Geographical Reallocation and Unemployment during the Great Recession: The Role of the Housing Bust

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Abstract

This paper quantitatively evaluates the hypothesis that the housing bust in 2007 decreased the geographical mobility of workers and exacerbated unemployment during the Great Recession. We document the following facts regarding this episode: i) \textit{ceteris paribus}, unemployment increased more in metropolitan statistical areas (MSAs) with larger housing busts, and ii) controlling for local labor productivity, out-migration rates were lower in MSAs with larger housing busts. We construct and calibrate a directed search model of local housing and labor markets and migration. The model is quantitatively consistent with the volatilities of unemployment and vacancies, gross and net flows across MSAs, and the correlation between net flows and local unemployment prior to the Great Recession. A housing bust decreases the amount of home equity and makes it harder to afford a new house after moving due to a down payment requirement. Some households in low productivity regions that would have moved absent the housing bust decide to stay and look for jobs in their local labor market. The model accounts for 88% of the increase in the dispersion of unemployment across MSAs and the entire decline in net migration during the 2007-2009 recession. We find that this mechanism explains around 17% of the rise in unemployment after accounting for the fall in labor productivity.

\textbf{JEL Codes:} E24, E32, E44, J61, J63, J64, R12, R23

\textbf{Keywords:} housing bust, local labor markets, migration, mismatch, unemployment

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1 Introduction

The recent economic downturn in the U.S. was the most severe since the Great Depression. House prices plummeted in 2007, and the unemployment rate increased from 5% in January 2008 to 10.1% in October 2009—the largest level since the 1980s. At the same time, the dispersion of unemployment rates across metropolitan statistical areas (MSAs) doubled. In this paper, we propose a mechanism through which a decline in house prices can reduce the extent of geographical reallocation and affect local and aggregate unemployment. We develop a model of housing and labor markets and use it to quantify the effect of the housing bust on the level of unemployment and its dispersion across regions.

How does a decline in house prices affect geographical reallocation and the labor market? In this paper, we focus on a financial friction: the down payment requirement in purchasing a home. When house prices fall, the amount of home equity declines, making it harder to afford the down payment on a new house after moving. To the extent that households care about owning a house, the decline in house prices affects their migration decisions. Some households that would normally move out of regions with low productivities may stay and look for jobs in distressed labor markets. Thus, the aggregate job finding rate decreases, which results in higher aggregate unemployment.

The decline in house prices during the housing bust is asymmetric across MSAs, ranging from 5% to 50%. Furthermore, the decline in local labor productivity across MSAs, measured as output per worker, is also asymmetric and ranges from 0% to 9%. Exploiting these variations, we document the following facts: First, controlling for the decline in labor productivity, the unemployment rate increased more in MSAs that experienced larger housing busts. Second, controlling for local labor productivity, MSAs with larger housing busts had smaller out-migration rates. These facts suggest that the housing bust in 2007 may have affected geographical migration and local unemployment rates.

To quantitatively evaluate the importance of the proposed mechanism in explaining worker flows and local and aggregate unemployment in the context of the recent housing bust and recession, we develop a computationally tractable directed search model of housing and labor markets with two MSAs. In the model, MSAs are subject to local labor productivity shocks. Households decide where to live and work, whether to be homeowners or renters, and how much to save and consume. There is a down payment requirement to purchasing a home. Homeowners are required to sell their houses in order to migrate to the other MSA, whereas renters are not subject to this constraint.

We calibrate our model to match national gross and net migration rates, the homeowner-
ership rate, median leverage, average time to sell a house, and aggregate statistics related to the labor market. The calibrated model is quantitatively consistent with the volatilities of unemployment and vacancies, the correlation between leverage and mobility rates, the cyclical properties of gross migration, and the negative correlation between local unemployment and net flows prior to 2007.

We then use the model to simulate the 2007-2009 recession. We first group MSAs in the U.S. into two categories according to the decline in house prices in 2007. The decline in labor productivity is larger in the group with the larger housing bust. We feed into the model the observed declines in labor productivities and house prices. A decline in local labor productivity decreases wages and job finding probabilities as firms find it less profitable to post vacancies. This effect is more pronounced in the region with the larger productivity decline and creates a difference across regions in wages and job finding probabilities. Thus, the asymmetry in labor productivity increases the benefit of migration for households in the low productivity region. The decline in house prices decreases home equity and makes it harder to afford the down payment on a new house after moving, which results in a decline in migration. Due to the asymmetry in the housing bust, this effect is more pronounced in the region with the larger housing bust. Compared to a recession without a housing bust, more unemployed workers look for jobs in the region with lower productivity. This increases the aggregate unemployment rate and the dispersion of the unemployment rate across the two regions.

We find that the model captures well the decline in net and gross migration rates. In the model, average net flows decline from a pre-recession annual rate of 0.8% to 0.2%. We document that, in the data, net flows decreased from 0.8% in 2006 to 0.3% in 2009. Turning to gross flows, the model predicts a decline from 4.3% to 3.3%, compared to a decline from 4.3% to 3.8% in the data. The model accounts for 88% of the increase in the dispersion of unemployment rates across these MSAs: The model predicts an increase in the difference of unemployment rates of 2.2 percentage points. In the data, this difference rises by 2.5 percentage points. We find that, through its effect on geographical reallocation, the housing bust explains about 0.5 percentage point of the rise in aggregate unemployment during the recession. This corresponds to 17% of the rise in unemployment after accounting for the decline in labor productivity.

The possibility that the housing bust in 2007 might affect aggregate unemployment by reducing migration has been recognized in the popular media.\(^1\) This argument, however, was

\(^1\)For example, the New York Times reports: “Experts said the lack of mobility was of concern on two fronts. It suggests that Americans were unable or unwilling to follow any job opportunities that may
largely dismissed due to an empirical observation documented by Kaplan and Schulhofer-Wohl (2010): the decline in inter-state migration during the 2007-2009 recession is small (around 0.5 percentage point). The decline in the job-related migration rate is about a third of the overall decline.

Our findings show that the housing bust in 2007 has had a nontrivial effect on local and aggregate unemployment and that this effect is driven by changes in net and gross migration rates that are consistent with the data. The key to resolving the apparent contradiction between our results and the aforementioned literature is the idea that the decline in migration observed during the recession may not represent the total decline in migration caused by the housing bust. The decline in labor productivity is asymmetric across MSAs and one might expect, absent the housing bust, net and gross migration to increase. Our model allows us to compute these counterfactual migration rates. We find that, absent the housing bust, gross migration would have increased to 5.3% and net migration would have increased to 1.6%. We conclude that the decline in migration observed in the data constitutes only half of the decline attributable to the housing bust.

To study the effect of the housing bust on the labor market, we are confronted with the challenge of developing a model of housing and labor markets with endogenous asset accumulation and aggregate shocks. Dynamic models with such features are difficult to solve, even numerically. In the presence of search frictions, computation of the model becomes even more complex. The main source of the difficulty is the fact that one needs to keep track of the entire distribution of households over assets, employment status, wages, etc., an infinite dimensional object. To overcome this issue, we use the structure of the labor market with directed search as in Menzio and Shi (2010a) and Menzio and Shi (2011). We build on their idea by also modeling the housing market with directed search. As in Head and Lloyd-Ellis (2010) and Hedlund (2011), trade in the housing market is facilitated by real estate companies. We show that the market structure in our model admits a block-recursive equilibrium (BRE), a particular recursive equilibrium where the endogenous distributions generated by the model are not part of the state space. This property of the equilibrium makes the computation feasible, without having to resort to approximations.

Related Literature  This paper is related to several strands of the literature. The closest study to ours is Sterk (2010), who also studies the effect of the housing bust in 2007 on have existed around the country, as they have in the past. And the lack of movement itself, they said, could have an impact on the economy, reducing the economic activity generated by moves.” (NYT (2009), http://www.nytimes.com/2009/04/23/us/23census.html). Also see Kocherlakota (2010).
the labor market during the recent recession. Davis, Faberman, and Haltiwanger (2010), Elsby, Hobijn, and Sahin (2010), and Hall (2010), among others, have noted that the Beveridge curve—the empirical relationship between unemployment and vacancies—exhibited a rightward shift after 2008, implying that, for a given level of vacancies, the level of aggregate unemployment is higher than predicted by the historical relationship between unemployment and vacancies. Sterk (2010) argues that the housing bust, by affecting the acceptance of job offers that require relocation, can account for this empirical pattern. We differ from him in our explicit modeling of different local labor markets and in our focus on the local properties of the Great Recession and the housing bust. Unlike in Sterk (2010), we allow households to own or rent. The data show that more than 40% of homeowners who move are renters after a year, suggesting that the option of renting after moving is quantitatively important for understanding the effect of the housing bust on migration.\(^2\)

On the empirical front, there is a large and growing literature that investigates the effect of declining house prices on the mobility of workers. Chan (2001) finds that declining house prices significantly constrain the mobility rates of homeowners, in particular, those with high loan-to-value ratios. Henley (1998), using data from the U.K., reaches a similar conclusion. More recently, Ferreira, Gyourko, and Tracy (2010), Ferreira, Gyourko, and Tracy (2011), and Schulhofer-Wohl (2010) study the relationship between leverage and residential mobility in the context of the recent housing bust. These studies differ in how they identify long-distance and permanent moves in the data. While Ferreira, Gyourko, and Tracy (2010) and Ferreira, Gyourko, and Tracy (2011) find that negative equity reduces mobility by as much as 30%, Schulhofer-Wohl (2010) finds no effect. Finally, using data from the Boston condominium market in the 1990s, Genesove and Mayer (1997) document the differences between homeowners with different amounts of leverage in the price posted for the house and the time it takes to sell it. They find that homeowners with 100% leverage post 4% higher prices compared to homeowners with 80% leverage, and consequently, selling the house takes 15% longer.\(^3\)

Our paper is also related to the empirical literature that studies “matching efficiency” and “mismatch” in the labor market. Barnichon and Figura (2011a) study the sources of the movements in matching efficiency, the efficiency of the labor market to bring together

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\(^2\) Another explanation of the high level of unemployment observed during the 2007−2009 is the extension of unemployment benefits. Nakajima (2011), Rothstein (2011), and Mitman and Rabinovich (2011) provide a quantitative evaluation.

\(^3\) More broadly, the idea that homeownership can effect the labor market has long been recognized. There is a substantial empirical and theoretical literature studying this link. A partial list includes Oswald (1996); Oswald (1997); Green and Hendershott (2001); Coulson and Fisher (2002, 2009); Halket and Vasudev (2010); Guler and Taskin (2011); Lkhagvasuren (2011); and Winkler (2010).
unemployed workers and vacancies. They find that, until 2006, most of the movements in matching efficiency are due to compositional changes in the pool of unemployed workers, whereas the decline in matching efficiency in 2008-2009 is largely due to the dispersion in labor market conditions, the fact that tight labor markets coexist with slack ones. This is consistent with the findings in this paper that the housing bust generates a large dispersion in economic conditions across regions, thereby decreasing aggregate matching efficiency. Sahin, Song, Topa, and Violante (2011) formalize the notion of mismatch and provide a framework to measure the extent to which mismatch has caused higher unemployment during the recent recession. Mismatch is measured as the difference between the observed allocations of unemployment and vacancies across different markets and the optimal allocations obtained from a social planner’s problem. They find that mismatch accounts for 0.6 to 1.7 percentage points of the recent rise in the unemployment rate.

On the theoretical front, our model builds on the island-economy framework of Lucas and Prescott (1974). Head and Lloyd-Ellis (2010) study a two-island economy with rental and ownership markets. Their focus is on the effect of the illiquidity in the homeownership markets on the different mobility rates of homeowners and renters and its implications for local and aggregate unemployment. Our analysis differs from theirs, as we focus on the role of mortgage leverage to explain the stylized facts from the Great Recession. This requires deviation from a steady-state analysis by incorporating aggregate shocks to the productivity of regions and allowing for asset accumulation.

Finally, this paper is part of a recent literature that employs directed search models of the labor market to study economies with heterogeneity and aggregate shocks (e.g., Menzio and Shi (2010a), Menzio and Shi (2010b), Schaal (2010), and Kaas and Kircher (2011)). Along this dimension, most closely related is Hedlund (2011), who develops a directed search model of the housing market and studies the implications of search frictions on house price dynamics.

The rest of the paper is organized as follows. Section 2 provides information about the data and documents the stylized facts regarding local labor and housing markets and

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4Mismatch captures the idea that the characteristics of workers (e.g. geography or skill) may not coincide with the needs of firms.
6Another paper that studies the interaction between housing and labor markets is Rupert and Wasmer (2009), who study a model with a location choice and use it to explain differences in mobility and unemployment rates between the U.S. and Europe.
7Other search models of the housing market include Wheaton (1990); Diaz and Jerez (2010); Head, Lloyd-Ellis, and Sun (2010); Piazzesi and Schneider (2009); Burnside, Eichenbaum, and Rebelo (2011); and Albrecht, Gautier, and Vroman (2010)
migration that motivate our analysis. Section 3 presents the model. Section 4 provides the
details of our calibration and its fit along targeted and untargeted dimensions of the data.
Section 5 uses the model as a measurement tool to quantify the effect of the housing bust
on the labor market. Finally, Section 6 concludes.

2 Empirical Findings

In this section, we document the behavior of labor productivity, unemployment, house prices,
and population flows across MSAs during the 2007 – 2009 recession. We construct an annual
panel data set of 351 MSAs containing information on output, employment, unemployment,
population and population flows, and house prices. The details of the data are given in
Appendix A.

Dispersion of the Decline in House Prices across MSAs: We start by documenting
the large dispersion in the size of the housing bust across MSAs. The size of the housing bust
is defined as the percentage decline in house prices between the peak house price preceding
2007 and the trough after the bust. Between 2007 and 2008, house prices declined by 28.7%
on average. The standard deviation of this decline across MSAs is 14.7%. To give a more
detailed picture, Figure 6 plots the histogram of this measure. While there are regions in
which house prices declined by less than 10%, there are also a fair number of MSAs where the
decline in house prices exceeded 40%. If the housing bust had an effect on the labor market
during the 2007 – 2009 recession through its effect on local labor markets, one would expect
to see this effect more pronounced in MSAs with larger housing busts. Later in this section,
we exploit this variation to provide evidence for the mechanism pursued in this paper.

Dispersion of the Decline in Labor Productivity across MSAs: At the core of
this paper is the idea that a decline in house prices reduces geographical reallocation by
distorting the migration choice of homeowners. The importance of migration for the behavior
of aggregate unemployment, however, depends on the amount of regional disparities across
local labor markets. Motivated by this observation, we now document the change in local
and aggregate labor productivity (output per worker) during the recent recession. Consistent
with previous studies, our data suggests that aggregate labor productivity declined by around
3% at the start of the recession. Strikingly, there is substantial variation across MSAs: the
standard deviation of the decline from 2007 to 2008 (weighted by employment) is 2%.

8See Figure 7 for a histogram of the decline in local labor productivity.
Dispersion of Unemployment Rates across MSAs:  How did the differences in labor productivity translate into local unemployment rates? Using monthly data on unemployment rates, Figure 8 plots the standard deviation of unemployment rate across MSAs for the period 2001 – 2010. Dispersion of unemployment rates were relatively stable around 1.2% until the recent recession. The figure illustrates that the recent recession is characterized by large differences in local labor market conditions: starting in 2008, the standard deviation increased to around 2.5%.9 Being unprecedented over the sample period, it is important to understand the causes of this increase. One possible explanation, which we explore using the model in Section 3, is that the housing bust in 2007 distorted the migration decisions of households and caused many unemployed households look for jobs in regions with low job finding rates, which ultimately led to an increase in the dispersion of unemployment rates.

The proposed mechanism has two implications that we now explore in the data. First, it implies that, even after controlling for the decline in labor productivity, unemployment should increase more in MSAs with larger house price declines. Second, it implies that, controlling for local labor productivity, MSAs with larger housing busts should have smaller out-migration rates. The substantial heterogeneity in the decline of labor productivity and the decline in house prices allow us to test these implications.

Housing Bust and the Rise in Unemployment:  If the housing bust contributed to unemployment during the recent recession via the proposed mechanism, one should expect to see a larger increase in local unemployment in regions with larger housing busts. We now look at the statistical relationship between the increase in unemployment rates during the recent recession and the decline in house prices. In the last column of Table 1, we regress the change in local unemployment rate during the recession on the change in house prices. The coefficient is positive and significant at 5%, and suggests that a 10 percentage point decline in house prices is associated with a further increase in local unemployment of 0.2 percentage points.10

The correlation between the increase in unemployment and the decline in house prices can be spurious. More specifically, a larger regional slump can both cause a larger fall in house prices and a larger increase in local unemployment.11 One should control for potential

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9 Sahin, Song, Topa, and Violante (2011) also report an increase in the dispersion using a different measure.
10 For completeness, the second column regresses the level of local unemployment rate during the recession period (2007 – 2009) on the size of the housing bust. The coefficient is positive and significant at 5%. According to the coefficient estimate, an MSA that experienced a 10% larger decline in house prices suffers almost 0.3% more unemployment.
11 Alternatively, a larger decline in house prices could cause a larger decline in productivity (Midrigan and Philippon (2011)).
omitted variable bias arising from the exclusion of labor productivity. Indeed, we find that the correlation between the decline in labor productivity and the decline in house prices is \(-0.21\) and significant, suggesting that part of the unemployment differences across MSAs may be driven by the differences in labor productivity. We now investigate if this correlation is entirely driven by the differences in labor productivities.

Table 1: Rise in Unemployment and the Decline in House Prices

<table>
<thead>
<tr>
<th>Dep. Variable</th>
<th>unemployment rate $u_{i,t}$</th>
<th>change in unemployment rate $\Delta u_{i,t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>size of housing bust</td>
<td>0.027***</td>
<td>0.022***</td>
</tr>
<tr>
<td>$\Delta \log p_{i,t}$</td>
<td>(0.009)</td>
<td>(0.0045)</td>
</tr>
</tbody>
</table>

Note: Numbers in parentheses are standard errors. Observations are weighted by MSA employment. *** = significant at 1%. Table shows the statistical relationship between unemployment and the size of the housing bust. Second column regresses unemployment after 2008 on the size of the housing bust, whereas the last column regresses the change in unemployment between 2007 and 2008 on the size of the housing bust. Source: See the data appendix for a detailed description of the data.

Table 2 reports the results to several regressions. In each regression, observations are weighted by MSA employment.\(^\text{12}\) In Column (2), we regress the change in unemployment on the change in labor productivity and two interaction terms: the interaction of the change in labor productivity with the size of the housing bust and the interaction of the change in local labor productivity with the square of the size of the housing bust. These terms are intended to capture the variation across MSAs in the increase in unemployment as a response to the same decline in local labor productivity. More specifically, if MSAs with larger housing busts systematically differ from others in how much unemployment increased as a response to the same decline in local labor productivity, these differences will be captured by the interaction terms. The coefficient on the linear term is positive but not significant, whereas the coefficient on the second interaction term is negative and significant at 5%. Column (3) adds the size of the housing bust to the specification. The coefficient on the linear and quadratic interaction terms are positive and negative, respectively, and both are significant at 5%. To visualize the effect of the interaction terms, in Figure 9, we plot the corresponding effect against the decline in house prices for both specifications. The figure shows that, holding the change in

\(^{12}\)Results are qualitatively the same if we do not weigh the observations.
labor productivity constant, unemployment increased more in MSAs that experienced larger housing busts.\footnote{For completeness, Column (1) shows a simpler specification: we regress the change in log unemployment to the change in labor productivity. The coefficient on labor productivity is negative and significant, showing that unemployment increased more in MSAs that experienced a larger fall in labor productivity. This implies that local unemployment responds to local labor productivity conditions.}

| Table 2: Rise in Unemployment and the Decline in House Prices |
|---------------------------------|-------------|-------------|
| Dependent Variable: $\Delta \log u_i$ | (1) | (2) | (3) |
| change in labor productivity | $-0.986^{**}$ | $-0.038$ | $-2.065^*$ |
| $\Delta \log z_i$ | $(0.323)$ | $(1.428)$ | $(1.240)$ |
| size of the housing bust | $0.004^{***}$ | | $(0.0006)$ |
| $hbust_i$ | | | |
| linear interaction | $0.096$ | $0.169^*$ | |
| $\Delta \log z_i \times hbust_i$ | $(0.099)$ | $(0.092)$ | |
| quadratic interaction | $-0.384^{***}$ | $-0.339^{**}$ | |
| $\Delta \log z_i \times (hbust_i/10)^2$ | $(0.146)$ | $(0.134)$ | |

Note: Numbers in parentheses are standard errors. Observations are weighted by MSA employment. $^{***} =$ significant at 1%, $^{**} =$ significant at 5%, $^* =$ significant at 10%. Table shows the statistical relationship between the rise in unemployment during the recent recession and the following variables: the decline in house prices during the housing bust, the decline in local labor productivity, and the interaction of these two.

**Housing Bust and Migration:** This paper argues that the housing bust affects the labor market by decreasing migration. We now document migration rates during the 2007 – 2009 recession.

Table 3 documents gross and net flows for the period for which we have access to the IRS data (2004 – 2009). The results point to non-negligible declines in both migration rates. Gross migration rate across MSAs falls from 4.3% before the housing bust to a low of 3.8% during 2008 – 2009. Kaplan and Schulhofer-Wohl (2010) report a similar decline in inter-state migration rate during the recession. Net migration plummeted during the recession.
the housing bust and the recent recession, an average MSA in U.S. gained or lost around 0.8% of its population in a given year. For the period 2008 – 2009, this number declined to 0.3%, less than half of the pre-recession level. The magnitude of the decline is surprising given the large dispersion across local labor markets and the importance of migration for local labor markets (Blanchard and Katz (1992)). The decline in migration rates alone are not sufficient, however, to conclude that they were caused by housing bust in 2007. This is because net migration is pro-cyclical (Saks and Wozniak (2011)). To bring further evidence, we now turn from aggregate numbers to an investigation of worker flows at the MSA level and how they are related to size of the local housing bust.

Table 3: Gross and Net Migration Rates (2004 – 2009)

<table>
<thead>
<tr>
<th>Year</th>
<th>Gross Migration Rate</th>
<th>Net Migration Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004 – 2005</td>
<td>4.2%</td>
<td>0.7%</td>
</tr>
<tr>
<td>2005 – 2006</td>
<td>4.3%</td>
<td>0.9%</td>
</tr>
<tr>
<td>2006 – 2007</td>
<td>4.2%</td>
<td>0.7%</td>
</tr>
<tr>
<td>2007 – 2008</td>
<td>4.1%</td>
<td>0.5%</td>
</tr>
<tr>
<td>2008 – 2009</td>
<td>3.8%</td>
<td>0.3%</td>
</tr>
</tbody>
</table>

Note: Second column reports gross migration rate for each year. This is defined as the average of gross inflow rate and gross inflow rate. In the last column, we show average net migration rate, defined as the average of the absolute value of net flows across MSAs (weighted by population).

Another implication of the mechanism proposed in this paper is that outflows should respond less to adverse local conditions in MSAs with larger housing busts. In Table 4, we investigate the relationship between population flows and local labor productivity during the recent recession. Column (1) regresses out-migration rate on the “productivity gap” – the difference between aggregate (log) labor productivity and MSA level labor productivity.
A larger gap means that local labor productivity is lower. The coefficient is positive and significant, showing that MSAs with lower labor productivity tend to have larger outflows. In Column (2), we add the interaction of the productivity gap with the size of the housing bust. The interaction term is intended to capture differences across MSAs in how outflows are related to local conditions. In particular, we are interested in documenting if, after controlling for local labor productivity, MSAs with larger housing busts have more or less outflows compared to others. The coefficient on the interaction term is negative and significant, and lends support to the hypothesis that a decline in house prices constrains the mobility of homeowners.\footnote{We also report regression results having the inflow rate as the dependent variable. These are reported in Columns (3) and (4). We do not find any effect of the housing bust on inflow rates.}

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>productivity gap</td>
<td>0.0091***</td>
<td>0.0216***</td>
<td>0.0051**</td>
<td>0.0000</td>
</tr>
<tr>
<td>( \bar{z}<em>{i,t} = \log z_t - \log z</em>{i,t} )</td>
<td>(0.0019)</td>
<td>(0.0040)</td>
<td>(0.0020)</td>
<td>(0.0047)</td>
</tr>
<tr>
<td>interaction: prod. gap \times h. bust</td>
<td>$-0.0434^{***}$</td>
<td>0.0138</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \bar{z}<em>{i,t} \times (\text{hbust}</em>{i} / 100) )</td>
<td>(0.0125)</td>
<td>(0.0144)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>size of the housing bust</td>
<td>0.0246***</td>
<td>0.0099*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{hbust}_{i} / 100 )</td>
<td>(0.0023)</td>
<td>(0.0038)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Numbers in parentheses are standard errors. Observations are weighted by MSA population. *** = significant at 1%, ** = significant at 5%, * = significant at 10%.

**Summary of Findings**

The results of this section can be summarized as follows:

- The 2007 – 2009 recession is characterized by a large dispersion in economic conditions across MSAs. In particular, the standard deviation of unemployment has doubled during this episode.

- The increase in unemployment is systematically higher in MSAs that experienced in MSAs with larger housing busts. This is true even after controlling for differences in
the decline in labor productivity.

- Net migration rate declined to a level less than 50% of its pre-recession level. Gross migration across MSAs also declined during the recession period.
- Among places with the same labor productivity, MSAs with larger housing busts had smaller out-migration rates.

Motivated by these observations, we now turn to our model economy with local housing and labor markets. We use the model to study the effect of the housing bust in 2007 on migration, and local and aggregate unemployment.

3 A Quantitative Model of Labor and Housing Markets and Geographical Reallocation

The previous section documented several facts regarding local housing and labor markets, and migration patterns during the 2007 − 2009 recession. We now present the quantitative model of geographical reallocation. The model sheds light on the mechanisms behind these facts. Our focus is on the importance of a financial friction: the down payment requirement for purchasing a house. We show that a calibrated version of our model can account for these facts. Finally, we use the model to quantify the effect of the housing bust on aggregate unemployment.

3.1 Agents and Markets

The economy consists of two MSAs: MSA $A$ and MSA $B$. The MSAs are populated by a continuum of households of measure one, a continuum of firms and housing market intermediaries (real estate managers, leasing companies, and construction companies) with positive measure. Time is discrete and continues forever: $t = 0, 1, 2, \ldots$

Households: Households are ex-ante identical and have a periodical utility function given by $u(c, l, h, \chi)$,

$$u : \mathbb{R}_+ \times \{0, 1\} \times \{0, 1\} \times [\chi, \chi] \to \mathbb{R}$$

defined over consumption ($c$), leisure ($l$), housing status ($h$), and preference for their current MSA of residence ($\chi$). Leisure takes a value of 0 and 1, denoting the household is employed and unemployed, respectively. Similarly, housing status takes value 0 and 1, denoting that
the household is a renter and a homeowner, respectively. The preference of the household for the other MSA is given by $-\chi$. Households on which MSA to live and work, whether to purchase a house or live in a rental one, and how much to save and consume. Each household maximizes the expected sum of periodical utilities discounted at the discount rate $\beta \in (0, 1)$.

**Firms and the Labor Market:** Each firm in MSA $i \in \{A, B\}$ operates a constant returns to scale technology that, if matched with a worker, turns one unit of labor into $z_i$ units of consumption. The labor productivity $z_i$ is the same for all firms in a given MSA, but differs across the two MSAs. Labor productivity in an MSA follows a Markov process, and takes values in $Z = \{z_1, z_2, \ldots, z_N\}$ according to the transition matrix $\Upsilon_Z$.

Households and firms meet and produce output in a frictional labor market. The labor market is organized in a continuum of submarkets that differ in the wage contract that is offered. More specifically, we only allow for fixed-wage contracts. When a firm meets a worker in submarket $w$, the firm offers the worker an employment contract that pays the worker a wage of $w$ every period until the match ends exogenously with probability $\delta$. Firms and workers fully commit to this contract. Consequently, each submarket is characterized by the wage offered $w$.

Search in the labor market is directed: Firms choose the wage to offer to potential workers and the number of vacancies to post. Each vacancy requires the payment of a posting cost, $k$. Similarly, households decide in which submarket to look for jobs. We denote by $\theta_i^w(w; \psi)$ the market tightness of submarket $w$—the ratio of the number of vacancies created by firms in submarket $w$ to the number of workers that are looking for jobs in the same submarket in MSA $i$. Once in a submarket, workers find jobs with probability $\pi_i[\theta_i^w(w; \psi)]$, and firms find workers with probability $q_i[\theta_i^w(w; \psi)] = \pi_i[\theta_i^w(w; \psi)]/\theta_i^w(w; \psi)$. $\psi$ denotes the aggregate state of the economy and is described below in detail.

**Housing Market Intermediaries and the Structure of the Housing Market:** There are 3 types of companies in the housing market: construction companies, real estate managers (REMs), and leasing companies. Construction companies operate a constant returns to scale technology that turns $\mu_i$ units of the consumption good into 1 unit of housing in MSA $i$. Newly constructed houses can be sold to households or to leasing companies to be used as rental units. $\mu_i$ is assumed to be constant over time. We model the housing bust as an unexpected decrease in construction costs.

We assume that households cannot trade houses among each other. Instead, housing transactions are facilitated by REMs in that they buy houses from sellers and sell them to
buyers. Both of these markets are subject to search frictions. Households and REMs meet in a frictional housing market. Similar to the labor market, housing market is organized in a continuum of submarkets. Each submarket is characterized by the transaction price of the house, $p$.

Search is directed in the housing market. Renters that would like to buy a house decide on the price at which they are willing to buy, and search a house in that submarket. There is a down payment requirement to purchasing a house: households are allowed to buy a house at price $p$ only if their assets suffice to cover $\alpha$ fraction of the house price; i.e. $a \geq \alpha p$. REMs with a house decide on the selling price and post a vacancy accordingly. We define the market tightness of submarket $p$, $\theta_{b}(p;\psi)$, as the ratio of vacancies posted by REMs at price $p$ to the number of households that are looking for a house at this price. Once in a submarket, households buy a house with probability $\pi_{b}[\theta_{b}(p;\psi)]$. The probability of an REM of meeting a household is given by $q_{b}[\theta_{b}(p;\psi)] = \pi_{b}[\theta_{b}(p;\psi)]/\theta_{b}(p;\psi)$. The directed nature of the search ensures that upon meeting an REM, the household is willing to buy the house at price $p$.

On the other side of the housing markets, homeowners that would like to sell their houses choose a selling price, $p$. REMs that would like to buy houses decide the price to buy and look for a sellers who is willing to sell at this price. The market tightness on this side of the housing market is denoted by $\theta_{s}(p;\psi)$. The probability of trade for a seller is given by $\pi_{s}[\theta_{s}(p;\psi)]$, and the probability of trade for REM is given by $q_{s}[\theta_{s}(p;\psi)] = \pi_{s}[\theta_{s}(p;\psi)]/\theta_{s}(p;\psi)$.

Leasing companies buy houses from construction companies and turn these into rental houses at no cost. They then rent it out to households for one period to obtain a rent of $\rho(\psi)$. The market for rental units is perfectly competitive. Finally, rental units depreciate: A rental unit is destructed every period with probability $\gamma$.

**Financial Markets:** Financial markets are incomplete. Households can save and borrow using a risk-free bond. The risk-free bond yields a constant interest rate $r$. When borrowing, homeowners and renters face (exogenous) different borrowing limits $a_{1}$ and $a_{0}$, where $a_{1}$ is the borrowing limit of homeowners and $a_{0}$ is the borrowing limit of renters. The borrowing limit is tighter for renters, $a_{0} > a_{1}$.

Renters can use a mortgage to buy a house. As we mentioned previously, households can purchase a house at price $p$, as long as their assets are larger than $\alpha p$. The portion of the house price that is not paid at the time of purchase is borrowed at interest rate $r$. Households can then roll over their debt by paying the interest only or lower their balance by paying more every period. The details of the mortgage arrangement will be further explained in the
problem of the household.

### 3.2 Timing of Events

As mentioned earlier, the introduction of assets into a model with aggregate shocks in a search model raises the dimensionality of workers’ and firms’ problems. In principle, the aggregate state variables include the two labor productivity shocks, \( z_A \) and \( z_B \), the fraction of households in one MSA, as well as the distribution of employment, wages, assets, preference shocks, and housing status. The latter is critical because one must keep track of an infinite dimensional object in the state space, rendering the dynamics of the model almost intractable. Fortunately, the structure of the model gives rise to a block-recursive equilibrium, an equilibrium in which firms’ and workers’ problems are independent of the distribution. We present the model in its general form and allow the distribution to be part of the state space. We discuss in the next section what conditions give rise to this property.

At the beginning of each period, the state of the economy, \( \psi \), is given by

\[
\psi = (z_A, z_B, n_A, \Gamma_A, \Gamma_B)
\]

The first two elements of \( \psi \) denote the labor productivities in MSAs \( A \) and \( B \), respectively. \( n_A \) is the fraction of people in MSA \( A \), and \( \Gamma_i : \mathbb{R} \times \{0, 1\} \times \mathbb{R}_+ \times \{0, 1\} \times [\underline{\chi}, \overline{\chi}] \rightarrow [0, 1] \) is a function denoting the measure of households in MSA \( i \) over assets, employment status, wages, housing tenure, and preference shock.

Each period is divided into 5 stages: separation, housing market transactions, migration, search in the labor market, and production. During the separation stage, employed households exogenously move into unemployment with probability \( \delta \).

After shocks are realized, housing markets open. If a homeowner wants to sell his house, he chooses the price to sell and looks for a buyer. If the homeowner successfully sells his unit, he becomes a renter for one period, but may enter the housing market next period. If a renter wants to purchase a house, he chooses at which price to look for a house and visits the corresponding submarket for a seller.

Upon completing housing transactions, households decide whether to remain in the current MSA or migrate to the other MSA. Only unemployed renters and unemployed homeowners that sold their houses within this period are allowed to migrate.

After migration, labor markets open. Firms post vacancies at various wages and unemployed households choose at what wage to look for a job. Job search is local: Only residents
of MSA $i$ are allowed to apply for jobs in MSA $i$.

During the consumption-savings stage, an unemployed household collects $b$ units of the consumption good as unemployment benefits. Employed households in MSA $i$ produce $z_i$ units of output and are paid their wage $w$. Households then decide on their consumption and savings and renters pay out the MSA-specific rent $\rho^i(\psi)$.

### 3.3 Problem of the Household

In this section, we present the Bellman equations that govern the decision problems of households. The value functions are measured at the beginning of the consumption-savings stage—the last stage in a period. We use auxiliary value functions to denote the value functions at the job search stage and define them when necessary. We first consider the problem of an unemployed renter.$^{15}$

#### 3.3.1 Consumption-Savings Problem

We start by describing the problem at the consumption-savings stage. The search in the housing market for buyers and sellers is also explained in this section. We then turn to the search problem in the labor market.

**Unemployed Renters:** Equation (1) presents the problem of an unemployed renter at the consumption-savings stage:

$$U^A(a, h = 0, \chi; \psi) = \max_{c, a'} u(c, l = 1, h = 0, \chi) + \beta \mathbb{E} \left\{ \max_{p, a'_p \geq \alpha p} \pi_b(p; \psi')D^A(a' - p, h = 1, \chi'; \psi') \right. \\
+ \left. [1 - \pi_b(p; \psi')] \max \left\{ D^A(a', h = 0, \chi'; \psi'), D^B(a', h = 0, -\chi'; \psi') \right\} \right\}$$

(1)

$$a + b = c + \rho(\psi) + \frac{a'}{1 + r}, \quad a' \geq a_0.$$

Here, $U^A(a, h = 0, \chi; \psi)$ is the value of being unemployed in MSA $A$ to a renter ($h = 0$) with assets $a$, and preference for current MSA ($\chi$). The household chooses current consumption ($c$) and savings ($a'$), subject to the budget constraint that we present and discuss below, and obtains an instantaneous utility of $u(c, l = 1, h = 0, \chi)$ and goes to the next period. At the

$^{15}$We only present the problem of households in MSA $A$. The problem is analogous for a household in MSA $B$.  

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beginning of the next period, housing markets open. Since he is a renter, he has the option of purchasing a house. He chooses the purchasing price $p$, subject to the down payment constraint, $a' \geq \alpha p$. For that price, his expected payoff is

$$\pi_b(p; \psi')D^A(a' - p, h = 1, \chi'; \psi') + [1 - \pi_b(p; \psi')] \max \{D^A(a', h = 0, \chi'; \psi'), D^B(a', h = 0, -\chi'; \psi')\}$$  (2)

The first term reflects the fact that he finds a seller with probability $\pi_b(p; \psi')$ and obtains the ownership of the house upon paying the transaction price. In that case, his assets are given by $a' - p$. He enters the migration stage as a homeowner and is not allowed to migrate. After the migration stage comes the job search stage. He looks for a job his current MSA ($A$). This delivers an expected payoff of $D^A(a' - p, h = 1, \chi'; \psi')$. The value function $D$ denotes the value of searching for a job in MSA $A$ and will be defined below.

The second term in (2) reflects the fact that the household does not find a seller with complementary probability. In that case, he is still a renter and can choose to migrate. If he decides to stay, he obtains the value of looking for a job in MSA $A$, $D^A(a', h = 0, \chi'; \psi')$, and migration delivers a value of looking for a job in $B$, $D^B(a', h = 0, \chi'; \psi')$.

We now turn to the budget constraint facing the unemployed renter. His resources for the period are given by his assets and the unemployment benefit he collects. He uses these to finance current consumption $c$, pay for the rent $\rho(\psi)$. He can borrow and save at interest rate $r$, subject to the borrowing constraint that $a' \geq a_0$.

**Unemployed Homeowners:** Equation (3) shows the problem of an unemployed homeowner:

$$U^A(a, h = 1, \chi; \psi) = \max_{c, a'} u(c, l = 0, h = 1, \chi)$$

$$+ \beta \mathbb{E} \left\{ \max_p \pi_s(p; \psi') \max \{D^A(a' + p, h = 0, \chi'; \psi'), D^B(a' + p, h = 0, -\chi'; \psi')\} \right.$$  

$$+ [1 - \pi_s(p; \psi')] D^A(a', h = 1, \chi'; \psi') \right\}$$  (3)

$$a + b = c + \frac{a'}{1 + r}, \quad a' \geq a_0$$

Here, $U^A(a, h = 1, \chi; \psi)$ is the value of being unemployed in MSA $A$ to a homeowner ($h = 1$) with assets $a$, and preference for current MSA ($\chi$). The household chooses current consump-
tion $(c)$, and savings $(a')$, subject to the budget constraint, and obtains an instantaneous utility of $u(c, l = 1, h = 1, \chi)$ and goes to the next period. At the beginning of the next period, housing markets open. The household has the option of selling the house. He decides the selling price $p$ that then determines the probability of finding a buyer. For that price, the expected payoff is given by:

$$
\max_p \pi_s(p; \psi') \max \{ D^A(a' + p, h = 0, \chi'; \psi'), D^B(a' + p, h = 0, -\chi'; \psi') \} + [1 - \pi_s(p; \psi')] D^A(a', h = 1, \chi'; \psi') \tag{4}
$$

The first term reflects the fact that the household finds a buyer with probability $\pi_s(p; \psi')$ and transfers the ownership of the house upon receiving the payment. In that case, his assets are given by $a' + p$. Consequently, he enters the migration stage as a renter and decides whether to move to the other MSA or remain in the current location. Similar to the renter’s problem, $D^A(a' + p, h = 0, \chi'; \psi')$ and $D^B(a' + p, h = 0, \chi'; \psi')$ measure the value of looking for a job in MSAs $A$ and $B$, respectively.

The second term in (4) reflects the fact that the household does not find a buyer with complementary probability. In that case, he is a homeowners and is not allowed to migrate. As a consequence, he searches for an employer in the current location, MSA $A$, and obtains a value of $D^A(a', h = 1, \chi'; \psi')$.

Equation (4) highlights the option value of migration. By selling the house, the household obtains the option of migration. Clearly, the more the household wants to migrate, the sooner he would like to sell the house. As a result, differences between labor markets and household’s preference for his current location will affect the price posting decision.

**Employed Renters:** We now turn to the problem of employed households. We start by describing the decision problem of an employed renter. Equation 5 shows the Bellman
equation for an employed renter in MSA $A$:

$$W^A(w, a, h = 0, \chi; \psi) = \max_{c, a'} u(c, l = 0, h = 0, \chi)$$

$$+ \beta \mathbb{E}\left\{ (1 - \delta) \left( \max_{a' \geq a_0} \pi_b(p; \psi') W^A(a' - p, h = 1, \chi'; \psi') + [1 - \pi_b(p; \psi')] W^A(a', h = 0, \chi'; \psi') \right) 
+ \delta \left( \max_{a' \geq a_0} \pi_b(p; \psi') D^A(a' - p, h = 1, \chi'; \psi') 
+ [1 - \pi_b(p; \psi')] \max \{ D^A(a' - p, h = 1, \chi'; \psi'), D^B(a' - p, h = 1, \chi'; \psi') \} \right) \right\}$$

(5)

$$a + w = c + \rho^A(\psi) + \frac{a'}{1 + r}, \quad a' \geq a_0$$

Here, $W^A(w, a, h = 0, \chi; \psi)$ is the value of being employed at wage $w$ in MSA $A$ to a renter ($h = 0$) with assets $a$, and preference for current MSA $\chi$. The household chooses current consumption ($c$), and savings ($a'$), subject to the budget constraint, and obtains an instantaneous utility of $u(c, l = 0, h = 0, \chi)$ and goes to the next period. At the beginning of the next period, job destruction shock $\delta$ and productivity shocks are realized. The second line in (5) captures the event that the household keeps his job. The household has the option of buying a house in the housing market. He decides the buying price $p$ (subject to the down payment constraint) that then determines the probability of finding a seller. For that price, the expected payoff is given by:

$$\pi_b(p; \psi') W^A(a' - p, h = 1, \chi'; \psi') + [1 - \pi_b(p; \psi')] W^A(a', h = 0, \chi'; \psi')$$

(6)

Here, the first term measures the payoff associated with buying the house: upon finding a seller, the buyer pays the house price and becomes an employed renter. With complementary probability, $1 - \pi_b(p; \psi')$, he doesn’t find a seller and remains as an employed renter. Being employed, he is not allowed to migrate or look for another job, and skips these two stages to obtain a value of $W^A(a' - p, h = 1, \chi'; \psi')$ if he is an owner or $W^A(a', h = 0, \chi'; \psi')$ if he is still a renter.

The last two lines in 6 capture the event that the household loses his job and becomes unemployed. For this household, the rest of the problem looks very similar to the problem of an unemployed renter: The household decides whether to purchase a house or not. In the case of a successful purchase, the household searches for a job locally. In the other case, he decides whether to move or not and then looks for a job locally.
The budget constraint facing an employed renter is very similar to the one facing an unemployed renter, the difference being the labor income \( w \) instead of the unemployment benefits.

**Employed Homeowners:** Equation 7 shows the Bellman equation for an employed owner in MSA \( A \):

\[
W^A(w, a, h = 1, \chi; \psi) = \max_{c, a'} u(c, l = 0, h = 1, \chi) + \beta \mathbb{E} \left\{ (1 - \delta) \left( \max_p \pi_s(p; \psi') W^A(a' + p, h = 0, \chi'; \psi') + [1 - \pi_s(p; \psi')] W^A(a', h = 1, \chi'; \psi') \right) \\
+ \delta \left( \max_p \pi_s(p; \psi') \max \{ D^A(a' + p, h = 0, \chi'; \psi'), D^B(a' + p, h = 0, \chi'; \psi') \} \\
+ [1 - \pi_s(p; \psi')] D^A(a', h = 1, \chi'; \psi') \right) \right\}
\]

\( a + w = c + \frac{a'}{1 + r}, \quad a' \geq a_1 \)

Here, \( W^A(w, a, h = 0, \chi; \psi) \) is the value of being employed at wage \( w \) in MSA \( A \) to a renter (\( h = 0 \)) with assets \( a \), and preference for current MSA \( \chi \). The household chooses current consumption \( (c) \), and savings \( (a') \), subject to the budget constraint, and obtains an instantaneous utility of \( u(c, l = 0, h = 1, \chi) \) and goes to the next period. At the beginning of the next period, job destruction shock \( \delta \) and productivity shocks are realized. The second line in (7) captures the event that the household keeps his job. The household has the option of selling the house in the housing market. Selling delivers a payoff of \( W^A(a' + p, h = 0, \chi'; \psi') \) and not selling delivers a payoff of \( W^A(a', h = 1, \chi'; \psi') \).

If the household becomes unemployed, by setting a selling price \( p \), he may try to sell the house and get the option value of migration \( \max \{ D^A(a' + p, h = 0, \chi'; \psi'), D^B(a' + p, h = 0, \chi'; \psi') \} \) or not sell it and get a payoff of \( D^A(a', h = 1, \chi'; \psi') \).

### 3.3.2 Job Search Problem

So far, we have described the problem of employed and unemployed homeowners and renters at the consumption and savings stage as well as in the housing market. Here, we describe the search problem of households in the labor market. Recall that by assumption only unemployed households are allowed to search for jobs and that job search is local: households
in an MSA can only apply for vacancies in the same MSA.

Compared to a random matching technology, where there is a single market tightness in the labor market, $\theta(\psi)$, there is a continuum of wages and corresponding market tightnesses in this model, due to the directed nature of search. Households search in which submarket (at what wage) to look for jobs. Submarkets are indexed by the fixed wage $w$, and the market tightness in this submarket is given by $\theta_i^A(w;\psi)$. Correspondingly, $\pi_t[\theta_i^A(w;\psi)]$ denotes worker’s job finding probability as a function of the applied wage $w$.

The value of searching in the local labor market for a household with assets $a$ and housing status $h$ is given by

$$D^A(a,h,\chi;\psi) = \max_w \pi_t[\theta_i^A(w;\psi)] W^A(w,a,h,\chi;\psi) + (1 - \pi_t[\theta_i^A(w;\psi)]) U^A(a,h,\chi;\psi) \quad (8)$$

**Policy Functions:** We now introduce the notation for optimal policy rules as they will be used in the definition of recursive equilibrium. The optimal savings rule to the savings decision of employed and unemployed households are denoted by $g^W_i$ and $g^U_i$, respectively. Optimal house buying price is denoted by $p^*_b$, and optimal house selling price is denoted by $p^*_s$. Optimal migration decision is denoted by $m^*_i$. Finally, by $w^*_i$ we denote the optimal solution to the job search problem in (8).

### 3.4 Problem of the Firm

We now turn to firms in the labor market. Firms post vacancies to hire workers. Each vacancy lasts for one period. Recall that search is directed, so that when a firm decides to post a vacancy, it also decides in which submarket to post it. Our contract space only allows for fixed wage contracts, therefore vacancies are indexed by the offered wages, $w$. The value to the firm of being matched with a worker paying wage $w$ in MSA $i \in \{A, B\}$ can be written as:

$$J^i(w;\psi) = z_i - w + \frac{1 - \delta}{1 + r} E J^i(w;\psi') \quad (9)$$

Posting a vacancy requires the payment of a cost, $k$. The value of creating a vacancy in MSA $i$ with wage $w$ is given by:

$$V^i(w;\psi) = -k + q_i[\theta_i^A(w;\psi)] J^i(w;\psi), \quad (10)$$
where $q_l$ denotes the probability of finding a worker at wage $w$, and is a function of the labor market tightness $\theta_i^l (w; \psi)$. When the value of creating one vacancy at wage $w$ is strictly positive, the firm finds it optimal to create infinite vacancies. When it is strictly negative, no vacancies are created in submarket $w$. When the value is zero, then firm’s profit is independent of the number of vacancies it creates in submarket $w$.

We assume free entry of firms. Therefore, in any submarket visited by a positive measure of workers, the following must hold:

$$k \geq q_l \left[ \theta_i^l (w; \psi) \right] J^i (w; \psi)$$

(11)

with complementary slackness. That is, (11) must hold with equality if $\theta_i^l (w; \psi) > 0$. When we focus on block-recursive equilibrium, we will focus on equilibria where there is a positive number of entrants every period.

### 3.5 Problem of Housing Market Intermediaries

Our work borrows from the insights of Menzio and Shi (2010a) and extends the notion of block-recursivity to the housing market. In what follows, we will describe the structure of the housing market that gives rise to the existence of such an equilibrium. As we will see in the next section, this requires a combination of directed search and free-entry conditions in every market. The introduction of the housing market intermediaries makes the existence possible. We have three types of firms in the housing market: real estate managers (REM), leasing companies, and construction companies.

REMs with a vacant house try to sell it to buyers. They get a payoff of $p$ when they succeed in selling a house at price $p$, but get no flow payoff from having vacant houses. Therefore, the value of holding a vacant house in MSA $i \in \{A, B\}$ to a real estate manager is:

$$R^i (\psi) = \max_p q_b \left[ \theta_b^i (p; \psi) \right] p + \left( 1 - q_b \left[ \theta_b^i (p; \psi) \right] \right) \frac{1}{1 + r} \mathbb{E} R^i (\psi'),$$

(12)

where $\theta_b^i (p; \psi)$ is the market tightness for the housing submarket with price $p$. The subscript $b$ indicates that this is the side of the housing market where households are buyers. Equation (12) holds that REMs choose the price $p$ at which they are willing to sell the house and are successful in doing so with probability $q_b \left[ \theta_b^i (p; \psi) \right]$. They cannot find a buyer with complementary probability and the house remains for one period.

We now turn to the other side of the housing market. In order to buy houses from sellers
in the housing market, REMs post vacancies by paying a cost $\kappa$. As in the other markets, search is directed so that when REMs decide to post vacancies they also decide the price at which they are willing to buy. There is full commitment to the posted price, so that whenever an REM meets a homeowner, the housing unit is transferred to the REM at price $p$. The value of posting a vacancy for an REM in MSA $i \in \{A, B\}$ at price $p$ is given by

$$Q^A(p; \psi) = -\kappa + q_s \left[ \theta^s(p; \psi) \right] \left[ R^A(\psi) - p \right]$$

(13)

We assume free entry of REMs. Therefore, a free entry condition similar to (11) holds for all the submarkets in the selling market that are visited by a positive measure of homeowners. This is given by

$$\kappa \geq q_s \left[ \theta^s(p; \psi) \right] \left[ R^A(\psi) - p \right]$$

(14)

with (14) holding with equality whenever $\theta^s(p; \psi) > 0$.

Leasing companies operate in a competitive rental market. Rental contract is for one period. At the end of every period, a constant fraction $\gamma$ of rental houses depreciate. Depreciation is discrete; i.e., these rental houses are completely destructed. Leasing companies get the rental rate, $\rho(p; \psi)$, as a flow payoff, until the unit is destructed. Thus, the value of holding a rental house to leasing companies is given by

$$L^i(\psi) = \rho^i(\psi) + \frac{1 - \gamma}{1 + r} \mathbb{E} L^i(\psi')$$

(15)

Depreciation is important to ensure that every period new houses are built, which in turn is important to make the free entry conditions hold with equality. We elaborate on this issue when we discuss the existence of a block-recursive equilibrium.

Finally, we turn to construction companies. Construction companies can build a new house immediately at cost $\mu_A$. They then have the choice to become REMs and try to sell these units to renters or to become leasing companies and rent out the house. As long as the value of holding a house as a REM or as a leasing company exceeds the cost of constructing a new one, there will be new construction. This setup introduces two additional free entry conditions:

$$\mu_i \geq R^i(\psi)$$

(16)

$$\mu_i \geq L^i(\psi)$$

(17)
3.6 Equilibrium

We now define a recursive equilibrium for this economy. We denote the set of housing service types as $H = \{0, 1\}$ and the set of MSAs as $I = \{A, B\}$. The set of MSA level productivity shocks are given by $Z = [\underline{z}, \overline{z}]$. Define $W = [b, z_N]$ to be the set of possible wages, and $A = [\underline{a}, \overline{a}]$ to be the set of possible assets.\footnote{It is easy to prove that there are endogenous bounds on the set of possible wages that will be offered and thus on the set of assets that will be realized in equilibrium. The assumption of bounded sets is in that sense not an assumption but a result.} Let $\Xi$ denote the set of preference shocks. Finally, let $\Psi$ denote the possible realizations of the aggregate state.

**Definition 1.** A Recursive Equilibrium comprises

- a set of value functions for households: \( \{W^i : W \times A \times H \times \Xi \times \Psi \to \mathbb{R}, U^i : A \times H \times \Xi \times \Psi \to \mathbb{R}, D^i : A \times H \times \Xi \times \Psi \to \mathbb{R} \}_{i \in I} \),

- a set of policy functions for households: \( \{g^i_W : W \times A \times H \times \Xi \times \Psi \to \mathbb{R}, g^i_U : A \times H \times \Xi \times \Psi \to \mathbb{R}, p^*_s : W \times A \times H \times \Xi \times \Psi \to \mathbb{R}^+, m^* : A \times \Xi \times \Psi \to \mathbb{R}^+, w^* : A \times H \times \Xi \times \Psi \to \mathbb{R}^+ \}_{i \in I} \),

- value function for firms: \( \{J^i : W \times \Psi \to \mathbb{R} \}_{i \in I} \),

- value functions for intermediaries in the housing market: \( \{R^i : \Psi \to \mathbb{R}, L^i : \Psi \to \mathbb{R} \}_{i \in I} \),

- market tightness function in the labor market, \( \{\theta^l_i : W \times \Psi \to \mathbb{R}^+ \}_{i \in I} \),

- market tightness functions in the housing market \( \{\theta^s_i : \mathbb{R}^+ \times \Psi \to \mathbb{R}^+, \theta^b_i : \mathbb{R}^+ \times \Psi \to \mathbb{R}^+ \}_{i \in I} \),

- a transition probability function for the aggregate state of the economy $\Phi : \Psi \times \Psi \to [0, 1]$; such that

- **Households maximize:** Given the market tightness functions, the value functions solve (1), (3), (5), (7), (8), and $g^i_W, g^i_U, p^*_s, m^*$ and $w^*$ are the associated policy functions.

- **Firms and housing market intermediaries maximize:** $J$ solves 9, and $R$ and $L$ satisfy (12) and (15), respectively.

- **Free entry of firms:** Given the value function of firms, $J$, the market tightness function $\theta^l$ satisfies (11).
iv) **Free entry of real estate managers:** Given the value function of housing intermediaries, (14) holds.

v) **Free entry of construction companies:** (16) and (17) are satisfied.

vi) **Law of motion for the aggregate state space:** $\Phi$ is derived from the policy functions of households and the probability distribution for the productivity distribution.

### 3.7 Existence of a Block-Recursive Equilibrium

Solving a recursive equilibrium outside of the steady-state requires solving functional equations in which the functions depend on the entire distribution of workers across MSAs, assets, employment states, housing tenure, and the preference shock. This requires keeping track of an infinite dimensional object. In general, this feature makes the problem difficult to solve, even numerically. In the presence of search frictions, this class of models become even more complex. To address this difficulty, we utilize the notion of block recursivity. We now define this property and show the existence of an equilibrium with this property. We then elaborate on the usefulness of this result and discuss the structure of the market that makes this possible.

**Definition 2.** A **Block Recursive Equilibrium (BRE)** is a recursive equilibrium such that the value functions, policy functions, and market tightness functions depend on the aggregate state of the economy, $\psi$, only through the stochastic shocks, $(z_A, z_B)$, and not through any endogenous distributions generated within the economy ($\Gamma_A, \Gamma_B$, or $n_A$), and free entry conditions (11), (14), (16), and (17) are satisfied with equality.

**Proposition 1.** There exists a Block Recursive Equilibrium.

**Proof.** See Appendix B.

Let us now elaborate on the usefulness of Proposition 1. In general, there is no easy way to compute an equilibrium of this model because of the high dimensionality of the state space. A commonly used approach in the literature is to approximate the distribution with several moments and conjecture a law of motion for these. One can iterate on this conjecture to make it consistent with the policy rule of the households.\footnote{More precisely, one solves household’s decision problem given this law of motion to obtain optimal policy rules. Simulating data from the model with these policy rules, one can obtain the implied law of motion for aggregate variables and compare it to the conjectured law of motion. The conjecture is revised until the procedure converges.} Note that this
procedure already adds a large number of (continuous) state variables: we need to add at least homeownership and unemployment rates in the two MSAs, fraction of population in one MSA, mean assets and mean wages in both MSAs. Typically, having a good description of the evolution of aggregate variables requires second order moments. This approach renders our model impossible to compute.\footnote{Solving only the household problem, taking as given the market tightness functions, takes on average 15 hours on a cluster with 20 cores.}

Proposition 1 asserts that there exists a block-recursive equilibrium. Moreover its proof reveals that it is possible to compute the market tightness functions in the housing and labor markets without solving household’s decision problem. Having solved for these, it is straightforward to solve the decision problem of the households. This makes it possible to solve the model in two steps: First, solve for the market tightness functions using the free entry conditions. Second, solve the household’s problem taking as given the market tightness functions.

It is important to note that the endogenous distribution of households matters for the evolution of the economy: migration decisions, job search decisions, house buying and selling decisions all depend on individual characteristics. Therefore, the response of the aggregate variables (e.g. unemployment rate, homeownership rate, etc,) to shocks will depend crucially on the distribution of households (across assets, wages, etc.) at the time the shock hits the economy. In particular, if it turns out to be the case that households with large amounts of leverage optimally decide not to move, then the dynamics of aggregate unemployment would look very different in an economy with high leverage in houses then in an economy with lower leverage. Block-recursive equilibrium is an equilibrium in which prices do not depend on endogenous distributions generated in the economy, but the evolution of important aggregate variables do.

As stressed in Menzio and Shi (2010a), the directed nature of the search technology is important for Proposition 1. The reason is the following: If the search is random (but there is still price posting), then the firm needs to forecast what type of worker will apply and show up. The necessity arises because the type or worker affects the probability of the job being accepted. To appropriately compute expectations, it needs to therefore know the entire distribution of households. A similar problem arises in a setting with random search and bargaining, which is the more commonly used approach in the literature. The surplus of the match depends on the type of the worker that meets the firm as a homeowner might have a very different threat point than a renter or someone with a lot of wealth will have a better outside option. As a result, the outcome of the bargaining—the wage—depends on the type of
the worker. To compute the expected profit of a vacancy, the firm needs to know the exact
distribution of workers. Free entry of firms is also important as it pins down the relationship
between the offered wage and the probability of finding a worker, hence the corresponding
tightness of the submarket.

The directed search in the housing market does a similar job. With random search, real
estate managers in the housing market would need to forecast what type of a buyer would
show up as this determines the willingness to pay for the house. This, again, requires the
knowledge of the entire distribution. In directed search, buyers (sellers) go to the submarket
of their choice, so once they meet a seller (buyer) trade happens with probability 1. An
alternative structure to ours could be to have directed search in the housing market, but
have households trade among each other. That is, sellers post houses and buyers look for
houses. Although this is perhaps a more realistic setup, one needs free entry conditions to
pin down market tightnesses in the housing market. The introduction of housing market
intermediaries and construction companies give rise to three free entry conditions as we have
shown above. Free entry is critical for the existence of block-recursive equilibrium.\textsuperscript{19}

4 Calibration

We now turn to the calibration of the model. We calibrate the model to match a number of
targets related to the labor and housing markets, mobility patterns, and wealth distribution
before the housing bust in 2007. This model has only labor productivity shocks. Before
turning to address the 2007 – 2009 recession, we evaluate the model’s performance along a
number of untargeted dimensions such as business cycle statistics, cyclicality of migration
rates, and the correlation between net flow rates and local unemployment rates. We then
use the model to study the 2007 – 2009 recession.

4.1 Functional Forms

Let us introduce functional forms for the utility function and the matching probabilities.
The utility function takes the following form

\[
\begin{align*}
&u(c,l,h,\chi) = (c + \lambda (1 - l) + \phi h)^{1-\sigma} + \chi
\end{align*}
\]

\textsuperscript{19}Also important is the constant returns to scale in the matching function. If there were increasing or
decreasing returns to scale, then meeting probabilities could not be expressed as a function of market tightness
only and the distribution of households across different submarkets would also affect meeting probabilities.
where $\phi_h$ is the consumption services from the housing type $h$, and $\lambda$ is the value of home production and leisure. If $l = 1$, this means the household is currently employed and $\lambda (1 - l) = 0$. If $l = 0$, the household is currently unemployed and gets a flow consumption of $\lambda$ from home production. Note that home production is not tradable, so it directly enters the utility function as a perfect substitute for the consumption good.

Following Menzio and Shi (2011) and Schaal (2010), we pick the CES contact rate functions

$$
p(\theta) = \theta (1 + \theta \gamma)^{-1/\gamma}, \quad q(\theta) = (1 + \theta)^{-1/\gamma}
$$

for the labor and housing markets. $\gamma_l, \gamma_b, \gamma_s$ denote the matching function parameters for the labor market, and the buying and selling sides of the housing buying market, respectively. We assume that $\gamma_b = \gamma_s$.

### 4.2 Stochastic Process for Labor Productivity

We need to calibrate the stochastic processes for labor productivity. This amounts to calibrating $Z$, and its transition function $\Upsilon_Z$.

We use our previously constructed measure of local labor productivity, as explained in Section 2, and estimate the following specification to obtain the persistence and variance of local labor productivity shocks at annual frequency:

$$
\log z_{i,t} = \alpha + \rho \log z_{i,t-1} + \epsilon_{i,t}
$$

Since our data is annual, we convert point estimates to monthly numbers. Results are reported in Table 5.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\rho$</th>
<th>$\sigma_\epsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0.98</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Note: Results to the estimation of an $AR(1)$ process on the log of local labor productivity (output per worker).

$^{20}$Apart from providing a good fit to the data, a constant returns to scale matching function is needed for the existence of a block-recursive equilibrium.
We discretize the process for local labor productivity using the Rouwenhorst method with 4 grid points.\footnote{A more common alternative in the literature is the Tauchen method. A drawback of this method is that it requires a lot of grid points to approximate well a highly persistent process. Computational concerns limit our choice of the number of grid points. Rouwenhorst method, on the other hand, performs a better job with fewer grid points. We verify using simulated data that 4 grid points suffice to provide a good fit.}

### 4.3 Calibration Strategy

Calibration proceeds in two steps. In the first step, we set some parameters exogenously. These are either parameters that have direct counterparts in the data or they can be taken from previous studies because they are not model dependent. The second step follows a simulated method of moments.

**Parameters Calibrated a Priori:** A period in the model corresponds to a month. We set the monthly interest rate $r$ to match an annual interest rate of 3\% \(r = (1 + 3\%)^{1/12} - 1 \approx 0.25\%). Risk aversion coefficient in the utility function, $\sigma$, is set to 2. The down payment requirement for buying a home is set to 10\%. This is lower than the typical 20\% used in most of the literature on housing, but is consistent with the financial developments in the housing market before the housing bust in 2007. Average replacement rate in the unemployment insurance system is around 40\%. Consistent with this, $b$ is set to 0.4. Monthly job destruction rate $\delta$ is set to 3.4\%, as reported in \textit{Shimer (2005)}.\footnote{This is constructed in \textit{Shimer (2005)} using data on employment, short-term unemployment, and the hiring rate. The number reported is the average of monthly separation rate over the period 1951 to 2003.} Table 6 summarizes the parameters of the model. The top panel presents parameters that are calibrated outside the model and the bottom panel presents those that are calibrated within the model.

**Parameters Calibrated with Simulated Method of Moments:** Parameters in the bottom panel of Table 6 are estimated by the Simulated Method of Moments. This requires a set of moments from the data that will inform us on the parameters of the model. For a given set of parameter values, a simulation of the model will generate statistics that can be compared to the moments in the data. The parameters are then chosen to minimize the distance between the model-generated statistics and the targets in the data. The distance is defined as the percentage deviation from the target and use the identity matrix as the weighting matrix. We now explain the targeted moments in the data in detail.

We start by describing housing market related moments. We target a homeownership...
rate of 69\%, and an average time to sell of 3.5 months.\textsuperscript{23} Vacancies to sell a house in the model last for one period. We define the model counterpart of time to sell as the inverse of selling probability, \(1/p_s\). To calibrate the elasticity of the matching function, we need a moment that relates the posted price to time to sell. To that end, we target the findings of Genesove and Mayer (1997)–that homeowners with 100\% leverage post prices that are about 4\% higher than homeowners with 80\% leverage. They report that the corresponding time to sell is 15\% lower for the highly leveraged homeowners.

We now elaborate on this part of calibration further as this is a novel approach. There are two parameters about search in the housing market that need to be calibrated: vacancy posting cost \(\kappa\) of REMs and the “elasticity” parameters in the matching function, \(\gamma_s = \gamma_b\). Average time to sell is intimately linked to \(\kappa\) as changes in this parameter shift the entire market tightness functions (and thus the relationship between selling price and the probability of trade). On the other hand, the elasticity parameter governs how the probability of trade is affected by a change in selling price. The decision problem of the household provides a mapping between asset position and the optimal selling price. By taking the composition of this decision rule with the market tightness function that maps price to time to sale, we can construct the model counterpart of the relationship between leverage and time to sell. This moment depends tightly on the elasticity parameter and is therefore used to calibrate it.

How can the model generate a negative relationship between assets and time to sell? There are two forces in the model. Households with lower assets have a higher propensity to move for job-related reasons. This is because their marginal utility of consumption is higher compared to households with more liquid wealth. On the other hand, households with less assets (and thus more leverage) find it harder to afford a new house after moving and end up renting for many periods. This is due to the financial friction in the model–the down payment constraint. Because households obtain a higher utility from owning (also a calibrated parameter), highly leveraged households have a motive to post higher selling prices (and sell slower) than households with lower leverage. The relationship between leverage and time to sell is a result of this tradeoff. It turns out that the model does generate the right relationship quantitatively.

This strategy is somewhat analogous to the standard one in the search literature that is used to calibrate the vacancy posting cost and matching the function parameter. For

\textsuperscript{23}This is the homeownership rate in the U.S. right before the onset of the housing bust. Source: Census web site http://www.census.gov/hhes/www/housing/hvs/charts/files/fig05.pdf. Average time to sell is taken from the National Association of Realtors web site. Different sources report different numbers that range between 2.5 and 5.5 months at times with “good” housing markets.
example, Shimer (2005) uses average job finding probability and the correlation between the job finding rate and market tightness. The former is (mostly) informative about average job finding rate and the latter is informative about the correlation. Average time to sell is analogous to job finding probability and the elasticity of time to sell with respect to leverage is somewhat analogous to the elasticity of job finding probability with respect to market tightness.

### Table 6: Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-calibrated:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>10%</td>
<td>down payment requirement</td>
</tr>
<tr>
<td>$r$</td>
<td>0.25%</td>
<td>monthly interest rate</td>
</tr>
<tr>
<td>$b$</td>
<td>0.4</td>
<td>unemployment benefits</td>
</tr>
<tr>
<td>$\zeta_0$</td>
<td>0</td>
<td>Consumption flow from renting</td>
</tr>
<tr>
<td>$\delta$</td>
<td>3.4%</td>
<td>job destruction probability</td>
</tr>
<tr>
<td>Within-the-model:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\zeta_1$</td>
<td>0.20</td>
<td>Consumption flow from owning</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.984</td>
<td>discount rate</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.51</td>
<td>home production</td>
</tr>
<tr>
<td>$k$</td>
<td>0.75</td>
<td>vacancy posting cost for firms</td>
</tr>
<tr>
<td>$\gamma_l$</td>
<td>1.80</td>
<td>labor market elasticity</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.12</td>
<td>vacancy posting cost for REMs</td>
</tr>
<tr>
<td>$\gamma_b, \gamma_s$</td>
<td>0.80</td>
<td>housing market matching functions</td>
</tr>
<tr>
<td>$\rho_\psi$</td>
<td>0.991</td>
<td>persistence of the preference shock</td>
</tr>
<tr>
<td>$\sigma_\psi$</td>
<td>0.002</td>
<td>standard deviation of preference shocks</td>
</tr>
<tr>
<td>$\mu$</td>
<td>48</td>
<td>housing construction cost</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.05</td>
<td>depreciation rate of rental houses</td>
</tr>
</tbody>
</table>

Note: Table reports the calibrated parameter values of the model. Upper panel reports parameters calibrated a priori. Lower panel reports the parameters calibrated within the model.

Finally, we target the ratio of average house prices to average monthly earnings in the model. To compute the empirical value of this moment, we compute the ratio of average house price to average monthly wages for each year over the period 2001 – 2005. We then average this time series to obtain an average of 48.

Turning to the labor market dimension of the model, we target an average job finding rate of 45% and a correlation at the quarterly frequency between (log) job finding rate and market tightness of 0.94. Both targets are the same as in Shimer (2005). In the model presented above, there are multiple submarkets in the labor market at any time, and thus
multiple job finding rates at any point in time. Unlike in Shimer (2005), the elasticity cannot be calibrated prior to solving the model: we need to solve the model to obtain the average of job finding probabilities across different submarkets, weighted by the number of workers that apply there. The resulting series from the model are monthly. We obtain quarterly series by taking the average over 3 months. The quarterly time series for (log) job finding probabilities and (log) market tightnesses are then filtered with an HP-filter using a scaling parameter of 1600. The model counterpart of the correlation is computed using the detrended series.

The model has also predictions about mobility rates. We target two mobility related moments: average gross mobility and average net mobility in the U.S. Gross mobility is defined as the average of population inflow and outflow rates; i.e. 
\[
grossmobility = \frac{\sum_t \sum_i |inflow_{i,t} + outflow_{i,t}|}{2NT},
\]
where \( N \) is the number of MSAs for which we have data on population flows and \( T \) is the number of years our data spans. Net mobility is defined as the average of absolute values of population net flows; i.e. it is given by 
\[
netmobility = \frac{\sum_t \sum_i |inflow_{i,t} - outflow_{i,t}|}{NT}.
\]
Empirical values for gross and net migration are computed from the IRS data that we described in Section 2. We use data for the period 2004 – 2007 to exclude the recession period. We find an average gross migration rate of 4.3% and an average net migration rate of 0.8%.

The key parameters that help us match the net and gross migration rates are the persistence and variance of preference shocks. In the model, net migration occurs because of labor productivity differences across the two MSAs: households tend to relocate to the MSA with better labor productivity because it is easier to find jobs and also wages are higher. Unemployed households would like to look for a job in the MSA with higher labor productivity. Absent any other motive, the model generated net mobility is the same as gross mobility. Preference shocks make people move for non-labor-market-related reasons and makes them move in both directions at the same time. That is, we observe populations that lose workers also attracts new workers. This breaks the relationship between net migration and gross migration. If the persistence of preference shocks gets larger, households do not respond to local labor market differences as much and net migration rate decreases. Yet, gross mobility is large because households move whenever their preferences dictate so. This intuitive discussion suggests that the persistence of preference shocks helps us match the difference between gross and net migration rates. Increasing the variance of preference shocks on the other hand, increases the gross migration rate as households are hit by larger preference shocks.

\footnote{Alternatively, one can use the change in the unemployment rate in the model to infer average job finding probability.}
shocks. By choosing the persistence and variance of preference shocks, we can calibrate the model to exactly match gross and net migration rates.

We target a median leverage of 67%. This is computed from the 2004 wave of the Survey of Consumer Finances (SCF). Leverage in the model is computed as the ratio of household debt and average house selling price in household’s location.

Table 6 shows the resulting parameter values and Table 7 summarizes the targets and the fit of the model with respect to the targeted moments.

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homeownership rate</td>
<td>69%</td>
<td>69%</td>
</tr>
<tr>
<td>Average time to sell</td>
<td>3.5</td>
<td>3.5</td>
</tr>
<tr>
<td>Genesove and Mayer (1997)</td>
<td>15%</td>
<td>15%</td>
</tr>
<tr>
<td>Job finding probability</td>
<td>0.45</td>
<td>0.44</td>
</tr>
<tr>
<td>Elasticity of job finding probability</td>
<td>0.95</td>
<td>0.94</td>
</tr>
<tr>
<td>Volatility of job finding rate/labor productivity</td>
<td>5.5</td>
<td>5.5</td>
</tr>
<tr>
<td>Median Leverage</td>
<td>67%</td>
<td>67%</td>
</tr>
<tr>
<td>House price/monthly wage</td>
<td>48</td>
<td>48</td>
</tr>
<tr>
<td>Gross mobility</td>
<td>4.3%</td>
<td>4.3%</td>
</tr>
<tr>
<td>Net mobility</td>
<td>0.8%</td>
<td>0.8%</td>
</tr>
</tbody>
</table>

Note: Table reports calibration targets and their values from the model. See the discussion in the text for a detailed information on the targets.

4.4 Model’s Fit on Non-Targeted Moments

Before using the model to address the 2007 – 2009 recession, we evaluate the model’s performance along a number of untargeted dimension of the data.

4.4.1 Volatility of Labor Market Variables

An important aspect of our study is the movements of labor market variables over time. We now show that our model with only labor productivity shocks can explain a number of co-movements in U.S. We simulate the model for a large number of periods and compute volatilities, cross-correlations, and auto-correlations of series such as unemployment, vacancies, and market tightness. Empirical counterparts to these variables are taken from Hagedorn and Manovskii (2011) (Table 1, Panel 3). They report standard deviations, autocorrelations, and cross-correlations at quarterly frequency. We compute the model counterparts of the
empirical moments by averaging monthly data generated from the model and detrending the resulting series with an HP-Filter with a smoothing parameter of 1600. Results are reported in Table 8. In the top panel, we report the numbers in Hagedorn and Manovskii (2011). The bottom panel reports data generated by the model.

Table 8: Business Cycle Statistics

<table>
<thead>
<tr>
<th></th>
<th>$u$</th>
<th>$v$</th>
<th>$v/u$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data (Hagedorn-Manovskii, 2011, CPS)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard Deviation (rel. to z)</td>
<td>7.15</td>
<td>9.54</td>
<td>16.3</td>
</tr>
<tr>
<td>Auto correlation</td>
<td>0.92</td>
<td>0.92</td>
<td>0.93</td>
</tr>
<tr>
<td>Correlation with $z$</td>
<td>−0.63</td>
<td>0.72</td>
<td>0.70</td>
</tr>
<tr>
<td><strong>Model (quarterly, detrended with HP-Filter)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard Deviation relative to $z$</td>
<td>6.28</td>
<td>10.2</td>
<td>12.5</td>
</tr>
<tr>
<td>Auto correlation</td>
<td>0.76</td>
<td>0.57</td>
<td>0.75</td>
</tr>
<tr>
<td>Correlation with $z$</td>
<td>−0.45</td>
<td>0.84</td>
<td>0.95</td>
</tr>
</tbody>
</table>

Note: Table compares business cycle statistics from the data (upper panel) at quarterly frequency to their model counterparts (lower panel). Monthly data generated from the model is averaged to obtain quarterly data, which is then filtered with an HP-filter with a smoothing parameter of 1600.

The model does quite well at replicating the volatilities in the data. As for autocorrelations, the model does reasonably well except for the autocorrelation of vacancies in the data. Turning to co-movements with labor productivity, the model is able to match the procyclicality of vacancies and market tightness and the counter-cyclicality of unemployment. As reported in the previous search literature, the correlations are sometimes larger than in the data. We should note that our model generates more realistic co-movements compared to the existing literature.\(^{26}\)

\(^{25}\)Hagedorn and Manovskii (2011) report the standard deviations. When we report the standard deviations, we adjust their numbers and report the ratio of the standard deviation of a series to the standard deviation of labor productivity. We do the same thing in the model.

\(^{26}\)Hagedorn and Manovskii (2011) argues that the model generates more realistic correlations with labor productivity if the value of home production is stochastic.
4.4.2 Labor Market Conditions and Population Flows

We now turn to study the relationship between the labor market and population flows. We compare 3 predictions of our model to the data: standard deviation of aggregate gross and net flows, cyclicality of gross migration, and correlation between local labor market conditions and local population flows.

Table 9 reports the volatility of aggregate gross and net migration rates from the model and the data. In the model, worker flows are more volatile than the data. Table 10 shows the regression coefficient of outflow rates on local productivity gap. Productivity gap for an MSA is defined as the deviation of local productivity in a year from aggregate productivity ($\log(z_t) - \log(z_{i,t})$), and is defined such that a positive value for productivity gap means that the productivity of the MSA is lower than aggregate productivity in the economy. The model is able to replicate the negative relationship between outflows and productivity gap. The correlation is stronger in the data than it is in the model.

Table 9: Volatility of Gross and Net Flows

<table>
<thead>
<tr>
<th></th>
<th>Standard Deviation</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross Migration</td>
<td>0.35</td>
<td>0.22</td>
<td></td>
</tr>
<tr>
<td>Net Migration</td>
<td>0.20</td>
<td>0.12</td>
<td></td>
</tr>
</tbody>
</table>

Finally, since this paper is about the co-movement of migration with the business cycle, we turn to analyze the cyclicality of gross migration. Table 11 reports the regression coefficient of the log of gross migration rate on log unemployment. We use the numbers reported in Davis, Fisher, and Veracierto (2010) on MSA level gross migration rates. We regress the log of this on aggregate unemployment. Our model is able to generate a pro-cyclical gross migration rate. There are two main forces in the model that affect the cyclicality of gross migration. On the one hand, there is a composition effect: in our model only unemployed workers are allowed to migrate. Since in a recession there are more unemployed households, this tends to increase migration in a recession, resulting in a counter-cyclical gross migration rate. On the other hand, households care less about preference shocks during a recession. This happens because of the functional form of the utility function. Preference shocks enter the utility function in a separable fashion. This implies that, fixing the preference shock, the
marginal benefit of migration is constant over time. However, moving is costly and entails a wealth loss because it involves selling the house, and selling takes place in a frictional housing market. Hence, there is a tradeoff between moving for preference related reasons and avoiding the loss. Since marginal utility of consumption is higher in a recession, this tends to make the migration rate pro-cyclical.\textsuperscript{27} It turns out that the effect of the utility function dominates the composition effect resulting in a pro-cyclical gross migration rate that is consistent with the data.

Table 10: Local Productivity and Population Flows

<table>
<thead>
<tr>
<th>outflow(_i,t)</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>productivity gap</td>
<td>−0.011</td>
<td>−0.0047</td>
</tr>
<tr>
<td>(\log (z_t) - \log (z_{i,t}))</td>
<td>(0.0019)</td>
<td></td>
</tr>
</tbody>
</table>

Table 11: Cyclicality of Gross Flows

<table>
<thead>
<tr>
<th>(\log grossflow_t)</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\log unemployment)</td>
<td>−0.009</td>
<td>−0.011</td>
</tr>
<tr>
<td>(\log (u_t))</td>
<td>(0.004)</td>
<td></td>
</tr>
</tbody>
</table>

4.4.3 Transitions in Housing Status for Migrants

We compute the transition rates across different housing status for a migrant. This moment is about flows from one housing status (owner and renter) to another conditional on migration, and is closely related to how easy it is in the model to acquire a unit after a move. The empirical values are constructed using the PSID as this is the only data set that tracks households over time (as opposed to housing units) and contains information on housing expenditures over time and the differences in health expenditures between high and low income households. For example, see Hall and Jones (2007) and Ozkan (2010).
tenure and migration in every wave. Probabilities are constructed for a sample of prime-aged (25-60) head of households. Table 12 presents the model generated transition rates and their empirical counterparts. The model does quite well in capturing the fact that in the data persistence is larger for renters than it is for homeowners.

<table>
<thead>
<tr>
<th>Table 12: Housing Tenure Transitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>Renter-Renter transition probability</td>
</tr>
<tr>
<td>Owner-Owner transition probability</td>
</tr>
</tbody>
</table>

5 Housing Bust and The Great Recession

We now turn to quantifying the effect of the housing bust in 2007 to local and aggregate unemployment during the recent recession. In order to study the effects of the housing bust on local labor markets, we need to group the MSAs in two groups as there are more than 300 MSAs in the U.S. but only two in the model. Since we are mostly interested in the role of the housing bust, we group the MSAs into two categories based on the size of the housing bust in 2007: MSA A contains all the MSAs in our dataset for which house prices declined by less than 35%. MSA B contains all the other MSAs. Out of the 341 MSAs for which we have data on house prices, 250 belong to MSA A. In 2007, the share of total employment in the United States employed in MSA A is about 66.5% and the share of population in MSA A is 65.9%.

To study the effect of an asymmetric housing bust (asymmetric across MSAs) on local and aggregate labor markets, we run two simulations. In the first simulation, which we label as the factual simulation, we feed into the model the labor productivity and house prices as observed in the data for these MSAs. Our data suggests that labor productivity declined by 1% in MSA A and by 5% in MSA B. In 2007, house prices declined by 19.6% in MSA A and 46.8% in MSA B. Unlike the decline in labor productivity, the housing bust is modeled as a one time, unanticipated, and permanent decline in housing construction costs, \( \{ \mu_i \}_{i=1,2} \). We choose the decline in these parameters so that the model matches the decline in house prices in both MSAs.

In a second simulation, which we call the counter-factual simulation, we feed in the
model the same realizations of labor productivity for MSA A and MSA B. The parameters for housing construction costs do not change in this simulation and, as a result, house prices decline only marginally. Figures (1)-(4) plot local and aggregate unemployment for these simulations.

Figure 1 shows local unemployment. The upper panel shows local unemployment rates in the simulation with house price and labor productivity shocks, whereas the bottom panel shows local unemployment rates in the counter-factual simulation (labor productivity shocks only). In both panels, we plot the deviation of unemployment from the level before the recession. The model predicts a rise in local unemployment of around 1.5% in MSA A (low productivity MSA) and 4% in MSA B. In the data, the rise is around 4.5% and 7% for MSAs A and B, respectively.

The bottom panel in Figure 1 highlights the role of the housing bust on local unemployment. Without the housing bust, the model predicts a rise in local unemployment of 2% and 2.5% for MSAs A and B, respectively. Thus, the housing bust increases the unemployment rate further in MSA B (large housing bust region) and decreases it in MSA A (small housing bust region).

Why is the effect of housing bust on local unemployment asymmetric? How does a decline in house prices amplify the effect of a local productivity shock on unemployment in regions with large housing busts and act in the opposite direction in regions with smaller housing busts? To explain these phenomena, we turn to the policy function of homeowners for moving. Recall that households move in the model for two reasons: preference shocks and differences across regions in the local labor market (labor productivity). Figure 2 plots the optimal policy rule for migration of a homeowner with 80% leverage that currently resides in MSA B. The x-axis is the productivity of MSA A and the y-axis is the preference of the household for MSA B, her current residence. Migration decisions follow a cutoff rule: Fixing labor productivity of the other MSA, not surprisingly, only households below a cutoff preference enter the housing market to sell their houses and migrate. Furthermore, this cutoff preference increases with the labor productivity of the other MSA. The top line shows the cutoff preference before the housing bust, whereas the bottom line shows the one after the housing bust. As the figure shows, the cutoff preference shifts down, suggesting that many households that would have moved, decide to stay (the region labeled as “locked-in households”).
Figure 1: Local Unemployment and the Housing Bust

Note: Figure shows the unemployment rate for MSA A and MSA B. We plot the deviation from the unemployment rate before the recession. Upper panel shows the simulation with house price shock as well as labor productivity shock. Lower panel plots the case where the model is hit by labor productivity shock only.
Figure 2: Cutoff Rule for Migration

Note: Figure shows the migration decision of a household with 80% leverage living in MSA B. X-axis is the labor productivity in MSA A, and the y-axis is the preference of the household for the current residence, MSA B. Migration decision is characterized by a cutoff rule that is increasing in the productivity of the other MSA. The cutoff shifts down when house prices in MSA B fall. The shaded area designates the set of households that do not move due to the housing bust. We label them as locked-in households.

As a consequence, many unemployed homeowners that would be looking jobs in the other MSA, now look for jobs in MSA B. This causes local unemployment in MSA B to rise more as a response to a labor productivity decline. At the same time, it causes unemployment in MSA A to rise less, since the households that would be unemployed and looking for jobs in MSA A are now still in MSA B. This mechanism results in an increase in the dispersion of unemployment across these two MSAs: MSA A faces a lower decline in labor productivity, and the housing bust decreases the effect of the housing bust on local unemployment, whereas
MSA B faces a higher decline in labor productivity, and the housing bust amplifies the effect on local unemployment.

Figure 3: Dispersion of Unemployment and the Housing Bust

Note: Figure compares the prediction of the model for the difference in unemployment rates between MSA B and MSA A with the data. Solid line shows the difference that arises as a result of labor productivity and house price shock. Dashed-dotted line shows the effect of the decline in labor productivity without a housing bust. Dashed line is data.

Although the model does not generate the entire increase in local unemployment rates, it generates a difference in unemployment rates that is very similar to the data. We now focus on the ability of the model to explain the increase in the dispersion of unemployment. Figure 3 plots the evolution of the difference of local unemployment rates between MSA A and MSA B. Dashed line shows the data. In the solid line, we plot the difference in the simulation with both types of shocks, whereas in the dashed-dotted line we show the difference in the counter-factual simulation. We conclude that the housing bust substantially increases the
model’s ability to capture the rise in the dispersion of unemployment rates across MSAs.

Figure 4: Aggregate Unemployment and the Housing Bust

Note: In the solid line, we show the rise in aggregate unemployment as a response to house price and labor productivity shock. The dashed line shows the rise with labor productivity shock but without housing bust. Finally, the dashed-dotted line shows the data.

The increase in the geographic dispersion of unemployment rate has consequences for aggregate unemployment. This is because more households are now searching in the MSA with lower labor productivity, and thus with lower job finding rate. Figure 4 shows the evolution of the aggregate unemployment rate for the two simulations. The asymmetric decline in labor productivity, accompanied by an asymmetric decline in house prices, (shown in solid line) results in an increase in unemployment by around 2.5 percentage points. The blue line reveals that the increase in the aggregate unemployment rate would have been around 2 percentage points had there been no housing bust. The effect of the decline in labor productivity on unemployment is 2 percentage points, leaving the remaining 3% rise unexplained. We conclude that the effect of the housing bust on aggregate unemployment is 0.5 percentage points. Thus, the housing bust accounts for 17% (0.5% out of 3%) of the rise in unemployment that is not explained by labor productivity.

The possibility that the housing bust in 2007 could affect aggregate unemployment by
reducing migration has been long recognized. It was, however, largely dismissed due to several empirical observations: First, as documented by Kaplan and Schulhofer-Wohl (2010), the decline in inter-state migration during the 2007 – 2009 recession is small; around 0.5 percentage point. The decline in job-related migration rate is about a third of the overall drop. Second, it has been argued that net flows are too small compared to gross flows to be quantitatively relevant for unemployment dynamics in local labor markets. We now turn to address these issues.

Table 13: Net Migration: Data vs. Model

<table>
<thead>
<tr>
<th>Year</th>
<th>Data</th>
<th>Model</th>
<th>w/o Housing Bust</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005-2006</td>
<td>0.9%</td>
<td>0.8%</td>
<td>0.8%</td>
</tr>
<tr>
<td>2006-2007</td>
<td>0.7%</td>
<td>0.8%</td>
<td>0.8%</td>
</tr>
<tr>
<td>2007-2008</td>
<td>0.5%</td>
<td>0.3%</td>
<td>1.6%</td>
</tr>
<tr>
<td>2008-2009</td>
<td>0.3%</td>
<td>0.2%</td>
<td>1.1%</td>
</tr>
</tbody>
</table>

Note: Table compares the net migration rates in the data and the model. The last column shows the net migration rate predicted by the model in the absence of a housing bust.

Our findings show that the housing bust in 2007 has had a nontrivial effect on local and aggregate unemployment and this effect is driven by changes in net and gross migration rates that are consistent with the data. Table 13 compares the net migration rates in the model and the data. We find that the model predicts a decline to 0.2% in net migration from a pre-recession level of 0.8%. IRS data show a decline from a pre-recession average of 0.8% to 0.3%. Considering that the decline in net migration is not targeted in our model, this fit is remarkable. The last column of Table 13 shows the net migration rates predicted by the model in the absence of the housing bust. Interestingly, the model predicts a rise in net migration. This means that the effect of the housing bust on geographical reallocation is larger than the decline observed in the data. In other words, the decline in migration observed during the Great Recession is about 50% of the decline caused by the housing bust.

Figure 5 shows the model’s prediction for gross migration rate. The dashed-dotted line shows the prediction of the model. The model predicts a fall by around 1 percentage point from its pre-recession level. The counter-factual simulation (shown in solid line) indicates that, absent the housing bust, gross migration would have increased to 5.3%.

The model predicts a rise in both migration rates because the decline in labor productivity during the Great Recession is asymmetric across MSAs, a fact that increases the benefit of
migration, and thus migration rate in the absence of additional frictions coming from the housing market. We conclude that the observed decline in migration (both gross and net) in the data constitutes only half of the decline caused by the housing bust. This is what makes it possible to observe a rather small decline in migration after a large housing bust, but yet estimate a non negligible effect on local and aggregate unemployment.

Figure 5: Gross Migration and the Housing Bust

Note: Figure shows the gross migration rate predicted by the model. The blue line shows the decline in migration as a result of the housing bust and decline in labor productivity. The green line shows the counter-factual migration rate, that would have obtained absent the housing bust. The model predicts a rise in migration. The difference between the two lines is the true effect of the housing bust on migration.

6 Conclusions and Future Work

We have developed a tractable general equilibrium model of local housing and labor markets and migration with heterogeneous agents and used it to study the effect of the housing bust
in 2007 on local and aggregate labor markets during the Great Recession. Our analysis suggests that the housing bust is responsible for a variety of facts regarding the behavior of migration and unemployment during this episode, in particular the increase in the dispersion of unemployment across regions. We found that the housing bust accounts for 17% of the rise in unemployment that is unaccounted for by the decline in labor productivity. The results are driven by declines in net and gross migration rates that are similar to the data.

The effect of the housing bust on labor market is due to a fundamental friction in the economy: the down payment requirement. Reduction in house prices reduces the home equity for households, and makes it harder for them to afford new houses after moving. It is because they prefer owning to renting that this distorts their migration decisions. The fact that the decline in the extent of worker reallocation occurs at a time when there are large differences across MSAs in terms of labor productivity generates a nontrivial effect on aggregate unemployment.

The results lead to further questions relevant for policy makers since unemployment remains stubbornly high despite the recovery in labor productivity. Should the government subsidize the down payment to facilitate migration? What is the optimal policy for the down payment requirement during a recession that is accompanied by a housing bust? How do the results to these questions depend on the amount of leverage in the economy?

Another question relevant to policy makers is the extension of unemployment benefits. Is it possible that by extending the length of unemployment benefits, the government further distorted the location choice of homeowners? If so, how important is this for unemployment and welfare? The framework developed here is suitable to study these issues. We wish to take up these questions in future research.

References


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7 Figures

Figure 6: Variation in the Decline of House Prices Across MSAs during the Housing Bust in 2007

Source: Federal Housing Finance Agency (FHFA) and authors’ calculations. Figure plots the histogram of the size of the housing bust for MSAs. For each MSA, the size of the housing bust is defined as the percentage decline in house prices from the peak before 2007 and the trough during the housing bust in 2007.
Figure 7: Variation in the Decline of Labor Productivity Across MSAs during the 2007–2009 Recession

Note: Figure plots the histogram of the decline in labor productivity across MSAs between 2007 and 2008.
Figure 8: Dispersion of Unemployment Rates Across MSAs

Source: Bureau of Labor Statistics (BLS) and authors’ calculations. Figure plots the standard deviation of unemployment rate across MSAs (weighted by employment). Dispersion of unemployment doubled during the 2007-2009 recession and is at an all time high. For each MSA, BLS reports seasonally unadjusted unemployment rate at monthly frequency. The fluctuations before the 2007-2009 recession are likely to be due to seasonal factors.
Figure 9: Housing Bust and the Response of Local Unemployment to Adverse Labor Productivity Shocks

Note: Figure plots how a local housing bust changes the response of local unemployment to local labor productivity. The blue solid line is calculated using point estimates in Column (2) in Table 2, whereas the green dashed line uses point estimates in Column (3) in Table 2. Both lines show that, holding the change in labor productivity constant, unemployment increased more in MSAs that experienced larger housing busts.
Figure 10: Gross Migration and Dispersion of Labor Productivity

Note: Figure shows the time series of standard deviation of log of labor productivity across states and inter-state migration rate. See the data section for details on labor productivity data at the state level. Inter-state migration rate is taken from Kaplan and Schulhofer-Wohl 2010.
A Data Appendix

Annual output data at the MSA level are available at the Bureau of Economic Analysis (BEA) website for the period 2001:2009.\footnote{http://bea.gov/regional/gdpmetro/} We use real GDP by metropolitan area in millions of chained 2005 dollars (all industry total). Employment and unemployment data are taken from the Bureau of Labor Statistics (BLS). These variables are also available at monthly frequency. We construct a measure of local labor productivity for each MSA as the ratio of output to employment. Population data are taken from the “Regional Economic Accounts” of the Bureau of Economic Analysis (Table CA1-3). These are available at \url{http://www.bea.gov/regional/reis}. Population numbers reported are mid-year estimates. Quarterly data on house prices are obtained from the Federal Housing Finance Agency. We use all-transaction indexes (estimated using sales price and appraisal data). Annual estimates are computed as the average of quarterly observations.

Using county-county migration data based on tax return records of the Internal Revenue Service (IRS), we construct data on MSA-level population gross inflows, gross outflows, and net flows. These data are available from the IRS web site for the period 2004 – 2009. For each year, IRS reports population inflows and outflows for all counties. These files report the origin and the destination counties, and the number of migrants in two units: “returns” and “personal exemptions”. We follow \textit{Davis, Fisher, and Veracierto (2010)} and use the exemptions data, as these data approximates the migrant population as opposed to the number of households as in returns data.\footnote{For a more detailed description of the IRS data, see \textit{Davis, Fisher, and Veracierto (2010)}.} Gross inflows into an MSA are computed as the sum of all inflows into any county in that MSA from any other county in other MSAs. Gross outflows are computed analogously. Inflow and outflow rates are computed as the ratio of flow to the population in that year. Finally, net flow rate is defined as the difference between gross inflow rate and gross outflow rate.

B Existence of a Block-Recursive Equilibrium

To prove Proposition 1, we proceed in two steps. We first show that the functional equations and the corresponding free-entry conditions for firms and housing market intermediaries admits a solution, where the dependance of the value functions, market tightness functions, and the rental rate on the aggregate state is through exogenous shocks only. More formally, we show that there exists a set of market tightness functions \( \{ \theta_i^f(w; \psi), \theta_s^f(p; \psi), \theta_b^f(p; \psi) \}_{i \in A,B} \),
rental rate \( \rho(\psi) \), and value functions of firms and housing market intermediaries \( \{J_i, L_i, R_i\}_{i \in \{A,B\}} \), which depend on \( \psi \) only through exogenous shocks \( (z_i, \mu_i)_{i \in \{A,B\}} \), and not through any endogenous distribution \( \Gamma_A \) or \( \Gamma_B \). That is, we can reduce the state space \( \psi \) into exogenous shocks, \( (z_i, \mu_i)_{i \in \{A,B\}} \). In the second stage, we collapse the problem of households into one big functional equation and show that it is a contraction. We also show that the functional equation maps the set of functions that do not depend on the endogenous distribution into the same set, provided that the market tightness functions and the rental rate are independent from the endogenous distribution as well. This shows that there is a solution to the problem of households that, together with the value functions, market tightness functions, and rental rates of the first step, constitute a block-recursive equilibrium of the economy.

**B.1 Market Tightness Functions**

- Define \( J(\mathcal{W} \times \mathcal{Z}) \) as the set of continuous and bounded functions \( J \) such that \( J : \mathcal{W} \times \mathcal{Z} \to \mathbb{R} \) and denote \( T_J \) as an operator associated with (9). It is easy to verify that \( T_J \) maps \( J \) into \( J \). Applying Blackwell’s sufficiency conditions, we can show that the operator \( T_J : J \to J \) is a contraction. Denote the fixed point of \( T_J \) as \( J^* \in J \).\(^{30}\)

- Plugging in \( J^* \) into the free-entry condition for the labor market, (11), we get the labor market tightness function \( \theta^*_l(w; \psi) \) as only a function of wage and labor productivity shock \( z \).

\[
\theta^*_l(w; z) = \begin{cases} 
q^{-1} \left( \frac{k}{J^*(w; z)} \right) & \text{if } w \in \mathcal{W}(z) \\
0 & \text{o/w}
\end{cases}
\]

- Similarly, we define \( R(\mathcal{P}_b \times M) \) as the set of continuous and bounded functions mapping \( \mathcal{P}_b \times M \) to \( \mathbb{R} \), and \( L(M) \) as the set of continuous and bounded functions from \( M \) to \( \mathbb{R} \). It is easy to show that the operator associated with (12) maps functions from \( R(\mathcal{P}_b \times M) \) into \( R(\mathcal{P}_b \times M) \) if \( \theta^*_b(\cdot) \) depends on \( \psi \) only through \( \mu \). Similarly, one can show that the operator associated with (15) maps functions from \( L(M) \) into \( L(M) \) if \( \psi^*_i(\cdot) \) depends on \( \psi \) only through \( \mu \). Standard contraction mapping argument can be applied to establish the existence of fixed points, \( R^* \) and \( L^* \), of operators (12) and (15), respectively.

- Using the free-entry conditions for the housing markets, (14) and (16), and plugging

\(^{30}\)The assumption of full commitment to a constant wage contract of workers guarantees \( J(\mathcal{W} \times \mathcal{Z}) \) be a space of bounded and continuous functions.
them into the operators (12) and (15), respectively, one can solve for the market tight-ness functions \( \theta^*_s(p; \mu) \) and \( \theta^*_b(p; \mu) \)

\[
\theta^*_s(p; \mu) = \begin{cases} 
\xi_s^{-1} \left( \frac{\kappa}{R^*(p; \mu)} \right) & \text{if } p \in \mathcal{P}_s(\mu) \\
0 & \text{o/w}
\end{cases}
\]

\[
\theta^*_b(p; \mu) = \begin{cases} 
\xi_b^{-1} \left( \frac{\mu-(1+r)^{-1}\mathbb{E}_{\psi'|\psi}[\mu'|\mu]}{p-(1+r)^{-1}\mathbb{E}_{\psi'|\psi}[\mu'|\mu]} \right) & \text{if } p \in \mathcal{P}_b(\mu) \\
0 & \text{o/w}
\end{cases}
\]

Using the free-entry condition (17) in the operator (15), we get the rental rate of the economy \( \rho(\psi) \) as a function of \( \mu \).

\[
\rho^*(\mu) = \mu - \frac{1 - \gamma}{1 + r} \mathbb{E}_{\psi'|\psi}[\mu' | \mu]
\]

### B.2 Households’ Value Function

- First, we reformulate the value functions of households as one function \( V : I \times E \times A \times \{0, 1\} \times \Psi \rightarrow \mathbb{R} \) such that \( V(1, 1, w, a, h; \psi) = W_A(w, a, h; \psi) \), \( V(0, 1, w, a, h; \psi) = W_B(w, a, h; \psi) \), \( V(1, 0, w, a, h; \psi) = U_A(a, h; \psi) \), and \( V(0, 0, w, a, h; \psi) = U_B(a, h; \psi) \).

- Using the above value function, \( V \), we can define the labor market surplus function as the following:

\[
\tilde{\Delta}(i, a, h; \psi) = \max_{w \in \mathcal{W}(z_i)} ip_l(\theta^*_l(w; z_i)) \{ V(1, 1, w, a, h; \psi) - V(0, 1, w, a, h; \psi) \} + (1 - i) p_l(\theta^*_l(w; z_i)) \{ V(1, 0, w, a, h; \psi) - V(0, 0, w, a, h; \psi) \}
\]

- Similarly, the option value of migration can be written as

\[
\tilde{M}(i, a, h; \psi) = (1 - h)[i \max \tilde{\Delta}(1, a, h; \psi), \tilde{\Delta}(0, a - \nu, h; \psi)] + (1 - i) \max \tilde{\Delta}(1, a - \nu, h; \psi), \tilde{\Delta}(0, a, h; \psi)]
\]

- We define a set of functions \( \mathcal{V} : I \times E \times \mathcal{W} \times A \times \{0, 1\} \times \mathbb{Z}_2 \times \mathbb{M}^2 \rightarrow \mathbb{R} \) and \( T_V \) such
that

\((TVV)(i,e,w,a,h;(z_0, z_1, \mu_0, \mu_1))\)

\[= \left(1-e\right) \left(1-h\right) \left\{ \max_{a' \geq a_0} u(c,0,0) + \beta \mathbb{E}_{\psi|\psi} \left[ \max_{p \in \mathcal{P}(\mu_1),a' \geq p} \theta^*_a(p; \hat{\mu}_1) \times \left\{ \tilde{\Delta}(1,a' - p,1; \hat{\psi}) + V(0,1,w,a' - p,1; \hat{\psi}) - \tilde{M}(1,a',0; \hat{\psi}) \right\} + \tilde{M}(1,a',0; \hat{\psi}) \right] \right\}^{\delta} \]

\[+ \left(1-e\right) h \left\{ \max_{a' \geq a_1} u(c,0,1) + \beta \mathbb{E}_{\psi|\psi} \left[ \max_{p \in \mathcal{P}(\mu_1)} \theta^*_a(p; \hat{\mu}_1) \times \left\{ \tilde{M}(1,a' + p,0; \hat{\psi}) - \tilde{\Delta}(1,a',1; \hat{\psi}) \right\} + \tilde{\Delta}(1,a',1; \hat{\psi}) \right] \right\} \]

\[+ e h \left\{ \max_{a' \geq a_1} u(c,1,0) + \beta \mathbb{E}_{\psi|\psi} \left[ \left(1-\delta\right) \max_{p \in \mathcal{P}(\bar{\mu}_1)} \theta^*_a(p; \bar{\mu}_1) \times \right. \left\{ V(1,1,w,a' - p,1; \hat{\psi}) - V(1,1,w,a',0; \hat{\psi}) \right\} + V(1,1,w,a',0; \hat{\psi}) \right\} \]

\[+ \left(1-i\right) \left(1-e\right) \left(1-h\right) \left\{ \max_{a' \geq a_0} u(c,0,0) + \beta \mathbb{E}_{\psi|\psi} \left[ \max_{p \in \mathcal{P}(\bar{\mu}_0),a' \geq p} \theta^*_a(p; \bar{\mu}_0) \times \left\{ \tilde{\Delta}(0,a' - p,1; \hat{\psi}) + V(0,1,w,a' - p,1; \hat{\psi}) - \tilde{M}(1,a',0; \hat{\psi}) \right\} + \tilde{M}(1,a',0; \hat{\psi}) \right\} \right\}^{\delta} \]

\[+ \left(1-e\right) h \left\{ \max_{a' \geq a_1} u(c,0,1) + \beta \mathbb{E}_{\psi|\psi} \left[ \max_{p \in \mathcal{P}(\bar{\mu}_0)} \theta^*_a(p; \bar{\mu}_0) \times \left\{ \tilde{M}(0,a' + p,0; \hat{\psi}) - \tilde{\Delta}(0,a',1; \hat{\psi}) \right\} + \tilde{\Delta}(0,a',1; \hat{\psi}) \right] \right\} \]

\[+ e h \left\{ \max_{a' \geq a_1} u(c,1,0) + \beta \mathbb{E}_{\psi|\psi} \left[ \left(1-\delta\right) \max_{p \in \mathcal{P}(\bar{\mu}_0)} \theta^*_a(p; \bar{\mu}_0) \times \right. \left\{ V(0,1,w,a' - p,1; \hat{\psi}) - V(0,1,w,a',0; \hat{\psi}) \right\} + V(0,1,w,a',0; \hat{\psi}) \right\} \]

\[+ \left(1-i\right) \left(1-e\right) \left(1-h\right) \left\{ \max_{a' \geq a_0} u(c,0,0) + \beta \mathbb{E}_{\psi|\psi} \left[ \max_{p \in \mathcal{P}(\bar{\mu}_0),a' \geq p} \theta^*_a(p; \bar{\mu}_0) \times \left\{ \tilde{\Delta}(0,a' - p,1; \hat{\psi}) + V(0,1,w,a' - p,1; \hat{\psi}) - \tilde{M}(1,a',0; \hat{\psi}) \right\} + \tilde{M}(1,a',0; \hat{\psi}) \right\} \right\} \]
where
\[ a + ew + (1 - e)b = c + (1 - h) \{ i\rho^*(\mu_1) + (1 - i) \rho^*(\mu_0) \} + \frac{a'}{1 + r} \]

We can show that the operator \( T_V \) maps a function from \( \mathcal{V} \) into \( \mathcal{V} \), where \( \mathcal{V} \) is the set of continuous and bounded functions on the appropriate domain, assuming a bounded and continuous utility function. From the definition of \( \tilde{\Delta}(\cdot, \cdot) \) and \( \tilde{M}(\cdot, \cdot) \), it is clear that if \( V \in \mathcal{V} \), then \( \tilde{\Delta}(i, a, h; \psi) = \tilde{\Delta}(i, a, h; z_i, \mu_i) \) and \( \tilde{M}(i, a, h; \psi) = \tilde{M}(i, a, h; z_i, \mu_i) \).

Substituting \( \tilde{\Delta}(i, a, h; z_i, \mu_i) \) and \( \tilde{M}(i, a, h; z_i, \mu_i) \) into the definition of \( T_V \), we get \( T_V : \mathcal{V} \rightarrow \mathcal{V} \).

\( \mathcal{V} \) is a complete metric space. Verifying Blackwell’s sufficiency conditions, one can easily show that \( T_V : \mathcal{V} \rightarrow \mathcal{V} \) is a contraction. Therefore, there exists a fixed point, \( V^* \in \mathcal{V} \). This is a solution to households problem, and depends on the aggregate state only through \( (z_A, z_B, \mu_A, \mu_B) \).