Supply-Side Policies and the Zero Lower Bound[*]

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Abstract

This paper examines how supply-side policies may play a role in fighting a low aggregate demand that traps an economy at the zero lower bound (ZLB) of nominal interest rates. Reductions in mark-ups or future increases in productivity triggered by supply-side policies generate a wealth effect that pulls current consumption and output up. Since the economy is at the ZLB, increases in interest rates do not undo this wealth effect, as is the case outside the ZLB. We illustrate this mechanism with a two-period New Keynesian model. We discuss possible objections to this set of policies and the relation of supply-side policies with more conventional monetary and fiscal policies.

Keywords: Zero lower bound, supply-side policies, New Keynesian models.

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1. Introduction

This paper shows how supply-side policies can play a role in economies trapped at the zero lower bound (ZLB) of nominal interest rates. The argument is straightforward: any policy that raises future output -for instance, by liberalizing the goods market and reducing mark-ups or by removing regulations that lower productivity- generates a wealth effect that increases consumption and decreases desired savings. Thus, supply-side measures address the core of the problem of the ZLB, the weakness of current aggregate demand. Supply-side policies are helpful precisely because there is a shortfall in aggregate demand.

This point provides formal support for proposals of structural reforms in countries, such as those in the Euro zone, that have suffered from the dire consequences of debt crises and the ZLB.[1] Far from being a call for "more of the same," supply-side policies, such as reforming labor market institutions, liberalizing service sectors to strengthen competition, or improving vocational education, can be part of a coherent strategy to fight stagnation.

Our point is different from the traditional "grow-out-of-debt" argument that, as a country grows, its debt burden becomes proportionally smaller. While that argument is trivially true as an accounting proposition, its formulation usually fails at specifying how to get that growth going. Our paper illuminates, in comparison, which mechanism works to deliver the desired result.

The possibility of using supply-side policies to cure the maladies of the ZLB is not a reason for inaction along other fronts. Fiscal and monetary policy can be used in a coordinated fashion. For instance, fiscal policy can be directed toward expenditures, such as investments on infrastructure or R&D, that, beyond pulling aggregate demand today, raise future productivity. Our position is, more modestly, that supply-side policies should not be forgotten and that, in many economies, they may be one of the most powerful tools left around.

Think, for instance, about the cases of countries such as Portugal or Spain that are members of the Euro zone. Without their own currency, these countries cannot rely on monetary policy. Similarly, policies such as exchange rate depreciation or tariffs, which may

[1] Technically, Euro zone countries are not at the ZLB, since the nominal interest rates are slightly positive. However, the ECB will not let the short-term nominal interest rate fall further. The rigidity of the nominal interest rate is all we need to deliver the results here. In fact, having a slightly positive nominal interest rate when the natural interest rate should be negative makes the situation even worse.

push aggregate demand up, are out of the question while the currency union is maintained. At the same time, fiscal policy is limited by a growing level of sovereign debt and the cost of servicing it (see the evidence in Ilzetzki, Mendoza, and Végh, 2010, that fiscal multipliers in high-debt countries are zero). With monetary and fiscal policy off the table, supply-side policies are among the last men standing.

Fortunately, these countries also have a sufficient number of "low-hanging fruits" in terms of supply-side reforms. Anyone familiar with the deep inadequacies of the Spanish labor market or with the regulations in many sectors of its economy cannot but forecast considerable gains out of structural reforms. One interesting aspect of our argument is that it does not depend on a permanent change in the growth trend of the economy, something that after 25 years of endogenous growth theory is still a policy chimera, but only on the possibility of raising the level of output. As long as we generate a wealth effect that is significant, supply-side policies will play a positive role. Thus, we are much more sanguine about the role of supply-side policies in Euro zone countries than in the U.S. or the United Kingdom where, arguably, there are less productivity gains to be picked up.

We illustrate the previous paragraphs with a two-period New Keynesian model. Prices are fixed in the first period but can be changed, at a cost, in the second period. This nominal rigidity makes output partially demand-determined. The representative household consumes, supplies labor, holds money, and saves. When the (gross) nominal interest rate is above 1, the household holds money to diminish transaction costs and saves in an uncontingent nominal bond. When the nominal interest rate is 1, the household is indifferent between holding money or bonds. Because of a nominal rigidity, prices cannot adjust fast enough and the real interest rate is too high to induce sufficient consumption in the first period.

Then, if we increase productivity in the second period or, alternatively, we lower the market power of firms, future output and consumption will rise. Because of the Euler equation of consumption, higher future consumption is followed by either higher interest rates and/or higher consumption today. Since, at the ZLB, the nominal rates are stuck at zero, this wealth effect of higher future output causes higher consumption and hours worked today.

Part of our reasoning is close to that of Krugman (1998), who used a drop in future productivity caused by population aging as the cause of the ZLB. In our paper, we revert

the direction of the change in future output and think about it as a policy option.[2] Our alternative channel of increased competition is, as far as we are aware, original to us.

A possible motive for why this point is not discussed more often is that increments in productivity in the current period make the problem of the ZLB worse. Higher productivity today means that the current weak demand is satisfied with even less hours worked. That is why we focus on reductions in mark-ups or future productivity gains, both of which do not suffer from this problem. Most policies that increase productivity have long implementation lags. In practice, when we talk about supply-side policies, we are talking about future productivity increases (and stronger competition in the goods market has positive effects in the short run even if it was implemented in the first period).

Our argument of a wealth effect that, when the interest rate does not respond, increases current consumption and hours worked resembles the mechanism in the "news" literature (Jaimovich and Rebelo, 2009). Instead of using a class of preferences that control for the wealth effect, as in Jaimovich and Rebelo, we rely on the absence of changes in the interest rate. More generally, there is a common thread that any positive wealth effect, regardless of its origin, helps demand today. For instance, fiscal policies that decrease future government spending achieve the same objective of raising future private consumption.[3]

Finally, we highlight that the model we present is of interest in itself. It allows us to easily find an exact solution and to characterize it. Also, it embodies all the classical results about the ZLB highlighted in the literature. Our quest for simplicity puts us close to Mankiw and Weinzierl (2011). Our emphasis and goals are, however, different. We incorporate an explicit labor supply decision, monopolistic competition, a role for money through transaction costs, and partial price rigidity in the second period. These features are relevant for the economics of the mechanism we explore. For example, our specification of the transaction costs makes it transparent when the demand for money is satiated and the different forces that affect

[2] Rogoff (1998), in his discussion of Krugman's paper, makes en passant the same point that future productivity gains are a solution to the ZLB problem, but without linking it to a policy strategy.

[3] There is also a connection with the literature on uncertainty shocks (see Bloom, 2009 or Fernández-Villaverde *et al.,* 2011). Reductions in future uncertainty increase the desire to consume today because they lower precautionary saving. In the absence of a response of the interest rate caused by the ZLB, those effects would be much bigger than in the standard case.

it.[4] Monopolistic competition is required to talk about variations in the market power of firms. On the other hand, we have a simpler set of policy tools, since our objective is not to assess fiscal or monetary policy (although it would be easy to incorporate additional policy instruments). Our investigation is also related to Eggertsson and Krugman (2012), who study the role of deleveraging in pushing the economy into a ZLB.

The rest of the paper is organized as follows. In section 2 we present our model and in section 3 we discuss its equilibrium. Section 4 outlines a parametric specification that leads us, in section 5, to some numerical results. In section 6 we discuss objections to our argument and we conclude in section 7. The appendix contains extra algebra.

2. Model

We fix a monetary environment with two periods, $t \in \{1, 2\}$. Three mechanisms motivate the presence of money. First, money reduces, up to some level, the transaction costs required to reach a given consumption. This structure, introduced by Sims (1994), generates a demand for money when interest rates are positive and when they are at the ZLB. Second, money is a store of value between periods. If the ZLB binds, the household is indifferent between using nominal bonds or money as a saving vehicle and the household can hold more money in equilibrium than it would otherwise need to minimize its transaction costs. Third, money in period 2 appears in the utility function. This captures the idea that money is valuable in the long run. Furthermore, it sets up a terminal condition to induce the household to hold money at the end of period 2.[5] Nominal rigidities appear in two forms. First, prices in the first period are predetermined (for instance, because firms set their prices in the past). Second, firms have to pay a cost to change their prices in the second period. Nominal rigidities make output partially demand-determined and give the ZLB a real bite.

To keep the model analytically tractable, we abstract from three important aspects. First,

[4] Also, it introduces a new channel, unexplored in the literature, where changes in the transaction technology have an effect on when the economy is at the ZLB.

[5] This extra utility term makes the problem more symmetric between the two periods. When the household holds money in the first period, it gets the reduction in transaction cost and an asset that pays off in the second period. In the absence of money-in-the-utility, in the second period, money would yield only a reduction in transaction cost. The asymmetry between the two periods would induce a large movement in the price level that would hide the channel we are interested in.

we do not have uncertainty. Second, we generate a ZLB through a discount factor bigger than 1. We could imagine that the discount factor is a random variable and that firms determine their prices before observing the realization of the factor.[6] Third, fiscal policy is trivial.

2.1. Household

There is a representative household with preferences:

$$\frac{c_1^{1-\sigma}}{1-\sigma} - \frac{l_1^{\psi}}{\psi} + \beta\left(\frac{c_2^{1-\sigma}}{1-\sigma} - \frac{l_2^{\psi}}{\psi} + \gamma \log \frac{m_2}{p_2}\right)$$

where c_t is consumption at time t, l_t is labor supply, and $\frac{m_2}{p_2}$ are real balances (nominal money m_2 divided by the price level p_2). We assume $\sigma > 1$, that is an elasticity of intertemporal substitution lower than 1 (as most of the empirical estimates), but, since we have finite periods, we do not assume that $\beta < 1$. We do not bound labor supply by 1 either. This can be accomplished by the right choice of units.

The budget constraints for the household are:

$$\left(1 + s\left(v_1\right)\right) p_1 c_1 + m_1 + \frac{b}{R} = p_1 w_1 l_1 + p_1 F_1 + p_1 T$$

and

$$\left(1 + s\left(v_2\right)\right) p_2 c_2 + m_2 = p_2 w_2 l_2 + p_2 F_2 + m_1 + b$$

where b is an uncontingent nominal bond, R is the gross nominal interest rate, w_t is the wage in period t, F_t denotes profits from firms, and T is transfers from the government.

The function $s\left(\cdot\right)$ parameterizes the transaction costs (in resource terms) of consuming c_t when the real balances of money used for transactions in the period are $\frac{m_t}{p_t}$ as a function of velocity $v_t = \frac{p_t c_t}{m_t}$. The transaction cost function is nonnegative, twice continuously differentiable, and there exists a velocity $\overline{v} > 0$, which we refer to as the satiation level, such that $s(\overline{v}) = s'(\overline{v}) = 0$, and $s'\left(\cdot\right) \geq 0$.

[6] The first draft of our model had that precise feature. We disregarded it as an unnecessary complication that did not add much economic insight.

The FOCs of the household's problem are:

$$
\begin{aligned}
c_1^{\sigma} l_1^{\psi-1} &= \frac{w_1}{1+s(v_1)+s'(v_1)v_1} \\
c_2^{\sigma} l_2^{\psi-1} &= \frac{w_2}{1+s(v_2)+s'(v_2)v_2} \\
\frac{R-1}{R} &= s(v_1)' v_1^2 \\
\gamma = c_2^{1-\sigma} & \frac{\frac{1}{v_2} - s(v_2)' v_2}{1+s(v_2)+s'(v_2)v_2}
\end{aligned}
$$

and

$$
\frac{1}{c_1^{\sigma}} = \beta \frac{1}{c_2^{\sigma}} R \frac{p_1}{p_2} \frac{1+s(v_1)+s'(v_1)v_1}{1+s(v_2)+s'(v_2)v_2}
$$

five conditions that, together with the two budget constraints, determine the seven choices of the household. The first two conditions are the static optimality conditions that equate the ratio of marginal utilities of leisure and consumption with their relative prices (wages over the marginal cost of consumption once we consider transaction costs). The third equation tells us that the household holds cash until its marginal return (in terms of reduction of transaction costs) is equal to its opportunity cost given by R. The fourth equation is the demand for money in the second period. The final equation is the Euler equation for bond holdings.

At the ZLB, we have $0 = s(v_1)' v_1^2$, which indicates that the opportunity cost of holding money has been reduced to zero. Since $\frac{p_1 c_1}{m_1} \neq 0$, $s(v_1)' = 0$, that is, the household is satiated in its need for money in period 1 for transaction costs. Conversely, outside the ZLB, $s(v_1)' > 0$.

2.2. The Final Good Producer

There is one final good produced using intermediate goods with the production function:

$$
y_t = \left(\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{1}
$$

where ε is the elasticity of substitution.

The final good producer is perfectly competitive and maximizes profits subject to the production function (1), taking as given intermediate goods prices p_{ti} and the final good

price p_t. Thus, the input demand functions are:

$$y_{it} = \left(\frac{p_{it}}{p_t}\right)^{-\varepsilon} y_t \qquad \forall i,$$

and the price level $p_t = \left(\int_0^1 p_{it}^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}$.

2.3. Intermediate Goods Producers

Each intermediate firm produces differentiated goods out of labor with a technology $y_{it} = A_t l_{it}$, where l_{it} is the labor input rented by the firm and A_t is productivity. Therefore, the real marginal cost of all intermediate goods producers is $mc_t = \frac{w_t}{A_t}$.

The monopolistic firms face nominal rigidities. Prices in period 1, p_1, are fixed. At time 2, they reoptimize their prices to p_{i2}, but they pay an adjustment cost per unit of goods sold:

$$AC_i = \frac{\phi}{2}\left[\frac{p_{i2}}{p_{i1}} - 1\right]^2.$$

This Rotemberg setup introduces rigidities in the second period without keeping track of distributions (as would happen with Calvo pricing) or to solve a menu cost problem.

Hence, prices p_{i2} are chosen to maximize

$$\max_{p_{i2}} \left(\frac{p_{i2}}{p_2} - mc_2 - \frac{\phi}{2}\left[\frac{p_{i2}}{p_{i1}} - 1\right]^2\right) y_{i2}$$
$$\text{s.t. } y_{i2} = \left(\frac{p_{i2}}{p_2}\right)^{-\varepsilon} y_2.$$

The solution of that problem leads to an aggregate pricing condition:

$$1 - \varepsilon + \varepsilon mc_2 - \phi\frac{p_2}{p_1}\left[\frac{p_2}{p_1} - 1\right] + \varepsilon\frac{\phi}{2}\left[\frac{p_2}{p_1} - 1\right]^2 = 0.$$

2.4. Government

The government policy is extremely simple. It issues m units of currency in the first period, which it rebates back to the representative household as transfers. Then, the budget constraint of the government is $m = T$ and, by clearing in the money market, $m = m_2 = m_1$.

2.5. Aggregation

Standard algebra and symmetry in the firm's behavior gives us $(1+s(v_1))\, c_1 = A_1 l_1$ and

$$(1+s(v_2))\, c_2 = \left(1-\frac{\phi}{2}\left[\frac{p_2}{p_1}-1\right]^2\right) A_2 l_2$$

Using the consumption-labor optimality condition of the household, we get:

$$mc_t = \frac{w_t}{A_t} = \frac{c_t^{\sigma} l_t^{\psi-1}\left(1+s(v_t)+s'(v_t)\, v_t\right)}{A_t}$$

3. Equilibrium

Given a feasible policy sequence $\{m, T\}$ and an initial p_1, an equilibrium is an allocation and prices c_1, l_1, v_1, R, c_2, l_2, p_2, and v_2 that solve:

$$1-\varepsilon+\varepsilon\frac{c_2^{\sigma} l_2^{\psi-1}\left(1+s(v_2)+s'(v_2)\, v_2\right)}{A_2} - \phi\frac{p_2}{p_1}\left[\frac{p_2}{p_1}-1\right] + \varepsilon\frac{\phi}{2}\left[\frac{p_2}{p_1}-1\right]^2 = 0 \tag{2}$$

$$(1+s(v_1))\, c_1 = A_1 l_1 \tag{3}$$

$$(1+s(v_2))\, c_2 = \left(1-\frac{\phi}{2}\left[\frac{p_2}{p_1}-1\right]^2\right) A_2 l_2 \tag{4}$$

$$\frac{1}{c_1^{\sigma}} = \beta\frac{1}{c_2^{\sigma}} R \frac{p_1}{p_2}\frac{1+s(v_1)+s'(v_1)\, v_1}{1+s(v_2)+s'(v_2)\, v_2} \tag{5}$$

$$s(v_1)' v_1^2 = \begin{cases} \frac{R-1}{R} \text{ if } R>1 \\ 0 \text{ if } R=1 \end{cases} \tag{6}$$

and

$$\gamma = c_2^{1-\sigma}\frac{\frac{1}{v_2}-s(v_2)' v_2}{1+s(v_2)+s'(v_2)\, v_2} \tag{7}$$

plus the definition $v_t = \frac{p_t c_t}{m}$, $t \in \{1,2\}$.

The previous equilibrium conditions display a convenient recursive structure. Given p_1, we use (2), (4), and (7) to find p_2, c_2, and l_2. This recursive structure is derived from the fact that, beyond p_1 and m, we do not have any state variable in the model. Thus, prices and quantities in period 2 are not a function of any variable from period 1. In particular, they are independent of whether the ZLB binds or not.

If prices are flexible, $\phi = 0$, and there are no transaction costs in the second period, we have $l_2 = \frac{c_2}{A_2}$, $p_2 = \frac{m}{\gamma c_2^\sigma}$, and

$$c_2 = A_2^{\frac{\psi}{\sigma+\psi-1}} \left(\frac{\varepsilon - 1}{\varepsilon}\right)^{\frac{1}{\sigma+\psi-1}}.$$

While in the calibrated model below we will assume that $\phi > 0$ and that $s(v_2) > 0$, both frictions will be small and the expressions above will nearly hold. These three equations embody several messages:

1. In $t = 2$, the economy presents a classical dichotomy: quantities are determined by preferences and technology, and the price level by money supply and consumption.

2. p_2 is proportional to m conditional on c_2.

3. A higher A_2 raises consumption and lowers hours worked and velocity: $\frac{p_2 c_2}{m} = \frac{1}{\gamma c_2^{\sigma-1}}$.

With the variables in the second period, we solve (3), (5), and:

$$\frac{R-1}{R} = s(v_1)' v_1^2$$

for c_1, l_1, and R. If $R > 1$, we are done. Otherwise, we fix $R = 1$ and find:

$$c_1 = \left(\frac{1}{\beta}\frac{p_2}{p_1}\left(1 + s(v_2) + s'(v_2)\, v_2\right)\right)^{\frac{1}{\sigma}} c_2 \tag{8}$$

and $l_1 = \frac{c_1}{A_1}$.

Given p_1, equation (8) reveals that we are at the ZLB when $p_2 c_2^\sigma$ is too small. In this situation, the household satisfies intertemporal optimality by reducing c_1, which translates into less labor. When prices are fully flexible, any problem caused by a low $p_2 c_2^\sigma$ is undone by reductions in p_1. Thus, our model illustrates how the ZLB is a problem because prices today are not fully flexible. Furthermore, we have the paradox of thrift: a higher β lowers c_1 and, with it, output. Equation (8) shows the mechanism: as a response to $p_2 c_2^\sigma$ that is too small and with a fixed p_1, we can either lower c_1 or raise p_2 or c_2. We argue that increases in c_2 are a possibility that has often been overlooked.

4. A Parametric Specification

To make further progress, we specify the transaction cost function in terms of the velocity v_t:

$$s(v_t) = \begin{cases} 0 \text{ if } 0 < v_t < \sqrt{\frac{\alpha_1}{\alpha_0}} \\ \alpha_0 v_t + \alpha_1 \frac{1}{v_t} - 2\sqrt{\alpha_0 \alpha_1} \text{ if } v_t \geq \sqrt{\frac{\alpha_1}{\alpha_0}} \end{cases}$$

This function has two parts. For velocities sufficiently small, it is zero, as the demand for money has been satiated. But when we reach a threshold ($\sqrt{\frac{\alpha_1}{\alpha_0}}$), the transaction cost grows in a convex fashion.

The interpretation is simple. A low velocity means that there is a large quantity of money in relation to the nominal price of consumption. Hence, the transaction cost is zero and cannot be reduced further (we could translate the whole function by a constant α_3 if we want to keep a positive minimum transaction cost). After the threshold, there is relatively little money and the household must use resources in executing transactions. Convexity is a natural assumption. The functional form is the same as in Schmitt-Grohé and Uribe (2011), except for the flat part before the satiation point.

With this transaction function we get the demand for money:

$$\frac{m}{p_1} = \frac{c_1}{\sqrt{\frac{\alpha_1}{\alpha_0} + \frac{1}{\alpha_0} \frac{R-1}{R}}}$$

that shows that money holdings increase with consumption and decrease with the nominal interest rate. It is also the case that if $R > 1$, $v_1 > \sqrt{\frac{\alpha_1}{\alpha_0}}$.

At the ZLB, any holding of money that satisfies:

$$\frac{m}{p_1} \geq \frac{c_1}{\sqrt{\frac{\alpha_1}{\alpha_0}}}$$

is compatible with an equilibrium because, at the margin, the household is holding money just as a store of value. We assume that the actual holdings of money are determined by clearing in the money market and the previous equation holds with equality. At the ZLB, $s(v_1) = s(v_1)' = 0$, that is $v_1 = \sqrt{\frac{\alpha_1}{\alpha_0}}$.

We can come back to equation (8) and rewrite it as:

$$c_1 = \left(\frac{1}{\beta}\frac{1}{p_1}\frac{m}{\gamma}\left(1 - \alpha_0 (v_2)^2 + \alpha_1\right)\right)^{\frac{1}{\sigma}}.$$

This expression tells us that anything that reduces money velocity in period 2, such as a higher c_2, lifts consumption in period 1.

5. Some Numerical Results

We offer now some numerical results that illustrate the forces at work. This is not a formal calibration (we are not aiming to match any moment of the data), but a quantitative exercise to better understand the model.

We proceed as follows. First, we select parameter values. Second, we will introduce three variations of the benchmark economy. These variations will be helpful in interpreting our results. Third, we will present a case where technology and market power are constant over time, which will tell us how the economy behaves in the absence of policy changes. Fourth, we will implement different exercises to show how increases in future consumption raise consumption today. We will close by revisiting some classical results.

We start, then, by setting up a calibrated utility function:

$$-c_1^{-1} - \frac{l_1^2}{2} + 1.2\left(-c_2^{-1} - \frac{l_2^2}{2} + \log\frac{m_2}{p_2}\right)$$

that is, we fix $\sigma = 2$, $\psi = 2$, $\beta = 1.2$, and $\gamma = 1$. The values of σ and ψ are standard in the business cycle literature, γ is a normalization of the units of currency, and β is a large number to induce the ZLB to bind.

The transaction cost function of money is:

$$s(v_t) = \begin{cases} 0 \text{ if } 0 < v_t < \sqrt{0.75} \\ 0.4v_t + 0.3\frac{1}{v_t} - 2\sqrt{0.12} \text{ if } v_t \geq \sqrt{0.75} \end{cases}$$

where we pick the parameters to generate a small transaction cost (for instance, in the second period of our example, of less than 0.25 percent of output; in the first period the ZLB binds

and the costs are zero). The parameter controlling the elasticity of substitution among goods is $\varepsilon = 10$ (a conventional value[7]), and the price adjustment cost $\phi = 1$ (which implies an adjustment cost of 0.44 percent of the second period output). Finally, we set $m = 1.1$ (this generates a p_2 slightly bigger than 1) and $p_1 = 1$, around 7.6 percent higher than in a flexible prices equilibrium.[8] With these parameter choices, we find a unique equilibrium in our numerical exercises.

To facilitate the interpretation of the results, we compare our benchmark model with three alternative versions (that we derive in the appendix) that are nested within it. First, we eliminate money ($m = 0$), monopolistic competition ($\varepsilon = \infty$), and price rigidities ($\phi = 0$). This is a neoclassical environment with an analytical solution that helps us to think in the right order of magnitude for each variable. This version is also Pareto efficient, so it can be read as the solution to the social planner's problem. We call this version model I. Second, we reintroduce monopolistic competition ($\varepsilon = 10$), but without money ($m = 0$) or rigidities ($\phi = 0$). The presence of a mark-up gets us closer to the results of the model presented in the previous section. We call this version model II. Third, we reintroduce money ($m = 1.1$) and the ZLB, but prices are still flexible ($\phi = 0$). We call this version model III. For reference, we call the model presented in the previous section model IV.

Our first step is to compute case I, where $A_1 = A_2 = 1$ and $\varepsilon_1 = \varepsilon_2 = 10$. The results are in table 1, where $1 + r$ is the real interest rate (defined as the rate of return of a real bond). The second column shows the results for model I. There we see the convenience of our parameterization: consumption and labor are equal to 1 in both periods and the real interest rate is the inverse of the discount factor ($0.833 = 1/1.2$). In the absence of money, price levels and the nominal interest rate are not defined. In the third row, we move to model II. Market power works as a consumption tax that decreases consumption and labor in both periods by 3.5 percent. The real interest rate is unchanged.

[7]When analyzing cases where the elasticity of substitution changes across periods, we will call ε_1 the elasticity of period 1 and ε_2 the elasticiy of period 2.

[8]By setting p_1 "too high," we ensure that output is below what it would be given preferences, technology, and flexible prices. In the old disequilibrium models of the 1970s, there was an alternative case when p_1 was too low, often called the "repressed inflation case." This case will resurface later in this section.

Table 1: Case I, A_1= $A_2 = 1$ and $\varepsilon_1 = \varepsilon_2 = 10$				
	Model I	Model II	Model III	Model IV
c_1	1	0.965	0.965	0.931
l_1	1	0.965	0.965	0.931
p_1	-	-	0.929	1
c_2	1	0.965	0.946	0.947
l_2	1	0.965	0.948	0.953
p_2	-	-	1.095	1.094
$1 + r$	0.833	0.833	0.848	0.914
R	-	-	1	1

The fourth row is model III, where we introduce money but prices are fully flexible. With our parameters, the ZLB is binding ($R = 1$). However, the ZLB is irrelevant because p_1 and p_2 adjust to deliver the "right" real interest rate. Since the transaction costs are zero in the first period, the allocation in that period is the same as in model II. In the second period, the transaction costs are not zero, and they induce a reduction in consumption (by 2.0 percent) and labor (by 1.8 percent). While the transaction costs paid (0.2 percent) are small, they create a wedge that lowers consumption. The price levels, 0.929 and 1.095, adjust the real interest rate to (nearly) the value of the case without money, 0.848. The slight difference comes from the transaction costs on the Euler equation.

Finally, in the fifth row, we have the complete model (model IV). Consumption in period 1 is 3 percent lower than in model III. With p_1 fixed, the real interest rate goes down only to 0.914 and households want to save more. But the only way in which the savings market can clear is by a reduction in consumption in period 1, which is achieved by a fall in demand that, given the nominal rigidity, lowers production. Consumption and labor in the second period are higher than in model III because the positive price adjustment cost ϕ makes p_2 rise less than in model III. A lower price level induces more consumption and labor. Labor rises to pay for the adjustment cost. As a final remark, note that even if we are at the ZLB, the economy still experiences inflation, just not enough to lower the real interest rate sufficiently.

Table 2: Case II, $A_1 = 1$, $A_2 = 1.05$ and $\varepsilon_1 = \varepsilon_2 = 10$

	Model I	Model II	Model III	Model IV
c_1	1	0.965	0.965	0.936
l_1	1	0.965	0.965	0.936
p_1	-	-	0.938	1
c_2	1.033	0.997	0.981	0.982
l_2	0.984	0.950	0.936	0.944
p_2	-	-	1.041	1.039
$1+r$	0.889	0.889	0.901	0.962
R	-	-	1	1

We move now to case II where supply-side policies have increased A_2 by 5 percent, to 1.05. In model I, labor is a decreasing function of technology. Hence, as A_2 goes up, l_2 falls. The contrary is true for c_2. In comparison, in the first period, the allocation is the same since we are not more productive at time 1. The real interest rate increases to 0.889 to induce the household to save enough to clear the asset market. The results for model II are similar. Model III does not change much with respect to case I. The economy is at the ZLB, but allocations in the first period are not affected when compared with model II. Prices behave differently: p_1 is somewhat higher and p_2 is slightly lower than in case I. A higher productivity brings down the marginal cost and, with it, the optimal price of monopolistic producers in period 2. Since inflation adjusts the real interest rate to (nearly) the value it takes in model II, p_1 also changes.

Finally, model IV shows us the main mechanism in this paper: the impact of higher future productivity on consumption in period 1. Consumption increases in the second period to 0.982 while labor falls to 0.944 with respect to case I. Similarly, p_2 is now only 1.039. Next, the most important implication: a higher c_2 brings about a higher c_1. The effect is not large because of the higher r induced by a lower p_2. However, this is a simple numerical example to illustrate our argument. We would need a fully-fleshed-out business cycle model to evaluate how big of an increase in productivity we would require to get a sizable impact.

Table 3: Case III, A_1= 1, $A_2 = 1.30$ and $\varepsilon_1 = \varepsilon_2 = 10$

	Model I	Model II	Model III	Model IV
c_1	1	0.965	0.965	0.954
l_1	1	0.965	0.965	0.954
p_1	-	-	0.982	1
c_2	1.191	1.150	1.150	1.133
l_2	0.916	0.885	0.885	0.882
p_2	-	-	0.830	0.848
$1+r$	1.182	1.182	1.183	1.179
R	-	-	1	1.001

In case III (table 3), we set A_2 to 1.30 to show that, with a sufficiently large increase in future productivity, we leave the ZLB (although barely so, in our numerical example). Although this may seem like a large number, again, we are just dealing with a numerical example (and there are cases where structural reforms, as in Spain in 1959, yielded increases in productivity much higher than 30 percent, see Prados de la Escosura, Rosé, and Sanz Villarroya, 2011).

Table 4: Case IV, A_1= 1, $A_2 = 1$ and ε_1= 10, $\varepsilon_2 = 100$

	Model I	Model II	Model III	Model IV
c_1	1	0.965	0.965	0.936
l_1	1	0.965	0.965	0.936
p_1	-	-	0.938	1
c_2	1	0.997	0.981	0.980
l_2	1	0.997	0.982	0.982
p_2	-	-	1.041	1.043
$1+r$	0.833	0.888	0.901	0.959
R	-	-	1	1

In table 4, we report case IV where, instead of affecting productivity, supply-side policies strengthen competition in the economy and reduce the mark-ups (for example, with an

aggressive enforcement of antitrust law). We follow the influential work of Blanchard and Giavazzi (2003) when they explored the potentialities of goods market de-regulation in Europe by modelling such policy as changing ε_1 (the parameter controlling market power) from 10 to $\varepsilon_2 = 100$. This reduced-form approach is justified because, for our purpose, we do not need to be explicit about why firms enjoy market power. Since there was no monopolistic competition to begin with, model I is unaffected with respect to case I. In models II and III, the second period allocation gets much closer to the first best. Finally, in model IV, since we are at the ZLB and p_1 is fixed, as c_2 increases, the Euler equation implies that c_1 increases by 0.5 percent. The expansionary impact of increased competition works even if ε_1 increases to 100 as well (we omit the corresponding table for concision). The result is interesting because in the case of improvements in productivity, increases in A_1 may actually reduce l_1. This "paradox of productivity" does not apply to reductions in the market power of firms.[9]

We close this section with two additional experiments that revisit two classical topics. First, we divide adjustment costs by 10: ϕ is now only 0.1. The results in table 5 confirm the old argument by DeLong and Summers (1986), recently revisited by Werning (2011), that increasing price flexibility (but short of reaching full price flexibility) may not help. Due to higher price flexibility, consumption and labor are lower in the first period. The concrete channel in our model, though, is different: with higher price flexibility, prices rise too fast, not fall too fast as in DeLong and Summers. With $\phi = 0.1$, p_2 responds more to demand conditions in period 2. A higher p_2 lowers c_2 and with it, c_1 (although in our calibration the effect is minimal as we already start with a low ϕ: labor in period 1 goes from 0.93089 to 0.93079, because of rounding, the drop does not appear in table 5). Or, in other words, more flexible prices in the second period lower demand and with it output, generating less consumption and output in the first period as well. Welfare implications are more nuanced because bigger price flexibility brings the allocation closer to first best and wastes less resources in adjusting prices. In our example, welfare goes up when $\phi = 0.1$.

[9] As we will see in our next experiment, reductions in market power may also be a better policy tool than reductions in price stickiness (for example, by changing commercial and labor law to allow for more frequent contract or collective bargaining agreement renegotiations), which can often deliver negative results.

Table 5: $\phi = 0.1$

	Model I	Model II	Model III	Model IV
c_1	1	0.965	0.965	0.931
l_1	1	0.965	0.965	0.931
p_1	-	-	0.929	1
c_2	1	0.965	0.946	0.946
l_2	1	0.965	0.948	0.949
p_2	-	-	1.095	1.095
$1+r$	0.833	0.833	0.848	0.913
R	-	-	1	1

Second, we increase m by 9 percent from 1.1 to 1.2 (see table 6). In our model, this is a permanent increase in the monetary base. Prices in model III rise around 9 percent in both periods, but, since we have price flexibility, the allocations remain unchanged. More interesting is the response of model IV. Now p_1 is below the value it would have under price flexibility. Even if we stay at the ZLB, c_1 goes up to 0.971. This experiment demonstrates that the ZLB is damaging when p_1 is too high and how permanent increases in money ease the problems involved with the bound (Auberbach and Obstfeld, 2005).

Table 6: $m = 1.2$

	Model I	Model II	Model III	Model IV
c_1	1	0.965	0.965	0.971
l_1	1	0.965	0.965	0.971
p_1	-	-	1.014	1
c_2	1	0.965	0.946	0.940
l_2	1	0.965	0.948	0.963
p_2	-	-	1.195	1.206
$1+r$	0.833	0.833	0.848	0.829
R	-	-	1	1

We could perform other experiments to show classical results in the ZLB literature such

as that an increase in A_1 lowers l_1 or that an increase in β lowers c_1. After our previous explanations, though, we skip the details.

6. Possible Objections

There are four main objections to our argument: two we do not think are important and two that we think are. The first objection is to ask why we want to embark on supply-side reforms, whose outcomes are uncertain and perhaps small, when we have at hand monetary and fiscal policies. The ZLB comes about because future nominal output is too low. Monetary policy can fix that problem by increasing p_2, either through a commitment to temporarily higher inflation (Krugman, 1998, Eggertsson and Woodford, 2003) or through lump-sum transfers of cash (Auberbach and Obstfeld, 2005). Similarly, as shown by Correia *et al.* (2010), fiscal policy can neutralize the ZLB and achieve first best by using taxes to replicate the optimal path for the prices.[10]

Unfortunately, monetary and fiscal policies may not be at hand. Monetary policy can be offline, either because of political pressures (for instance, a central bank reluctant to engage in expansionary monetary policy) or institutional arrangements (a monetary union). Fiscal policy often has few degrees of freedom (partisan divisions within a polity, high debt-to-output ratios, constitutional limits, etc.). Therefore, supply-side policies become a second line of defense (or, in the case where monetary and fiscal policy still work, a complementary one). Furthermore, supply-side policies may alleviate the negative consequences of monetary or fiscal policies designed to fight the ZLB today. For example, they may generate higher future tax revenues to pay down the debt incurred by expansionary fiscal policy.

The second objection is to ask why we emphasize the importance of increases in future output when we are at the ZLB. Should not a government want to increase future output regardless of whether we are at the ZLB? Yes, it should if these increases are free. However, these increases are usually costly, either in economic terms (we need to build a new bridge or learn a new technology, more competition may reduce incentives to R&D) or politically (the reforms decrease the rents of some groups). But when we are at the ZLB, these structural

[10] See also Christiano, Eichenbaum, and Rebelo (2011) and Woodford (2011) for investigations of the effects of fiscal policy at the ZLB.

reforms have a higher than normal rate of return. Not only do we obtain more consumption tomorrow, we also fight the demand problems today. Outside the ZLB, increases in future productivity are undone, in terms of consumption today, by an increase in the interest rate that ensures market clearing in the current period. At the ZLB, that effect disappears and current consumption rises. Thus, reforms that are too expensive either economically or politically in normal times can become desirable at the ZLB.

The third objection (and the first important one) is that increases in current productivity may lead to the economic situation to deteriorate. As we saw before, we have an Euler equation that pins down c_1 as a function of future variables and l_1 is whatever quantity is required to produce c_1. Thus, a higher productivity today lowers employment. We do not find this objection compelling. First, most increases in productivity today are permanent (or at least they have high persistence) and the contractionary impact today has to be compensated against the wealth effect on c_1.[11] Second, most structural reforms, such as reorganizing labor markets, take some time before having an impact. Thus, any policy action today is unlikely to have much effect on current productivity. Finally, as we saw in our experiment 4, stronger competition in the economy does not suffer from this problem.

The last objection is whether increases in future productivity are a feasible policy instrument. After two decades of research on endogenous growth, we do not hold a magic wand to miraculously beget higher output. It may well be the case that increases in future productivity are not part of the feasible set of actions for a government or that the increases that a government can induce are too small to make a difference. There are two counter-arguments to this objection. First, we do not seek an increase in the growth trend of an economy. This is probably beyond the reach of policy. A wealth effect works even if we just generate a one-shot increase in productivity, a more realistic goal. Second, the economies of countries such as Spain have so many areas of inefficiency (the labor market being the paradigmatic case) that increases in productivity after some reforms are much more likely than in the U.S. or the United Kingdom. Similarly, making markets more competitive in some European economies, which have many service sectors shielded from market forces, is quite possible.

[11] Also, in our model, even if labor goes down, welfare is increasing. In a more realistic environment, for example, one with heterogeneous agents and liquidity constraints, welfare changes can go in either direction depending on the details of the economy.

7. Conclusions

In this paper we have argued that supply-side policies can play a role in fighting situations where an economy is at a ZLB. While we do not want to overemphasize the power of these policies, we should not forget about them either. Our results suggest the need for middle-size business cycle models in the style of Christiano, Eichenbaum, and Evans (2005) modified to incorporate an explicit ZLB to measure how big the potential impact from these policies is and how they complement more traditional monetary and fiscal policies. We leave that investigation for future research.

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8. Appendix (Not for publication)

8.1. Algebraic Derivations for Model I: No Money, Perfect Competition

The FOCs of the household are:

$$c_t^{\sigma} l_t^{\psi-1} = w_t$$
$$\frac{1}{c_1^{\sigma}} = \beta \frac{R}{c_2^{\sigma}}$$

where R is now a real interest rate.

The problem of the firms is just $w_t = A_t$ and market clearing $c_t = A_t l_t$. Then:

$$(A_t l_t)^{\sigma} l_t^{\psi-1} = A_t \Rightarrow l_t = A_t^{\frac{1-\sigma}{\sigma+\psi-1}}$$

and

$$R = \frac{1}{\beta}\left(\frac{c_2}{c_1}\right)^{\sigma} = \frac{1}{\beta}\left(\frac{A_2}{A_1}\right)^{\frac{\psi\sigma}{\sigma+\psi-1}}$$

8.2. Algebraic Derivations for Model II: No Money, Monopolistic Competition

The FOCs of the household are still the same as before, but now the optimality condition for the firm is $mc_1 = mc_2 = \frac{\varepsilon-1}{\varepsilon}$. Since $mc_t = \frac{w_t}{A_t}$, we get:

$$w_t = \frac{\varepsilon-1}{\varepsilon} A_t$$

By market clearing, $c_t = A_t l_t$. Then:

$$(A_t l_t)^{\sigma} l_t^{\psi-1} = \frac{\varepsilon-1}{\varepsilon} A_t \Rightarrow l_t = \left(\frac{\varepsilon-1}{\varepsilon}\right)^{\frac{1}{\sigma+\psi-1}} A_t^{\frac{1-\sigma}{\sigma+\psi-1}}$$

and

$$R = \frac{1}{\beta}\left(\frac{c_2}{c_1}\right)^{\sigma} = \frac{1}{\beta}\left(\frac{A_2}{A_1}\right)^{\frac{\psi\sigma}{\sigma+\psi-1}}$$

8.3. Algebraic Derivations for Model III: Flex Prices

Now we introduce money and transaction costs, but prices are fully flexible. The FOCs of the household (see next subsection for the Lagrangian of the household):

$$
\begin{aligned}
c_1^{\sigma} l_1^{\psi-1} &= \frac{w_1}{1+s(v_1)+s'(v_1)v_1} \\
c_2^{\sigma} l_2^{\psi-1} &= \frac{w_2}{1+s(v_2)+s'(v_2)v_2} \\
\frac{R-1}{R} &= s(v_1)' v_1^2 \\
\gamma &= c_2^{1-\sigma} \frac{\frac{1}{v_2} - s(v_2)' v_2}{1+s(v_2)+s'(v_2)v_2} \\
\frac{1}{p_1 c_1^{\sigma}} &= \beta \frac{R}{p_2 c_2^{\sigma}} \frac{1+s(v_1)+s'(v_1)v_1}{1+s(v_2)+s'(v_2)v_2}
\end{aligned}
$$

The problem of the producers is still $mc_1 = mc_2 = \frac{\varepsilon-1}{\varepsilon}$ and we get:

$$
mc_t = \frac{w_t}{A_t} = \frac{c_t^{\sigma} l_t^{\psi-1}\left(1+s(v_t)+s'(v_t)v_t\right)}{A_t}
$$

Therefore:

$$
\begin{aligned}
\frac{c_1^{\sigma} l_1^{\psi-1}\left(1+s(v_1)+s'(v_1)v_1\right)}{A_1} &= \frac{\varepsilon-1}{\varepsilon} \\
(1+s(v_1))\, c_1 &= A_1 l_1 \\
\frac{c_2^{\sigma} l_2^{\psi-1}\left(1+s(v_2)+s'(v_2)v_2\right)}{A_2} &= \frac{\varepsilon-1}{\varepsilon} \\
(1+s(v_2))\, c_2 &= A_2 l_2 \\
\frac{1}{c_1^{\sigma}} &= \beta \frac{1}{c_2^{\sigma}} R \frac{p_1}{p_2} \frac{1+s(v_1)+s'(v_1)v_1}{1+s(v_2)+s'(v_2)v_2} \\
s(v_1)' v_1^2 &= \begin{cases} \frac{R-1}{R} \text{ if } R>1 \\ 0 \text{ if } R=1 \end{cases}, \; R=1 \text{ otherwise} \\
\gamma &= c_2^{1-\sigma} \frac{\frac{1}{v_2} - s(v_2)' v_2}{1+s(v_2)+s'(v_2)v_2}.
\end{aligned}
$$

Note that we have a recursive structure, with a second period block

$$\frac{c_2^{\sigma} l_2^{\psi-1}\left(1+s\left(v_2\right)+s'\left(v_2\right)v_2\right)}{A_2}=\frac{\varepsilon-1}{\varepsilon}$$

$$\left(1+s\left(v_2\right)\right)c_2=A_2 l_2$$

$$\gamma=c_2^{1-\sigma}\frac{\frac{1}{v_2}-s\left(v_2\right)'v_2}{1+s\left(v_2\right)+s'\left(v_2\right)v_2}$$

and a first period one:

$$\frac{c_1^{\sigma} l_1^{\psi-1}\left(1+s\left(v_1\right)+s'\left(v_1\right)v_1\right)}{A_1}=\frac{\varepsilon-1}{\varepsilon}$$

$$\left(1+s\left(v_1\right)\right)c_1=A_1 l_1$$

$$\frac{1}{c_1^{\sigma}}=\beta\frac{1}{c_2^{\sigma}}R\frac{p_1}{p_2}\frac{1+s\left(v_1\right)+s'\left(v_1\right)v_1}{1+s\left(v_2\right)+s'\left(v_2\right)v_2}$$

$$s\left(v_1\right)'v_1^2=\begin{cases}\frac{R-1}{R}\text{ if }R>1\\ 0\text{ if }R=1\end{cases},\ R=1\text{ otherwise}$$

8.4. Algebraic Derivations for Model IV: Nominal Rigidities

The Lagrangian associated with the problem of the household is:

$$\begin{aligned}&\frac{c_1^{1-\sigma}}{1-\sigma}-\frac{l_1^{\psi}}{\psi}+\beta\left(\frac{c_2^{1-\sigma}}{1-\sigma}-\frac{l_2^{\psi}}{\psi}+\gamma\log\frac{m_2}{p_2}\right)\\&+\lambda_1\left(w_1 l_1+F_1+T-\left(1+s\left(v_1\right)\right)c_1-\frac{m_1}{p_1}-\frac{1}{p_1}\frac{b}{R}\right)\\&+\beta\lambda_2\left(w_2 l_2+F_2+\frac{m_1}{p_2}+\frac{b}{p_2}-\left(1+s\left(v_2\right)\right)c_2-\frac{m_2}{p_2}\right)\end{aligned}$$

The FOCs are:

$$
\begin{aligned}
c_1 &: \; c_1^{-\sigma} = \lambda_1 \left(1 + s\left(v_1\right) + s'\left(v_1\right) v_1\right) \\
l_1 &: \; l_1^{\psi - 1} = \lambda_1 w_1 \\
c_2 &: \; c_2^{-\sigma} = \lambda_2 \left(1 + s\left(v_2\right) + s'\left(v_2\right) v_2\right) \\
l_2 &: \; l_2^{\psi - 1} = \lambda_2 w_2 \\
m_1 &: \; \lambda_1 \frac{1}{p_1}\left(1 - s\left(v_1\right)' v_1^2\right) = \beta \lambda_2 \frac{1}{p_2} \\
m_2 &: \; \frac{\gamma}{m_2} = \lambda_2 \frac{1}{p_2}\left(1 - s\left(v_2\right)' v_2^2\right)
\end{aligned}
$$

and

$$
R : \; \lambda_1 \frac{1}{p_1} = \beta \lambda_2 \frac{R}{p_2}
$$

We can combine the first four conditions:

$$
\begin{aligned}
c_1^{\sigma} l_1^{\psi - 1} &= \frac{w_1}{1 + s\left(v_1\right) + s'\left(v_1\right) v_1} \\
c_2^{\sigma} l_2^{\psi - 1} &= \frac{w_2}{1 + s\left(v_2\right) + s'\left(v_2\right) v_2}
\end{aligned}
$$

Then, with the last fifth and the seventh:

$$
\begin{aligned}
\left(1 - s\left(v_1\right)' v_1^2\right) = \beta \frac{\lambda_2}{\lambda_1} \frac{p_1}{p_2} &= \frac{1}{R} \Rightarrow \\
\frac{R - 1}{R} &= s\left(v_1\right)' v_1^2
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\gamma}{m_2} &= \lambda_2 \frac{1}{p_2}\left(1 - s\left(v_2\right)' v_2^2\right) \Rightarrow \\
\gamma &= \frac{c_2^{-\sigma}}{1 + s\left(v_2\right) + s'\left(v_2\right) v_2} \frac{m_2}{p_2}\left(1 - s\left(v_2\right)' v_2^2\right) \Rightarrow \\
\gamma &= c_2^{1-\sigma} \frac{\frac{1}{v_2} - s\left(v_2\right)' v_2}{1 + s\left(v_2\right) + s'\left(v_2\right) v_2}
\end{aligned}
$$

Finally:

$$\frac{1}{c_1^\sigma} = \beta \frac{1}{c_2^\sigma} R \frac{p_1}{p_2} \frac{1 + s(v_1) + s'(v_1) v_1}{1 + s(v_2) + s'(v_2) v_2}$$

In the case where the ZLB binds, we have the same conditions except that now $0 = s(v_1)' v_1^2$.

Now, we exploit the recursive structure of the previous equations and solve for the second period choices:

$$1 - \varepsilon + \varepsilon \frac{c_2^\sigma l_2^{\psi-1} (1 + s(v_2) + s'(v_2) v_2)}{A_2} - \phi \frac{p_2}{p_1} \left[\frac{p_2}{p_1} - 1 \right] + \varepsilon \frac{\phi}{2} \left[\frac{p_2}{p_1} - 1 \right]^2 = 0$$

$$(1 + s(v_2)) c_2 = \left(1 - \frac{\phi}{2} \left[\frac{p_2}{p_1} - 1 \right]^2 \right) A_2 l_2$$

$$\gamma = c_2^{1-\sigma} \frac{\frac{1}{v_2} - s(v_2)' v_2}{1 + s(v_2) + s'(v_2) v_2}$$

Note

$$l_2 = \left(\frac{1 + s(v_2)}{1 - \frac{\phi}{2} \left[\frac{p_2}{p_1} - 1 \right]^2} \right) \frac{c_2}{A_2}$$

$$1 + s(v_2) + s'(v_2) v_2 = \frac{c_2^{1-\sigma}}{\gamma} \left(\frac{1}{v_2} - s(v_2)' v_2 \right)$$

Then:

$$s(v_2)' v_2^2 \frac{m}{p_2} = \frac{m}{p_2} \left[\alpha_0 v_2^2 - \alpha_1 \right]$$

and:

$$\frac{m}{p_2} - s(v_2)' v_2 c_2 = \frac{m}{p_2} \left(1 - \alpha_0 v_2^2 + \alpha_1 \right) = \frac{c_2}{v_2} \left(1 - \alpha_0 v_2^2 + \alpha_1 \right)$$

Also:

$$1 + s(v_2) + s'(v_2) v_2 =$$

$$1 + \alpha_0 v_t + \alpha_1 \frac{1}{v_t} - 2\sqrt{\alpha_0 \alpha_1} + \alpha_0 v_t - \alpha_1 \frac{1}{v_t} =$$

$$1 + 2\alpha_0 v_t - 2\sqrt{\alpha_0 \alpha_1}$$

and then

$$1+2\alpha_0 v_t-2\sqrt{\alpha_0\alpha_1}=\frac{c_2^{1-\sigma}}{\gamma}\frac{1}{v_t}\left(1-\alpha_0 v_t^2+\alpha_1\right)\Rightarrow c_2=\left(\gamma\frac{v_t+2\alpha_0 v_t^2-2\sqrt{\alpha_0\alpha_1}v_t}{1-\alpha_0 v_t^2+\alpha_1}\right)^{\frac{1}{1-\sigma}}$$

Finally, at the ZLB

$$\gamma=c_2^{1-\sigma}\frac{\frac{1}{v_2}-s\left(v_2\right)' v_2}{1+s\left(v_2\right)+s'\left(v_2\right)v_2}\Rightarrow 1+s\left(v_2\right)+s'\left(v_2\right)v_2=\frac{c_2^{1-\sigma}}{\gamma}\left(\frac{1}{v_2}-s\left(v_2\right)' v_2\right)$$

and then

$$\begin{aligned}
c_1 &= \left(\frac{1}{\beta}\frac{p_2}{p_1}\left(1+s\left(v_2\right)+s'\left(v_2\right)v_2\right)\right)^{\frac{1}{\sigma}} c_2 \\
&= \left(\frac{1}{\beta}\frac{p_2}{p_1}\frac{c_2^{1-\sigma}}{\gamma}\left(\frac{1}{v_2}-s\left(v_2\right)' v_2\right)\right)^{\frac{1}{\sigma}} c_2 \\
&= \left(\frac{1}{\beta}\frac{1}{p_1}\frac{m}{\gamma}\left(1-s\left(v_2\right)' v_2^2\right)\right)^{\frac{1}{\sigma}} \\
&= \left(\frac{1}{\beta}\frac{1}{p_1}\frac{m}{\gamma}\left(1-\alpha_0 v_2^2+\alpha_1\right)\right)^{\frac{1}{\sigma}}
\end{aligned}$$

We move now to the problem of the firms. Note that an equivalent problem for the intermediate good producer is:

$$\max_{p_{2t}}\left(\frac{p_{i2}}{p_2}\right)^{1-\varepsilon}-\left(\frac{p_{i2}}{p_2}\right)^{-\varepsilon}mc_2-\frac{\phi}{2}\left[\frac{p_{i2}}{p_{i1}}-1\right]^2\left(\frac{p_{i2}}{p_2}\right)^{-\varepsilon}$$

with FOC:

$$(1-\varepsilon)\left(\frac{p_{i2}}{p_2}\right)^{1-\varepsilon}\frac{1}{p_{i2}}+\varepsilon\left(\frac{p_{i2}}{p_2}\right)^{-\varepsilon}\frac{1}{p_{i2}}mc_2-\phi\frac{1}{p_{i1}}\left[\frac{p_{i2}}{p_{i1}}-1\right]\left(\frac{p_{i2}}{p_2}\right)^{-\varepsilon}+\varepsilon\frac{\phi}{2}\left[\frac{p_{i2}}{p_{i1}}-1\right]^2\left(\frac{p_{i2}}{p_2}\right)^{-\varepsilon}\frac{1}{p_{i2}}=0$$

Then, we can apply the symmetry of all firms ($p_{i2}=p_2$) to get:

$$1-\varepsilon+\varepsilon mc_2-\phi\frac{p_2}{p_1}\left[\frac{p_2}{p_1}-1\right]+\varepsilon\frac{\phi}{2}\left[\frac{p_2}{p_1}-1\right]^2=0.$$