# Job Search Models

Jesús Fernández-Villaverde

University of Pennsylvania

February 12, 2016

# Motivation

- We want to have dynamic models of the job market.
- Examples of questions we are interested in:
  1. Why is there unemployment?
  2. Why does unemployment fluctuate over the business cycle?
  3. Why does unemployment fluctuate in the lower frequencies?
  4. Why are unemployment rates different across countries?
  5. Is the unemployment level efficient?
  6. What are the effects of labor market regulation?
  7. What are the effects of UI?
  8. What determines the distribution of jobs and wages?
- Equilibrium models of unemployment.
- Labor market frictions.

# Search Models

- We will begin with a simple model of job search.
- Matching is costly. Think about getting a date.
- We can bring our intuition to the job market. Why?
- Useful to illustrate many ideas and for policy analysis.
- Contributions of:
    1. Stigler (1961).
    2. McCall (1970).
- Static problem versus sequential.

# Stigler's Model

- Risk-neutral agent.
- Easier to think as an agent asking for bids.
- Samples offers i.i.d. from $F(w)$.
- Decide ex-ante how many offers $n$ she is going to ask for.
- Each offer has a cost $c$.

# Optimal Number of Offers

- Remember that:

$$M_n = \mathbb{E} \min (w_1, w_2, ..., w_n) = \int_0^\infty (1 - F(w))^n \, dw$$

- Then, gain of additional offer is:

$$\begin{aligned} G_n &= M_{n-1} - M_n \\ &= \int_0^\infty (1 - F(w))^{n-1} \, dw - \int_0^\infty (1 - F(w))^n \, dw \\ &= \int_0^\infty (1 - F(w))^{n-1} \, dw - \int_0^\infty (1 - F(w))^{n-1} (1 - F(w)) \, dw \\ &= \int_0^\infty (1 - F(w))^{n-1} F(w) \, dw \end{aligned}$$

- Then $G_n$ is a decreasing function with $\lim_{n\to\infty} G_n = 0$.
- Optimal rule: set $n$ such that $G_n \geq c > G_{n+1}$.
- Basic problem of static decisions: What if I get the lowest possible price in my first offer?

# McCall's Model

- An agent searches for a job, taking market conditions as given.
- Preferences:

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t x_t$$

where

$$x_t = \begin{cases} = w \text{ if employed} \\ = z \text{ if unemployed} \end{cases}$$

- Interpretation of $w$ and $z$.

# Job Offers

- An unemployed agent gets every period one offer i.i.d. from distribution $F(w)$.
- Offer can be rejected (unemployed next period) or accepted (wage posting by firms).
- No recall of offers (no restrictive because of stationarity of the problem).
- Job last forever (neither quitting nor firing).
- Undirected search (alternative: directed search).

# Bellman Equations

- Value function of employed agent:

$$W(w) = w + \beta W(w)$$

Clearly: $W(w) = \frac{w}{1-\beta}$.

- Value function of unemployed agent:

$$U = z + \beta \int_0^\infty \max \{U, W(w)\} \, dF(w)$$

Then:

$$U = z + \beta \int_0^\infty \max \left\{ U, \frac{w}{1-\beta} \right\} dF(w)$$

- Lebesgue integral: discrete and continuous components.

# Reservation Wage Property

- There exist a reservation wage $w_R$

$$W(w_R) = U = \frac{w_R}{1-\beta}$$

  such that if $w \geq w_R$ the worker should accept the offer and reject otherwise.

- Then:

$$w_R = T(w_R) = (1-\beta)z + \beta \int_0^\infty \max\{w_R, w\}\, dF(w)$$

  that is a contraction (that is, $\lim_{N\to\infty} T^N(w_0) = w_R$ and $w_R$ is unique).

# Characterizing Strategy I

Note:

$$\frac{w_R}{1-\beta} = z + \beta \int_0^\infty \max\left\{\frac{w_R}{1-\beta}, \frac{w}{1-\beta}\right\} dF(w) \Rightarrow$$

$$\int_0^{w_R} \frac{w_R}{1-\beta} dF(w) + \int_{w_R}^\infty \frac{w_R}{1-\beta} dF(w) =$$
$$= z + \beta \int_0^{w_R} \frac{w_R}{1-\beta} dF(w) + \beta \int_{w_R}^\infty \frac{w}{1-\beta} dF(w) \Rightarrow$$

$$w_R \int_0^{w_R} dF(w) - z = \beta \int_{w_R}^\infty \frac{\beta w - w_R}{1-\beta} dF(w)$$

- Adding $w_R \int_{w_R}^\infty dF(w)$ to both sides:

$$w_R - z = \frac{\beta}{1-\beta} \int_{w_R}^\infty (w - w_R)\, dF(w)$$

# Characterizing Strategy II

- Interpretation

$$\underbrace{w_R - z}_{\text{Cost of Search one more time}} = \underbrace{\frac{\beta}{1-\beta}\int_{w_R}^{\infty}(w - w_R)\,dF(w)}_{\text{Expected Gain of one more search}}$$

- Sequential nature of the problem.
- Note that

$$\begin{aligned} g(w_R) &= \frac{\beta}{1-\beta}\int_{w_R}^{\infty}(w - w_R)\,dF(w) \\ g'(w_R) &= -\frac{\beta}{1-\beta}(1 - F(w_R)) < 0 \\ g''(w_R) &= \frac{\beta}{1-\beta}f(w_R) \geq 0 \end{aligned}$$

# Characterizing Strategy III

- Integrating by parts

$$\int_{w_R}^{\infty} (w - w_R)\, dF(w) = \int_{w_R}^{\infty} (1 - F(w))\, dw$$

- Then:

$$w_R - z = \frac{\beta}{1-\beta} \int_{w_R}^{\infty} (1 - F(w))\, dw$$

# Characterizing Strategy IV

Note that

$$
\begin{aligned}
w_R - z &= \frac{\beta}{1-\beta}\int_{w_R}^{\infty}(w-w_R)\,dF(w) + \frac{\beta}{1-\beta}\int_{0}^{w_R}(w-w_R)\,dF(w) \\
&\quad -\frac{\beta}{1-\beta}\int_{0}^{w_R}(w-w_R)\,dF(w) \\
&= \frac{\beta}{1-\beta}\int_{0}^{\infty}(w-w_R)\,dF(w) - \frac{\beta}{1-\beta}\int_{0}^{w_R}(w-w_R)\,dF(w) \\
&= \frac{\beta}{1-\beta}\int_{0}^{\infty}w\,dF(w) - \frac{\beta}{1-\beta}\left(w_R - \int_{0}^{w_R}(w-w_R)\,dF(w)\right) \\
&= \frac{\beta}{1-\beta}\mathbb{E}w - \frac{\beta}{1-\beta}w_R - \frac{\beta}{1-\beta}\int_{0}^{w_R}(w-w_R)\,dF(w)
\end{aligned}
$$

# Characterizing Strategy V

- Now:

$$w_R - z = \frac{\beta}{1-\beta}\mathbb{E}w - \frac{\beta}{1-\beta}w_R - \frac{\beta}{1-\beta}\int_0^{w_R}(w - w_R)\,dF(w) \Rightarrow$$

$$(1-\beta)(w_R - z) = \beta\mathbb{E}w - \beta w_R - \beta\int_0^{w_R}(w - w_R)\,dF(w) \Rightarrow$$

$$w_R - z = \beta(\mathbb{E}w - z) - \beta\int_0^{w_R}(w - w_R)\,dF(w)$$

- Integrating by parts $\int_0^{w_R}(w - w_R)\,dF(w) = -\int_0^{w_R}F(w)\,dw$ and then:

$$w_R - z = \beta(\mathbb{E}w - z) + \beta\int_0^{w_R}F(w)\,dw$$

# Comparative Statics

Factors that affect search strategy:

1. Value of unemployment $z$. Unemployment insurance: length and generosity of unemployment insurance vary greatly across countries. US replacement rate is 34%. Germany, France, and Italy the replacement rate is about 67%, with duration well beyond the first year of unemployment.

2. Distribution of offers. Let $\widetilde{F}(w)$ be a mean-preserving spread of $F(w)$. Then $\int_0^{w_R} \widetilde{F}(w)\, dw > \int_0^{w_R} F(w)\, dw$ for all $w_R$ and $\widetilde{w}_R > w_R$.

3. Minimum Wages: If the minimum wage is so high that it makes certain jobs unprofitable, less jobs are offered and job finding rates decline.

# Problems

Rothschild (1973): Where does the distribution $F(w)$ come from?
Diamond (1971): Why is the distribution not degenerate?.
Intuition:

1. In a model such as the previous one, workers follow a reservation wage strategy.
2. Hence, firms do not gain anything out of posting any $w > w_R$.
3. At the same time, firms will never hire anyone if they post $w < w_R$.
4. Therefore, $F(w)$ will have a unit mass at $w_R$. (Rothschild's Paradox).
5. Moreover (Diamond's Paradox):

$$w_R - z = \beta\left(\mathbb{E}w - z\right) + \beta\int_0^{w_R} F(w)\, dw \Rightarrow$$

$$w_R - z = \beta\left(w_R - z\right) \Rightarrow$$

$$w_R = z$$

# Answers

1. Exogenously given: different productivity opportunities.

2. Endogenous:

    1. Lucas and Prescott model of islands economy.

    2. Bargaining.

    3. Directed search.

# Lucas and Prescott (1974)

- Continuum of workers.
- Workers are risk neutral.
- A large number of separated labor markets (islands).
- There is a firm in each island subject to productivity shocks.
- Wage is determined competitively in each island.

# Firms

- Each island has an aggregate production function:

$$\theta f(n)$$

where $\theta$ is a productivity shock, $n$ is labor, and $f$ has decreasing returns to scale.

- $\theta$ evolve according to kernel $\pi(\theta, \theta')$.

- There is a stationary distribution of $\theta$.

# Worker

- At the beginning of the period, worker observes:
  1. Productivity $\theta$.
  2. Amount of worker on the island $x$.
  3. Distribution of islands in the economy $\Psi(\theta, x)$.
- They decide whether or not to move:
  1. If it stays, workers will get wage $w(\theta, x)$.
  2. If it moves, it does not work this period and picks which island to move to.

# Equilibrium within the Island

- Firms maximize:

$$w(\theta, x) = \theta f'(n(\theta, x))$$

- Markets clear:

$$n(\theta, x) \leq x + \text{arrivals}$$

# Value Function for the Worker

- The Bellman equation for the worker is given by:

$$v(\theta, x) = \max\left\{\beta v_u, w(\theta, x) + \beta \int v(\theta', x')\, d\theta\right\}$$

where $v_u$ is the value of search.

- Three cases:

1. $v(\theta, x) = \beta v_u$: some workers are leaving the market.
2. $v(\theta, x) > \beta v_u$ : no worker is leaving the market. Some may or may not arrive.
3. $v(\theta, x) < \beta v_u$ : cannot happen.

# Case 2

- No worker is leaving but some workers are arriving:

$$v_u = \int v\left(\theta', x'\right) d\theta$$

Thus:

$$v\left(\theta, x\right) = \theta f'\left(n\left(\theta, x\right)\right) + \beta v_u.$$

- No worker is leaving and no workers are arriving:

$$v\left(\theta, x\right) = \theta f'\left(n\left(\theta, x\right)\right) + \beta \int v\left(\theta', x'\right) d\theta \leq \theta f'\left(n\left(\theta, x\right)\right) + \beta v_u$$

# A New Expression

- Putting all these cases together:

$$v\left(\theta,x\right)=\max\left\{\beta v_u,\theta f'\left(n\left(\theta,x\right)\right)+\min\left\{\beta v_u,\beta\int v\left(\theta',x'\right)d\theta\right\}\right\}$$

- Functional equation on $v\left(\theta,x\right)$.
- Unique solution.

# Evolution of the Labor Force

- Some agents leave the market. Then $x' = n(\theta, x)$ solves:

$$\theta f'(n(\theta, x)) + \beta \int v(\theta', x')\, d\theta = \beta v_u$$

- No worker is leaving but some will arrive next period. Then $x'$ solves:

$$\int v(\theta', x')\, d\theta = v_u$$

- No worker is leaving and no workers will arrive next period. Then:

$$x' = x$$

# Stationary Distribution

- The evolution of $(\theta, x)$ is then governed by a function $\Gamma\left(\theta', x' \middle| \theta, x\right)$ that embodies the equations above.

- Then the stationary distribution solves:

$$\Psi(\theta, x) = \int \Gamma\left(\theta', x' \middle| \theta, x\right) \Psi(\theta, x)\, d\theta$$

- From the stationary distribution we can find $v_u$.

# Álvarez and Veracierto (1999)

- Now, instead of going to their favorite island, unemployed workers search for a new job randomly.
- Every period they find one island from distribution $\Psi(\theta, x)$.
- They decide whether to accept it or reject it.
- Endogenous distribution of wage offers.