Preliminary

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Time-Consistent Policy
can be simple triggers. They focus on the basic economics dictated by the state variables and

state variables such as capital, debt, income distribution, etc.

We ask how does Markov equilibrium look like in models with natural

Later: use „reputation mechanisms“ special to infinite-horizon economies.

Early literature (Kydland-Prescott (1977)) focused on finding the Markov

over it (e.g., the optimal taxation problem without commitment).

A persuasive problem: A decision maker at time \( t \) cares about the future,

Motivation
Earlier Work on Markov Equilibria

- Controlled accuracy. These methods are of the "black-box" type and they did not deliver papers; more recently, e.g., Klein and Ross-Rull (2001). Problem here: Numerical approach: Russell, Quadri, & Ross-Rull (1997) and related.


- Lots of work on finite-horizon models. Solution procedure: solve it.
... and so on...

First-order dynamics, one needs to solve for second-order dynamics; for a steady state, one needs to solve jointly for dynamics; to solve for a steady state, one needs to solve jointly for dynamics; Reason: to solve for

2. We show how to solve this functional equation: a much harder problem

... Quantitative Interpretations. It does not appear in the existing literature, and it allows quantitative and interpret the incentives facing the decision maker; this equation does allow us to

I. We derive a “generalized Euler equation”—CEE—allowing us to

... We show how to characterize and solve for the Markov equilibrium:

... Contributions
The methods are entirely general and widely applicable: optimal fiscal and monetary policy, dynamic political economy, dynamic oligopoly, partial intergenerational altruism.

- Often Markov has lower taxes & effects are large.
- Lower leisure in Pareto even with capital income taxes.
- Lump sum.
- Governments without commitment do not think of capital taxation as.
- Time consistency is relevant even in static contexts.

Pareto, Ramsey and Markov Equilibrium and find (among other things) optimally provide public goods over time. We compare the predictions of 3. We study a simple, canonical problem in public economics: how to...

Contributions
\[
[B - (I + N)] T = 0
\]
Balanced budget constraint:

\[
B(I - 1) + (I + N)f = C + I + 1B + C
\]
Resource constraint:

\[
[B(I - 1) + b] (I - 1) + b = I + 1B + C
\]
\[\text{s.t.} \]
\[
\begin{bmatrix}
\forall n \\
\forall \ell
\end{bmatrix}
\]
Households maximize

Our Economy: Public Goods Provision And Finance
Policy rule $\bar{\pi}$ is determined by the government's FOC (CEE).

Function $\mathcal{H}$ is determined so as to satisfy the FOCs for the household.

Functions $\pi$ and $\mathcal{H}$ are unknown—they are the key equilibrium objects.

The idea: the government and private agents expect the future value of the state, $Y$. Thus we need to find the two key equilibrium objects: given any current policy choice, $\bar{\pi}$, an endogenous variable $Y$, and any current variables $\xi$, the government compares the effects of

Analysis
\[
(\lor' \mathcal{Y}) \delta - (\lor' \mathcal{Y}) \mathcal{H} - \mathcal{Y}(\varphi - 1) + (1' \mathcal{Y} \mathcal{f}) = (\lor' \mathcal{Y}) \mathcal{O}
\]

\[
[\mathcal{Y} \varphi - (1' \mathcal{Y} \mathcal{f})] \lor = (\lor' \mathcal{Y}) \delta
\]

Satisfying

\[
(\lor' \mathcal{Y}) \delta = \mathcal{O}
\]

\[
(\lor' \mathcal{Y}) \mathcal{O} = \mathcal{O}
\]

There are two other auxiliary functions that are convenient to define:
Future government behavior influences how consumers save. Very important to note: $\Phi$ is a determinant of $H$. The expectations of $H$ is obtained by using $H$ into this equation for all $\left(\perp, Y\right)$.

$$\{[\phi - (I, (\perp, Y)\mathcal{H})_{\mathcal{T}}]((\perp, Y)\mathcal{H})\Phi - I] + I\}$$
$$\cdot \{[(\perp, Y)\mathcal{H})\Phi (\perp, Y)\mathcal{H}] \delta[(\perp, Y)\mathcal{H})\Phi (\perp, Y)\mathcal{H}] \mathcal{C}\} \omega_n \mathcal{G}$$
$$= [(\perp, Y)\delta (\perp, Y)\mathcal{C}] \omega_n$$

$$\cdot [(\phi - (I, (I + \mathcal{T}, Y)\mathcal{H})_{\mathcal{T}})(I + \mathcal{T} - I) + I] (I + \mathcal{T})_{\mathcal{C}} \omega_n \mathcal{G} = (I + \mathcal{T})_{\mathcal{C}} \omega_n$$

Function $\mathcal{H}$ satisfies the functional-against version of the FOC for savings. At:

The private sector's first-order conditions.
This is a recursive problem:

\[ \left( \bot, X \right) H \left. \right| \alpha \; \not\vdash \; \left. \left( \bot, X \right) \delta \left( \bot, X \right) C \right| \alpha \bar{\varepsilon} \max = (X) \alpha \]

Hence

\{ \left. \left( \bot, Y \right) H \right| \alpha \; \not\vdash \; \left. \left( \bot, Y \right) \delta \left( \bot, X \right) C \right| \alpha \bar{\varepsilon} \max = \left( \alpha, Y \right) \Phi \}

Equilibrium requirement

\[ \left. \left( \alpha, Y \right) H \right| \alpha \; \not\vdash \; \left. \left( \alpha, Y \right) \delta \left( \alpha, Y \right) C \right| \alpha \bar{\varepsilon} \max = (Y) \alpha \]

where

\[ \left. \left( \bot, Y \right) H \right| \alpha \; \not\vdash \; \left. \left( \bot, Y \right) \delta \left( \bot, Y \right) C \right| \alpha \bar{\varepsilon} \max \]

Govern's problem

The government's problem
A sequential formulation: derivation of the CEE
\( (\Psi \Phi) \) solves the functional FOC of the government.

\( (\Psi \Phi) \) solves the functional FOC of the private sector; and

functions \( \Phi \) and \( C \) (and \( g \)) such that

A time-consistent policy equilibrium is a set of smooth

Equilibrium: It holds for all \( \Psi \), and is a functional equation in

\[ 0 = \left\{ \left( \psi_0 \delta_i \delta_i^\top n + \chi_0 \delta_i \delta_i^\top \right) \frac{\psi_0}{\psi_0} + \chi_0 \delta_i^\top \delta_i^\top n + [\chi_0 \delta_i \delta_i - \chi_0 \delta_i \delta_i - \chi_0 \delta_i \delta_i - 1 + \chi_0 \delta_i] \right\} \psi_0 \delta_i 

+ \psi_0 \delta_i \delta_i^\top n + [\psi_0 \delta_i - \psi_0 \delta_i - \psi_0 \delta_i - \psi_0 \delta_i] \psi_0 \delta_i \delta_i 

Differentiation yields

The CEE
\[ 0 = \left[ n - \frac{6}{5} n \right] + \delta \left( \frac{\delta H}{\delta H} - 1 \right) \delta H + \{(q - \frac{4}{5} f + 1)^2 n \delta + n - \delta H + [n - \frac{6}{5} n] \delta \}

The "public economics" version of the CEE. Trade off wedges:

The "public economics" version of the CEE. Trade off wedges:

\[ 0 = \left\{ \left( \frac{\delta H}{\delta H} + \frac{\delta - \delta H}{\delta H} \right) \frac{\delta H}{\delta H} + \frac{\delta - \delta H}{\delta H} \right\} + [n - \frac{6}{5} n] \delta \]

\[ + \delta \delta n + [\delta - \delta H - \delta H - 1 + \frac{4}{5} f] \delta n \]

Interpretations
For all of these, smoothness is key: it allows successive differentiations of a virtue. 

\[ \text{Hope } y^\ast_u, y^\ast_0 \text{ converges (so far it has).} \]

- \[ (y^\ast_1)_u b^\ast_1 \text{ and } (y^\ast_0)_u b^\ast_0 = y^\ast_0 \text{ solve for ST-ST.} \]

- Assume going \( y^\ast_u \text{ converges for ST-ST.} \)

- \[ (y^\ast_1)_u b^\ast_1 = y^\ast_0 \text{ solve for ST-ST.} \]

\[ \text{Find steady state. Use Perturbation methods.} \]

\[ \text{For all } y: \text{ Only one functional equation in one unknown function } b(y). \]

\[ \{ (\rho/1-1)[(y)b]_u b - [(y)b]_u f \} \{ [(y)b]_u b - [(y)b]_u f \} \rho \in \mathbb{R} = [(y)b - (y)f]_u \]

To see the issues, let's look at simplest GEE: quasi-geometric discounting

**Computation**
Table 1: Parameterization of the Baseline Model Economy

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>$\phi_1$</td>
<td>0.8</td>
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<tr>
<td>$\phi'$</td>
<td>0.96</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.36</td>
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</tbody>
</table>

Parameter Values

\[ \theta_1^T Y \cdot V = (T'X)^f \]

Meanwhile, the production function is

\[ \delta \ln d + \gamma \ln (\omega - 1)(d - \omega) + c \ln \omega (d - \omega) = (\delta, \gamma, \omega) \]

We specify the period utility function as

Quantitative analysis: Baseline example
No intertemporal distortion, but static distortion. Ramsey has the right ratio between $C$ and $G$. Not Markov who ignores the effect of $T_1$ on $T_{1-1}$.

<table>
<thead>
<tr>
<th></th>
<th>Labor taxes only</th>
<th>Statistical Markov</th>
<th>Ramsey Pareto</th>
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<tr>
<td></td>
<td>y</td>
<td>y</td>
<td>y</td>
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<tr>
<td>1.000</td>
<td>0.700</td>
<td>0.719</td>
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<tr>
<td>2.959</td>
<td>2.959</td>
<td>2.005</td>
<td>0.245</td>
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<tr>
<td>3.017</td>
<td>2.005</td>
<td>0.350</td>
<td>0.397</td>
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<tr>
<td>0.252</td>
<td>0.245</td>
<td>-</td>
<td>T</td>
</tr>
<tr>
<td>0.297</td>
<td>0.397</td>
<td>-</td>
<td>T</td>
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</table>
A crucial tax. Small expenditures in Ramsey, and in Markov despite being non-distortions. to increase $Y$.

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<tbody>
<tr>
<td>0.812</td>
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<td>1.749</td>
<td>1.734</td>
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<tr>
<td>0.488</td>
<td>0.588</td>
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<tbody>
<tr>
<td>$T$</td>
<td>$T$</td>
<td>$c/c$</td>
<td>$k/y$</td>
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<tr>
<td>$v$</td>
<td>$v$</td>
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</thead>
<tbody>
<tr>
<td>Paret</td>
<td>Ramsey</td>
<td>Markov</td>
<td>Stats</td>
</tr>
<tr>
<td>Capital taxes only</td>
<td></td>
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</tbody>
</table>
Yesterday work effort, and it does want to increase tomorrow’s Y. Markov taxes less. Does not take into account the effect of the tax on

<table>
<thead>
<tr>
<th></th>
<th>0.256</th>
<th>0.350</th>
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<th>2.559</th>
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<tbody>
<tr>
<td>Y</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>0.255</td>
<td>0.256</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>C/G</td>
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<td></td>
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<tr>
<td>K/Y</td>
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<td>2.527</td>
<td>2.649</td>
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<tr>
<td>V</td>
<td></td>
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<td></td>
<td>0.669</td>
<td>0.693</td>
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<table>
<thead>
<tr>
<th>Markov</th>
<th>Ramsey</th>
<th>Pareto</th>
<th>Statistic</th>
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</thead>
<tbody>
<tr>
<td>Total</td>
<td>Income taxes</td>
<td></td>
<td></td>
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</tbody>
</table>
The class of problems for which these methods are relevant seems vast. (d) Often Markov has lower taxes & effects are large. (c) Lower leisure in Paretto even with capital income taxes. (b) Markov governments do not think of capital taxation as lump sum. (a) Time consistency is relevant even in static contexts. 3 We find interesting properties for the optimal provision of public goods. 2 We show how to solve this functional equation, a much harder problem than that of solving a standard Euler equation. 1 We derived a CEE to interpret the decision of time inconsistent agents.

Conclusions