Ramsey Fiscal Policy

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Optimal Fiscal Policy

- We can use dynamic equilibrium theory to think about the design and implementation of optimal policy.

- Reasons for a non-trivial problem: absence of a lump-sum tax.

- We will focus first in the case of full commitment: Ramsey problems.

- Two approaches:
  1. Primal approach: we search directly for allocations by maximizing a social planner’s problem subject to an implementability constraint. Then, we decentralize the allocation.
  2. Dual approach: we search directly for optimal taxes.
A Nonstochastic Economy

- Preferences:
  \[
  \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)
  \]

- Budget constraint:
  \[
  c_t + k_{t+1} + \frac{b_{t+1}}{R_t} = (1 - \tau^l_t) w_t l_t + \left[ 1 + \left( 1 - \tau^k_t \right) (r_t - \delta) \right] k_t + b_t
  \]

- Technology: representative firm
  \[
  c_t + g_t + k_{t+1} = F(k_t, l_t) + (1 - \delta) k_t
  \]

- Government:
  \[
  g_t = \tau^k_t (r_t - \delta) k_t + \tau^l_t w_t l_t + \frac{b_{t+1}}{R_t} - b_t
  \]
Competitive Equilibrium

A Competitive Equilibrium is an allocation \( \{c_t, l_t, k_t, g_t\}_{t=0}^\infty \), a price system \( \{\hat{w}_t, \hat{r}_t, \hat{R}_t\}_{t=0}^\infty \), and a government policy \( \{\hat{g}_t, \hat{r}_t^k, \hat{r}_t^l, \hat{b}_t\}_{t=0}^\infty \) such that:

1. Given prices and the government policy, households maximize.

2. Given prices, firms minimize costs.


Note that 3. plus the budget constraint of households deliver market clearing.
Ramsey equilibrium

- Fix a sequence of exogenously given government purchases $\{g_t\}_{t=0}^{\infty}$ (alternative: $g_t$ can be a choice variable given some utility from government consumption).

- A Ramsey equilibrium is the best competitive equilibrium given $\{g_t\}_{t=0}^{\infty}$, $k_0$, $b_0$, and bounds on $\tau_t^k$.

- Note that best is defined ex-ante.
Consolidating Budget Constraints

- Consolidate two consecutive budget constraints:

\[
\begin{align*}
ct + \frac{ct+1}{R_t} + \frac{kt+2}{R_t} + \frac{bt+2}{R_tR_{t+1}} &= \\
(1 - \tau_t^l) w_t l_t + (1 - \tau_{t+1}^l) \frac{w_{t+1}l_{t+1}}{R_t} + \left(1 + (1 - \tau^k_t) (r_t - \delta)\right) k_t \\
&\quad + \left(1 + (1 - \tau^k_{t+1}) (r_{t+1} - \delta)\right) \left(\frac{1}{R_t} - 1\right) k_{t+1} + b_t
\end{align*}
\]

- By no arbitrage: \( R_t = 1 + (1 - \tau^k_{t+1}) (r_{t+1} - \delta) \). Then:

\[
\begin{align*}
ct + \frac{ct+1}{R_t} + \frac{kt+2}{R_t} + \frac{bt+2}{R_tR_{t+1}} &= \\
\left(1 - \tau_t^l\right) w_t l_t + \left(1 - \tau_{t+1}^l\right) \frac{w_{t+1}l_{t+1}}{R_t} + \left((1 - \tau^k_t) r_t + 1 - \delta\right) k_t + b_t
\end{align*}
\]
Asset Pricing

- Define

\[ Q(t|0) = \prod_{i=1}^{t} R_{i-1}^{-1} \]

where clearly \( Q(0|0) = 1 \).

- Also, we have

\[ \frac{Q(t|0)}{Q(t+1|0)} = \frac{1}{\beta} \frac{u_c(t)}{u_c(t+1)} \]
Using asset prices to iterate on the budget constraint:

$$\sum_{t=0}^{\infty} Q(t|0) c_t = \sum_{t=0}^{\infty} Q(t|0) \left( 1 - r_t^l \right) w_t l_t + \left( 1 + \left( 1 - r_0^k \right) \left( r_0 - \delta \right) \right) k_0 + b_0$$

subject to

$$\lim_{T \to \infty} \left( \prod_{i=1}^{T} R_i^{-1} \right) k_{T+1} = \lim_{T \to \infty} Q(T-1|0) k_{T+1} = 0$$

$$\lim_{T \to \infty} Q(T|0) b_{T+1} = 0$$

Role of transversality conditions.
Necessary Conditions

- Necessary conditions for households:

\[ \beta^t u_c (t) - \lambda Q (t|0) = 0 \]
\[ -\beta^t u_l (t) - \lambda Q (t|0) (1 - \tau^l_t) w_t = 0 \]
\[ -\lambda Q (t|0) + \lambda Q (t + 1|0) \left( 1 + \left( 1 - \tau^k_{t+1} \right) (r_{t+1} - \delta) \right) k_t = 0 \]

- Given \( Q (0|0) = 1 \), we can find

\[ Q (t|0) = \beta^t \frac{u_c (t)}{u_c (0)} \]

and:

\[ \frac{u_l (t)}{u_c (t)} = (1 - \tau^l_t) w_t \]

- From firms’ problem:

\[ r_t = F_k (t) \]
\[ w_t = F_l (t) \]
Budget Constraint

- Substituting necessary conditions in the budget constraint of household:

\[
\sum_{t=0}^{\infty} \beta^t \frac{u_c(t)}{u_c(0)} c_t = \sum_{t=0}^{\infty} \beta^t \frac{u_c(t)}{u_c(0)} \frac{u_l(t)}{u_c(t)} l_t + \left(1 + \left(1 - \tau_0^k\right) (r_0 - \delta)\right) k_0 + b_0
\]

- Rearranging terms:

\[
\sum_{t=0}^{\infty} \beta^t \left(u_c(t) c_t - u_l(t) l_t\right) - u_c(0) \left\{ \left(1 + \left(1 - \tau_0^k\right) (r_0 - \delta)\right) k_0 + b_0 \right\} = 0
\]

\[
A(c_0, l_0, \tau_0^k, b_0)
\]

- You can think about extra term as an implementability constraint with associated lagrangian \(\Phi\).
Social Planner

- Define $W(c_t, l_t, \Phi) = (u(c_t, l_t) + \Phi(u_c(t)c_t - u_l(t)l_t))$

- We get the social planner’s objective function:
  \[
  \sum_{t=0}^{\infty} \beta^t W(c_t, l_t, \Phi) + \\
  \theta_t (F(k_t, l_t) + (1 - \delta)k_t - c_t - g_t - k_{t+1}) - \Phi A(c_0, l_0, \tau_0^k, b_0)
  \]

- Interpretation.

- Convex set?
Necessary Conditions

- If solution is interior:

\[
W_c(t) = \theta_t, \quad t \geq 1 \\
W_l(t) = -\theta_t F_l(t), \quad t \geq 1 \\
\theta_t = \beta \theta_{t+1} (F_k(t+1) + 1 - \delta), \quad t \geq 0 \\
W_c(0) = \theta_0 + \Phi A_c \\
W_l(0) = -\theta_0 F_n(0) + \Phi A_l
\]

- Playing with conditions:

\[
W_c(t) = \beta W_c(t + 1) (F_k(t + 1) + 1 - \delta), \quad t \geq 1 \\
W_l(t) = -W_c(t) F_l(t), \quad t \geq 1 \\
W_l(0) = [\Phi A_c - W_c(0)] F_l(t) + \Phi A_l
\]
Capital Taxation I: Basic Result

• Assume $\exists T \geq 0$ s.t. $g_t = g$ for $t \geq T$ and $\exists$ a Ramsey Equilibrium that converges to a steady state in finite time. Then:

$$W_c(ss) = \beta W_c(ss) (F_k(ss) + 1 - \delta)$$

or

$$1 = \beta (F_k(ss) + 1 - \delta)$$

• Now, note that in the steady state of any decentralized equilibrium:

$$\frac{Q(t|0)}{Q(t+1|0)} = \frac{1}{\beta u_c(ss)} = \frac{1}{\beta} = (1 - \tau^k_{t+1}) r_{ss} + 1 - \delta$$

• Now, note that $r_{t+1} = F_k(ss)$. Hence,

$$1 = \beta \left(1 + \left(1 - \tau^k_{t+1}\right) (r_{ss} - \delta)\right)$$
Capital Taxation II: Zero Capital in Steady State

- If we compare

\[ 1 = \beta (F_k(ss) + 1 - \delta) \]

with

\[ 1 = \beta \left( 1 + \left(1 - \tau_{t+1}^k \right) (r_{ss} - \delta) \right) \]

we see that, Ramsey implies:

\[ \tau_{t+1}^k = 0. \]


- Intuition and robustness.

- Relation with uniform taxation theorem and with the no taxation of intermediate goods.
Role of First Period Taxation

- Note that the first order condition of the objective function with respect to $\tau_0^k$ is
  \[ \Phi u_c(0) F_k(0) k_0 \]
  which is positive as long as $\Phi$ is positive.

- $\Phi$ represents the welfare cost of distorted margins induced by taxation.

- Optimal policy in first period $\Rightarrow$ war chest. Taxation of capital in first period is non-distorsionary.

- Relation with time inconsistency problem.

- Woodford’s timeless perspective.
Capital Taxation III: A Stronger Result

• Now, assume that \( u(c, l) = \frac{c^{1-\gamma}}{1-\gamma} + v(l) \).

• Then \( W_c(t) = c_t^{-\gamma} + \Phi(-\gamma c_t^{-\gamma}) = (1 - \gamma \Phi) c_t^{-\gamma} \) and:
  \[
  W_c(t) = \beta W_c(t+1) (F_k(t+1) + 1 - \delta) \Rightarrow \\
  (1 - \gamma \Phi) c_t^{-\gamma} = \beta (1 - \gamma \Phi) c_{t+1}^{-\gamma} (F_k(t+1) + 1 - \delta)
  \]

  which implies:
  \[
  \left(\frac{c_t}{c_{t+1}}\right)^{-\gamma} = \beta \left( F_k(t+1) + 1 - \delta \right)
  \]

• In the decentralize equilibrium:
  \[
  \left(\frac{c_t}{c_{t+1}}\right)^{-\gamma} = \beta \left( 1 + \left( 1 - \tau_{t+1}^k \right) (r_{ss} - \delta) \right)
  \]

• Hence, for \( t \geq 2 \Rightarrow \tau_{t+1}^k = 0 \).
Capital Taxation IV: Extensions

- Judd (1985).
Stochastic Economy

• We follow same notation than in the basic RBC model.

• Preferences:

\[
\max_{s} \sum_{t=0}^{\infty} \sum_{s} \beta^{t} \mu(s_{t}) u(c_{t}(s_{t}), l_{t}(s_{t}))
\]

such that:

\[
c_{t}(s_{t}) + k_{t+1}(s_{t}) + b_{t}(s_{t}) = \\
(1 - \tau_{l}(s_{t})) w_{t}(s_{t}) l_{t}(s_{t}) \\
+ \left[ 1 + (1 - \tau_{k}(s_{t})) (r_{t}(s_{t}) - \delta) \right] k_{t}(s_{t-1}) + R_{t}^{b}(s_{t}) b_{t}(s_{t-1})
\]

\[
k_{-1} \text{ given}
\]
Technology

- Production function

\[ F\left(k_t\left(s^{t-1}\right), l_t\left(s^{t}\right), s^t\right) \]

- Competitive pricing ensures that:

\[ r_t\left(s^{t}\right) = F_k\left(k_t\left(s^{t-1}\right), l_t\left(s^{t}\right), s^t\right) \]
\[ w_t\left(s^{t}\right) = F_l\left(k_t\left(s^{t-1}\right), l_t\left(s^{t}\right), s^t\right) \]

- Law of motion for capital:

\[ k_{t+1}\left(s^{t}\right) = i_t\left(s^{t}\right) + (1 - \delta) k_t\left(s^{t-1}\right) \]
Government

- Budget constraint:

\[
g_t(s^t) = b_t(s^t) - R_t^b(s^t)b_t(s^t) + \tau_t^l(s^t)w_t(s^t)l_t(s^t) + \tau_t^k(s^t)(r_t(s^t) - \delta)k(s^{t-1})
\]

with \(b_{-1}\) given.

- Policy:

\[
\pi = \{\pi_t(s^t)\}_{t=0}^{\infty} = \{\tau_t^l(s^t), \tau_t^k(s^t), R_t^b(s^t)\}_{t=0}^{\infty}
\]

Note: state contingent rule.
Ramsey Equilibrium

- Allocation rule: $x(\pi)$ maps policies into allocations (consumption, labor, capital).

- Price rules: $w(\pi)$ and $r(\pi)$ maps policies into prices.

- A Ramsey equilibrium is an allocation rule $x(\cdot)$, price rules $w(\cdot)$ and $r(\cdot)$ and a policy $\pi$ such that:
  1. $\pi$ maximizes household utility.
  2. households maximize for any $\pi'$.
  3. prices equate marginal productivities.
  4. Government budget constraint is satisfied.
Proposition

The allocation in a Ramsey Equilibrium solve the Ramsey problem:

\[
\max_{t=0}^{\infty} \sum_{s^t} \beta^t \mu \left( s^t \right) u \left( c_t \left( s^t \right), l_t \left( s^t \right) \right)
\]

s.t.

\[R.C.: \quad c_t \left( s^t \right) + g_t \left( s^t \right) + k_{t+1} \left( s^t \right) = F \left( k_t \left( s^{t-1} \right), l_t \left( s^t \right), s^t \right) + (1 - \delta) k_t \left( s^{t-1} \right)\]

and

\[I.C.: \quad \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \mu \left( s^t \right) \left( u_c \left( s^t \right) c_t \left( s^t \right) - u_l \left( s^t \right) l_t \left( s^t \right) \right) = u_c \left( s_0 \right) \left( R^k_0 \left( s_0 \right) k_{-1} + R^b_0 \left( s_0 \right) b_{-1} \right)\]
Social Planner Problem

\[ \max \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \mu(s^t) W(c_t(s^t), l_t(s^t), \Phi) \]

\[ + \theta_t(s^t) \left( F(k_t(s^{t-1}), l_t(s^t), s^t) + (1 - \delta) k_t(s^{t-1}) \right) \]

\[ - c_t(s^t) - g_t(s^t) - k_t(s^t) \]

\[ - \Phi u_c(s_0) \Psi(k_{-1}, b_{-1}, s_0) \]

where

\[ W(c_t(s^t), l_t(s^t), \Phi) = u(c_t(s^t), l_t(s^t)) + \Phi (u_c(s^t)c_t(s^t) - u_l(s^t)l_t(s^t)) \]

and

\[ \Psi(k_{-1}, b_{-1}, s_0) = R^k_0(s_0)k_{-1} + R^b_0(s_0)b_{-1} \]
Ramsey Equilibrium

\[ W_c(s^t) = \sum_{s_{t+1}} \beta^t \mu(s_{t+1}|s^t) W_c(s^{t+1}) (F_k(s^{t+1}) + 1 - \delta), \quad t \geq 1 \]

\[ -\frac{W_l(s^t)}{W_c(s^t)} = F_l(s^{t+1}), \quad t \geq 1 \]

\[ W_c(s_0) - \Phi u_{cc}(s_0) \Psi(k_{-1}, b_{-1}, s_0) = \beta \sum_{s^1} \mu(s^1|s_0) W_c(s^1) (F_k(s^1) + 1 - \delta) \]

\[ W_l(s_0) - \Phi \left\{ u_{cl}(s_0) \Psi(k_{-1}, b_{-1}, s_0) + u_c(s_0) \left( 1 - \tau^k_0(s_0) \right) F_{kl}(s_0) \right\} \]

\[ W_c(s_0) - \Phi u_{cc}(s_0) \Psi(k_{-1}, b_{-1}, s_0) = -F_l(s_0) \]
Decentralizing Ramsey

- We need to move from the allocation derived before to a policy \( \pi = \{\tau^l_t(s^t), \tau^k_t(s^t), R^b_t(s^t)\}_{t=0}^{\infty} \).

- First note that, from the solution of the necessary conditions, we can evaluate:

\[
\tau^l_t(s^t) = 1 - \frac{1}{F_l(s^t) u_c(s^t)}
\]

- What about the \( R^b_t(s^t) \) and \( \tau^k_t(s^t) \)?
We use
\[ u_c(s^t) = \beta \sum_{s_{t+1}|s^t} \mu(s_{t+1}|s^t) u_c(s^{t+1}) R_{t+1}^b (s^{t+1}) \]
\[ u_c(s^t) = \beta \sum_{s_{t+1}|s^t} \mu(s_{t+1}|s^t) u_c(s^{t+1}) R_{t+1}^k (s^{t+1}) \]
\[ R_{t+1}^k (s^{t+1}) = 1 + \left(1 - \tau_{t+1}^k (s^{t+1})\right) \left(F_k (s^{t+1}) - \delta\right) \]
plus the budget constraint of household for each state.

If there are \( N \) states period per period, we have \( N + 2 \) equations (there is one of the previous equations that disappears because of Walras law) in \( 2N \) unknowns \( R_t^b (s^t) \) and \( \tau_t^k (s^t) \) \( \Rightarrow N - 1 \) degrees of indeterminacy.
Origin of Indeterminacy

- Take budget constraint of household, multiply by $\beta^t \mu(s_{t+1}|s^t) u_c(s^t+1)$, sum up over $s_{t+1}$, and use necessary conditions on bonds, capital, and the fact that

$$b_t(s^t) = \sum_{t=\tau+1}^{\infty} \sum_{s^t} \beta^{t-\tau} \mu(s_t|s^\tau) \frac{u_c(s^t) c_t(s^t) - u_l(s^t) l_t(s^t)}{u_c(s^\tau)} - k_{\tau+1}(s^\tau)$$

to get an expression that does not depend on $R^b_t(s^t)$ and $\tau^k_t(s^t)$.

- Hence, we can rearrange policy in different equivalent ways.
Indeterminacy of Capital Taxes

If $R^b_t(s^t)$ and $\tau^k_t(s^t)$ satisfy the necessary conditions of the households, then so do $\hat{R}^b_t(s^t)$ and $\hat{\tau}^k_t(s^t)$ such that

$$
\beta \sum_{s_{t+1}|s^t} \mu(s_{t+1}|s^t) u_c(s^{t+1}) R^b_{t+1}(s^{t+1}) = \\
\beta \sum_{s_{t+1}|s^t} \mu(s_{t+1}|s^t) u_c(s^{t+1}) \hat{R}^b_{t+1}(s^{t+1}) \tag{1}
$$

$$
\beta \sum_{s_{t+1}|s^t} \mu(s_{t+1}|s^t) u_c(s^{t+1}) \tau^k_{t+1}(s^{t+1}) (F_k(s^{t+1}) - \delta) = \\
\beta \sum_{s_{t+1}|s^t} \mu(s_{t+1}|s^t) u_c(s^{t+1}) \hat{\tau}^k_{t+1}(s^{t+1}) (F_k(s^{t+1}) - \delta) \tag{2}
$$

$$
\tau^k_{t+1}(s^{t+1}) (F_k(s^{t+1}) - \delta) k_{t+1}(s^t) - R^b_{t+1}(s^{t+1}) b_t(s^t) = \\
\hat{\tau}^k_{t+1}(s^{t+1}) (F_k(s^{t+1}) - \delta) k_{t+1}(s^t) - \hat{R}^b_{t+1}(s^{t+1}) b_t(s^t) \tag{3}
$$
• Proof: for the first two conditions, equate marginal utilities in necessary conditions of the households. The last one is just an arbitrage condition.

• Two alternatives:

  1. Uncontingent debt.

  2. Uncontingent capital tax.

• However, we cannot have simultaneously 1. and 2. and implement a Ramsey equilibrium.
Ex-Ante Capital Tax

- Note that even if state-by-state capital taxes are not pinned down, the payments across states are determined.

Define

\[ Q(s_{t+1}|s^t) = \beta^t \mu(s_{t+1}|s^t) \frac{u_c(s_{t+1})}{u_c(s^t)} \]

- Then, we can find the ex-ante capital income tax rate:

\[ \tau_{t+1}^{e_k}(s^t) = \frac{\sum_{s_{t+1}|s^t} Q(s_{t+1}|s^t) \tau_{t+1}^k(s^{t+1}) \left( F_k(s^{t+1}) - \delta \right)}{\sum_{s_{t+1}|s^t} Q(s_{t+1}|s^t) \left( F_k(s^{t+1}) - \delta \right)} \]
• Result by Zhu (1992):

\[ P^\infty (\tau_{t+1}^{ek} (s^t) = 0) = 1 \Leftrightarrow P^\infty \left( \frac{W_c (s^t)}{u_c (s^t)} = \text{const.} \right) = 1 \]

• Note that for \( u (c, l) = \frac{c^{1-\gamma}}{1-\gamma} + v (l) \), we have

\[ \frac{W_c (s^t)}{u_c (s^t)} = \frac{(1 + \Phi (1 - \gamma)) c_t (s^t)^{-\gamma}}{c_t (s^t)^{-\gamma}} = \text{const.} \]

• For other functions, \( \tau_{t+1}^{ek} (s^t) \simeq 0 \) (Chari, Christiano, Kehoe, 1994).
Numerical Properties

- Three main characteristics:
  
  1. $\tau^l_t(s^t)$ fluctuates very little.
  
  2. $\tau^k_t(s^t)$ fluctuates a lot
  
  3. Public debt works as a shock absorber.

- Origin of welfare gains.

- What if we have balanced budget?