfy the worker and firm Bellman equations (5), (6), (7), (8), (9), (10), (11), (12)
2. Optimal search: The search effort  $S_t^E$  solves the maximization problem in (6) for  $s_t^E$ , and the search effort  $S_t^I$  solves the maximization problem in (7) for  $s_t^I$
3. Free entry: The value  $V_t(z^t)$  of a vacant firm is zero for all  $z^t$
4. Nash bargaining: The wage maximizes equation (13)
5. Law of motion for employment and eligibility status: Employment and the measure of eligible unemployed workers satisfy (2) , (3)
6. Budget balance: Tax revenue and benefits satisfy (4)

### 2.7.2 Characterization of Equilibrium

We characterize the equilibrium given policy via a system of equations that involves allocations only, and does not involve the value functions. This will be helpful in computing the optimal policy.

**Lemma 1** *Fix an initial condition and a policy  $(\tau, b_t(\cdot), e_t(\cdot))$ . Suppose that the sequence*

$$\Upsilon_t(z^t) = \{w_t(z^t), S_t^E(z^t), S_t^I(z^t), \theta_t(z^t), L_t(z^t), D_t(z^t), \\ W_t(z^t), U_t^E(z^t), U_t^I(z^t), J_t(z^t), V_t(z^t), \Delta_t(z^t), \Xi_t(z^t), \Gamma_t(z^t)\}$$

*is an equilibrium. Then the sequences  $\{w_t(z^t), S_t^E(z^t), S_t^I(z^t), \theta_t(z^t), L_t(z^t), D_t(z^t)\}$  satisfy:*

1. The laws of motion (2) , (3)
2. The budget equation (4)

3. *Modified worker Bellman equations (dependence on  $z^t$  is understood throughout)*

$$\begin{aligned} \frac{c'(S_t^E)}{f(\theta_t)} &= u(w_t) - u(h + b_t) + \\ &\quad (1 - e_t) \beta \mathbb{E}_t \left( c(S_{t+1}^E) + (1 - \delta - S_{t+1}^E f(\theta_{t+1})) \frac{c'(S_{t+1}^E)}{f(\theta_{t+1})} \right) \\ &\quad + e_t \beta \mathbb{E}_t \left( c(S_{t+1}^I) + (1 - S_{t+1}^I f(\theta_{t+1})) \frac{c'(S_{t+1}^I)}{f(\theta_{t+1})} - \delta \frac{c'(S_{t+1}^E)}{f(\theta_{t+1})} \right) \end{aligned} \quad (14)$$

$$\begin{aligned} \frac{c'(S_t^I)}{f(\theta_t)} &= u(w_t) - u(h + p) + \\ &\quad \beta \mathbb{E}_t \left( c(S_{t+1}^I) + (1 - S_{t+1}^I f(\theta_{t+1})) \frac{c'(S_{t+1}^I)}{f(\theta_{t+1})} - \delta \frac{c'(S_{t+1}^E)}{f(\theta_{t+1})} \right) \end{aligned} \quad (15)$$

4. *Modified firm Bellman equation*

$$\frac{k}{q(\theta_t)} = z_t - w_t - \tau + \beta (1 - \delta) \mathbb{E}_t \frac{k}{q(\theta_{t+1})} \quad (16)$$

5. *Nash bargaining condition*

$$\xi u'(w_t) k \theta_t = (1 - \xi) c'(S_t^E) \quad (17)$$

*Conversely, if  $\{w_t(z^t), S_t^E(z^t), S_t^I(z^t), \theta_t(z^t), L_t(z^t), D_t(z^t)\}$  satisfy (2)-(4) and (14)-(17), then there exist value functions such that  $\Upsilon_t(z^t)$  is an equilibrium.*

**Proof.** See Appendix A.1. ■

The conditions (14)-(17) are straightforward to interpret. Equations (14) and (15) state that the marginal cost of increasing the job finding probability for the eligible and ineligible workers, respectively, equals the marginal benefit. The marginal cost (left-hand side of each equation) of increasing the job finding probability is the marginal disutility of search for that worker weighted by the aggregate job finding rate. The marginal benefit (right-hand side of each equation) equals the current consumption gain from becoming employed plus the benefit of economizing on search costs in the future. Equation (16) gives a similar optimality condition for firms: it equates the marginal cost of creating a vacancy, weighted by the probability of filling that vacancy, to the benefit of employing a worker. Finally, (17) is a restatement of the first-order condition of the bargaining problem. It will be

clear in section 4 that the conditions (14)-(17) will play the role of incentive constraints in the optimal policy problem, analogous to incentive constraints in principal-agent models of unemployment insurance, e.g. Hopenhayn and Nicolini (1997).

### 3 Calibration

We calibrate the model to verify that it captures salient features of the US labor market, and is thus a useful one for studying optimal policy design. Unlike previous versions of the Pissarides model calibrated in the literature, e.g. Shimer (2005) and Hagedorn and Manovskii (2008), our model incorporates endogenous search intensity choices and stochastic benefit expiration. Moreover, the market tightness in our model is not equal to the vacancy-unemployment ratio; rather, it is the object defined in (1), which we do not directly observe in the data. Our calibration strategy will be correspondingly modified relative to the previous literature. We will calibrate the model to ensure that it is consistent both with aggregate US labor market data and with results from micro studies on the responsiveness of unemployment duration to benefit generosity. Next, we will verify that our model performs well empirically in matching non-targeted observations in the data, in particular the cyclical behavior of unemployment.

The model period is taken to be 1 week. We normalize mean weekly productivity to one. We assume a benefit scheme that mimics the benefit extension provisions currently in place within the US policy. We set the benefit level  $b = 0.4$  to match the average replacement rate of unemployment insurance. The standard benefit duration is 26 weeks; local and federal employment conditions trigger automatic 20-week and 33-week extensions. In the model we assume that  $e_t = 1/59$  when productivity is more than 3% below the mean,  $e_t = 1/46$  when productivity is between 1.5% and 3% below the mean, and  $e_t = 1/26$  otherwise. We set the welfare payment  $p = 0.05$  to match the amount of Food Stamp payments as a fraction of average weekly earnings.<sup>2</sup> We pick the tax rate  $\tau = 0.023$  so that the government balances its budget if the unemployment rate is 5.5%.

We assume log utility:  $u(x) = \ln x$ . For the cost of search, we assume the functional form

$$c(s) = \frac{A}{1+\psi} \left[ (1-s)^{-(1+\psi)} - 1 \right] - As \quad (18)$$

This functional form is chosen to ensure that the optimal search effort will always be strictly between 0 and 1. In particular, the functional form above guarantees that, for any  $A > 0$ ,

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<sup>2</sup>See the US Department of Health and Human Services Annual Report (2008).

we have  $c' > 0, c'' > 0$ , as well as  $c(0) = c'(0) = 0, c(1) = c'(1) = \infty$ .

For the matching function, we follow den Haan, Ramey, and Watson (2000) and pick

$$M(\mathcal{N}, v) = \frac{\mathcal{N}v}{[\mathcal{N}^\chi + v^\chi]^{1/\chi}}$$

The choice of the matching technology is likewise driven by the requirement that the job-finding rate and the job-filling rate always be strictly less than 1.<sup>3</sup> We obtain:

$$f(\theta) = \frac{\theta}{(1 + \theta^\chi)^{1/\chi}}$$

$$q(\theta) = \frac{1}{(1 + \theta^\chi)^{1/\chi}}$$

Following Shimer (2005), labor productivity  $z_t$  is taken to mean real output per person in the non-farm business sector. This measure of productivity is taken from the quarterly data constructed by the BLS for the time period 1951-2004. We also use the 1951-2004 seasonally adjusted unemployment series constructed by the BLS, and measure vacancies using the seasonally adjusted help-wanted index constructed by the Conference Board.

We set the discount factor  $\beta = 0.99^{1/12}$ , implying a yearly discount rate of 4%. The parameters for the productivity shock process are estimated, at the weekly level, to be  $\rho = 0.9895$  and  $\sigma_\varepsilon = 0.0034$ . The job separation parameter  $\delta$  is set to 0.0081 to match the average weekly job separation rate.<sup>4</sup> We set  $k = 0.58$  following Hagedorn and Manovskii (2008), who estimate the combined capital and labor costs of vacancy creation to be 58% of weekly labor productivity.

This leaves five parameters to be calibrated: (1) the value  $h$  of non-market activity; (2) the worker bargaining weight  $\xi$ ; (3) the matching function parameter  $\chi$ ; (4) the level coefficient of the search cost function  $A$ ; and (5) the curvature parameter of the search cost function  $\psi$ . We jointly calibrate these five parameters to simultaneously match five data targets: (1) the average vacancy-unemployment ratio; (2) the standard deviation of vacancy-unemployment ratio; (3) the average weekly job-finding rate; (4) the average duration of unemployment; and (5) the elasticity of unemployment duration with respect to benefits. The first four of these targets are directly measured in the data. For the elasticity of unemployment duration

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<sup>3</sup>The frequently used alternative is the Cobb-Douglas specification, which does not guarantee that the job-finding rate is always less than 1. In fact, for the large fluctuations in the vacancy-unemployment ratio observed in the data, commonly used calibrations of the Cobb-Douglas matching function would imply that the job-finding rate is frequently greater than 1; see e.g. Hall (2005).

<sup>4</sup>We use the same procedure of adjusting for time aggregation as Hagedorn and Manovskii (2008) to obtain the weekly estimates for the job finding rate and the job separation rate from monthly data.

with respect to benefits,  $\mathcal{E}_{d,b}$ , we use micro estimates reported by Meyer (1990) and target an elasticity of 0.9. Intuitively, given the first three parameters, the average unemployment duration and its elasticity with respect to benefits identify the parameters  $A$  and  $\psi$ , since these parameters govern the distortions in search behavior induced by benefits.

Table 1 reports the calibrated parameters and the matching of the calibration targets. Note that our calibration procedure implies a large value of  $h$ . In fact, the combined value of  $h$  and unemployment benefits is  $h+b = 0.974$ , while the mean equilibrium wage is  $w = 0.954$ . This might seem surprising, considering that empirical studies (e.g. Browning and Crossley (2001)) report a consumption drop for workers upon becoming unemployed. However, it is important to note that  $h$  includes the consumption value of leisure, which would not appear as consumption in the data.<sup>5</sup> Moreover, the utility of the unemployed *net of search costs* is always lower than that of the employed.

Tables 2 and 3 display the ability of the model to match the cyclical behavior of unemployment, vacancies, and the vacancy-unemployment ratio. Our model performs well in matching non-targeted moments (note that the only moment in Table 2 that was targeted was the standard deviation of the vacancy-unemployment ratio). In particular, our model generates a standard deviation of unemployment equal to 0.128, very close to the standard deviation of 0.125 observed in the data for the 1951-2004 period. Next, we confirm that our model is consistent with the unemployment experience of the 2007-2009 recession. Specifically, benefit duration was extended several times throughout that time period (see Whittaker (2008) for details). We mimic these extensions within the model by assuming that the expiration rate of benefits  $e_t$  falls to 1/79 in February of 2009, and falls again to 1/99 in November of 2009.<sup>6</sup> We assume that these policy changes are unanticipated by the workers and firms in the model. We simulate the model for the 2007-2011 time period with the extensions, taking the productivity series for this period from the data. Figure 1 shows the time series of unemployment in the data, the unemployment series predicted by the model, and the counterfactual unemployment series that the model would generate if the additional extensions had not occurred.<sup>7</sup> First, Figure 1 shows that the model generates persistently high unemployment rates of the magnitude close to the data. Second, it shows that the

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<sup>5</sup>In addition, studies such as Browning and Crossley (2001) include, in their sample of unemployed workers, a significant fraction who are ineligible for benefits. The model counterpart of the consumption of the unemployed would thus be some weighted average of  $h+b$  and  $h+p$ .

<sup>6</sup>Since there were more than two extensions enacted during the 2007-2009 time period, our experiment may be thought of as providing a lower bound on the rise in unemployment predicted by our model.

<sup>7</sup>This counterfactual series still includes the automatic 20-week and 33-week extensions, which are the same as in the calibration.

additional benefit extensions were quantitatively important to generating this result: had the extensions not been enacted, the model predicts a smaller rise in unemployment and a much faster recovery.

## 4 Optimal Policy

### 4.1 Optimal Policy Definition

We assume that the government is utilitarian: it chooses a policy to maximize the period-0 expected value of worker utility, taking the equilibrium conditions as constraints.

**Definition 2** *A policy  $\tau, b_t(z^t), e_t(z^t)$  is feasible if there exists a sequence of  $z^t$ -measurable functions  $\{w_t(z^t), S_t^E(z^t), S_t^I(z^t), \theta_t(z^t), L_t(z^t), D_t(z^t)\}$  such that (2), (3), (14)-(17) hold for all  $z^t$ , and the government budget constraint (4) is satisfied.*

**Definition 3** *The optimal policy is a policy  $\tau, b_t(z^t), e_t(z^t)$  that maximizes*

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} L_t(z^t) u(w_t(z^t)) + \left( \frac{D_t(z^t)}{1-e_t(z^t)} \right) u(h + b_t(z^t)) + \\ \left( 1 - L_t(z^t) - \frac{D_t(z^t)}{1-e_t(z^t)} \right) u(h + p) - D_{t-1}(z^{t-1}) c(S_t^E(z^t)) - \\ (1 - L_{t-1}(z^{t-1}) - D_{t-1}(z^{t-1})) c(S_t^I(z^t)) \end{array} \right\} \quad (19)$$

*over the set of all feasible policies.*

The government's problem can be written as one of choosing a policy  $\tau, b_t(z^t), e_t(z^t)$  together with functions  $\{w_t(z^t), S_t^E(z^t), S_t^I(z^t), \theta_t(z^t), L_t(z^t), D_t(z^t)\}$  to maximize (19) subject to (2), (3), (14)-(17) holding for all  $z^t$ , and subject to the government budget constraint (4). We find the optimal policy by solving the system of necessary first-order conditions for this problem. The period- $t$  solution will naturally be state-dependent: in particular, it will depend on the current productivity  $z_t$ , as well as the current unemployment level  $1 - L_{t-1}$ , and current measure of benefit-eligible workers  $D_{t-1}$  with which the economy has entered period  $t$ . However, in general the triple  $(z_t, 1 - L_{t-1}, D_{t-1})$  is not a sufficient state variable for pinning down the optimal policy, which may depend on the entire past history of aggregate shocks. In the appendix, we show that the optimal period  $t$  solution is a function of  $(z_t, 1 - L_{t-1}, D_{t-1})$  as well as  $(e_{t-1}, \mu_{t-1}, \nu_{t-1}, \gamma_{t-1})$ , where  $e_{t-1}$  is the previous period's benefit expiration rate and  $\mu_{t-1}, \nu_{t-1}, \gamma_{t-1}$  are Lagrange multipliers on the constraints (14),(15),(16), respectively, in the maximization problem (19). The tuple  $(z_t, 1 - L_{t-1}, D_{t-1}, e_{t-1}, \mu_{t-1}, \nu_{t-1}, \gamma_{t-1})$  captures the dependence of the optimal  $b_t, e_t$  on the history  $z^t$ . The fact that the  $z_t, 1 - L_{t-1}$

and  $D_{t-1}$  are not sufficient reflects the fact that the optimal policy is time-inconsistent: for example, the optimal benefits after two different histories of shocks may differ even though the two histories result in the same current productivity and the same current unemployment level. Intuitively, the government might want to induce firms to post vacancies - and workers to search - by promising low unemployment benefits, but has an ex post incentive to provide higher benefits, so as to smooth worker consumption, after employment outcomes have realized. Including the variables  $e_{t-1}, \mu_{t-1}, \nu_{t-1}, \gamma_{t-1}$  as state variables in the optimal policy captures exactly this trade-off. Note that we assume throughout the paper that the government can fully commit to its policy. In Appendix A.2 we explain the method used to solve for the optimal policy.

## 4.2 Optimal Policy Results

We now investigate how the economy behaves over time under the optimal policy. To this end, we simulated the model both under the current benefit policy and under the optimal policy. Table 4 reports the summary statistics, under the optimal policy, for the behavior of unemployment benefit levels  $b$  and potential benefit duration  $1/e$ . Benefits are higher and expire faster under the optimal policy than under the current policy. The optimal tax rate under the optimal policy is  $\tau = 0.018$ , lower than under the current policy.

The key observation is that, over a long period of time, the correlation of optimal benefits with productivity is positive: both benefit levels and potential benefit duration are procyclical in the long run and, in particular, negatively correlated with the unemployment rate. Moreover, this result is not driven by any balanced budget requirement, since we allow the government to run deficits in recessions.

In order to understand the mechanism behind this behavior of the optimal policy, in Figure 2 we plot the optimal benefit policy function  $b_t(z, 1 - L_{t-1}, D_{t-1}, e_{t-1}, \mu_{t-1}, \nu_{t-1}, \gamma_{t-1})$  as a function of current  $z$  and last period's  $1 - L$  only, keeping  $D_{t-1}, e_{t-1}, \mu_{t-1}, \nu_{t-1}$  and  $\gamma_{t-1}$  fixed at their average values. The optimal benefit level is decreasing in current productivity  $z$  and decreasing in unemployment  $1 - L$ . The intuition for this result is that the optimal benefit is lower in states of the world when the marginal social benefit of job creation is higher, because lower benefits are used to encourage search effort by workers and vacancy creation by firms. The marginal social benefit of job creation is higher when  $z$  is higher, since the output of an additional worker-firm pair is then higher. The marginal social benefit is also higher when current employment is lower, because the expected output gain of increasing  $\theta$  is proportional to the number of unemployed workers. Note, however, that although the social

gains from creating jobs are high when unemployment is high, the private gains to firms of posting vacancies do not directly depend on unemployment. As a consequence, optimal benefits are lower, all else equal, when current unemployment is high. Figure 3 illustrates the same result for the optimal duration of benefits: optimal benefit duration is lowest at times of high productivity and high unemployment. This shape of the policy function also implies that during a recession, there are two opposing forces at work - low productivity and high unemployment - which give opposite prescriptions for the response of optimal benefits. This gives an ambiguous prediction for the overall cyclicity of benefit levels and benefit duration.

In Figures 4 and 5 we analyze the dynamic response of the economy to a negative productivity shock under the optimal policy and compare it to the response under the current policy. In Figure 4 we plot the impulse response of the optimal policy to a productivity drop of 1.5% below its mean. Note that under the current policy, benefit duration does not change in response to the shock, since automatic extensions are only activated when productivity is more than 1.5% below the mean. The optimal benefit level initially jumps up, but then falls for about two quarters following the shock, and slowly reverts to its pre-shock level. The same is true of optimal benefit duration. Unemployment rises in response to the drop in productivity and continues rising for about one quarter before it starts to return to its pre-shock level. Note that the rise in unemployment is significantly lower than under the current benefit policy. The intuition for this optimal policy response is that the government would like to provide immediate insurance against the negative shock and, expecting future productivity to rise, would like to induce a recovery in vacancy creation and search effort. Thus, benefit generosity responds positively to the initial drop in productivity but negatively to the subsequent rise in unemployment, precisely as implied by Figures 2 and 3.

In Figure 5 we plot the response of other key labor market variables. As compared to the current benefit policy, the optimal policy results in a faster recovery of the vacancy-unemployment ratio, the search intensity of unemployed workers eligible for benefits, and the job finding rate. Wages also fall less, in percent deviation terms, under the optimal policy than they do under the current policy. This is due to the fact that the initial rise in benefits smooths the fall in wages through an increase in the worker outside option. The fact that wages fall less in percentage terms indicates that firm profits fall more. Despite the fall in contemporaneous profits, there is not a large fall in market tightness. The reason for this is that firms expect future benefits to fall. The figure thus illustrates that the labor market response depends not only on the contemporaneous benefit policy but also on agents'

expectations about future policy dynamics.

Tables 5 and 6 report the moments of key labor market variables when the model is simulated under the current policy and the optimal policy, respectively. As compared to the optimal policy, the optimal policy results in lower average unemployment and lower unemployment volatility. These results show that the optimal benefit policy stabilizes cyclical fluctuations in unemployment.

Finally, we compute the expected welfare gain from switching from the current policy to the optimal policy. We find that implementing the optimal policy results in a significant welfare gain: 0.67% as measured in consumption equivalent variation terms.

## 5 Discussion of the Results

### 5.1 The assumption of no savings

An important assumption made for transparency in this paper is that workers cannot save or borrow. We now briefly discuss how relaxing this assumption could affect our results. On one hand, if workers are allowed to hold wealth, cyclical variations in this wealth will affect how government-provided insurance should vary over the business cycle. Periods in which unemployed workers' wealth is lower would warrant higher unemployment benefits. Intuitively, this effect is similar to the effect that would arise if  $h$  varied over the business cycle. In particular, if workers are more liquidity-constrained in recessions than in booms, this would provide a motive for raising unemployment benefits in recessions (or raising their duration), with the potential to reverse our optimal policy results.

On the other hand, the presence of savings reduces the responsiveness of the worker outside option to unemployment benefits. As a result, both worker search effort and firm vacancy posting will respond less to policy than they would in the absence of savings. Therefore, in the presence of savings, inducing any given behavioral response requires a larger change in benefits than it would have required otherwise. This effect would reinforce the cyclical behavior of optimal benefits in our model, potentially making optimal benefits even more strongly pro-cyclical.

The overall effect of introducing savings in the model on the pro-cyclicality of optimal benefits is thus ambiguous. We believe that it is an important extension to investigate whether our results are robust to relaxing the no-savings assumption. Assessment of this robustness is research in progress.

## 5.2 The Hosios condition and its relationship to our model

A concern in the Diamond-Mortensen-Pissarides model with Nash bargaining is that the laissez-faire equilibrium is not constrained efficient. Even with risk-neutral workers, the Hosios (1990) condition requires that the worker bargaining weight be equal to the elasticity of the matching function in order to attain efficiency. If the Hosios condition is violated, there is a role for government intervention - such as unemployment benefits - even in the absence of insurance considerations. The Hosios condition is not directly applicable to our model, since when workers are risk-averse, output maximization is not equivalent to welfare maximization. Nevertheless, the question can be posed to what extent our optimal policy results are driven by corrections for the externality that an individual firm imposes on other firms when entering. To answer this question, we solve for the optimal policy both for extremely high and low values of the worker bargaining power, keeping all other parameters fixed at the benchmark calibration values. A comparison of the impulse responses shown in figures 4 and 10 shows that the shape of the optimal policy is robust to raising  $\xi$  to 0.72. The same holds for the overall pro-cyclicality of optimal benefits. This indicates that the violation of the Hosios condition is not the driving force behind our results.

## 5.3 The complementarity of benefit level and benefit duration

An important aspect of our analysis is the simultaneous treatment of optimal benefit level and optimal benefit duration. The key mechanism that allows us to disentangle the role of these two policy instruments is as follows. Both benefit levels and benefit expiration have an effect on the search behavior of the eligible unemployed workers. However, the benefit expiration rate  $e$  also has a direct effect on the *composition* of the unemployed worker pool - the fraction of the eligible among the unemployed - while the level  $b$  does not have a direct effect on this composition. Note that the ineligible unemployed workers supply strictly more search effort than the eligible, but also have strictly lower utility. In deciding the optimal level of  $e$ , the government trades off the benefit of higher aggregate search effort against the cost of lower average utility. This provides an intuitive argument for how the optimal levels of  $b$  and  $e$  are uniquely pinned down. If the difference in utility is very high relative to the difference in search effort, the optimal  $e$  may be zero. For example, this would be the case if  $h + p$  is sufficiently low, since the marginal utility of the ineligible then approaches infinity.

With regard to optimal business cycle behavior, our results indicate that optimal benefit levels and optimal benefit duration move in the same direction in response to a productivity

shock, and therefore operate as complements over the business cycle. To further emphasize this complementarity, we illustrate how the optimal policy would change if the government were restricted to change only one of these two policy dimensions. This may be relevant, for example, because benefit duration may be more flexible in practice than the benefit level. This also facilitates comparison to the existing policy, in which mostly the duration of benefits, rather than the level, changes over the business cycle. We conduct three alternative policy experiments. In the first, we fix the benefit level at its current level:  $b = 0.4$ , and allow only the duration to change over the business cycle. The results, reported in Figure 6, show that the optimal policy response is similar qualitatively to our benchmark: in response to a negative productivity shock, potential duration of benefits should initially rise, and then fall considerably below its initial level. However, both the initial rise in the potential duration and its subsequent decline are greater than in the benchmark optimal policy result. In the second experiment, we fix the benefit expiration rate at its current level of  $e = 1/26$  and compute the optimal benefit policy. Finally, in the third experiment, we ask how the benefit level should vary if benefits are not allowed to expire at all, i.e. if we fix  $e = 0$ . The results are shown in Figures 7 and 8. We find that the shape of the policy response is once again similar to the benchmark: benefits initially rise and then fall. However, both the initial rise and the subsequent decline are greater in magnitude than in the benchmark optimal policy experiment. In each of these cases, the government has one policy instrument at its disposal rather than two, and the optimal cyclical response of this policy instrument becomes stronger as a result.

## 5.4 Sensitivity analysis

We examine the robustness of our results to the parameterization of the model. We have calibrated the model parameters - in particular, the value of non-market activity and the worker bargaining power - to make the model's behavior consistent with US labor market volatility data. However, since several alternative calibrations exist in the literature (see e.g. Shimer (2005)), we conduct sensitivity analysis to determine whether our optimal policy results remain valid under alternative parameterizations. Below, we report the results of sensitivity experiments in which we change the values of selected parameters (e.g.  $h$ ) while keeping the remaining parameters unchanged at their benchmark calibrated values. Similar robustness results hold if we recalibrate the other parameters.

Figure 9 displays the optimal policy results when  $h$  is set to 0. Because the value of unemployment is now considerably lower, the optimal policy prescribes for benefits not to

expire at all, but the optimal response of the benefit level is similar to our benchmark. Figure 10 displays the results when worker bargaining power is increased to 0.72. Next, we adopt a calibration similar to Shimer (2005), in which we set  $h$  to 0 and the bargaining power of the workers to 0.72. The result is displayed in Figure 11; once again, optimal benefits do not expire, but the optimal response of the benefit level is the same as in our benchmark. The main qualitative features of our results, including the result that the optimal benefit scheme is pro-cyclical, do not depend on which calibration is used.

In addition, we have computed the optimal policy for different values of worker risk aversion: specifically, we have computed it for constant relative risk aversion utility, for values of relative risk aversion equal to  $1/2$  and 2. The results are displayed in Figures 12 and 13. Once again, the qualitative features of our results remain intact.

## 6 Conclusion

We analyzed the design of an optimal UI system in the presence of aggregate shocks in an equilibrium search and matching model. Optimal benefits respond non-monotonically to productivity shocks: while raising benefit generosity may be optimal at the onset of a recession, it becomes suboptimal as the recession progresses and inducing a recovery is desirable. We find that optimal benefits are pro-cyclical overall, counter to previous results in the literature. Adopting the optimal policy would yield significant welfare gains. Furthermore, we find that the optimal benefit policy, in addition to providing insurance to workers, results in the smoothing of unemployment over the business cycle.

Our paper has focused on the optimal cyclical behavior of UI benefits and thus serves to inform the ongoing policy debate on the desirability of benefit extensions in recessions. UI benefits are a worker-side intervention, as they affect the economy by changing the workers' value of being unemployed. An interesting extension would be to consider the optimal behavior of UI benefits in conjunction with firm-side interventions, such as hiring subsidies. Increasing hiring subsidies in recessions may be desirable as another instrument for stimulating an employment recovery. A potential concern with hiring subsidies, frequently articulated in policy debates, is the firm-side moral hazard they generate: firms could, for example, fire existing employees only to hire them again in order to receive hiring subsidies. A thorough investigation of the tradeoffs involved with such policies seems a fruitful extension for future work.

Finally, an important direction for future research is investigating the role of government

commitment. The ability of the government to commit matters because the behavior of agents in our model depends not only on the current policy, but also on their expectations about future policy. Throughout the paper, we have assumed that the government can fully commit to its policy. A government without commitment power might be tempted not to lower benefits when there are a lot of unemployed workers. It will therefore be interesting to characterize the time-consistent policy and compare it to the optimal policy in the presence of aggregate shocks.

## 7 Tables and Figures

Table 1: Internally Calibrated Parameters

Parameter	Value	Target	Data	Model
$h$ Value of non-market activity	0.574	Mean $v/(1-L)$	0.634	0.634
$\xi$ Bargaining power	0.130	St. dev of $\ln(v/(1-L))$	0.259	0.259
$\chi$ Matching parameter	0.492	Mean job finding rate	0.139	0.139
$A$ Disutility of search	0.0035	Unemployment duration	13.2	13.2
$\psi$ Search cost curvature	2.836	$\mathcal{E}_{d,b}$	0.9	0.9

*Note:*  $\mathcal{E}_{d,b}$  is the elasticity of unemployment duration with respect to benefits.

Table 2: Summary statistics - quarterly US data, 1951:1-2004:4

	$z$	$1-L$	$v$	$v/(1-L)$
Standard Deviation	0.013	0.125	0.139	0.259
	$z$	1	-0.302	0.460
Correlation	$1-L$	-	1	-0.919
Matrix	$v$	-	-	1
	$v/(1-L)$	-	-	-

*Note:* Standard deviations and correlations are reported in logs as quarterly deviations from an HP-filtered trend with a smoothing parameter of 1600.

Table 3: Summary statistics - calibrated model

	$z$	$1-L$	$v$	$v/(1-L)$
Standard Deviation	0.013	0.128	0.151	0.259
	$z$	1	-0.855	0.867
Correlation	$1-L$	-	1	-0.758
Matrix	$v$	-	-	1
	$v/(1-L)$	-	-	-

*Note:* Standard deviations and correlations are reported in logs as quarterly deviations from an HP-filtered trend with a smoothing parameter of 1600.

Table 4: Optimal benefit behavior

	Benefit level	Potential duration
	$b$	$1/e$
Mean	0.478	11.7
Standard deviation	0.010	0.059
Correlation with $z$	0.694	0.476
Correlation with $1 - L$	-0.331	-0.08
Correlation with $b$	1	0.962

Table 5: Model statistics simulated under the current US policy

	$z$	$1 - L$	$v/(1 - L)$	$\hat{f}$	$w$	$S^E$	$S^I$
Mean	1	0.058	0.634	0.139	0.954	0.503	0.667
Standard Deviation	0.013	0.128	0.259	0.152	0.010	0.045	0.002
	$z$	1	-0.855	0.914	0.895	0.926	0.888
	$1 - L$	-	1	-0.913	-0.918	-0.679	-0.923
	$v/(1 - L)$	-	-	1	0.997	0.729	0.992
Correlation	$\hat{f}$	-	-	-	1	0.697	0.998
Matrix	$w$	-	-	-	-	1	0.686
	$S^E$	-	-	-	-	-	1
	$S^I$	-	-	-	-	-	-
							1

*Note:* Means are reported in levels, standard deviations and correlations are reported in logs as quarterly deviations from an HP-filtered trend with a smoothing parameter of 1600.  $\hat{f}$  denotes the weekly job finding rate.

Table 6: Model statistics simulated under the optimal US policy

	$z$	$1 - L$	$v/(1 - L)$	$\hat{f}$	$w$	$S^E$	$S^I$
Mean	1	0.048	0.772	0.161	0.956	0.523	0.668
Standard Deviation	0.013	0.027	0.061	0.032	0.011	0.009	0.003
	$z$	1	-0.875	0.815	0.774	0.918	0.744
	$1 - L$	-	1	-0.937	-0.923	-0.653	-0.907
	$v/(1 - L)$	-	-	1	0.998	0.519	0.993
Correlation	$\hat{f}$	-	-	-	1	0.459	0.999
Matrix	$w$	-	-	-	-	1	0.419
	$S^E$	-	-	-	-	-	1
	$S^I$	-	-	-	-	-	-
							1

*Note:* Means are reported in levels, standard deviations and correlations are reported in logs as quarterly deviations from an HP-filtered trend with a smoothing parameter of 1600.  $\hat{f}$  denotes the weekly job finding rate.

Figure 1: Model simulations for the 2007-2009 recession

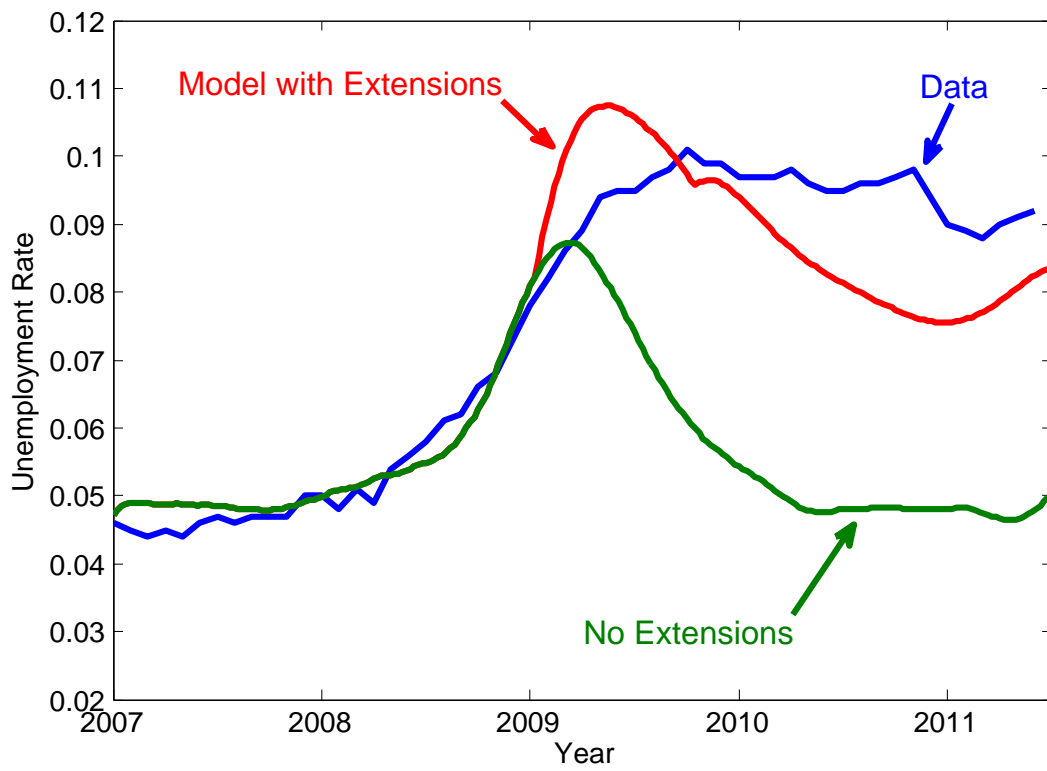


Figure 2: Optimal policy: benefit level

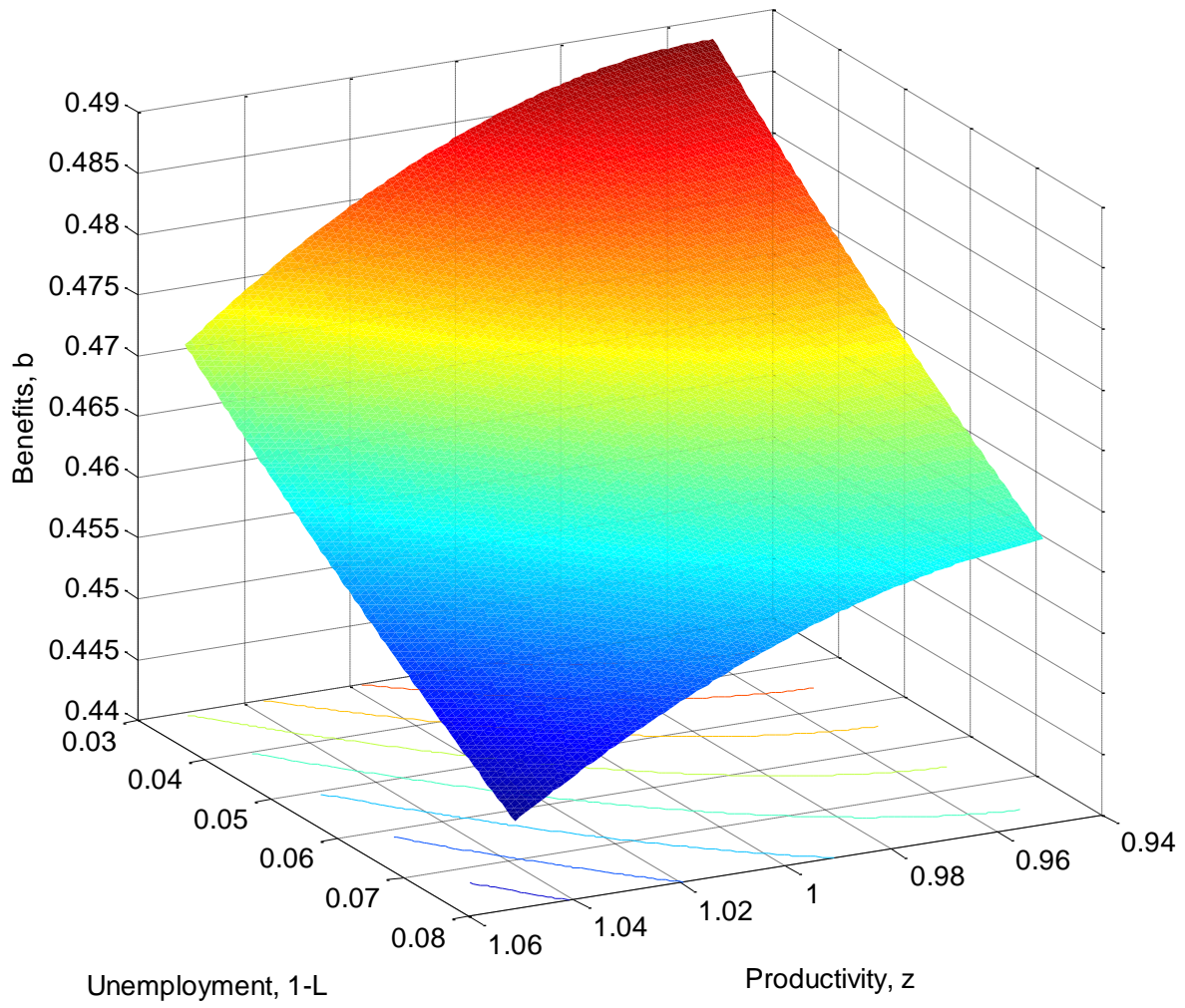


Figure 3: Optimal policy: benefit duration

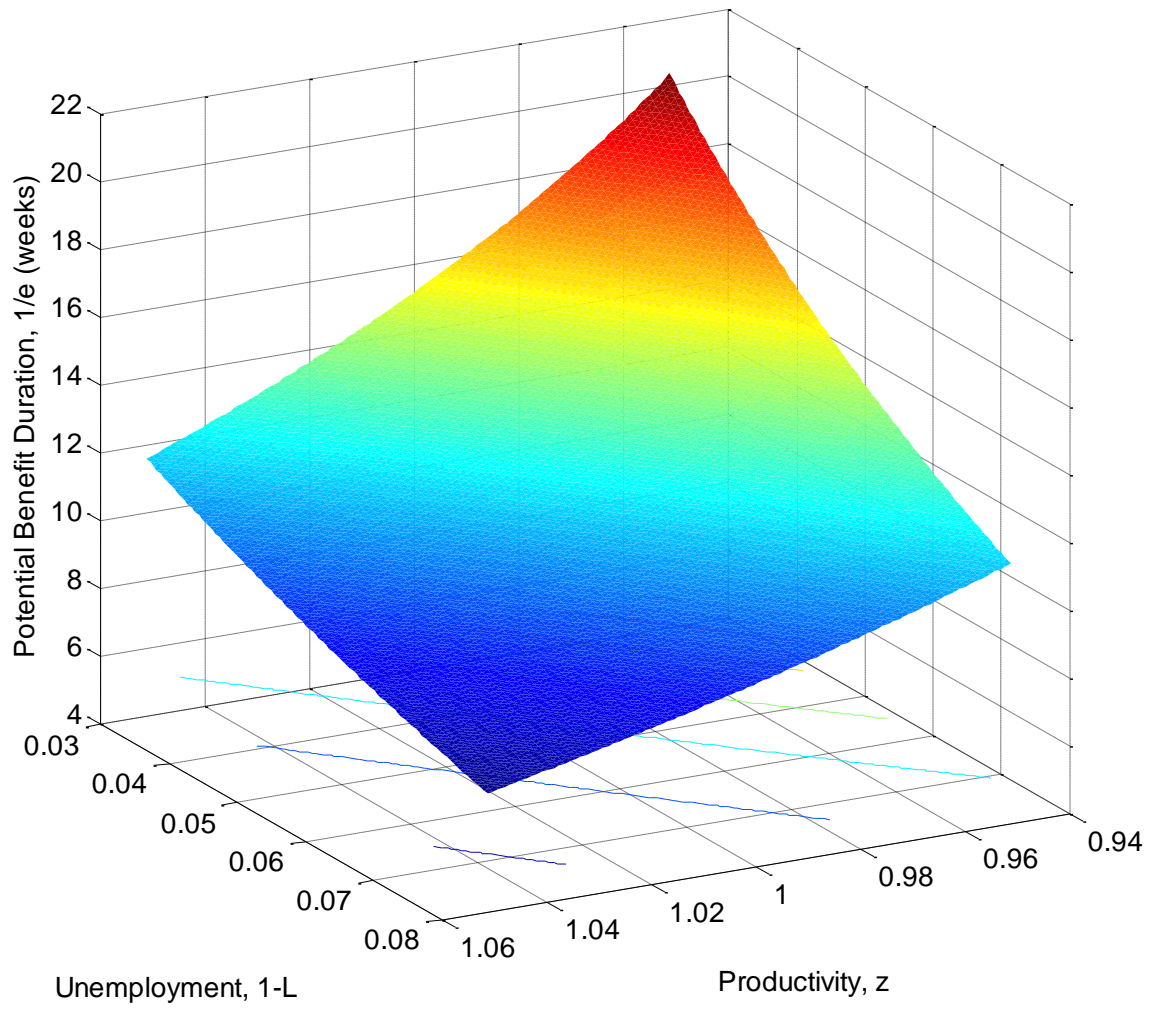


Figure 4: Responses to 1.5% drop in productivity

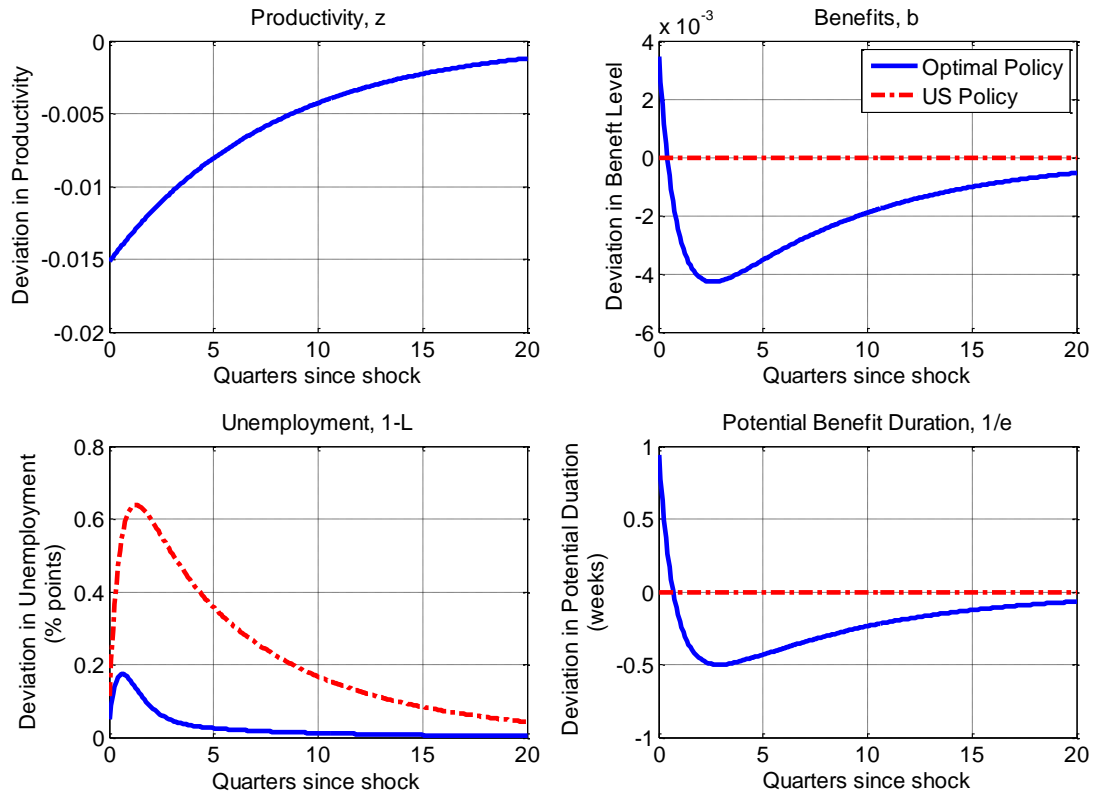


Figure 5: Responses to 1.5% drop in productivity

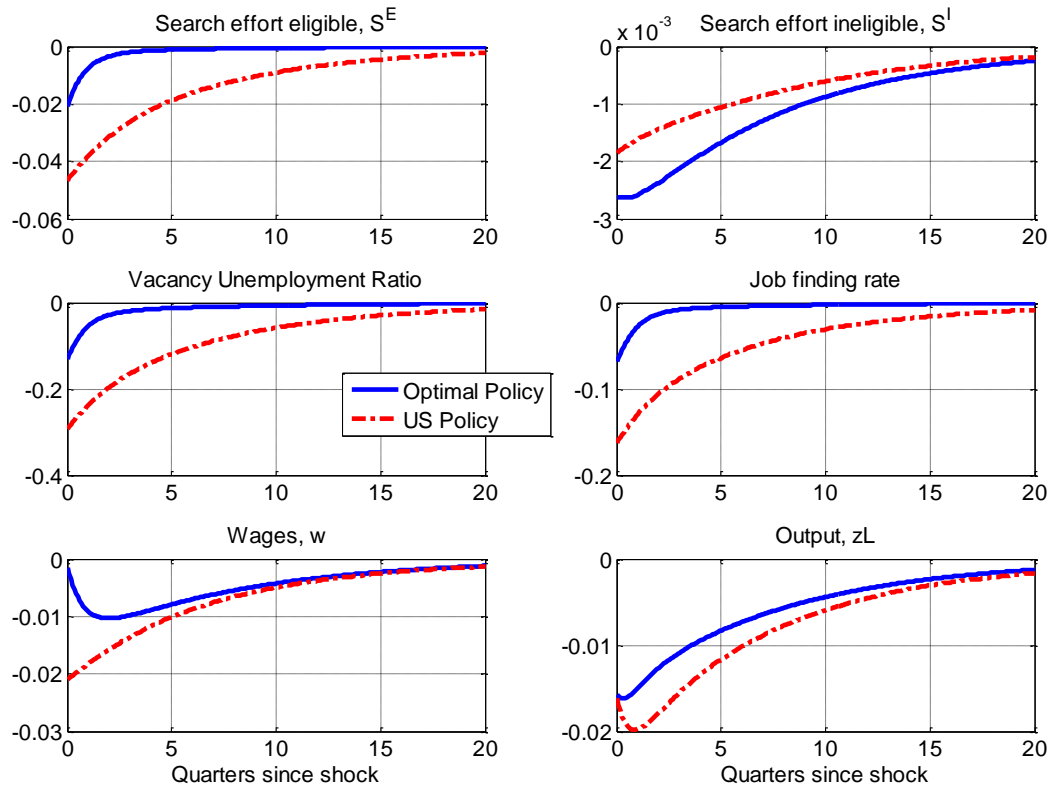


Figure 6: Response of duration to a 1.5% shock, fixing benefit level at  $b = 0.4$

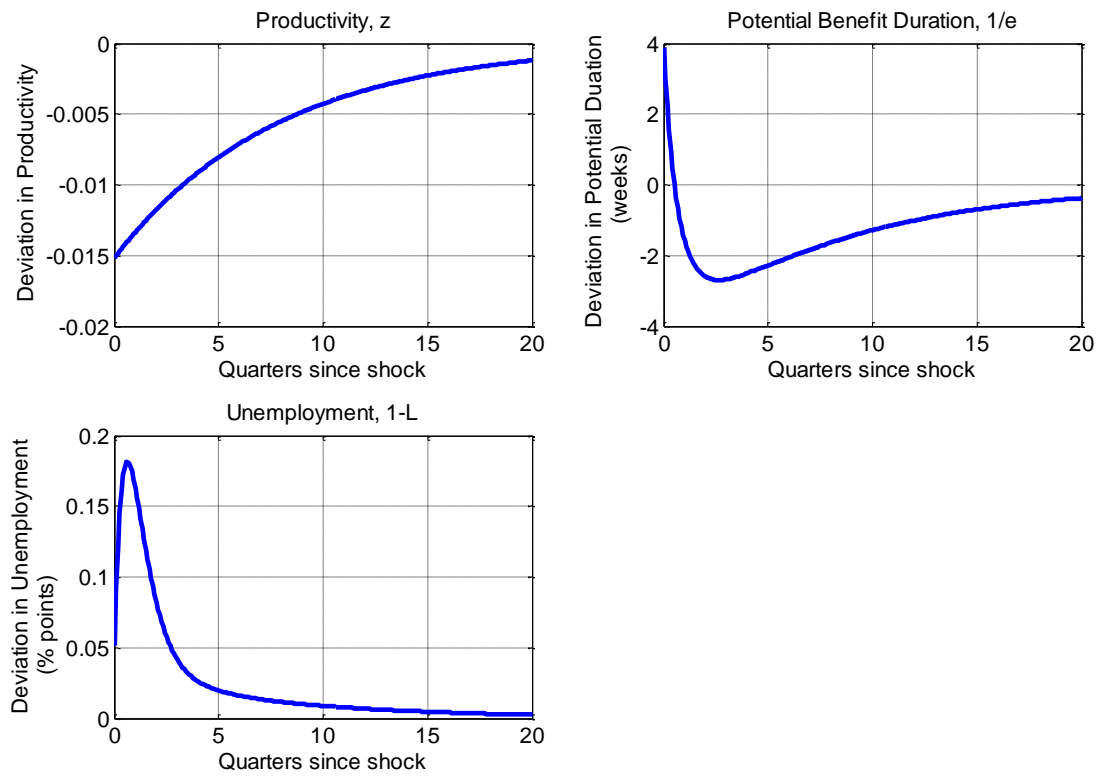


Figure 7: Response of benefit level to a 1.5% shock, fixing expected duration at 26 weeks

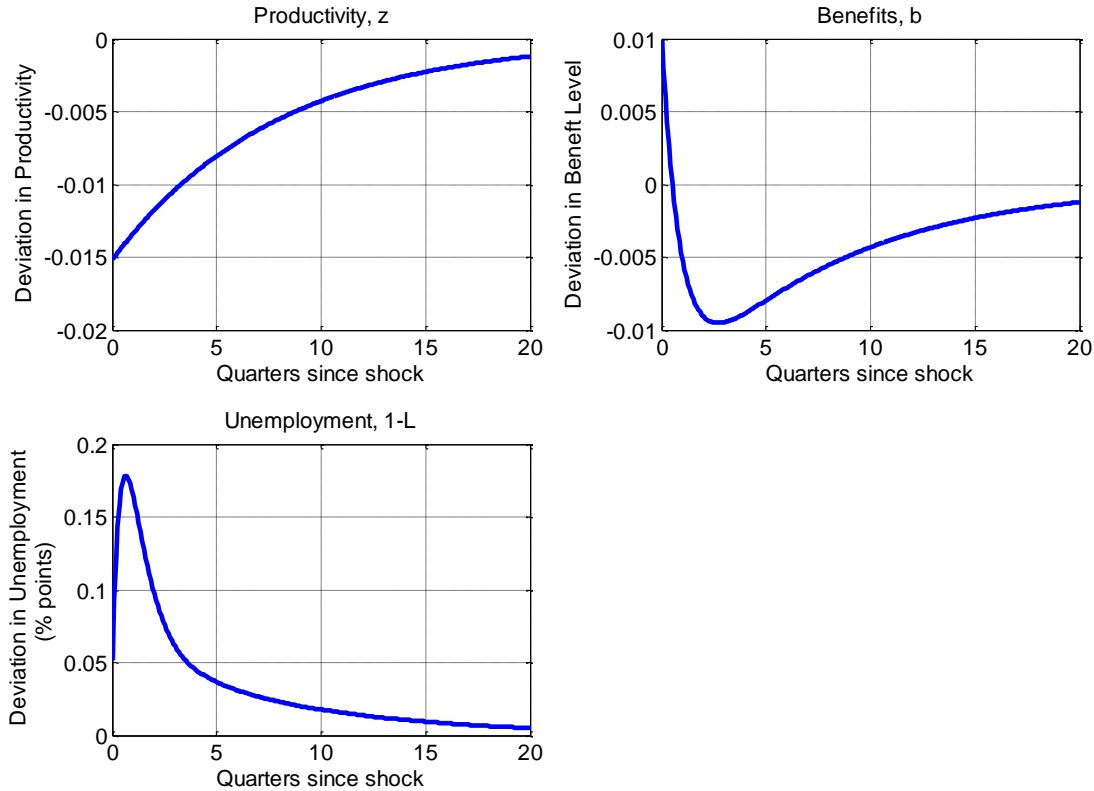


Figure 8: Response of benefit level to a 1.5% shock with no benefit expiration

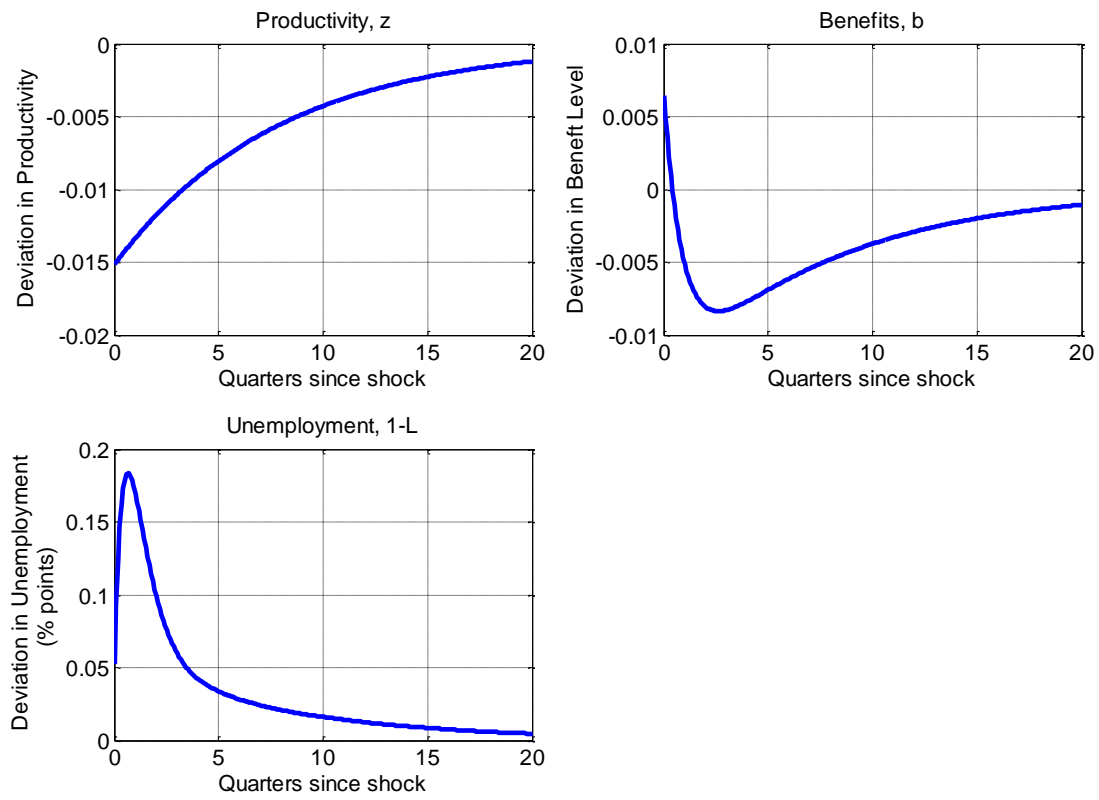


Figure 9: Response to a 1.5% shock with  $h = 0$

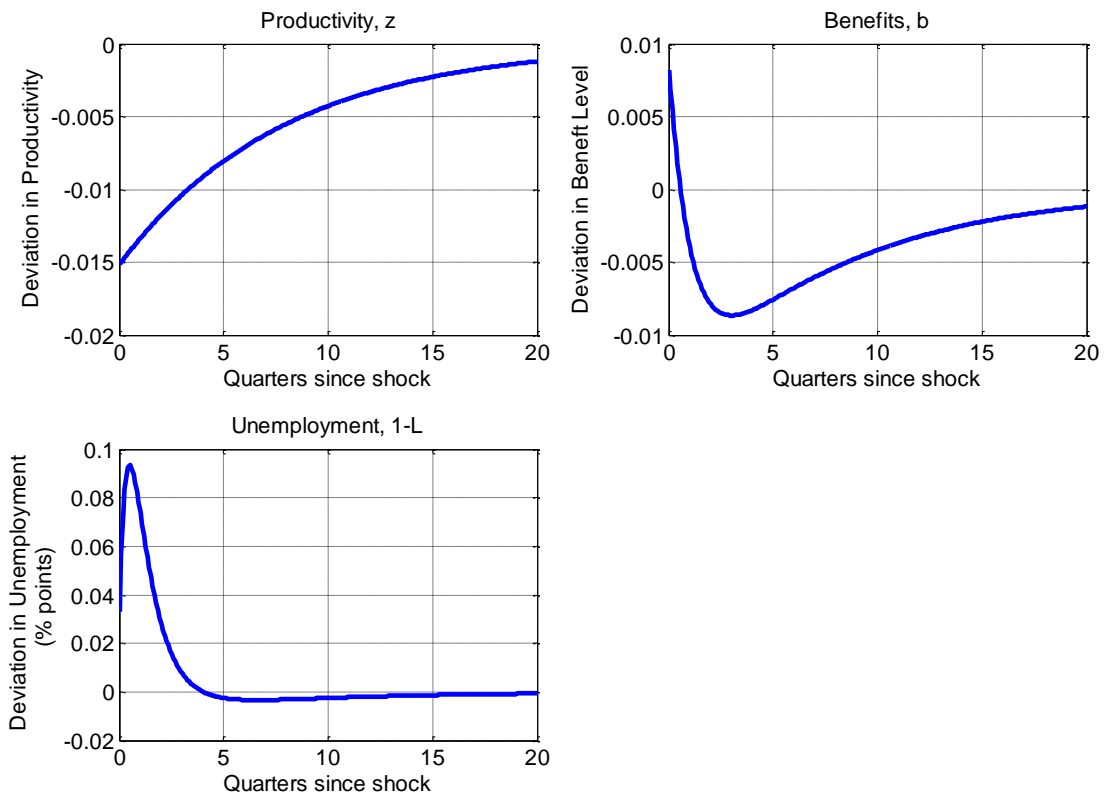


Figure 10: Response to a 1.5% shock with  $\xi = 0.72$

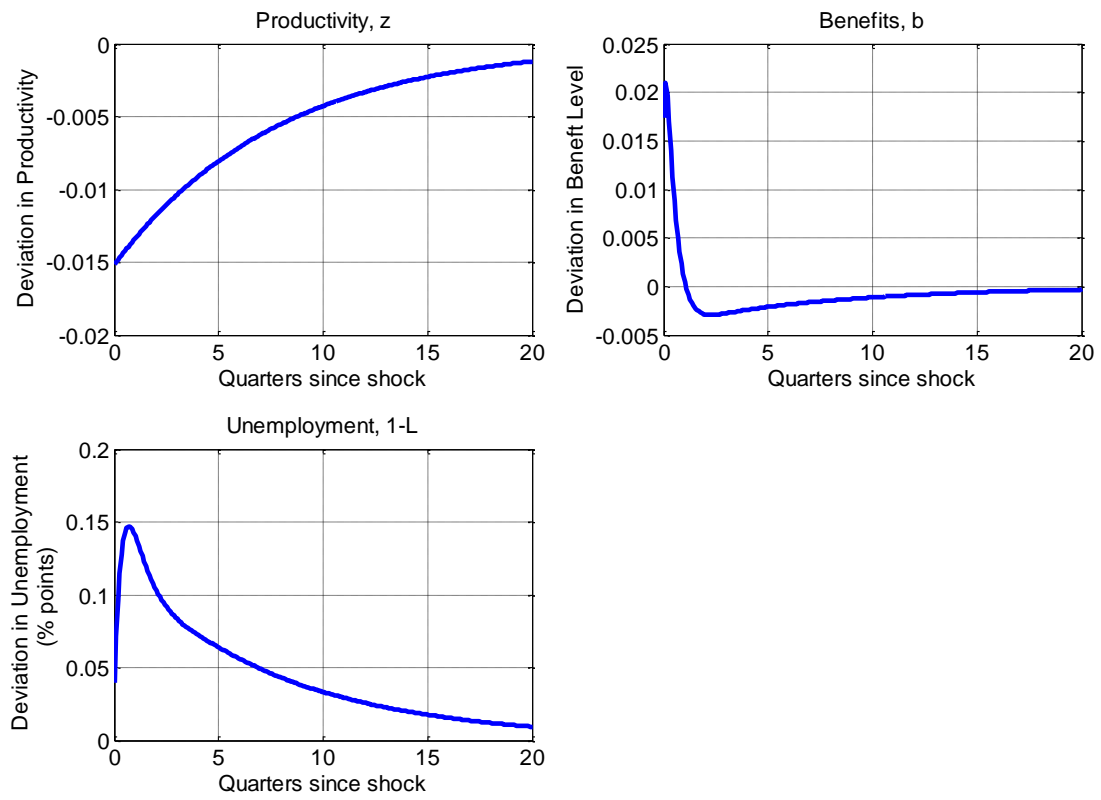


Figure 11: Response to a 1.5% shock with  $\xi = 0.72$ ,  $h = 0$

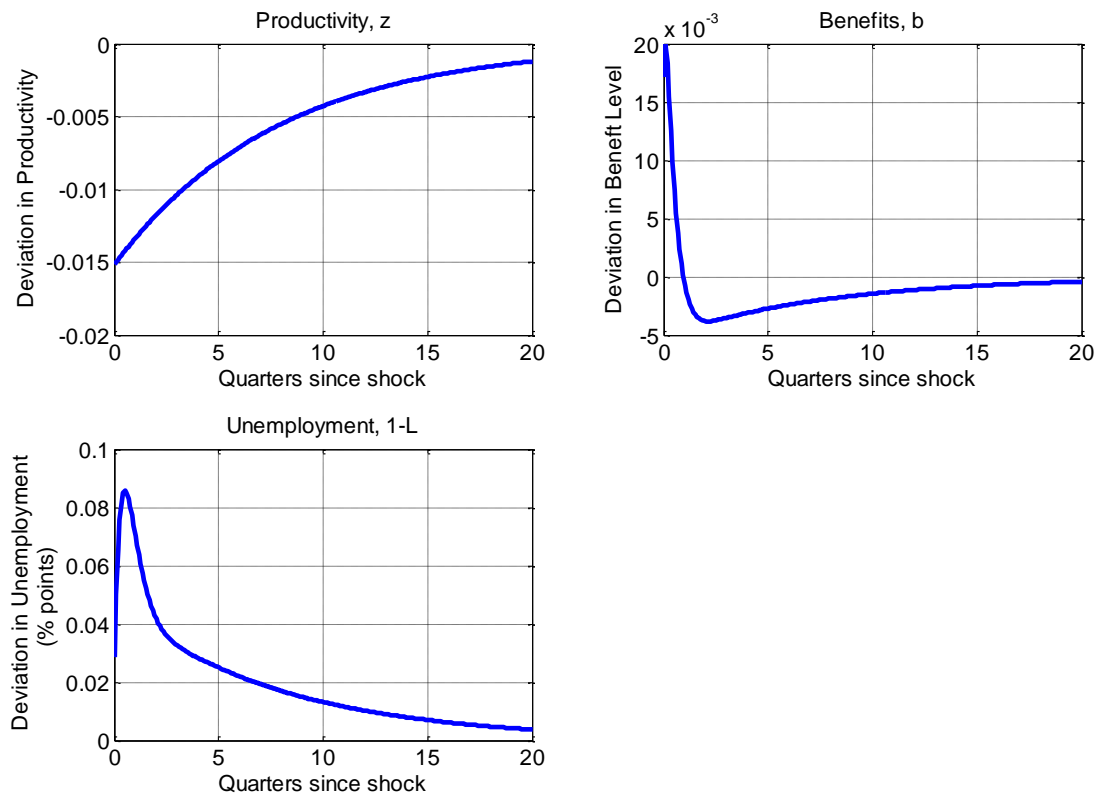


Figure 12: Response to a 1.5% shock under risk aversion of  $\sigma = 1/2$

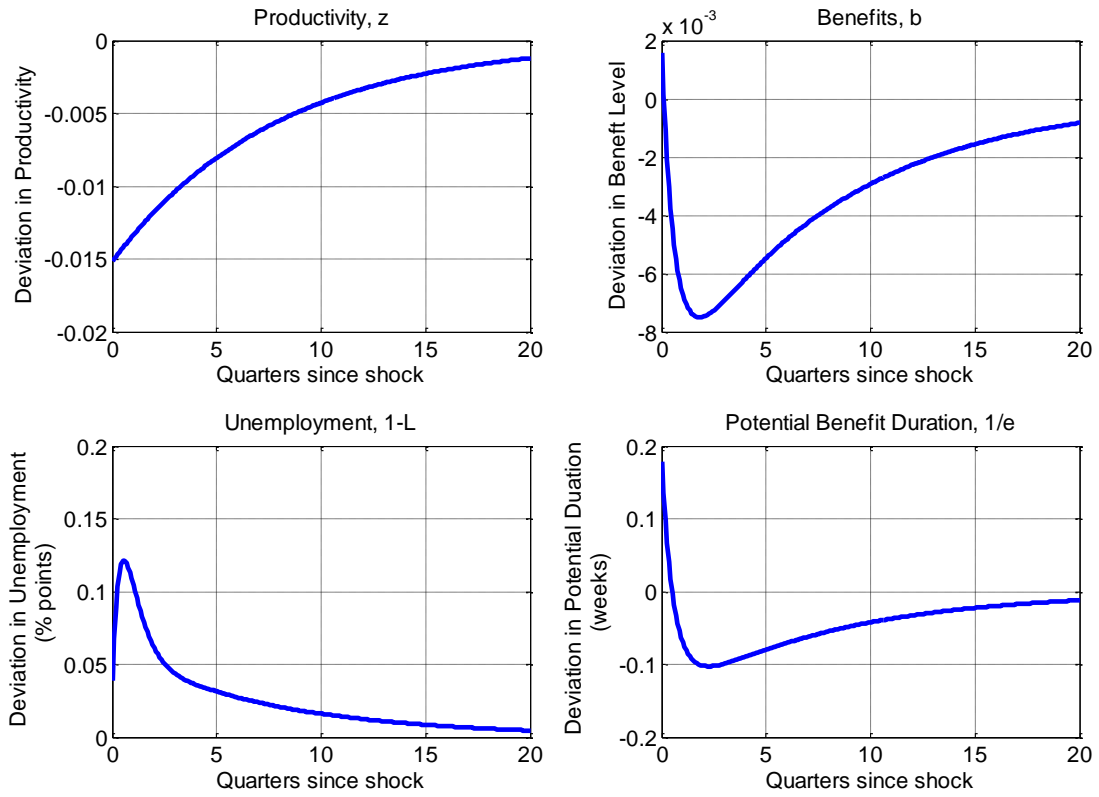
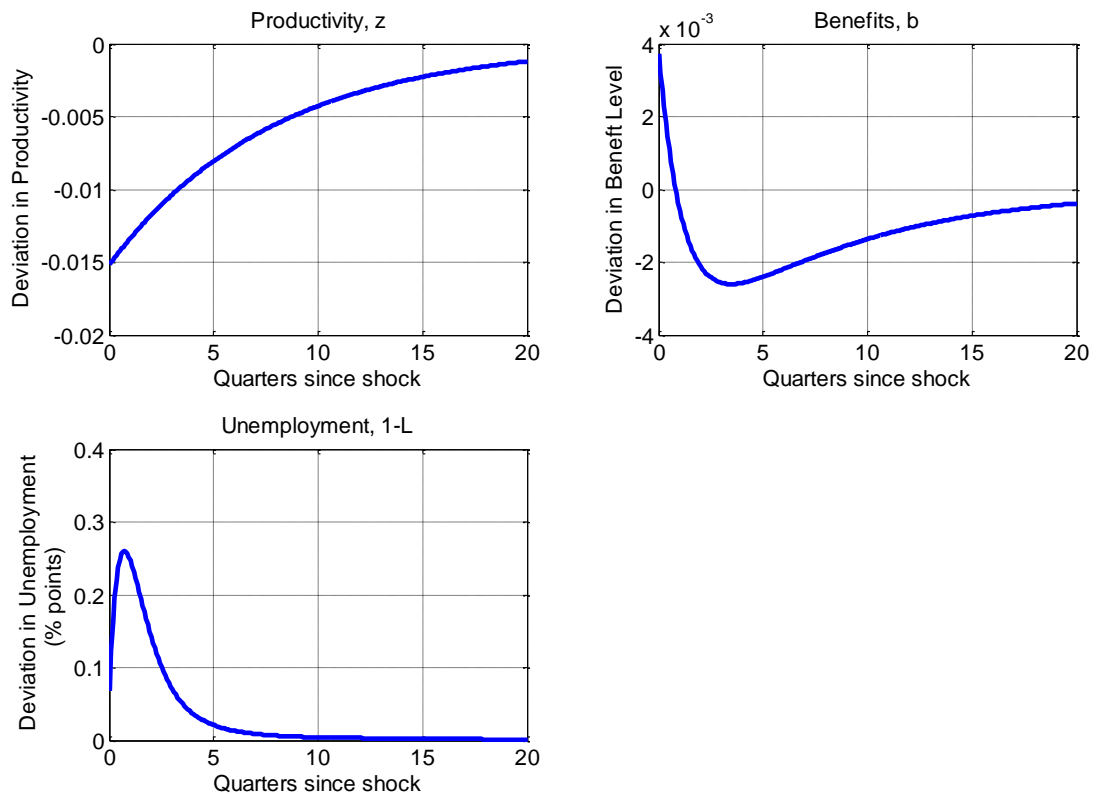


Figure 13: Response to a 1.5% shock under risk aversion of  $\sigma = 2$



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# A Appendix

## A.1 Characterization of Equilibrium

**Proof of Lemma 1.** First, observe that the necessary first-order conditions for optimal search effort are

$$\Delta_t = \frac{c'(S_t^E)}{f(\theta_t)} \quad (20)$$

$$\Xi_t = \frac{c'(S_t^I)}{f(\theta_t)} \quad (21)$$

Next, taking the differences of the workers' value functions from equations (5), (6), (7), we have

$$\begin{aligned} W_t - U_t^E &= c(S_t^E) + (1 - \delta - S_t^E f(\theta_t)) \Delta_t \\ &= c(S_t^E) + (1 - \delta - S_t^E f(\theta_t)) \frac{c'(S_t^E)}{f(\theta_t)} \end{aligned} \quad (22)$$

$$\begin{aligned} W_t - U_t^I &= c(S_t^I) + (1 - S_t^I f(\theta_t)) \Xi_t(z^t) - \delta \Delta_t \\ &= c(S_t^I) + (1 - S_t^I(z^t) f(\theta_t)) \frac{c'(S_t^I)}{f(\theta_t)} - \delta \frac{c'(S_t^E)}{f(\theta_t(z^t))} \end{aligned} \quad (23)$$

Next, we rearrange the expressions for worker surpluses (8), (9) to get

$$\begin{aligned} \Delta_t &= u(w_t) - u(h + b_t) \\ &\quad + \beta(1 - e_t) \mathbb{E}_t(W_{t+1} - U_{t+1}^E) + \beta e_t \mathbb{E}_t(W_{t+1} - U_{t+1}^I) \end{aligned} \quad (24)$$

$$\Xi_t = u(w_t) - u(h + p) + \beta \mathbb{E}_t(W_{t+1} - U_{t+1}^I) \quad (25)$$

Now, substituting (20) and (22) into the left and right hand sides of (24) gives (14); similarly, substituting (21) and (23) into the left and right hand sides of (25) gives (15).

Next, we derive the law of motion for the firm's surplus from hiring. By the free-entry condition, the value  $V_t(z^t)$  of a firm posting a vacancy must be zero. Equations (10) and (11) then simplify to:

$$J_t = (1 - \delta) [z_t - w_t - \tau + \beta \mathbb{E}_t J_{t+1}] \quad (26)$$

$$0 = -k + q(\theta_t) [z_t - w_t - \tau + \beta \mathbb{E}_t J_{t+1}] \quad (27)$$

which together imply

$$J_t = (1 - \delta) \frac{k}{q(\theta_t)} \quad (28)$$

$$\Gamma_t = \frac{k}{q(\theta_t)} \quad (29)$$

Equations (26) and (28) imply that  $\Gamma_t$  follows the law of motion  $\Gamma_t = z_t - w_t - \tau + \beta(1 - \delta)\mathbb{E}_t\Gamma_{t+1}$ , which, by (29), is precisely (16).

Finally, the first-order condition with respect to  $w_t$  for the Nash bargaining problem (13) is

$$\xi u'(w_t) \Gamma_t = (1 - \xi) \Delta_t \quad (30)$$

Substituting (29) and (20) into (30) and using the fact that  $f(\theta) = \theta q(\theta)$  yields (17).

The converse of the result holds since the value functions can be recovered via the corresponding Bellman equations. ■

## A.2 Solving for the Optimal Policy

The government is maximizing

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} L_t(z^t) u(w_t(z^t)) + \left( \frac{D_t(z^t)}{1-e_t(z^t)} \right) u(h + b_t(z^t)) + \left( 1 - L_t(z^t) - \frac{D_t(z^t)}{1-e_t(z^t)} \right) u(h + p) \\ -D_{t-1}(z^{t-1}) c(S_t^E(z^t)) - (1 - L_{t-1}(z^{t-1}) - D_{t-1}(z^{t-1})) c(S_t^I(z^t)) \end{array} \right\} \quad (31)$$

subject to the conditions (2), (3), (14). (15),(16),(17) holding for all  $z^t$ , and subject to the government budget constraint (4).

Let  $\pi(z^t)$  be the probability of history  $z^t = \{z_0, z_1, \dots, z_t\}$  given the initial condition  $z_{-1}$ . Denote by  $\eta$  the Lagrange multiplier on (4), and denote the Lagrange multipliers on (2), (3), (14). (15),(16),(17) by

$$\beta^t \pi(z^t) \lambda_t(z^t), \beta^t \pi(z^t) \alpha_t(z^t), \beta^t \pi(z^t) \mu_t(z^t), \beta^t \pi(z^t) \nu_t(z^t), \beta^t \pi(z^t) \gamma_t(z^t), \beta^t \pi(z^t) \phi_t(z^t),$$

respectively. In what follows, we suppress the dependence on  $z^t$  for notational simplicity. The first order necessary conditions with respect to  $b_t, e_t, w_t, S_t^E, S_t^I, L_t, D_t, \theta_t$ , respectively, are:

$$(D_t - (1 - e_t) \mu_t) u'(h + b_t) = \eta D_t \quad (32)$$

$$D_t [u(h + b_t) - u(h + p) - \eta b_t - \alpha_t] = \mu_t (1 - e_t) \left[ u(h + b_t) - u(h + p) - \frac{c'(S_t^I) - c'(S_t^E)}{f(\theta_t)} \right] \quad (33)$$

$$\gamma_t = (L_t + \mu_t + \nu_t) u'(w_t) - \phi_t \xi u''(w_t) k \theta_t \quad (34)$$

$$\begin{aligned} \phi_t (\xi - 1) c''(S_t^E) &= D_{t-1} [(\lambda_t - \alpha_t) f(\theta_t) - c'(S_t^E)] \\ &+ \frac{c''(S_t^E)}{f(\theta_t)} [\mu_{t-1} ((1 - e_{t-1}) (1 - S_t^I f(\theta_t)) - \delta) - \mu_t - \delta \nu_{t-1}] \end{aligned} \quad (35)$$

$$(1 - L_{t-1} - D_{t-1}) [c' - \lambda_t f(\theta_t) (S_t^I)] = \frac{c''(S_t^I)}{f(\theta_t)} [(\mu_{t-1} e_{t-1} + \nu_{t-1}) (1 - S_t^I f(\theta_t)) - \nu_t] \quad (36)$$

$$\lambda_t = u(w_t) - u(h + p) + \eta \tau + \beta \mathbb{E}_t \{ c(S_{t+1}^I) + \lambda_{t+1} (1 - \delta - S_{t+1}^I f(\theta_{t+1})) + \alpha_{t+1} \delta \} \quad (37)$$

$$\begin{aligned}
\alpha_t &= u(h + b_t) - u(h + p) - \eta b_t \\
&+ \beta(1 - e_t) \mathbb{E}_t \left\{ c(S_{t+1}^I) - c(S_{t+1}^E) + \lambda_{t+1} f(\theta_{t+1}) (S_{t+1}^E - S_{t+1}^I) + \alpha_{t+1} (1 - S_{t+1}^E f(\theta_{t+1})) \right\}
\end{aligned} \tag{38}$$

$$\begin{aligned}
&\phi_t \xi u'(w_t) k - f'(\theta_t) \left\{ \lambda_t [S_t^E D_{t-1} + S_t^I (1 - L_{t-1} - D_{t-1})] - \alpha_t S_t^E D_{t-1} \right\} - [\gamma_t - (1 - \delta) \gamma_{t-1}] \frac{kq'(\theta_t)}{(q(\theta_t))^2} \\
&= [\mu_t - \mu_{t-1} (1 - e_{t-1} - \delta) + \nu_{t-1} \delta] \frac{c'(S_t^E) f'(\theta_t)}{(f(\theta_t))^2} + [\nu_t - \nu_{t-1} - \mu_{t-1} e_{t-1}] \frac{c'(S_t^I) f'(\theta_t)}{(f(\theta_t))^2}
\end{aligned} \tag{39}$$

The first-order necessary condition for the optimal tax rate  $\tau$  is

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \{ \eta L_t(z^t) - \gamma_t(z^t) \} = 0 \tag{40}$$

To find the optimal policy given  $\eta$  and  $\tau$ , we solve the above system of difference equations (32)-(39) and (2), (3), (14). (15),(16),(17) for the optimal policy vector

$$\begin{aligned}
\Omega(z^t) &= \{ b_t(z^t), e_t(z^t), w_t(z^t), S_t^E(z^t), S_t^I(z^t), \theta_t(z^t), L_t(z^t), D_t(z^t), \\
&\lambda_t(z^t), \alpha_t(z^t), \mu_t(z^t), \nu_t(z^t), \gamma_t(z^t), \phi_t(z^t) \}
\end{aligned}$$

We then pick  $\eta$  and  $\tau$  so that (4) and (40) are satisfied.

Observe that the only period- $t-1$  variables that enter the period- $t$  first-order conditions are

$$L_{t-1}, D_{t-1}, e_{t-1}, \mu_{t-1}, \nu_{t-1}, \gamma_{t-1},$$

and no variables from periods prior to  $t-1$  enter the period- $t$  first-order conditions. This implies that  $(z_t, L_{t-1}, D_{t-1}, e_{t-1}, \mu_{t-1}, \nu_{t-1}, \gamma_{t-1})$  is a sufficient state variable for the history of shocks  $z^t$  up to and including period  $t$ . Specifically, fix  $\eta, \tau$ , and let  $(-)$  and  $(+)$  denote the previous period's variable and the next period's variable, respectively. Let

$$\Psi : (z, L_-, D_-, e_-, \mu_-, \nu_-, \gamma_-) \mapsto (b, e, w, S^E, S^I, L, D, \theta, \lambda, \alpha, \mu, \nu, \gamma, \phi)$$

be a function that satisfies

$$(D - (1 - e) \mu) u'(h + b) = \eta D \tag{41}$$

$$D[u(h+b) - u(h+p) - \eta b - \alpha] = \mu(1-e) \left[ u(h+b) - u(h+p) - \frac{c'(S^I) - c'(S^E)}{f(\theta)} \right] \quad (42)$$

$$\gamma = (L + \mu + \nu) u'(w) - \phi \xi u''(w) k \theta \quad (43)$$

$$\begin{aligned} \phi(\xi - 1) c''(S^E) = & D_- [(\lambda - \alpha) f(\theta) - c'(S^E)] \\ & + \frac{c''(S^E)}{f(\theta)} [\mu_- ((1 - e_-) (1 - S^I f(\theta)) - \delta) - \mu - \delta \nu_-] \end{aligned} \quad (44)$$

$$(1 - L_- - D_-) [c' - \lambda f(\theta) (S^I)] = \frac{c''(S^I)}{f(\theta)} [(\mu_- e_- + \nu_-) (1 - S^I f(\theta)) - \nu] \quad (45)$$

$$\lambda = u(w) - u(h+p) + \eta \tau + \beta \mathbb{E} \{ c(S^I_+) + \lambda_+ (1 - \delta - S^I_+ f(\theta_+)) + \alpha_+ \delta \} \quad (46)$$

$$\begin{aligned} \alpha = & u(h+b) - u(h+p) - \eta b \\ & + \beta(1-e) \mathbb{E} \{ c(S^I_+) - c(S^E_+) + \lambda_+ f(\theta_+) (S^E_+ - S^I_+) + \alpha_+ (1 - S^E_+ f(\theta_+)) \} \end{aligned} \quad (47)$$

$$\begin{aligned} \phi \xi u'(w) k - f'(\theta) \{ \lambda [S^E D_- + S^I (1 - L_- - D_-)] - \alpha S^E D_- \} - [\gamma - (1 - \delta) \gamma_-] \frac{k q'(\theta)}{(q(\theta))^2} \\ = [\mu - \mu_- (1 - e_- - \delta) + \nu_- \delta] \frac{c'(S^E) f'(\theta)}{(f(\theta))^2} + [\nu - \nu_- - \mu_- e_-] \frac{c'(S^I) f'(\theta)}{(f(\theta))^2} \end{aligned} \quad (48)$$

as well as

$$\begin{aligned} L = (1 - \delta) L_- + f(\theta) [S^E D_- + S^I (1 - L_- - D_-)] \\ D = (1 - e) [\delta L_- + (1 - s f(\theta)) D_-] \end{aligned} \quad (49)$$

$$\begin{aligned} \frac{c'(S^E)}{f(\theta)} = & u(w) - u(h+b) + (1 - e) \beta \mathbb{E} \left( c(S^E_+) + (1 - \delta - S^E_+ f(\theta_+)) \frac{c'(S^E_+)}{f(\theta_+)} \right) \\ & + e \beta \mathbb{E} \left( c(S^I_+) + (1 - S^I_+ f(\theta_+)) \frac{c'(S^I_+)}{f(\theta_+)} - \delta \frac{c'(S^I_+)}{f(\theta_+)} \right) \end{aligned} \quad (50)$$

$$\frac{c'(S^I)}{f(\theta)} = u(w) - u(h+p) + \beta \mathbb{E} \left( c(S^I_+) + (1 - S^I_+ f(\theta_+)) \frac{c'(S^I_+)}{f(\theta_+)} - \delta \frac{c'(S^E_+)}{f(\theta_+)} \right) \quad (51)$$

$$\frac{k}{q(\theta)} = z - w - \tau + \beta(1 - \delta) \mathbb{E} \frac{k}{q(\theta_+)} \quad (52)$$

$$\xi u'(w) k\theta = (1 - \xi) c'(S^E) \quad (53)$$

Then the sequence defined by

$$\Omega(z^t) = \Psi(z_t, L_{t-1}(z^{t-1}), D_{t-1}(z^{t-1}), e_{t-1}(z^{t-1}), \mu_{t-1}(z^{t-1}), \nu_{t-1}(z^{t-1}), \gamma_{t-1}(z^{t-1}))$$

satisfies the system (32)-(39) and (2), (3), (14). (15),(16),(17).

To find the optimal policy given  $\eta$ , we therefore solve the system of functional equations (41)-(53).