

Evaluating Default Policy: The Business Cycle Matters*

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Abstract

Legislation dealing with consumer default has consistently struggled with an important trade-off: more debt forgiveness directly benefits households but indirectly makes credit more expensive. Complicating the issue is that part of the risk households face is aggregate risk. This paper asks, “How does aggregate risk affect the consequences of eliminating or restricting default?” I find aggregate risk substantially reduces the welfare benefit of eliminating default, but its effect on restricting default depends crucially on the restrictions in place. In a calibrated general equilibrium life-cycle model, eliminating default results in an ex-ante welfare gain of 1.8% of lifetime consumption in steady state. Once the business cycle—the type of aggregate risk considered in this paper—is added, this gain drops to .5%. With or without aggregate risk, eliminating default greatly expands credit availability; however, when a protracted recession is possible, households use less credit unless they have a default option. While aggregate risk reduces the welfare gain of eliminating default this is not necessarily true for restricting default. A policy that pushes earnings-rich households into partial debt repayment (like a major 2005 reform) generates a gain of 2% with or without the business cycle. Moreover, with the policy instituted, eliminating default produces a welfare *loss* of .1%, which aggregate risk deepens to 1.5%. The reform improves credit markets while still preserving most of the insurance value of default. A different type of policy that restricts default to only be in recessions or expansions sharply reduces welfare relative to always allowing default (a loss of 1.4%) or never allowing it (a loss of 1.9%). The policy introduces uncertainty that makes credit expensive *and* keeps households from relying on the default option.

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1 Introduction

In both recent and past history, the merits and flaws of legislation dealing with consumer default have been subject to intense debate.¹ Central to the arguments has been the trade-off between debt relief and credit.² More debt relief directly benefits households but indirectly harms them through reduced access to credit: to cover losses due to default, creditors must charge a premium. Complicating the issue is that the circumstances of unfortunate debtors are often caused by events outside their control. Panics, financial crises, stock market crashes, and housing busts can leave otherwise prosperous, stable households in financial ruin.

The interaction between aggregate risk and consumer default is particularly relevant in light of recent US history. Preceded by twenty years of stable economic growth, the Bankruptcy Abuse Prevention and Consumer Protection Act of 2005 made debt-forgiveness significantly more difficult.³ Since then the economy has slipped into the most severe recession since the Great Depression. Far from being just coincidence, this sequence of events fits into a pattern where legislation responds to negative aggregate shocks with more debt relief and to positive shocks with less.⁴ Was the 2005 reform beneficial in light of the last recession? Would it have been if the economy had remained stable? How does law changing in response to aggregate shocks impact households? More generally, how does aggregate risk affect the welfare consequences of eliminating or restricting default? This paper seeks to answer these questions.

I find aggregate risk substantially decreases the welfare benefit of *eliminating* default. In a calibrated general equilibrium life-cycle model and absent aggregate risk, eliminating default improves welfare by 1.8% of lifetime consumption for newborn households. Once a business cycle—the type of aggregate risk considered in this paper—is added, this gain drops to .5%. Eliminating default in either the steady state or business cycle environment means that creditors offer any amount of debt at a risk-free rate. For their part, households avoid

¹ Bankruptcy laws changed frequently prior to 1900 and even since then have had major revisions in 1938, 1978, and 2005. Bankruptcy in its present form did not take shape until 1898, and even then it was highly controversial with efforts to repeal it made in 1903, 1909, and 1910 (Warren, 1935, p. 143).

² For instance, one Senator argued for a bill he introduced in 1885 saying, “At present interest rates . . . are from 8 to 20%, because of the doubt whether the creditor will have his fair share of the estate . . .” (quoted in Warren, 1935, p. 133). Another Senator argued against the bill saying, “it is no time to pass bills of this character—when every man in trade . . . is suffering from the depression . . .” (quoted in Warren, 1935, p. 132).

³The key provision of the reform is that households with above-median income may no longer file for Chapter 7 bankruptcy. Because Chapter 7 bankruptcy typically offers much more debt forgiveness than the only other widely-used form of consumer bankruptcy, Chapter 13, the reform severely restricted debt forgiveness for these households. The reform made a number of other significant changes (see White, 2007).

⁴I thank Satyajit Chatterjee for pointing this out to me. A recent example of this pattern, besides the 2005 reform, is the Emergency Economic Stabilization Act of 2008—passed in response to the latest recession—that authorized an ongoing mortgage-modification program (the Home Affordable Modification Program). There are also many older examples where “prosperity muted demands for relief” until a panic, drought, or war “eased . . . opposition to a bankruptcy law” (Coleman, 1974, p. 24, 28). Robe, Steiger, and Michel (2006) argue from a broad historical perspective that default penalties become lighter as risk becomes “more important” (p. 3).

taking out debt beyond what they can repay with probability one. This amount is much smaller in the business cycle because of the possibility, albeit remote, of a life-long recession which would severely depress aggregate capital and wages. With a default option, credit is more expensive and less abundant, but households can safely use all of it by defaulting if a protracted recession hits.

While including aggregate risk reduces the welfare gain of eliminating default, I find this need not be the case when *restricting* default. I consider two cases of default being restricted. First is a policy change designed to mimic the 2005 reform by forcing households with above-median earnings to repay a substantial fraction (but not all) of their debt. This reform increases welfare by around 2% with or without the business cycle. Further, with the reform instituted, households experience a welfare *loss* from eliminating default: .1% absent aggregate uncertainty and 1.5% with it. The reform increases repayment rates of the earnings rich, those with the greatest ability to repay, resulting in a large expansion of credit. At the same time, most of the insurance value of default is preserved: all households have access to some amount of default and earnings-poor households can easily default. This result suggests that “default” is not a problem but rather the amount of default allowed.

The second type of default restriction I consider is meant to capture aggregate risk’s effect on legislation by only allowing default in recessions or expansions. Surprisingly, the outcome of either policy is inferior to always having default (a loss of 1.4%) or never having it (a loss of 1.9%). The reason for this is that uncertainty about whether or not default will be allowed has two negative effects. First, it causes creditors to charge a default premium which households must always pay.⁵ Second, households must be prepared to never have the default option by limiting the amount of debt they take on. Consequently debt is expensive *and* households cannot rely on the default option.

Most research suggests that eliminating default would substantially improve welfare in the absence of uninsured “disaster” states.⁶ The result has proven surprisingly robust. In particular, it has been found in both partial and general equilibrium environments, for finitely-lived and infinitely-lived households, under different informational settings, and under alternative debt-pricing formulations.⁷ In addition, Athreya et al. (2009b) find the result holds for numerous specifications of earnings risk and preferences. Further, Chatterjee and Gordon (2011) consider multiple alternatives to bankruptcy law and find one that completely eliminates default is optimal. To my knowledge, the only exception in the literature is the work by Li and Sarte (2006) that shows accounting for general equilibrium effects can reduce the welfare benefit of eliminating default even to the point of making it a loss.⁸ A central contri-

⁵Consistent with the rest of the literature default premiums are paid upfront. If the premiums were charged *only* after the aggregate state was revealed, this effect would not be present.

⁶Examples include Athreya (2002), Livshits, MacGee, and Tertilt (2007), Athreya (2008), Athreya, Tam, and Young (2009a), Athreya, Tam, and Young (2009b), and Chatterjee and Gordon (2011).

⁷Throughout the paper “partial equilibrium” refers to allowing default-risk premia to adjust while holding fixed default-risk-free prices at the equilibrium values in the default economy. In the business cycle model this requires using the law of motion from the default economy to forecast prices.

⁸One potentially important caveat is that none of these papers have computed the transition. Chatterjee and Gordon (2011) compute the transition for eliminating *bankruptcy*—the focus of their paper—but not for eliminating *default*.

bution of the present paper is to show that aggregate risk is also an important determinant of the welfare consequences of eliminating default.

Research on the welfare gains of restricting access to default is more mixed and has focused on the 2005 reform. Athreya (2002), Li and Sarte (2006), and Nakajima (2008) have found modest changes in welfare and allocations from the reform.⁹ Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007) and Mitman (2011), on the other hand, have found sizable welfare gains. None of these papers have allowed for aggregate risk, and a contribution of the present paper is to take aggregate risk into account in studying how the 2005 reform—passed just before the Great Recession—affected households. I show that the reform likely made them better off. Another contribution of the paper is to examine default policy that restricts or permits default in response to aggregate shocks. I show that such a policy can be very harmful to households.

Little quantitative work has been done on default and business cycles. Nakajima and Ríos-Rull (2004) and Nakajima and Ríos-Rull (2005) examine whether bankruptcy amplifies or smooths aggregate shocks. In the first paper, the authors use a production economy where creditors make profits or losses and distribute them via a dividend. In the second, the authors use a storage economy but impose a special timing on the model to ensure creditors make zero profits loan-by-loan.¹⁰ While these papers examine default’s effect on aggregate dynamics, the present paper examines the effect of aggregate dynamics on default. Consequently, they are complementary. A technical contribution of the paper is to model the economy with aggregate uncertainty in a way that ensures creditors make zero profits loan-by-loan without imposing a special timing. This is done by allowing some households to have adjustable portfolios and offset losses from charge-offs.

The quantitative framework I use is an extension of Chatterjee et al. (2007) and Livshits et al. (2007) to include aggregate uncertainty. Recessions are modeled as a negative shock to total factor productivity, a large increase in earnings variance a la Storesletten, Telmer, and Yaron (2004), and a decline in exogenous labor supply (which is calibrated to match the standard deviation of hours worked). General equilibrium and the life-cycle are both included as the work of Li and Sarte (2006) and Livshits et al. (2007) suggests these are very important for evaluating default policy. Following Athreya et al. (2009b), I abstract from expenditure shocks (large negative shocks to asset positions).¹¹

⁹Athreya (2002) studied the Bankruptcy Reform Act of 1999, the main provisions of which would eventually become part of the Bankruptcy Abuse Prevention and Consumer Protection Act of 2005.

¹⁰Specifically, households must make their default decisions before the aggregate shock is realized.

¹¹ Livshits et al. (2007) have argued that expenditure shocks are quantitatively important for the evaluation of default policy. I abstract from expenditure shocks for three reasons. First, as Athreya et al. (2009b) argue, while catastrophic events (like large expenditure shocks) are sufficient to warrant loose default penalties, an important and practical question is whether they are also necessary. It is a practical question because debt forgiveness has often been made contingent on the hardships faced by households. Second, while households may not purchase insurance for expenditure shocks (which in part represent large medical bills), insurance is often available. Hence assuming these shocks are uninsurable, and that this would remain the case were default to be eliminated, is not entirely reasonable. Third, as Livshits et al. (2007) point out, expenditure shocks of a sufficiently large size necessitate default. That said, expenditure shocks would likely increase the welfare gains of low default-cost regimes relative to high (but not infinite) cost regimes, and I plan to explore

In a series of robustness exercises, I find the result that aggregate risk substantially *reduces* the welfare gain of eliminating default is quantitatively robust. Specifically, it is true for different earnings processes, mortality risk profiles, and economies of scale in household size. It is also true when a substantial portion of labor income is guaranteed. Further, the result holds even if default is not completely eliminated but just made very costly. While the reduction in the welfare gain is robust, the *level* of the welfare gain is not robust and can vary greatly. This is especially true when a substantial portion of earnings is guaranteed. In this case, the welfare gain from eliminating default can be significantly lower after including aggregate risk but still be very high in absolute terms.

The rest of the paper is organized as follows. Section 2 describes the models with and without aggregate uncertainty. Section 3 discusses the calibration and baseline model properties. Section 4 examines the consequences of eliminating default. Section 5 examines the consequences of restricting default. Section 6 concludes. Appendices include extended model and data descriptions, notes on computation, and robustness exercises.

2 Model

I first lay out the model without aggregate uncertainty and then modify it to include aggregate uncertainty. Time is discrete in both models.

2.1 Steady State Model

The steady state model is a completely standard model of consumer default with general equilibrium and the life cycle.

2.1.1 Demographics, Endowments, Technology, and Preferences

The economy is populated by a unit mass of households who die with certainty after T years. Households are endowed with one unit of time but differ in the productive efficiency e of their time endowment and in certain other characteristics s , *including age*. Characteristics lie in a finite set S and evolve according to a Markov chain $F(s'|s)$.¹² Efficiency is distributed iid conditional on characteristics according to a density function $f(e|s)$ which has support in \mathbb{R}^{++} for all s .

Households face an age-dependent conditional probability of survival ρ_s . Households who die are replaced by “newborn” households having zero assets, efficiency distributed according to $\hat{f}(e|s)$, and characteristics distributed according to $\hat{F}(s)$. The utility from death is normalized to zero.

this.

¹²Age evolves deterministically so that, if $t(s)$ denotes the age of a household, $F(s'|s) = 0$ whenever $t(s') \neq t(s) + 1$ (except if $t(s) = T$ in which case the transition is immaterial).

Preferences over consumption are given by

$$\sum_{t=1}^T \beta^{t-1} U(c_t, s_t) \quad (1)$$

where $\beta > 0$ is the discount factor and c is consumption. The period utility function is

$$U(c, s) = (c/\theta_s)^{1-\sigma} / (1 - \sigma), \quad \sigma > 0, \sigma \neq 1. \quad (2)$$

where θ_s is the age-dependent effective number of household members.¹³ When $\sigma = 1$, $U(c, s) = \log(c/\theta_s)$. Households do not value leisure.

A neoclassical production firm operates the production technology $K^\alpha N^{1-\alpha}$ with $\alpha \in (0, 1)$ that uses as inputs capital K rented at rate r and labor N hired at wage w . Capital depreciates at a constant rate $\delta \in (0, 1)$.

2.1.2 Legal Environment

The legal environment is designed to resemble Chapter 7 bankruptcy in US law. Households have a credit history $h \in \{0, 1\}$. Households in good standing, $h = 0$, have the right to file for bankruptcy. If they do, three things happen in the filing period: their debts are discharged in exchange for all their assets, they may not save and may not borrow, and a fraction $\chi \in [0, 1)$ of their income is given to creditors.¹⁴ In the period after filing, a household's credit history records that they filed for bankruptcy in the past, $h = 1$. This record is removed and their history shows $h = 0$ with probability $1 - \lambda$. For as long as a household is in bad standing, $h = 1$, a household is not allowed to borrow but may save.¹⁵ Households begin life with $h = 0$.

2.1.3 Asset Markets

Households do not directly hold claims to capital, but rather enter into debt or savings contracts with a financial intermediary who owns the capital stock and rents it to the production firm. I now describe these contracts.

From a household's perspective, a debt/savings contract looks just like a risk-free discount bond. The face value a' lies in a finite set A that includes zero and both positive and negative elements. I use the convention $a' \geq 0$ denotes savings and $a' < 0$ denotes borrowing. Because

¹³Since σ will be 2 in the calibration, θ_s will shift marginal utilities in the same way as in, for instance, Attanasio and Weber (1995) and Gourinchas and Parker (2002).

¹⁴There are a number of reasons to believe *some* income is transferred to creditors in the period of (but not after) default. First, as discussed in Livshits, MacGee, and Tertilt (2010, p. 174, footnote 12), the bankruptcy code incorporates "good faith" requirements that are not explicitly modeled here. Second, earnings are sometimes garnished before households file for bankruptcy with one estimate in Chatterjee and Gordon (2011) putting the net recovery rate on defaulted revolving debt at around 15%. Third, some households initially file for Chapter 13 (which results in a debt repayment plan) but subsequently file for Chapter 7.

¹⁵Musto (2004) finds credit opportunities are severely restricted while the record of a bankruptcy remains.

of default, each contract has a potentially different yield and so has a distinct price $q(a', s)$ that varies with all factors that can potentially influence next period's default decision. Since e is iid conditional on s , these are entirely summarized in characteristics s .¹⁶ The prices $q(a', s)$ for $a' \in A, s \in S$ define a "price schedule."

From the intermediary's perspective, a debt/savings contract is a repayment agreement that through pooling gives a certain return. In exchange for $q(a', s)a'$ of the consumption good, the intermediary expects a yield $\rho_s p(a', s)a'$ next period comprised of two parts. First, from households surviving to the next period, he expects to recover only $p(a', s) \in [0, 1]$ fraction of the debt because some may default. Second, he expects that only ρ_s fraction of households will survive to even have a chance of repaying.

In addition to contracts, the intermediary (but not households) has access to two other assets, capital and a risk-free discount bond. The bond B' has price \bar{q}_B and capital K' has return $1 + r - \delta$. Capital cannot be short sold and the bond is in zero net supply. Note that no arbitrage necessitates the bond's return be equated to the return on contracts. This pins down the price schedule as

$$q(a', s) = \bar{q}_B \rho_s p(a', s) \quad (3)$$

for $a' \neq 0$. Without loss of generality, $q(0, s)$ is taken to be zero.

2.1.4 The Household Problems

The household problems are as follows.¹⁷ Let $V(a, e, s, h)$ denote the value function of a household. A household in good standing $h = 0$ that can repay their debt solves

$$V(a, e, s, 0) = \max_{d \in \{0, 1\}} (1 - d) \cdot V^R(a, e, s) + d \cdot V^D(e, s) \quad (4)$$

where the value of repaying is

$$\begin{aligned} V^R(a, e, s) &= \max_{c, a'} U(c, s) + \beta \rho_s \mathbb{E}V(a', e', s', 0) \\ c + q(a', s)a' &= we + a \\ c &\geq 0, a' \in A \end{aligned} \quad (5)$$

and the value of defaulting is

$$\begin{aligned} V^D(e, s) &= \max_{c, a'} U(c, s) + \beta \rho_s \mathbb{E}V(0, e', s', 1) \\ c &= we(1 - \chi) \\ c &\geq 0, a' = 0. \end{aligned} \quad (6)$$

A household in good standing that cannot repay their debt must default.

¹⁶The contract can be made contingent on (a, e, s, h) . However since the default decision next period is a function of (a', e', s', h') , only a' and s matter. The credit history doesn't appear in $q(a', s)$ because debt, $a' < 0$, is only consistent with $h = 0$ (and $h' = 0$) and savings will not be defaulted upon regardless of h .

¹⁷When there is no risk of confusion I omit the conditional expectation's information set.

A household in bad standing $h = 1$ solves

$$\begin{aligned} V(a, e, s, 1) &= \max_{c, a'} U(c, s) + \beta \rho_s \lambda \mathbb{E}V(a', e', s', 1) + \beta \rho_s (1 - \lambda) \mathbb{E}V(a', e', s', 0) \\ c + q(a', s) a' &= we + a \\ c, a' &\geq 0, a' \in A. \end{aligned} \tag{7}$$

Let the policy functions be denoted $d(a, e, s, h)$, $a'(a, e, s, h)$, $c(a, e, s, h)$, where a household in bad standing is said to not default, i.e. $d(a, e, s, 1) = 0$.

2.1.5 The Intermediary's Problem

Details of the intermediary's problem are in Appendix A. The intermediary maximizes the net present value of financial income using contracts, capital, and the bond. He is indifferent over all feasible allocations if contracts are priced according to (3) and if capital's return equals the bond's return.

2.1.6 Equilibrium

I now give a simplified definition of equilibrium. An unsimplified definition, along with the characterizations that lead to this definition, are given in Appendix A.

A steady state equilibrium is a collection of prices r, w, q , recovery rates p , policy functions c, a', d , a value function V , a strictly positive capital stock K and labor supply N , and a distribution of households μ such that all of the following hold:

1. The policies and value function solve the household problems.
2. The capital stock and aggregate labor supply are given by the distribution:¹⁸

$$K = \int (a + d(a, e, s, h)(-a - \chi we)) d\mu / (1 + r - \delta) \tag{8}$$

$$N = \int e d\mu. \tag{9}$$

3. Prices satisfy

$$q(a', s) = \bar{q}_B \rho_s p(a', s) \tag{10}$$

$$\bar{q}_B = 1 / (1 + r - \delta) \tag{11}$$

$$r = \alpha (K/N)^{\alpha-1} \tag{12}$$

$$w = (1 - \alpha) (K/N)^\alpha. \tag{13}$$

4. Repayment probabilities are consistent: for all a, s_{-1} ,

$$p(a, s_{-1}) = \sum_s \int (1 - d(a, e, s, 0) + d(a, e, s, 0) \chi we / (-a)) f(e|s) de F(s|s_{-1}). \tag{14}$$

5. The distribution is invariant to household policies and stochastic transitions.

¹⁸A proof that this is the aggregate capital stock is given in Appendix A.

2.2 Business Cycle Model

I now modify the steady state model to include aggregate uncertainty. The model is setup recursively using $\mathcal{S} = (z, \mu)$ as the aggregate state where μ is a distribution of households and z is a productivity shock. The aggregate state evolves according to a law of motion Γ with $\mathcal{S}'_z = \Gamma(z', \mathcal{S})$ denoting next period's aggregate state conditional on a z' realization. Further, I use $\mathcal{G} = \Gamma(g, \mathcal{S})$ and $\mathcal{B} = \Gamma(b, \mathcal{S})$ to denote the states that will arise conditional on a g or b realization.

2.2.1 Demographics, Endowments, Technology, and Preferences

The production technology is now $zK^\alpha N^{1-\alpha}$. The productivity shock z takes on one of two possible values in $Z = \{g, b\}$ with $g > b$ and evolves according to a Markov chain $F(z'|z)$. Capital is rented at rate $r(\mathcal{S})$ and labor is hired at wage $w(\mathcal{S})$. As before, capital depreciates at a constant rate δ .

The household efficiency process is now allowed to vary with the aggregate state. Specifically, e is drawn from $f(e|s, z)$ and s evolves according to $F(s'|s, z')$. Similarly the distributions for newborn households are $\hat{f}(e|s, z)$ and $\hat{F}(s|z)$.

Household preferences over consumption are the same as before and leisure is still not valued. The assumptions on preferences, mortality risk, and endowments imply an exogenous stochastic process for aggregate labor supply N . It is assumed that the labor supply conditional on $z' = g$ is always weakly larger than the labor supply conditional on $z' = b$.

2.2.2 Legal Environment

The legal environment is the same as in steady state.

2.2.3 Asset Markets

As before households enter into debt or savings contracts with a financial intermediary, but now contracts are separated into two types: g -contingent and b -contingent. From the household's perspective, these contracts look like two Arrow securities a'_g and a'_b . The face value a'_z is to be delivered (if positive) or repaid (if negative) if and only if next period's productivity shock is z' . The price of a z' -contingent contract is denoted $q_{z'}(a', s; \mathcal{S})$. Hence there are two price schedules, $q_g(a', s; \mathcal{S})$ and $q_b(a', s; \mathcal{S})$.

From the intermediary's perspective, a contract costs $q_{z'}(a', s; \mathcal{S})a'$ and gives a certain yield $\rho_s p(a', s; \mathcal{S}'_{z'})a'$ contingent on a z' realization. The recovery rate $p(a', s; \mathcal{S}'_{z'})$ reflects that not only may households default, but that their decision depends on the aggregate state $\mathcal{S}'_{z'}$.

As before, the intermediary has access to capital K' and a risk-free discount bond B' with price $\bar{q}_B(\mathcal{S})$. Additionally, and following [Krusell, Mukoyama, and Şahin \(2010\)](#), the intermediary has access to an "aggregate complete" set of Arrow securities A'_g and A'_b with prices $\bar{q}_g(\mathcal{S})$ and $\bar{q}_b(\mathcal{S})$. Because capital's return is risky and the bond's return is risk-free, these

Arrow securities are redundant when the short-sale constraint on capital is not binding.¹⁹ All assets except capital are in zero net supply.

With the aggregate-complete set of Arrow securities, contract pricing is simple. Because a z' -contingent contract can be replicated by a z' -contingent Arrow security, no arbitrage dictates that

$$q_{z'}(a', s; \mathcal{S}) = \bar{q}_{z'}(\mathcal{S})\rho_s p(a', s; \mathcal{S}'_{z'}) \quad (15)$$

for $a' \neq 0$. Without loss of generality $q_{z'}(0, s; \mathcal{S}) = 0$. This shows the price of z' -contingent contract reflects the cost of transferring resources to that state $\bar{q}_{z'}(\mathcal{S})$, the probability of survival ρ_s , and the recovery rate $p(a', s; \mathcal{S}'_{z'})$ conditional on reaching that state. This contract pricing ensures that there is no cross-subsidization across different loan types and is therefore consistent with free entry by intermediaries.

While contracts are priced in this independent fashion, household access to these contracts is restricted. Specifically, a household of type s must choose (a'_g, a'_b) from a set $P(s)$.²⁰ In the calibrated model, there will be two groups of households separated, essentially, into the bottom 80% of labor income earners and the top 20%. The bottom 80% will have access to a fixed portfolio which for the sake of simplicity and clarity is a bond.²¹ For them $P(s) = \{(a'_g, a'_b) \in A \times A | a'_g = a'_b\}$. The top 20% on the other hand will have access to a bond for borrowing but can save using any (a'_g, a'_b) combination. For them $P(s) = \{(a'_g, a'_b) \in A \times A | a'_g = a'_b \text{ if } a'_g < 0 \text{ or } a'_b < 0\}$. These portfolio restrictions are similar to the ones in [Chien, Cole, and Lustig \(2011\)](#) where 10% of the population has freely adjustable portfolios, 20% use a fixed, weighted portfolio of bonds and equity, and the remaining 70% use a bond.

The portfolio restrictions are meant to capture that while a large menu of tradeable assets is available, only rich households seem to use them.²² In particular, [Kennickell \(2009\)](#) demonstrates that the top 20% of the income distribution (and especially the top 5%) hold a disproportionate share of their portfolio in businesses and other non-housing wealth while the bottom 80% hold primarily housing wealth.²³ Moreover, [Campbell \(2006\)](#) shows the portfolios of households with the least assets contain virtually only safe ones.²⁴ Together with the observation that interest on consumer credit is almost always tied to the prime

¹⁹Recall next period's labor supply is assumed to be weakly larger if g is realized than b is realized. This, together with $g > b$, ensures $r(\mathcal{G}) > r(\mathcal{B})$ so that capital is indeed risky.

²⁰I assume $A^+ \times A^+ \subset P(s)$ for some s having $\hat{F}(s|z) > 0$ for each z and $T \geq 2$. This technical assumption ensures some positive measure of households can always be incentivized to increase a'_g (in the aggregate) or a'_b by varying \bar{q}_g and \bar{q}_b .

²¹The restriction to a bond, $a'_g = a'_b$, rather than some other fixed portfolio such as capital, $a'_{z'} = (1 + r(\mathcal{S}'_{z'}) - \delta)k'$ for some k' and each z' , has three advantages. First, it is consistent with the theoretical structure of the model (where A is a fixed set). Second, in the computation it avoids interpolating the value function and price schedules in the a direction. Third, it allows for the natural borrowing limit to be written down in a straightforward fashion.

²²While imposing exogenous portfolio restrictions to capture this endogenous outcome is not ideal, it seems reasonable given that the model abstracts from informational costs and other potential barriers to entry in financial markets.

²³See p. 25 and Figure 27 of his paper. [Carroll \(2000\)](#) documents a similar fact for the wealth distribution.

²⁴See Figure 3 of his paper. He defines safe assets as "checking, saving, money market and call accounts, CDs, and U.S. savings bonds" (p. 1563).

rate (and not, for instance, the return on equity), the assumed portfolio restrictions seem plausible. That said, there will in general be large welfare gains from removing these portfolio restrictions. In Appendix D, I show one of the main results, that aggregate risk reduces the welfare gain of eliminating default, holds when there are no portfolio restrictions.

2.2.4 The Household Problems

Let $V(a, e, s, h; \mathcal{S})$ denote the value function of a household. Taking the law of motion and prices as given, households solve the following problems.

A household in good standing $h = 0$ that can repay its debt solves

$$V(a, e, s, 0; \mathcal{S}) = \max_{d \in \{0,1\}} (1 - d) \cdot V^R(a, e, s; \mathcal{S}) + d \cdot V^D(e, s; \mathcal{S}) \quad (16)$$

where the value of repaying is

$$\begin{aligned} V^R(a, e, s; \mathcal{S}) &= \max_{c, a'_g, a'_b} U(c, s) + \beta \rho_s \mathbb{E}V(a'_{z'}, e', s', 0; \mathcal{S}'_{z'}) \\ c + q_g(a'_g, s; \mathcal{S})a'_g + q_b(a'_b, s; \mathcal{S})a'_b &= w(\mathcal{S})e + a \\ c \geq 0, (a'_g, a'_b) &\in P(s) \end{aligned} \quad (17)$$

and the value of defaulting is

$$\begin{aligned} V^D(e, s; \mathcal{S}) &= \max_{c, a'_g, a'_b} U(c, s) + \beta \rho_s \mathbb{E}V(0, e', s', 1; \mathcal{S}'_{z'}) \\ c &= w(\mathcal{S})e(1 - \chi) \\ c \geq 0, (a'_g, a'_b) &= (0, 0). \end{aligned} \quad (18)$$

A household in good standing that cannot repay its debt must default.

A household in bad standing $h = 1$ solves

$$\begin{aligned} V(a, e, s, 1; \mathcal{S}) &= \max_{c, a'_g, a'_b} U(c, s) + \beta \rho_s \lambda \mathbb{E}V(a'_{z'}, e', s', 1; \mathcal{S}'_{z'}) \\ &\quad + \beta \rho_s (1 - \lambda) \mathbb{E}V(a'_{z'}, e', s', 0; \mathcal{S}'_{z'}) \\ c + q_g(a'_g, s; \mathcal{S})a'_g + q_b(a'_b, s; \mathcal{S})a'_b &= w(\mathcal{S})e + a \\ c, a'_g, a'_b \geq 0, (a'_g, a'_b) &\in P(s). \end{aligned} \quad (19)$$

Let the associated policy functions be denoted $d(a, e, s, h; \mathcal{S})$, $a'_{z'}(a, e, s, h; \mathcal{S})$, $c(a, e, s, h; \mathcal{S})$ where a household in bad standing is said to not default $d(a, e, s, 1; \mathcal{S}) = 0$.

2.2.5 The Intermediary's Problem

Details of the intermediary's problem may be found in Appendix A. Essentially, the intermediary maximizes the net present value of financial income discounted by Arrow security prices

$\bar{q}_{z'}(\mathcal{S})$. He does so using contracts, capital, a risk-free bond, and the two Arrow securities. He is indifferent over all feasible allocations if contract prices satisfy (15) and if

$$\begin{aligned} 1 &= \bar{q}_g(\mathcal{S})(1 + r(\mathcal{G}) - \delta) + \bar{q}_b(\mathcal{S})(1 + r(\mathcal{B}) - \delta) \\ \bar{q}_B(\mathcal{S}) &= \bar{q}_g(\mathcal{S}) + \bar{q}_b(\mathcal{S}). \end{aligned} \quad (20)$$

These are equivalent to

$$\begin{aligned} \bar{q}_g(\mathcal{S}) &= (1 - \bar{q}_B(\mathcal{S})(1 + r(\mathcal{B}) - \delta))/(r(\mathcal{G}) - r(\mathcal{B})) \\ \bar{q}_b(\mathcal{S}) &= (\bar{q}_B(\mathcal{S})(1 + r(\mathcal{G}) - \delta) - 1)/(r(\mathcal{G}) - r(\mathcal{B})). \end{aligned} \quad (21)$$

While not immediately apparent, the presence of adjustable portfolios permits the financial intermediary to make zero profits in every state (not just in expectation). A proof of this fact is given in Appendix A.²⁵ Because the intermediary makes zero profits, a theory of how any gains or losses are distributed across households does not need to be developed.

2.2.6 Equilibrium

I now give a definition of equilibrium that has been substantially simplified. The unsimplified definition, as well as the characterizations leading to it, are in Appendix A. A recursive competitive equilibrium is a collection of price functions $r, w, \bar{q}_g, \bar{q}_b, \bar{q}_B, q_g, q_b$, recovery rates p , policy functions, c, a'_g, a'_b, d , a value function V , a capital stock K and labor supply N as functions of the aggregate state, and a law of motion Γ such that the following hold:

1. The policies and value function solve the household problems.
2. The aggregate capital stock and labor supply are given by the distribution and the capital stock is strictly positive:

$$K(\mathcal{S}) = \int (a + d(a, e, s, h; \mathcal{S})(-a - \chi w(\mathcal{S})e))d\mu/(1 + r(\mathcal{S}) - \delta) > 0 \quad (22)$$

$$N(\mathcal{S}) = \int e d\mu. \quad (23)$$

3. Prices satisfy

$$q_{z'}(a', s; \mathcal{S}) = \bar{q}_{z'}(\mathcal{S})\rho_s p(a', s; \mathcal{S}'_z) \quad (24)$$

$$\bar{q}_g(\mathcal{S}) = (1 - \bar{q}_B(\mathcal{S})(1 + r(\mathcal{B}) - \delta))/(r(\mathcal{G}) - r(\mathcal{B})) \quad (25)$$

$$\bar{q}_b(\mathcal{S}) = (\bar{q}_B(\mathcal{S})(1 + r(\mathcal{G}) - \delta) - 1)/(r(\mathcal{G}) - r(\mathcal{B})) \quad (26)$$

$$r(\mathcal{S}) = z\alpha(K(\mathcal{S})/N(\mathcal{S}))^{\alpha-1} \quad (27)$$

$$w(\mathcal{S}) = z(1 - \alpha)(K(\mathcal{S})/N(\mathcal{S}))^\alpha. \quad (28)$$

²⁵Intuitively, for zero profits to obtain, the intermediary's capital income must exactly offset his contract obligations. This implies contract obligations must have an identical return structure to capital. In general, there will not exist a *fixed* portfolio of assets that will deliver this because the presence of default makes contract obligations, and in particular charge-offs, vary in a non-trivial way. By allowing flexible portfolios, some households can be induced to change the ratio a'_g/a'_b in the aggregate and so make contract obligations and capital have the same return structure. This is accomplished by varying the bond price \bar{q}_B , which controls the relative price \bar{q}_g/\bar{q}_b .

4. Repayment probabilities are consistent: for all a, s_{-1} and \mathcal{S} ,

$$p(a, s_{-1}; \mathcal{S}) = \sum_s \int \left(\frac{1 - d(a, e, s, 0; \mathcal{S}) + d(a, e, s, 0; \mathcal{S})\chi w(\mathcal{S})e/(-a)}{d(a, e, s, 0; \mathcal{S})\chi w(\mathcal{S})e/(-a)} \right) f(e|s, z) de F(s|s_{-1}, z). \quad (29)$$

5. All asset markets clear and the intermediary makes zero profits which is equivalent to

$$\begin{aligned} & (1 + r(\mathcal{B}) - \delta) \sum_{a'} \int \rho_s p(a', s; \mathcal{G}) a' \mathbf{1}[a' = a'_g(a, e, s, h; \mathcal{S})] d\mu \\ & = (1 + r(\mathcal{G}) - \delta) \sum_{a'} \int \rho_s p(a', s; \mathcal{B}) a' \mathbf{1}[a' = a'_b(a, e, s, h; \mathcal{S})] d\mu. \end{aligned} \quad (30)$$

6. The law of motion is consistent with stochastic transitions and household policies.

The least obvious equation is (30). This ensures the only asset the intermediary needs to use (besides contracts) is capital. To see this, suppose there was no mortality risk, no default, and that portfolios mimicked capital, $a'_{z'} = (1 + r(\mathcal{S}'_{z'}) - \delta)k'$ for some k' and each z' . In this case, (30) becomes

$$(1 + r(\mathcal{B}) - \delta) \int (1 + r(\mathcal{G}) - \delta) k' d\mu = (1 + r(\mathcal{G}) - \delta) \int (1 + r(\mathcal{B}) - \delta) k' d\mu \quad (31)$$

which always holds. To carry household savings into the next period, all the intermediary must do is choose capital holdings K' equal to $\int k' d\mu$ which makes the bond and Arrow securities unnecessary. With default, mortality risk, and flexible portfolios, some adjustments must be made to reflect that savings are contingent and that only the aggregate portfolio must resemble capital.

The only “deep” prices in the model are the factor prices, r and w , and the risk-free discount bond price \bar{q}_B . Essentially by no arbitrage, the Arrow security prices \bar{q}_g, \bar{q}_b and price schedules q_g, q_b can be written as functions of these, as is evident when looking at equations (24), (25), and (26). The bond price is used to clear the asset markets, as summarized in equation (30), by controlling the relative cost of saving using a'_g versus using a'_b .²⁶

3 Calibration and Baseline Properties

This section discusses the calibration and baseline model properties. The model period is a year.

²⁶Importantly, as \bar{q}_B approaches $1/(1 + r(\mathcal{B}) - \delta)$ from above, the Arrow price \bar{q}_g and hence the price schedule q_g approach zero. This makes saving using a'_g become arbitrarily cheap driving up the left hand side of (30). At the same time, the Arrow price \bar{q}_b approaches $1/(1 + r(\mathcal{B}) - \delta)$ meaning saving using a'_b does not become arbitrarily cheap. This keeps the right hand side of (30) bounded from above. As \bar{q}_B approaches $1/(1 + r(\mathcal{G}) - \delta)$, the reverse is true.

3.1 Functional Forms

I first describe the functional form for the efficiency process and then for portfolio availability.

3.1.1 Efficiency

I select the efficiency process to capture three potentially important features of the data. First is that earnings persistence and variance change over the life cycle. This feature is demonstrated in [Karahan and Ozkan \(2010\)](#) where it is shown earnings shocks are less persistent early in life. In the model, persistence and contract pricing are tightly linked, so this could prove important. Second is that the variance of persistent earnings shocks increases in recessions and decreases in expansions as demonstrated in [Storesletten et al. \(2004\)](#). As default provides a way of intratemporally smoothing consumption, this fluctuation in intratemporal dispersion could also prove important. Third is that the earnings distribution has a thick right tail. By selecting an efficiency process that generates this right tail, the model will also be able to match the concentration of wealth in the data. This is important because the wealthiest households will not be directly affected by default policy. Consequently, general equilibrium effects of changes in default policy would likely be overstated were these households missing.

To account for these features of the data, I use two efficiency processes with working households stochastically transitioning between them, as well as a separate process for retirement. The efficiency process for the majority of working households and all newborns is governed by

$$\begin{aligned} e_{h,z} &= \psi_z \phi_h \exp(u_h + \varepsilon_h) \\ u_h &= \gamma_h u_{h-1} + \eta_{h,z}, u_0 = 0 \\ \eta_{h,z} &\sim N(0, \sigma_{\eta,h,z}^2), \varepsilon_h \sim N(0, \sigma_{\varepsilon,h}^2) \end{aligned} \tag{32}$$

where h denotes age. This process has a deterministic component ϕ_h , a persistent component u_h , a transitory component ε_h , and an “aggregate labor supply shifter” ψ_z . As labor is supplied inelastically, the supply shifter is used to match the cyclical volatility of hours worked.²⁷ Note that the persistence is governed by γ_h which is age-dependent, as are the variances of the persistent and transitory shocks. Also note the variance of the persistent shock $\sigma_{\eta,h,z}^2$ is a function of z (the steady state variance is $\sigma_{\eta,h,1}^2$).

While most working households use this “log process,” they have a probability π_{bw} (bw for “blue to white” collar) of transiting to the process

$$\begin{aligned} e_{h,z} &= \psi_z \phi_h v \\ v &\sim \left(\frac{v - \underline{v}}{\bar{v} - \underline{v}} \right)^\xi \text{ with support } [\underline{v}, \bar{v}]. \end{aligned} \tag{33}$$

²⁷Note that the literature tends to focus on residual earnings, i.e. what is left after running a first-stage regression on log earnings using a complete set of time dummies. The model process is consistent with this in that running such a regression would remove ψ_z (and the wage w) which will be common across all households.

This “super rich” process also has an aggregate, deterministic, and transitory component, however the transitory component is drawn from a different distribution governed by three parameters, \underline{v} , \bar{v} , and ξ . The functional form for the v distribution is taken from Chatterjee et al. (2007) who in the spirit of Castañeda, Díaz-Giménez, and Ríos-Rull (2003) posit a low-probability state where earnings are very high but transitory. As Chatterjee et al. (2007) argue, such a state provides households with the “opportunity and incentive” to accumulate large amounts of wealth.²⁸ Households revert to the log process with probability π_{wb} and draw u_h from $N(0, \sigma_{\eta,1,z}^2)$ and ε_h from $N(0, \sigma_{\varepsilon,1}^2)$.²⁹ The transition is deterministic (i.e. $\pi_{wb} = 1$) in the last period of working life.

When households retire, their efficiency is³⁰

$$e_{h,z} = \kappa_F \psi_z \phi_J \exp(u_J) + \kappa_G \psi_z \quad (34)$$

where J is the first period of retirement. This retirement process is very similar to the one in Livshits et al. (2007) and Athreya et al. (2009a). The proportional component κ_F reflects earnings made over a household’s lifetime, and the guaranteed component κ_G provides a fraction of average earnings. Mean efficiency is normalized to one in steady state. By assumption all households are under the log process in the last period of working life $J - 1$ and take one last draw of η to arrive at u_J .

3.1.2 Portfolio Availability

Recall that the portfolio $P(s)$ available to households is allowed to vary with their characteristics s . I now let $P(s)$ equal $\{(a'_g, a'_b) \in A \times A | a'_g = a'_b \text{ if } a'_g < 0 \text{ or } a'_b < 0\}$ for “super-rich” households and $P(s)$ equal $\{(a'_g, a'_b) \in A \times A | a'_g = a'_b\}$ for all others.³¹ This means the typical household will use only a risk-free bond, but a few earnings-rich households will, in addition to a risk-free bond, have access to freely adjustable portfolios for savings.

3.2 Parameter Values and Baseline Properties

For the steady state calibration, I adopt most targets and many parameters from Chatterjee et al. (2007). The coefficient of relative risk aversion σ is 2. The capital share of income α is 0.36 and the depreciation rate of capital δ is .10. The average duration of a bad credit record $1/(1 - \lambda)$ is taken to be ten years implying $\lambda = 0.9$. Wealth and earnings targets are

²⁸The only way to equate marginal utilities, given the high income and transitory nature of the state, is to save a large amount. In the calibration, the iid assumption on v , together with a large value for \bar{v} and a small value for ξ , delivers this.

²⁹They draw from the distribution for newborn households both for simplicity and because the persistent variance is large (reflecting a large cross-sectional variance of the persistent shock).

³⁰I follow Krusell, Mukoyama, Şahin, and Smith (2009) and model retirement income as home production to avoid explicitly modeling a government.

³¹The technical assumption on $P(s)$ dictates some $\epsilon > 0$ measure of households is born super rich.

reported in Table 1. I target a debt-output ratio of .0067 and a fraction in debt of 6.7. For the filing rate, I target .50%.³²

Households begin life at age 20, begin retirement at 65, and live to at most 85. The profiles of residual earnings persistence and variance ($\gamma_h, \sigma_{\eta,h,1}^2, \sigma_{\varepsilon,h}^2$) are the non-parametric estimates of Karahan and Ozkan (2011).³³ The persistence γ_h begins low at around .7 before increasing to unity by age 40 and mostly staying there until retirement. For countercyclical earnings variance, there are no age-specific estimates available, so I use estimates from Storesletten et al. (2004) assuming the ratio $\sigma_{\eta,h,g}/\sigma_{\eta,h,b}$ is age-independent and equal to .59 and that $.5\sigma_{\eta,h,b} + .5\sigma_{\eta,h,g} = \sigma_{\eta,h,1}$.³⁴ The retirement parameters (κ_F, κ_G) are set to (.35, .15) giving an average replacement rate of roughly 50%.³⁵ The deterministic component of efficiency ϕ_h and mortality risk profile ρ_s are calibrated using estimates from Hubbard, Skinner, and Zeldes (1994).³⁶ The earnings profile ϕ_h follows a hump-shape over the life cycle, nearly doubling from age 20 to its peak at age 48 before almost returning to its age-20 value by retirement. The effective household-size profile θ_s is calibrated using the “mean” equivalence scale in Fernández-Villaverde and Krueger (2007) and the profile of household size calculated from the CPS by Bick and Choi (2011).³⁷ All the profiles are reported in Appendix B.

The productivity process is assumed to be symmetric with an expected duration of expansions and recessions at 3 years. This implies $F(g|g) = F(b|b) = 2/3$. A standard deviation of 2.24% is targeted for the Solow residual implying the technology shock takes on $g = 1.0224$ or $b = 0.9776$.³⁸ The aggregate labor supply shifter ψ_z is calibrated, once the rest of the

³²This is the average number of filings per household from 1999-2003, .93%, prorated to only account for the number of filings due to earnings shocks which Chatterjee et al. (2007) put at 53.5% based on Chakravarty and Rhee (1999). The literature has used a wide range of targets from .29% in Chatterjee et al. (2007) to 1.2% in Athreya et al. (2009a).

³³The sample in Karahan and Ozkan (2011) is for ages 24 to 60 and I use nearest-neighbor extrapolation to recover the values for households aged 23 or less and 60 or older. I view this extrapolation procedure as conservative because the authors report in their earlier working paper, Karahan and Ozkan (2010), that the shape of the profiles is even stronger for ages 20 to 65. A last detail, the authors allow for time loading factors on the persistent and transitory shock variance, and I use the average of the loadings from 1988 to 1992.

³⁴The value of $\sigma_{\eta,g}/\sigma_{\eta,b}$ is .59 in Storesletten et al. (2004).

³⁵Although Athreya et al. (2009a) and Livshits et al. (2010) set $(\kappa_F, \kappa_G) = (.35, .2)$, they do not have the earnings rich. The lower value for κ_G (the guaranteed portion of *mean* earnings) of .15 reflects a mean/median income that is approximately 75% of its value when including the income-rich (roughly 1.2 compared to 1.6).

³⁶Hubbard et al. (1994) estimate separate deterministic profiles for household heads with less than 12 years of education (NHS), 12-15 years (HS), and 16+ years (COL) of education. I average the profiles of these three types for the year 1986 assuming 13% of the population is NHS, 48% is HS, and 39% is COL. This breakdown of educational attainment is from the 2010 Current Population Survey for ages 30-34 (see Table 1 at <http://www.census.gov/hhes/socdemo/education/data/cps/2010/tables.html>).

³⁷Specifically, I assume $\theta_s = f(N_s)$ where N_s is the average number of household members and f is the mean equivalence scale in Fernández-Villaverde and Krueger (2007) linearly-interpolated to be continuous. The values for θ_s are very similar to the ones in Livshits et al. (2007). The benchmark calibration over-predicts the hump in consumption, but in a robustness exercise I show $\theta_s = 1$ makes the results even stronger (see Appendix D Table 18). This suggests estimating θ_s to match the consumption profile would not significantly alter the results.

³⁸This is the unconditional standard deviation estimated in Cooley (1995) (who uses a labor share of .4).

calibration is set, to match the 1.74% standard deviation of log hours worked reported in Castañeda, Díaz-Giménez, and Ríos-Rull (1998). This results in $(\psi_g, \psi_b) = (1.025, .975)$ with the steady state value ψ_1 normalized to 1.

There are 7 remaining parameters: 1 preference parameter β , 1 technology parameter χ , and 5 efficiency-process parameters $(\underline{v}, \bar{v}, \xi, \pi_{bw}, \pi_{wb})$. These are used to minimize the weighted distance between the steady state model and target statistics. The steady state model is computed with grid search and backward induction in Fortran. For details of the computation see Appendix C.

The results from the calibration are listed in Table 1. The model does fairly well at reproducing the targets. Specifically the right tails of both the earnings and wealth distributions are matched closely as is the capital-output ratio. As in virtually all the bankruptcy literature, the calibration has difficulty jointly matching the filing and debt statistics.³⁹ Having some income transferred to creditors in the period of default (via the default cost χ) helps a lot in matching these statistics, as does some flexibility with the efficiency process.

The model with aggregate uncertainty is computed using the method of Krusell and Smith (1998). The distribution is summarized with two of its moments, aggregate wealth and labor supply, and an equity premium is included as a state variable.⁴⁰ The household problem is solved with grid search, backward induction, and linear interpolation of the aggregate moments. For results to be comparable between the steady state and business cycle versions, the same number and placement of grid points are used in both models (in both the asset and efficiency direction). The model is simulated non-stochastically as in Young (2010), and the asset markets are cleared in each period of the simulation. The resulting approximate law of motion makes accurate price forecasts 1-step ahead with R^2 values above .997 and maximum errors below .13% as well as 50-steps ahead with R^2 values above .995 and maximum errors below .37%. For additional details, the reader is referred to Appendix C.

Table 2 reports the cyclical properties of the model and US economies with a discussion of the levels of model aggregates deferred to the next section. The data are described in Appendix B. Filings are too volatile and too countercyclical; however, default in the model is based entirely on earnings shocks. Other reasons for default, such as divorce, health bills, and lawsuits, are probably not as correlated with the business cycle nor as volatile. The volatility of consumption is too low. The labor supply volatility is exactly as targeted, and output and investment are close to their US counterparts. Output is persistent but not as persistent as in the data.

³⁹Three approaches have been used to try to overcome this problem. First is the approach used by Chatterjee et al. (2007) and Sánchez (2007) who calibrate the earnings process to match some earnings statistics. Second is the approach used in Athreya et al. (2009b) who take estimates from the literature but use a stochastic punishment for default. Third is the approach used in Livshits et al. (2007) who take estimates from the literature but make debt partially secured through a “garnishment” technology and posit transaction costs for debt. The approach I use here is a mixture of the first and the third.

⁴⁰Including an equity premium as a state variable is fairly common in the literature. For instance, Storesletten, Telmer, and Yaron (2007) include $\mathbb{E}_{\mathcal{S}}(R(\mathcal{S}'_z)) - \bar{q}_B(\mathcal{S})^{-1}$. The benefit of doing this is that it makes market clearing during the simulation much faster.

Statistic	Target	Model	Parameter*	Value
Targets Determined Independently				
Coefficient of Relative Risk Aversion	2	2	σ	2
Capital Share of Income	.36	.36	α	.36
Depreciation Rate of Capital	.10	.10	δ	.10
Avg. Duration of a Bad Credit History	10	10	λ	.9
Avg. Duration of a Business Cycle	3	3	$F(z z)$	2/3
Solow Residual Standard Deviation (%)	2.24	2.24	$g - 1, 1 - b$.0224
Targets Determined Jointly				
Capital-Output Ratio	3.08	3.08	ξ	1.59e-3
Debt-Output Ratio $\times 100$.67	.69	χ	.12
Population Filing (%)	.50	.18		
Population in Debt (%)	6.7	10.5	β	.940
Wealth Gini	.80	.83		
Wealth Mean/Median	4.03	4.13	π_{wb}	.588
Percentage of Wealth held by Top 5%	57.8	69.5	\bar{v}	1429.3
Percentage of Wealth held by Top 20%	81.7	83.6	\underline{v}	2.67
Percentage of Wealth held by 40-20%	12.2	9.8		
Percentage of Wealth held by 60-40%	5.0	4.9		
Percentage of Wealth held by 80-60%	1.3	1.8		
Earnings Gini	.61	.50		
Earnings Mean/Median	1.57	1.59	π_{bw}	.153
Percentage of Earnings held by Top 5%	31.1	30.5		
Percentage of Earnings held by Top 20%	60.2	55.4		
Percentage of Earnings held by 40-20%	22.9	19.1		
Percentage of Earnings held by 60-40%	13.0	12.5		
Percentage of Earnings held by 80-60%	4.0	7.8		

*Parameters are listed beside statistics they strongly influence.

Table 1: Model Targets, Statistics, and Parameters

Variable (x)	Stdev x	Stdev x/	Correlation of lagged x with y				
	in (%)	Stdev y	x(-2)	x(-1)	x	x(+1)	x(+2)
US (1960-2004)							
Output (y)	2.44	1.00	0.01	0.58	1.00	0.57	-0.01
Consumption	1.69	0.69	0.02	0.59	0.92	0.59	0.10
Investment	7.19	2.94	0.04	0.53	0.89	0.38	-0.26
Hours Worked	1.60	0.66	-0.23	0.26	0.82	0.69	0.09
Defaulting Pop	10.25	4.20	0.17	0.00	-0.03	0.26	0.56
Model							
Output (y)	2.96	1.00	-0.10	0.08	1.00	0.08	-0.09
Consumption	0.82	0.28	-0.28	-0.11	0.84	0.45	0.23
Investment	8.14	2.75	-0.05	0.12	0.99	-0.01	-0.16
Labor Supply	1.74	0.59	0.03	0.18	0.95	-0.14	-0.26
Defaulting Pop	12.34	4.17	0.27	0.15	-0.69	-0.21	-0.42
Debt*	6.54	2.21	0.28	0.25	0.08	-0.79	-0.58
Population in Debt*	4.50	1.52	0.29	0.25	0.07	-0.83	-0.54

*Debt is negative net worth. US counterparts to my knowledge are not available.

Table 2: Business Cycle Properties: Model and Data

4 Eliminating Default

Overall, the calibrated model is broadly consistent with US data. I now explore the consequences of eliminating default both in terms of welfare and allocations, and see how these change in the presence of aggregate risk.⁴¹ The benchmark economy, that is the one with default, is referred to as the CD economy for “consumer default.” The economy where default has been eliminated is referred to as the ND economy for “no default.” It is the limit of default economies as default becomes infinitely costly.⁴² The ND economy is just a “natural borrowing limit” economy, as in [Aiyagari \(1994\)](#), where households are able to borrow at a risk-free rate (because $p = 1$) as much as they can repay with certainty. This limit is the net present value of the worst possible labor income stream.

⁴¹I do not account for the transition, and there are two reasons for this. First is that when a policy change results in capital decumulation, accounting for the transition tends to improve welfare (this is the case, for instance, in [Chatterjee and Gordon, 2011](#)). As will be seen shortly, eliminating default does result in capital decumulation. However, the capital decumulation is larger in the business cycle than in the steady state environment. Hence, I conjecture that accounting for the transition will only strengthen the main result that aggregate risk reduces the welfare gain of eliminating default. Second is that it is not conceptually clear how to account for the transition from one ergodic distribution to another, and I am unaware of any paper that has done so.

⁴²One could think of this as χ converging up to 1 or some dead-weight cost (not modeled) levied on households. The actual form of the cost does not matter as the ND economy will not have default in equilibrium. Table 6 confirms computationally that the CD economy converges to the ND economy as $\chi \uparrow 1$.

While in most studies the use of a natural borrowing limit rather than some tighter exogenously fixed limit is of secondary importance, here it is the consequence of eliminating default. Consequently, the natural limit is extremely important. There are then three issues to consider, namely, what is the limit in the theory, in the computation, and in the data. In the theory, the use of a log-efficiency process implies efficiency, and hence labor income, can be arbitrarily close to zero prior to retirement. However, because some labor income is *guaranteed* in retirement, the natural limit will *not* be zero (as long as $\kappa_G > 0$). In the computation, the log process is discretized with the method of Tauchen (1986) using a large “coverage.”⁴³ This makes the lowest efficiency realization prior to retirement, .0043, very close to zero. In the data, Carroll (1992) documents that non-capital household income, *including* transfer income, falls to (or very close to) zero between .30% and .65% of the time for working-age households. Importantly, this sample does not appear to reflect measurement error.⁴⁴ At the same time, part of Social Security income in retirement seems to indeed be guaranteed.⁴⁵ Consistent with the data, the process I use has near-zero-earnings events but allows for truly guaranteed earnings in retirement in both the theory and computation.⁴⁶

4.1 Steady State

In steady state, the elimination of default results in a large increase in debt and wealth inequality and a significantly lower capital-output ratio. This is borne out in Table 3 which lists key wealth and debt statistics for both models. Most striking are the debt statistics: the population in debt increases by 60% from 11% to 17% and the debt-output ratio increases by 500% from .007 to .044. The increased indebtedness translates into a 3.5% decline in the capital-output ratio and is paired with a sharp increase in wealth inequality.

Why is there so much more debt once default has been eliminated? Households in both economies have *incentive* to borrow because of earnings uncertainty, impatience, and a hump-shaped earnings profile. However, the ND economy gives households the *opportunity* to borrow large amounts while the CD economy does not. One way to see this is to consider borrowing limits in the two economies. While there is no “borrowing limit” in the CD

⁴³The method specifies a way of approximating an AR1 process with a (finite state) Markov chain given bounds for the states, i.e. the coverage. The coverage I use is $\pm 6.25\bar{\sigma}_{\eta,1}/\sqrt{1-\bar{\gamma}^2}$ for the persistent shock and $\pm 3\bar{\sigma}_\varepsilon$ for the transitory shock where a bar denotes the numerical average across ages. The average values are $\bar{\gamma} = .946$, $\bar{\sigma}_{\eta,1} = .200$, and $\bar{\sigma}_\varepsilon = .241$. The coverage of $\pm 6.25\bar{\sigma}_{\eta,1}/\sqrt{1-\bar{\gamma}^2}$ for the persistent shock reflects a coverage of $\pm 5\bar{\sigma}_{\eta,b}/\sqrt{1-\bar{\gamma}^2}$ when using the standard deviation in recessions.

⁴⁴Carroll (1992) argues that these observations are not the result of measurement error for two reasons. First, when not-self-employed households report zero (non-capital) income, typically they experienced unemployment, injury, or health problems in the same period or just prior. Second, Duncan and Hill (1985) find outliers for annual income generally correspond to actual experience, not measurement error.

⁴⁵The most secure part may be Supplemental Security Income. Under this program an individual is eligible for benefits if they are 65 or older, legally reside in the US, have income that is not too high, and apply for benefits. Certain other individuals are also eligible. See <http://www.ssa.gov/ssi/text-eligibility-ussi.htm>.

⁴⁶That said, robustness tests are conducted with respect to the lower bound on efficiency. As mentioned in the introduction, it is found the result that aggregate risk reduces the welfare gain of eliminating default is robust. However, the levels of the gains are not robust. See Table 16 in Appendix D.

Statistic	CD	ND
Capital-Output Ratio	3.08	3.00
Debt-Output Ratio $\times 100$.69	4.37
Percentage in Debt	10.5	17.2
Wealth Gini	.83	.88
Wealth Mean/Median	4.13	4.95
Percentage of Wealth held by Top 5%	69.5	72.6
Percentage of Wealth held by Top 20%	83.6	86.9
Percentage of Wealth held by 40-20%	9.8	9.4
Percentage of Wealth held by 60-40%	4.9	4.1
Percentage of Wealth held by 80-60%	1.8	1.0

Table 3: CD and ND Steady State Comparison

economy, there is a maximum loan size that can be taken out by a household of type s : $\max_{a'} q(a', s)(-a')$. Similarly, no household in the ND economy would ever take out a loan worth more than $q(\bar{A}(s), s)(-\bar{A}(s))$ where $\bar{A}(s)$ is the natural borrowing limit (for type s).

Figure 1 compares these borrowing limits by age (averaging out the other components of s). As is clear, the maximum loan size in the ND economy is uniformly and typically *much* higher than in the CD economy. Because of this, and because households have incentive to use debt, the ND economy is much more indebted. Figure 1 also reveals that the elimination of default has a differential effect on the life-cycle profile of borrowing limits. While there are increases for each age, the largest occurs late in life and the smallest when young. Moreover, the shapes are very different with the CD limit tracking the earnings profile and the ND limit sharply increasing until retirement and thereafter sharply decreasing.

Three effects give the natural borrowing limit its shape. The first effect drives the borrowing limit up and is that as retirement approaches, the guaranteed earnings in retirement are discounted less, increasing their net present value. This increase in the net present value translates into an increase in the borrowing limit because the lowest efficiency realization prior to retirement is close to zero. The second effect drives the net present value downward. This is that once retirement begins, each year reduces the number of periods left of retirement income. The last effect causes the jump at around age 65 and is that once households enter retirement, they face no more uncertainty. This drastically increases the natural limit for all but the unluckiest households.

The borrowing limits in the default economy are shaped by two effects that are quite different from the effects shaping the natural limit. First is that retired households have little incentive to borrow because, by assumption, there is *no* uncertainty. Consequently, the punishment from default, which to a large extent is exclusion from credit markets, is very low. Because households would readily default, creditors are unwilling to extend much credit. Second is the composition effect caused by a hump-shaped earnings profile. *Conditional* on a level of debt and earnings, the incentives to default and repay do not vary much over the

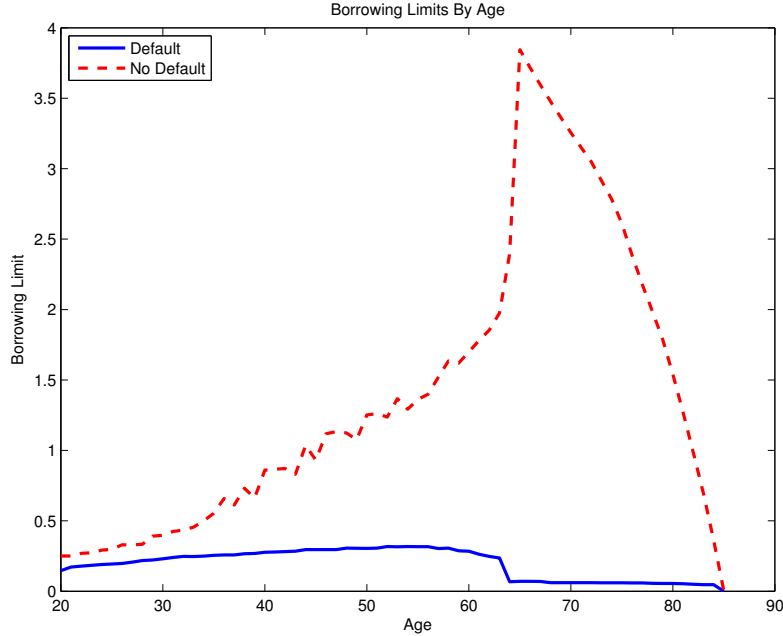


Figure 1: Borrowing Limits in the Default and No Default Economies

life cycle. However, else equal, earnings-rich households are less likely to default. Because average earnings follow a hump shape prior to retirement, so does the average limit.

The differences in credit, prices, and the availability of a default option have important ramifications for consumption smoothing. To see this, it is useful to consider how the mean and variance of log consumption evolve over the life cycle. Figure 2 plots these for the CD and ND economies. Note that the mean is higher and the variance lower in the ND economy until late in life. The interpretation of this is that households in the ND economy borrow heavily when young to smooth consumption while households in the CD economy do not (and cannot) borrow as heavily. In the ND economy, this large amount of debt must eventually be repaid whereas in the CD economy the debt is not as large and can be discharged. This lowers the mean and increases the variance of log consumption later in life in the ND economy. While credit is uniformly better in the ND economy, default does allow households to smooth consumption to some extent. However, the default option has little direct value to young households: to benefit from it, a household must first be indebted.

I now turn attention to welfare. To measure the welfare gain of eliminating default, I use the percent increase in consumption that a newborn household would need in every state of the CD economy to be indifferent between living in the CD economy and moving to the ND

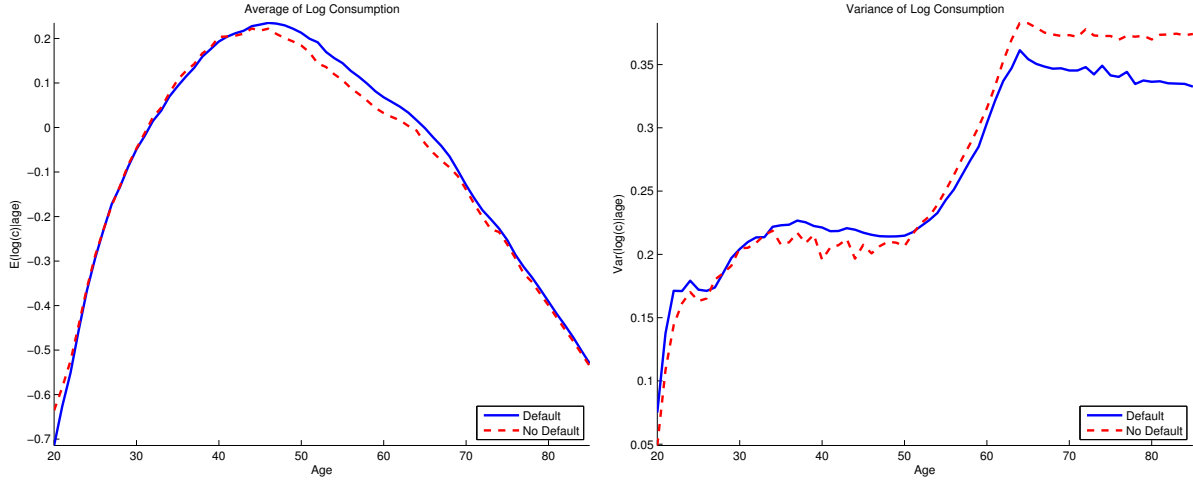


Figure 2: Log Consumption in Steady State

economy.⁴⁷ This consumption-equivalent variation measure is computed as

$$\omega = \left(\left(\frac{\sum_s \hat{F}(s) \int V_{ND}(0, e, s, 0) \hat{f}(e|s) de}{\sum_s \hat{F}(s) \int V_{CD}(0, e, s, 0) \hat{f}(e|s) de} \right)^{1/(1-\sigma)} - 1 \right) \quad (35)$$

where V_X denotes the value function from the X economy. If $\omega > 0$, then eliminating default is welfare improving. For a more complete picture of the welfare effects, I also report the percentage of the population in favor of the policy change and the welfare gains for various subsets of the population using consumption-equivalent variation.⁴⁸

The increased ability to borrow once default is eliminated results in large welfare gains. In general equilibrium, the gain is 1.82% of lifetime consumption. In partial equilibrium, i.e. holding fixed r , w , and \bar{q}_B , it is in fact much larger at 3.94%. Moreover, in partial equilibrium 100% of the population prefer the move. However, taking general equilibrium effects into account results in substantial disagreement over the policy change: only 56.8% of households now favor it.

To understand who gains and loses from the policy change, it is useful to break out the welfare gains by age both in partial and general equilibrium. This is done in Figure 3. In partial equilibrium, the gains begin high for young households, decline monotonically, and

⁴⁷This is the same welfare measure used in Livshits et al. (2007), Athreya et al. (2009b), and many others.

⁴⁸When calculating welfare gains for different groups, I assume the policy is changed *after* households have made their default decisions but *before* they have made their savings decisions. Defining $\omega(a, e, s, h) = ((V_{ND}(a, e, s, h)/V_{CD}(a, e, s, h = 0))^{1/(1-\sigma)} - 1)$, the welfare gain for a subset I of the population is calculated as $(\int_I \omega(a, e, s, h) d\mu) / \mu(I)$. Here, μ is the steady state distribution from the CD economy but with households who would have defaulted transited to $(a, e, s, h) = (0, e, s, 0)$. The population in favor is $\mu(\{\omega(a, e, s, h) > 0\})$. The timing convention, from Chatterjee and Gordon (2011), ensures no unanticipated losses or gains are inflicted on the intermediary.

approach zero in retirement. The decline is due to diminishing incentives for borrowing: young households have life-cycle reasons to borrow and face uncertainty, middle-aged households have only uncertainty, and retired households have neither. Once general equilibrium effects are accounted for, the consumption gains drop sharply for almost all households. This is especially true for young households. Further, now the decline is not monotonic. Because the CD economy has a higher capital-output ratio than the ND economy, households at every age lose labor income from the price changes. At the same time, households with savings gain capital income. Because the primary source of income for most households is derived from labor (by necessity of $\alpha = .36$), the first effect dominates and welfare tends to be lower in general equilibrium.

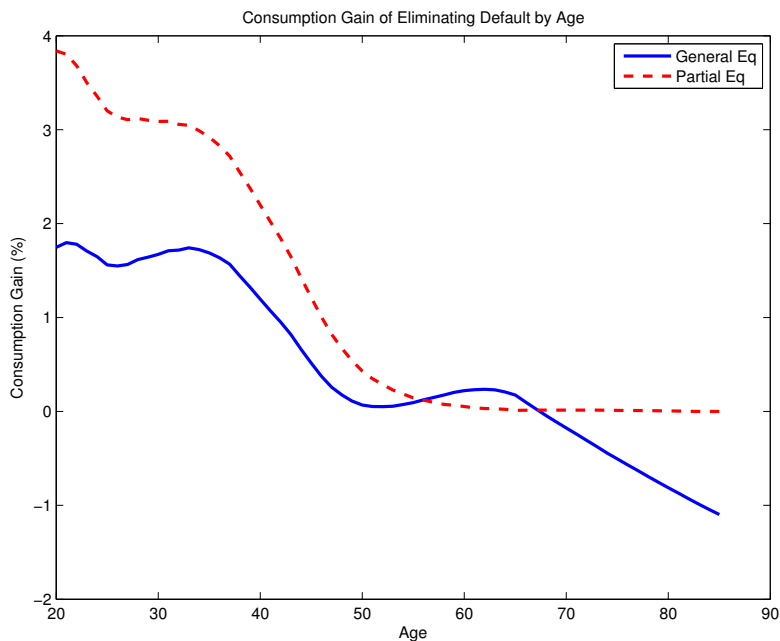


Figure 3: Welfare Gains of Eliminating Default By Age in Steady State

4.2 Business Cycle

Consistent with findings from the literature, the results from the steady-state model suggest eliminating default expands credit drastically and results in large welfare gains for young households (with mixed effects for the rest of the population due to changes in factor prices). Now I turn to the model with aggregate risk to see whether the same conclusions are reached.

Implications of aggregate risk for aggregates in the CD and ND economies are substantially different. Table 4 lists how the levels of key aggregates change after including aggregate risk. As one would expect, the capital-output ratio increases for the ND economy. This is expected because of precautionary savings: the increased risk causes households to increase

their buffer stock of capital. Interestingly, this is *not* the case in the CD economy despite having tighter borrowing limits. In fact, the capital-output ratio only increases by .02%. The debt statistics tell a similar story: the debt-output ratio falls by 38% in the ND economy but only 24% in the CD economy, and the percentage in debt declines 20% in the ND economy but only 14% in the CD economy. At the same time, the number of households filing for bankruptcy increases by 47%.

Statistic	Business Cycle		% Change	
	CD	ND	CD	ND
Capital-Output Ratio	3.08	3.03	0.02	0.90
Debt-Output Ratio $\times 100$	0.53	2.72	-23.7	-37.9
Percentage in Debt	9.00	16.53	-14.1	-20.4
Percentage Filing	0.26	-	46.6	-

Table 4: CD and ND Aggregates and Change from Steady State

Why does debt contract sharply in the ND economy but much less so in the CD economy? Figure 4, which plots borrowing limits by age, gives part of the answer.⁴⁹ At every age, the natural limit contracts, and this is especially true for the youngest households. The limit for newborns decreases by 40% from .250 to .149. At the same time, the amount households can borrow in the CD economy declines only slightly. For newborns, the limit decreases 8% from .146 to .135.

What drives the decline in the natural borrowing limit? To answer this, it is useful to consider the natural borrowing limit definitions for a newborn household. These are

$$\begin{array}{ll}
 \text{Steady State} & \text{Business Cycle} \\
 \min_{\{s_t\}_{t=2}^T} \sum_{t=2}^T \left(\prod_{j=2}^{t-1} \bar{q}_B \rho_{s_t} \right) w \underline{e}_{s_t} & \min_{\{s_t, z_t\}_{t=2}^T} \sum_{t=2}^T \left(\prod_{j=2}^{t-1} q_j \rho_{s_t} \right) w_t \underline{e}_{s_t, z_t} \\
 \text{s.t. } F(s_{t+1} | s_t) > 0 & \text{s.t. } F(s_{t+1} | s_t, z_{t+1}) > 0 \\
 s_1 \text{ given} & w_t = w(\mathcal{S}_t), q_t = \bar{q}_B(\mathcal{S}_t) \\
 & \mathcal{S}_{t+1} = \Gamma(z_{t+1}, \mathcal{S}_t) \\
 & s_1, \mathcal{S}_1 \text{ given}
 \end{array} \tag{36}$$

where \underline{e}_s denotes the lowest efficiency conditional on s and similarly for $\underline{e}_{s,z}$. As these definitions make clear, three factors can make the limit decrease: changes in the support of e ,

⁴⁹The borrowing limit definitions are analogous to the steady state ones. I calculate “average” price schedules by integrating out the aggregate state using an invariant distribution over nodes from the [Krusell and Smith \(1998\)](#) method (because linear interpolation between moments is used, the law of motion can be thought of as a probabilistic transition matrix with an invariant distribution). With these average price schedules in hand, I compute the CD economy’s borrowing limit as $\max_{a'} (q_g(a', s) + q_b(a', s))(-a')$ and the ND economy’s limit as $(q_g(\bar{A}(s), s) + q_b(\bar{A}(s), s))(-\bar{A}(s))$. Here, $\bar{A}(s)$ denotes the smallest value of a' that results in a household’s continuation value (with the aggregate state integrated out) being above a very negative number.

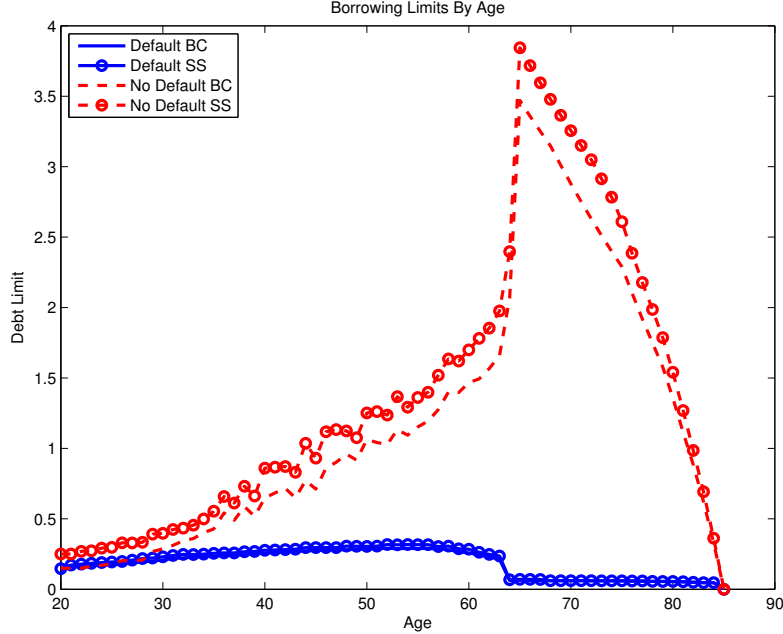


Figure 4: Borrowing Limits with and without Aggregate Uncertainty

changes in the support of s , and changes in the wage and bond prices. The support of e changes in the business cycle because of the supply shifter ψ_z . Specifically, $\psi_b = .975$ causes the limit to decrease by 2.5%. The support of s also changes slightly due to numerical precision.⁵⁰ However, by process of elimination, the factor prices must account for most of the change.

Is it possible that the principal reason for the decline in the natural borrowing limit is price changes? To check this, consider how the net present value of guaranteed retirement income, a close proxy for the natural limit, changes for young households in the deepest possible recession:

$$\frac{\sum_{t=46}^T (\prod_{j=2}^{t-1} q_j \rho_{s_t}) w_t \kappa_G}{\sum_{t=46}^T (\prod_{j=2}^{t-1} \bar{q}_B \rho_{s_t}) w \kappa_G} = .66 \quad (37)$$

where w_t and q_t reflect the lowest K , N and z possible for all t . Clearly, the decline is large: 34%.⁵¹ A key feature here is that in a protracted recession, capital decumulation causes both

⁵⁰The `erf` function in Intel Fortran (and Matlab) which I use to compute the normal cdf returns a value of zero for large negative numbers. As a consequence, the normal cdf used in the method of Tauchen (1986) essentially draws from a bounded normal with a support of around 8 standard deviations. Because of this and countercyclical earnings variance, the support effectively increases in recessions and households can reach the lowest efficiency state (slightly) faster in the business cycle.

⁵¹If one does the same exercise for the lowest K/N and z , one finds 49%. This is a less intuitive (because labor is not high in recessions) but perhaps better number to look at. The interpolation procedure ends up placing some weight on this state because the grid for N only has two points. This would make it seem that the grid matters a lot. However, I show in a robustness exercise that this is not the case. See Table 15.

w and q to decline. This lowers the natural limit for any age but especially so for young households because the decline in q is compounded.

It is worth briefly interpreting this result. When default is eliminated, creditors extend any amount of debt at a risk-free rate. While creditors offer any amount, households avoid taking on debt beyond what they can repay in the worst case scenario. In the steady state economy, the worst case scenario is bad efficiency shocks forever. In the business cycle, the possibility of a protracted recession makes the worst case scenario worse: households must now be prepared for the worst efficiency shocks *and* the worst wages and the highest interest rates (lowest bond prices). Hence they must further limit their debt exposure. This is the why the natural limit contracts.

While the ND economy's limit contracts sharply after including the business cycle, credit in the CD economy is roughly the same. To understand why, consider Figure 5, which plots the value from default V^D and repayment V^R for a typical newborn household before and after the inclusion of aggregate uncertainty. The presence of aggregate risk makes the utility from repaying fall. Else equal, this makes households more prone to default. However, the value from default also falls, making households less prone to default. What matters for credit is the difference between how much these fall. If the value of default falls less, as is the case here, then households default more and credit contracts. If however the value of repayment falls less, then households default less and credit expands. The slight credit contraction observed in Figure 4 indicates that the value of default falls less on average than the value of repayment.

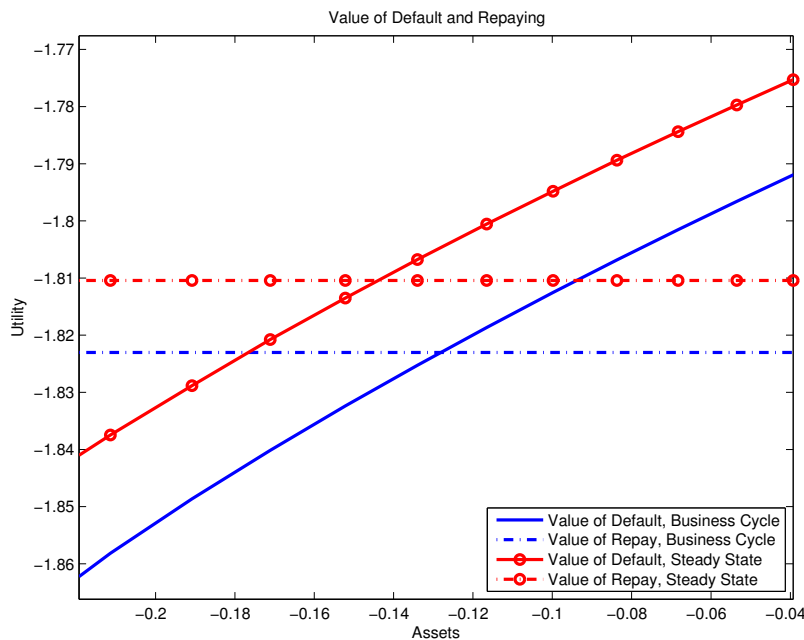


Figure 5: Value from Default and Repayment

The changes in credit markets, and specifically the sharp contraction in the ND economy,

affect consumption smoothing. This is made clear in Figure 6 which plots the variance of log consumption for young households. Both with and without aggregate uncertainty, the ND economy tends to have lower consumption variance. However, including the business cycle substantially increases the variance in the ND economy while leaving the CD economy’s variance roughly unchanged.

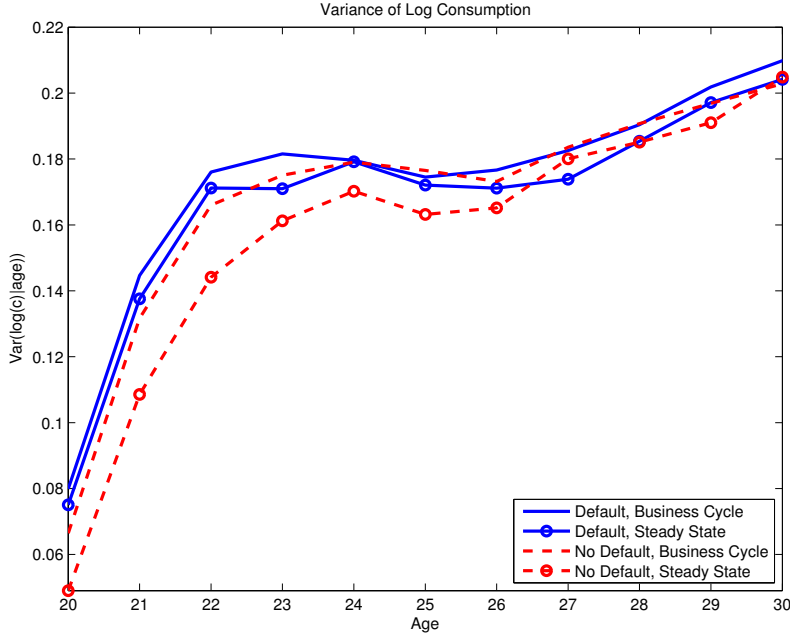


Figure 6: Variance of Log Consumption By Age

These results suggest eliminating default may not be as welfare-improving in the business cycle context as it appeared in steady state. To assess this, I now measure the welfare gain of eliminating default. As before, I ask what increase in consumption in the CD economy would be required to make a newborn household indifferent between the CD and ND economies; however, this is now slightly more involved to compute. In the computational work, the method of Krusell and Smith (1998) is used with linear interpolation of the aggregate moments (i.e. summary statistics of the distribution). Because of this, the law of motion can be thought of as a transition matrix with an implied invariant distribution over aggregate states. I use this long-run distribution over states to “integrate out” the aggregate state from the value functions and calculate welfare in the following way:

$$\omega = \left(\left(\frac{\sum_{z,s,m} F(z) \hat{F}(s|z) \int V_{ND}(0, e, s, 0; z, m) \Pi_{ND}(m|z) \hat{f}(e|s, z) de}{\sum_{z,s,m} F(z) \hat{F}(s|z) \int V_{CD}(0, e, s, 0; z, m) \Pi_{CD}(m|z) \hat{f}(e|s, z) de} \right)^{1/(1-\sigma)} - 1 \right)$$

where m denotes a moment and Π_X denotes the invariant distribution in economy X . As before, I also report the population in favor of the policy change and break out the welfare

gains by age.⁵²

Table 5 reports the welfare gains of eliminating default. Aggregate risk reduces the welfare gain of eliminating default to 0.49% of lifetime consumption, a decline of 1.33% from the steady state counterpart 1.82%. In partial equilibrium, i.e. holding fixed the law of motion and prices r, w and \bar{q}_B , the drop is larger at 1.69% with the gain going from 3.94% to 2.25%. Support for eliminating default increases slightly due to general equilibrium effects (recall the ND economy’s average capital is about 1% higher in the business cycle). In partial equilibrium the support drops from 100% to 80.3%.

	Aggregate Risk	
	Yes	No
General Eq.		
Welfare Gain (%)	0.49	1.82
Population in Favor (%)	57.0	56.8
Partial Eq.		
Welfare Gain (%)	2.25	3.94
Population in Favor (%)	80.3	100.0

Table 5: Welfare Gains of Eliminating Default

To aid understanding of how these gains are distributed over the life cycle, Figure 7 plots average gains by age. Consider first the gains in partial equilibrium. Note that aggregate risk causes a uniform decline in the gains with the largest drops occurring for young households. This shows the effect of credit becoming less effective in the ND economy. Now consider general equilibrium. As before, young households experience a significant welfare decline from eliminating default. However some households, specifically middle-aged households and very old retired households, have the opposite experience.

Figure 8 confirms and extends these findings. Using circles it plots those households whose views of default have *improved*—that is their welfare gain of eliminating default has decreased—by more than 5%, and using dots it plots households whose views have *deteriorated* by more than 5%.⁵³ Those who view default less favorably are concentrated among old households who are earnings and asset poor. These are households whose earnings are no longer mean-reverting, thus making credit ineffective for self-insurance, and for whom the

⁵²Because the mapping from aggregate moments to distributions is not one-to-one, calculating welfare for households who are not newborn—and hence for whom the joint distribution over wealth, efficiency, and credit history is not known—is not straightforward. To get a rough idea of the changes in welfare, I do the following: define $\tilde{V}_X(a, e, s, h) = \sum_{z, m} F(z) \Pi_X(m|z) V_X(a, e, s, h; z, m)$; define $\tilde{d}(a, e, s, h) = \max_{z, m} d(a, e, s, h; z, m)$; define $\omega(a, e, s, h) = ((\tilde{V}_{ND}(a, e, s, h) / \tilde{V}_{CD}(a, e, s, h = 0))^{1/(1-\sigma)} - 1)$; and compute the welfare gain for a subset I of the population as $\int_I ((1 - \tilde{d}(a, e, s, h)) \omega(a, e, s, h) + \tilde{d}(a, e, s, h) \omega(0, e, s, 0)) d\mu$ where μ is the *average* distribution along the simulated path of the CD economy. The population in favor is $\int ((1 - \tilde{d}(a, e, s, h)) \mathbf{1}[\omega(a, e, s, h) > 0] + \tilde{d}(a, e, s, h) \mathbf{1}[\omega(0, e, s, 0) > 0]) d\mu$.

⁵³Regions of the state space are only plotted if they are visited with positive probability.

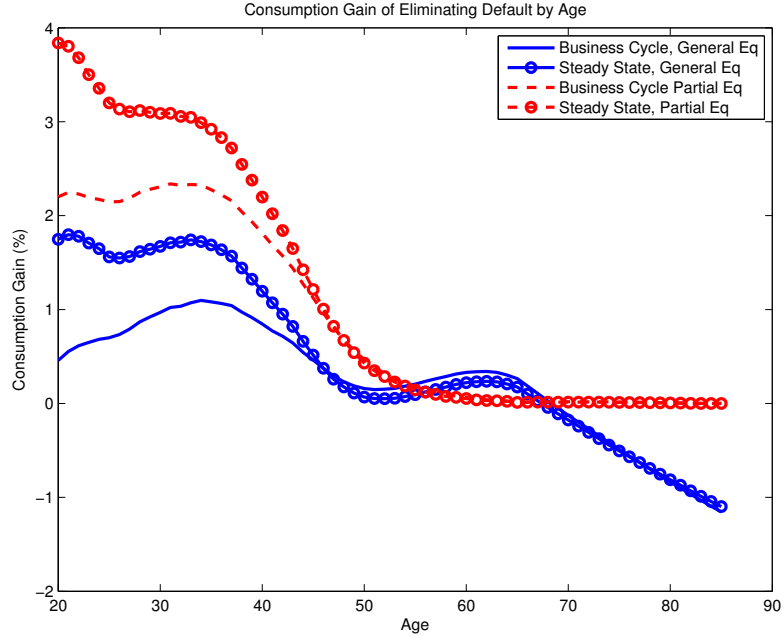


Figure 7: Welfare Gain By Age with and without the Business Cycle

value of labor income in retirement is especially important; since the no-default economy has higher wages in the business cycle, they like default less. Those who view default more favorably are concentrated among the young to middle aged, those with below-median earnings, and the asset poor. These are households for whom credit, and a lot of it, is especially important: credit is important because their future labor income is high in expectation, and a lot of credit is important because they may already be near the CD borrowing limit or have highly-persistent shocks. Absent aggregate risk, they were able to (and did) borrow large amounts in the ND economy to insure themselves. With aggregate risk, this is no longer an option.

One might be tempted to conclude the welfare loss and contraction of credit in the ND economy occurs because the natural borrowing limit is a special case. It definitely is a special case. However, aggregate risk causes economies with high levels of commitment to experience similar contractions of credit and reductions in welfare. To show this, I computed equilibrium for different levels of the default cost χ . The welfare gain of moving from the CD economy to each of these is reported in Table 6.

The business cycle significantly reduces the welfare gain of making default more costly for each level of χ greater than its benchmark value $\chi = .12$. Whereas the steady state gain from high default-cost regimes reaches as much as 7.5%, the gain in the business cycle never crosses 5.6%. Moreover, the regime with the highest gain in steady state is $\chi = .90$ while in the business cycle it is lower at .75. Paired with the reduced welfare gain in high default-cost regimes is a reduction in the amount of debt and an increase in the capital output-ratio. These results show that the qualitative features from making default infinitely costly (as it

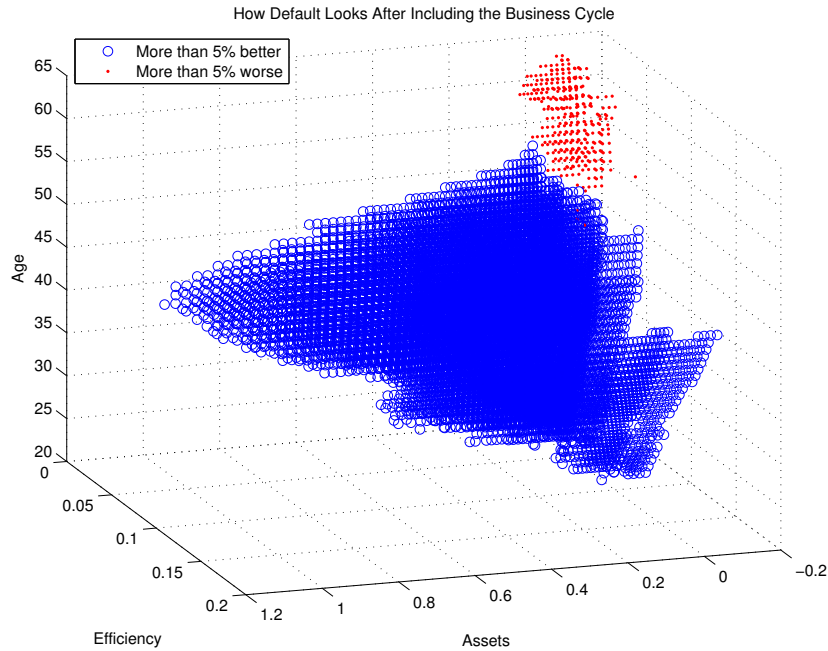


Figure 8: Business Cycle Effect on Welfare Gain of Eliminating Default

CD →	$\chi =$						$\chi = 1 - 10^x, x =$			ND
	.00	.25	.50	.75	.90	.99	-4	-8	-20	
Welfare										
Welfare Gain (%) SS	-0.9	1.9	5.3	7.4	7.5	7.0	6.5	4.9	1.8	1.8
Welfare Gain (%) BC	-0.7	1.8	4.6	5.6	4.2	4.1	3.9	3.6	0.5	0.5
Pop in Favor (%) SS	47.1	50.5	49.1	46.6	45.1	46.7	49.0	51.9	56.8	56.8
Pop in Favor (%) BC	44.7	49.6	48.9	44.6	39.7	41.3	44.1	48.9	57.0	57.0
Allocations										
Debt/Y × 100 SS	0.4	1.9	7.2	16.3	23.0	19.5	13.6	9.1	4.4	4.4
Debt/Y × 100 BC	0.4	1.5	5.6	12.0	14.4	11.5	7.8	5.7	2.7	2.7
Pop in Debt (%) SS	4.3	16.3	24.7	32.3	35.7	34.3	31.1	27.0	20.8	20.8
Pop in Debt (%) BC	3.5	14.9	22.0	28.2	29.5	27.8	24.9	22.3	16.5	16.5
Pop Filing (%) SS	1.04	.07	.04	.03	.02	8e-4	2e-6	6e-11	0	0
Pop Filing (%) BC	1.03	.10	.07	.04	.02	6e-4	1e-6	3e-9	0	0
K/Y SS	3.10	3.06	2.98	2.87	2.81	2.84	2.89	2.94	3.00	3.00
K/Y BC	3.09	3.06	3.00	2.93	2.91	2.93	2.96	2.99	3.03	3.03

Note: SS means “steady state” and BC means “business cycle.”

Table 6: Welfare Gains of Making Default More Costly

is in the ND economy) extend to the case of just making it more costly.

5 Restricting Default

The previous section analyzed how aggregate risk affected the consequences of eliminating default. I now consider how aggregate risk affects the consequences of restricting default. I consider two restrictions on default. The first is modeled after the 2005 reform, and the second only allows default in recessions or expansions.

5.1 The BAPCPA Reform

The Bankruptcy Abuse Prevention and Consumer Protection Act of 2005 (BAPCPA) is an important example of restricting default. The reform has made it typically impossible for households to file for Chapter 7 bankruptcy if they make above median income in their state.⁵⁴ At the same time, it did not eliminate debt forgiveness for these households because they may still file for Chapter 13. This chapter of the bankruptcy code allows households to obtain partial debt forgiveness by forfeiting future income for a number of years (with the benefit being that households may keep their assets).

This legal environment is mapped into the model in the following way. First, households with efficiency less than $\psi_z \tilde{e}$, where \tilde{e} is the median efficiency in steady state, may default as before with a fraction χ of their earnings transferred to creditors. Second, households with efficiency above $\psi_z \tilde{e}$ may forfeit a fraction $\chi_{13} \gg \chi$ of their earnings to creditors in the period of default in exchange for all their debt.⁵⁵ They subsequently enter into bad standing ($h = 1$) with a zero net asset position. These assumptions imply contract pricing in steady state is

$$q(a', s) = \bar{q}_B \rho_s \mathbb{E}_s(1 - d(a', e', s', 0) + d(a', e', s', 0)(\mathbf{1}_{e' \leq \tilde{e}} \chi + \mathbf{1}_{e' > \tilde{e}} \chi_{13}) w e' / (-a')), \quad (38)$$

with similar pricing in the business cycle. I set $\chi_{13} = .75$ to loosely represent a 15% annual contribution of earnings over the course of 5 years.⁵⁶

⁵⁴There are ways to get around this, especially through quitting your job, since the law treats “income” as average income over the six months prior to filing. The law also introduced other changes (increased filing costs of Chapter 7 bankruptcy, changed how much of disposable income must be used for Chapter 13 repayments, and others) which I do not consider. For a comprehensive list of changes, see [White \(2007\)](#).

⁵⁵This formulation uses a below-median earnings test rather than a below-median income test. However, because a in the model is net worth, $a < 0$ can be thought of zero assets and $-a$ debts implying households have no capital income. A satisfactory modeling of Chapter 13 discharge would require separating assets from debts, allowing for endogenous labor choice, and having debt-level contingent repayment plans as is done in [Li and Sarte \(2006\)](#). This would take the present paper too far afield. The approach I use captures an important feature of Chapter 13, namely that debt is forgiven in exchange for a substantial portion of income, while maintaining tractability.

⁵⁶In the law, households in this situation would have to contribute 100% of “disposable income,” income exceeding allowances defined by the IRS, for 5 years. In terms of what creditors get, χ_{13} might be too low as earnings will tend to mean-revert. However it is not clear this is too low as labor supply might be reduced in

What are the positive effects of BAPCPA? Table 7 reports the capital-output ratio and debt and filing statistics after the policy change alongside their CD and ND counterparts. While BAPCPA roughly doubles total filings, it lowers filings of above-median earnings households by around 80%. Somewhat surprisingly, once aggregate risk has been accounted for, the BAPCPA economy has the most households in debt, the lowest capital-output ratio, *and* the most filings.⁵⁷ However, the debt-output ratio is less than in the ND economy because it contracts more in the presence of aggregate risk.

Economy	K/Y	Debt/Y	% in Debt	% Filing	% Filing $e \leq \psi_z \tilde{e}$
Steady State					
CD	3.08	.0069	10.5	0.18	0.09
ND	3.00	.0437	20.8	0.00	0.00
BAPCPA	3.00	.0581	20.4	0.39	0.37
Business Cycle					
CD	3.08	.0053	9.0	0.26	0.16
ND	3.03	.0272	16.5	0.00	0.00
BAPCPA	3.01	.0197	18.2	0.45	0.43

Table 7: Effects of Policy Changes on Allocations

To understand these effects, it is useful to look again at the average borrowing limits. These are plotted in Figure 9 for the BAPCPA, CD and ND economies. The most striking feature is that the BAPCPA limit is hump-shaped and, until around age 55, larger than the limit in both the CD and ND economy. In fact once the business cycle is accounted for, newborn households in the BAPCPA economy can borrow more than twice the natural limit. It is also clear that the BAPCPA reform causes credit to expand more sharply for middle-aged households than for other households.

Why does the reform cause credit to expand, and why is this effect strongest for middle-aged households? The key insight for answering these questions is that an increase in default costs has both a direct and indirect effect. The direct effect is that when default is more costly, households are naturally less likely to default, and this expands credit. The indirect effect is that, when credit opportunities are better in the *future*, households are less likely to default now because they do not want to lose access to this expanded credit: by defaulting, a household would lose the ability to borrow for 10 years on average. At every age, the direct

the 5 year period or households might temporarily lower their income and file for Chapter 7. In terms of the utility cost to defaulters, χ_{13} might be too high as the default period is typically one of low consumption.

⁵⁷Clearly these are testable long-run implications. While the most recent recession has likely contributed to the filing numbers, it does appear Chapter 7 filings are returning to their pre-2005 levels. See Figure 11 in Appendix B. Note that although the filings nearly double, filings in the model are only caused by earnings shocks. If shocks resulting in large amounts of debt, such as uninsured hospital bills or lawsuits, were included in the model, the filings would likely not rise as much: a household's only option might be to file regardless of how costly it is.

effect will be present for *some* households, specifically those making above-median earnings who now must pay χ_{13} rather than χ when they default. However, because of a hump-shaped earnings profile, the direct effect will be present for *most* middle-aged households. This causes a large expansion in credit for these households. At the same time, the indirect effect is strongest for young households. These households look forward and see a large expansion in credit in middle age. This makes them much less prone to default.

This argument also suggests a reason for why the BAPCPA reform increases total filings but decreases filings of above-median earnings households. The direct effect of higher default costs makes households less likely to default. However, this direct effect only applies to some households. At the same time, all households enjoy expanded credit, and, consequently, become more indebted. Because the benefit from default is increasing in indebtedness, households whose default costs have not increased end up defaulting more often.

An important but less obvious feature in Figure 9 is how aggregate risk causes the debt limit to contract in the BAPCPA economy. The largest contractions are seen for middle-aged households, for whom the median-earnings restriction tends to bind, while for younger households or those approaching retirement the contraction is virtually nonexistent. Consistent with the findings for the high default-cost regimes in Table 6, the business cycle causes credit to contract when default is costly. However, here, default is only more costly for the earnings rich. These households experience worse credit in the business cycle while credit for the earnings poor is roughly unchanged.

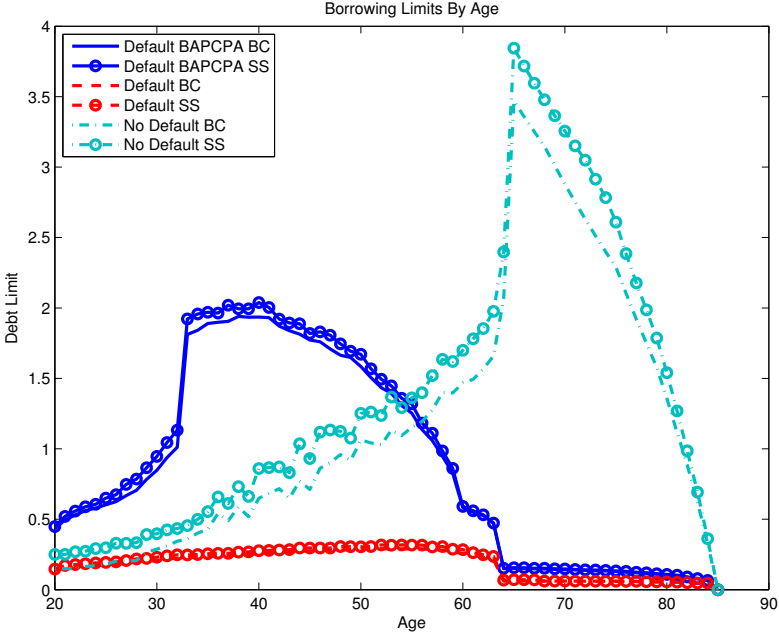


Figure 9: Borrowing Limits with the BAPCPA Reform

Was restricting access to default through the 2005 reform a good idea from a welfare perspective? The models with and without aggregate uncertainty suggest so. The top half of

Table 8 lists the welfare gains of moving from the CD economy to the BAPCPA one. With or without aggregate uncertainty there is a large gain of around 2%. In fact, the gain is slightly *higher* after accounting for aggregate risk. Despite these seemingly large improvements in welfare, there is still substantial disagreement with only a slight majority favoring the change.

	Aggregate Risk	
	Yes	No
CD to BAPCPA		
Welfare Gain (%)	2.00	1.95
Population in Favor (%)	52.7	51.2
BAPCPA to ND		
Welfare Gain (%)	-1.49	-0.13
Population in Favor (%)	45.8	78.1

Table 8: Welfare Gains of the 2005 Reform

One might ask, now that bankruptcy has been restricted, should the law go further and eliminate default altogether? The model suggests not. The bottom portion of Table 8 reports the welfare gains of moving from the BAPCPA economy to the ND economy. Absent aggregate uncertainty, this policy change results in a welfare *loss* of 0.1%. Despite this, around 80% of households prefer the move. Once the business cycle is included, this majority disappears and the welfare loss deepens to 1.5% of lifetime consumption.

The key difference between eliminating default and restricting default through the 2005 reform is that the reform is targeted: it gives those with little earnings, from whom creditors would not receive much anyway, a fresh start. At the same time, those with more ability to pay back must repay. In this way, the BAPCPA reform preserves most of the insurance value of default while substantially improving credit.

5.2 Aggregate-State Contingent Default Policy

I now consider a different type of default restriction that allows default only in recessions. As discussed in the introduction, this is meant to capture the pattern of legislation responding to negative aggregate shocks with more debt forgiveness and to positive shocks with less. For completeness, I also consider the case where default is only allowed in expansions. The policy that allows default in recessions ($z = b$) is referred to as CDR; the policy where default is only allowed in expansions ($z = g$) is referred to as CDE.

A priori, one might expect that either economy would look like a convex combination of the CD and ND economies. Surprisingly, the aggregate statistics reported in Table 9 do not bear this out. In particular, the CDR and CDE economies have the highest capital-output ratio and the least amount of debt.

The proximate cause of this lack of debt is revealed in Figure 10 which plots borrowing limits for the CD, ND, and CDR economies (the CDE and CDR limits look very similar).

Economy	K/Y	Debt/Y	% in Debt	% Filing
CD	3.08	.0053	9.0	0.26
ND	3.03	.0272	16.5	0.00
CDR	3.09	.0036	7.8	0.12
CDE	3.09	.0017	6.3	0.03

Table 9: Effects of Policy Changes on Allocations with Aggregate Risk

In the CDR economy, the borrowing limit for newborn households is virtually zero and until age 63 lies below the CD economy’s limit. In retirement, the limit is higher than in the CD economy, but it is still much smaller than in the ND economy.

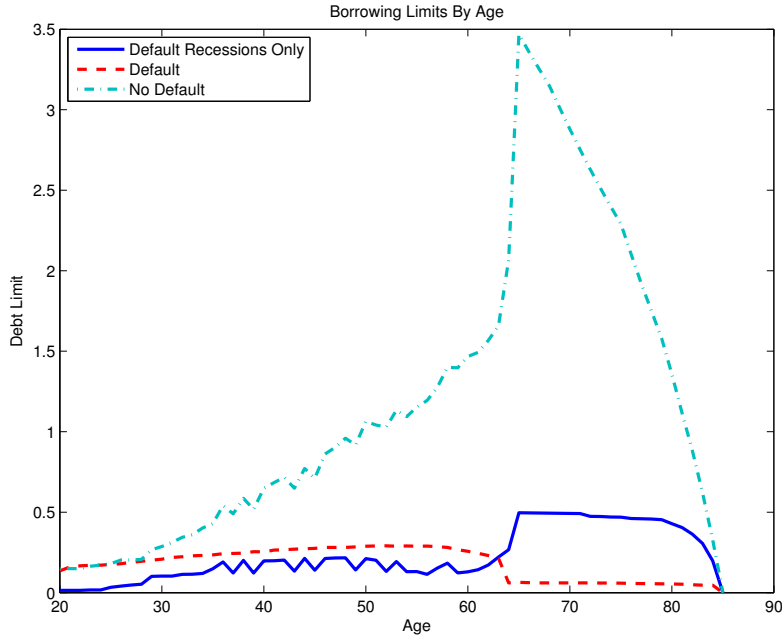


Figure 10: Borrowing Limits with Default in Recessions Only

To understand why the CDR limit is so tight, first note that in one possible scenario, expansion for a lifetime, households will never be allowed to default. Consequently, a natural borrowing limit is in effect because households will not borrow more than the net present value of the lowest possible earnings stream conditional on this sequence of aggregate shocks. Now note that households with the lowest possible earnings stream are very prone to default if a recession occurs. This fact, together with $\bar{q}_g(\mathcal{S}) \approx \bar{q}_B(\mathcal{S})F(g|g)$ in the calibrated model, implies the price of a bond for these households is approximately 2/3 of the risk-free price $\bar{q}_B(\mathcal{S})$.⁵⁸ Because they must borrow at this low price, the net present value of the worst

⁵⁸Let \underline{s} denote the persistent state associated with the worst earnings stream. Then $p(a', \underline{s}; \mathcal{B}) \approx 0$ for

	$z = b$	$z = g$
CD to Default in z Only		
Welfare Gain (%)	-1.40	-1.41
Population in Favor (%)	54.4	47.7
ND to Default in z Only		
Welfare Gain (%)	-1.90	-1.91
Population in Favor (%)	42.5	45.7

Table 10: Welfare Gains of the z -contingent Default Policy

earnings stream is drastically lowered: a unit of labor income from age 65 is worth roughly $.4 = .98^{45}$ at age 20 when discounted by $\bar{q}_B(\mathcal{S})$; this drops to only $.5 \times 10^{-8} = (.98 * 2/3)^{45}$ when discounted by $\bar{q}_B(\mathcal{S}) * 2/3$. Because future earnings are discounted heavily, and because hardly any earnings are guaranteed until retirement, the limit in the CDR economy is very tight and especially so for young households.

Given the tight borrowing constraints, one would expect that both the CDR and CDE economies are worse than either the CD or ND economy in terms of welfare. This is confirmed in Table 10 which reports the welfare gains of allowing default in recessions or expansions only. Relative to the CD economy, either policy change results in a welfare loss of around 1.4%; relative to the ND economy, the loss is higher at 1.9%. The move to default only in recessions is viewed more favorably than a move to default only in expansions, but the difference is slight.

One caveat to these results deserves mention. In this model (and, to my knowledge, in all the quantitative literature), the default-risk premium is paid entirely upfront. This modeling assumption matters here because each period households pay the premium even if they never default. If losses due to default were recouped within each period, the results would likely be different.

6 Conclusion

It was found that aggregate risk substantially lowers the welfare benefit of eliminating default. Without the ability to discharge debt, households limit their debt to what they can repay in the worst of circumstances. A business cycle makes the worst circumstances worse by introducing the possibility of a protracted recession. This possibility causes households to limit their debt more than in steady state unless they have a default option, reducing their ability to self-insure. When default is allowed, households can safely use all available credit, even with the possibility of a protracted recession, by defaulting if such a recession occurs. This benefit of having a default option is, by necessity, missing in a steady state environment.

virtually any $a' < 0$ because households are very likely to default. Using this and $\bar{q}_g(\mathcal{S}) \approx \bar{q}_B(\mathcal{S})F(g|g)$, one has $q_g(a', \underline{s}; \mathcal{S}) + q_b(a', \underline{s}; \mathcal{S}) = \bar{q}_g(\mathcal{S}) + \bar{q}_b(\mathcal{S})p(a', \underline{s}; \mathcal{B}) \approx \bar{q}_g(\mathcal{S}) \approx \bar{q}_B(\mathcal{S})F(g|g) = 2/3 \bar{q}_B(\mathcal{S})$.

While aggregate risk reduces the welfare benefit of eliminating default, aggregate risk's effect on restricting default depends on the restrictions in place. A policy change resembling the 2005 reform substantially increased welfare, and, unlike eliminating default, improved welfare *more* in the business cycle environment. A key aspect of the policy is that it only makes default more costly for some households, those with above-median earnings. This substantially improves credit while preserving most of the insurance value of default by allowing earnings-poor households to easily default.

In contrast, a different type of default restriction, one that allows default only in recessions or expansions, introduced uncertainty that resulted in substantially lower welfare. Specifically, it introduced a small probability that households would never be able to default. This kept households from borrowing more than what they could repay with probability one. At the same time, it introduced a fairly high probability that households would be able to default. This made credit expensive. Consequently, the economy was worse than always having default, in which case households can rely on the default option, and worse than never having default, in which case credit is cheap.

The welfare gains from restrictive default regimes in this paper should be viewed as upper bounds for a number of reasons. First, this paper treated aggregate risk as a productivity shock, countercyclical earnings variance, and a shock to aggregate labor supply. This is a narrow view. More broadly, aggregate risk could include risk to entitlement programs as well as deflationary risk. These types of aggregate risk would likely reduce the welfare gains of harsh default regimes. Second, as experience has shown, regardless of legislation some households will always default. Severely punishing these households can entail significant social and fiscal costs which are not modeled here. Third, the only idiosyncratic risk in this paper came from earnings shocks. Including expenditure shocks, such as uninsured medical expenses, would also likely reduce the welfare gains from harsh regimes.

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A Extended Model Description

This appendix contains extended model descriptions for both the steady state and business cycle models. The objective is to arrive at the simplified definitions of equilibrium presented in the main text.

A.1 Steady State Model

A.1.1 The Intermediary’s Problem

The financial intermediary’s problem is to maximize the net present value of financial wealth using contract holdings, capital, and a bond. This present value is discounted using the bond’s return. It is useful to think of a contract as a pair (a', s) and the intermediary’s portfolio of contract holdings as a mapping $l : A \times S \rightarrow \mathbb{R}$. To aggregate the intermediary’s contract holdings, I introduce two functions

$$C(m) := \sum_{a, s_{-1}} m(a, s_{-1}) \rho_{s_{-1}} p(a, s_{-1}) a \quad (39)$$

and

$$C'(m) := \frac{1}{\bar{q}_B} \sum_{a',s} m(a',s)q(a',s)a'. \quad (40)$$

In this notation, $C(l)$ gives the aggregate yield of portfolio l while $C'(l)$ gives the aggregate price of the portfolio l deflated by the bond's price (ensuring $C(l) = C'(l)$ in equilibrium). With this notation, the intermediary's problem can be written

$$\begin{aligned} P(K, B, l) = \max_{K' \geq 0, B', l'} & D + \bar{q}_B P(K', B', l') \\ D + \bar{q}_B C'(l') + K' + \bar{q}_B B' = & C(l) + K(1 + r - \delta) + B \end{aligned} \quad (41)$$

Let the policy functions for the intermediary be denoted $K'(K, B, l)$, $B'(K, B, l)$, and $l'(K, B, l)$.

A.1.2 Equilibrium

A steady state equilibrium is a collection of prices r, w, q, \bar{q}_B , repayment rates p , policy functions c, a', d, K', B', l' , value functions V, P , aggregates K, N, B, l , and a distribution of households μ such that all of the following hold:

1. Household policies c, a', d solve their problem.
2. Financial intermediary policies K', B', l' solve their problem.
3. Capital holdings are positive: $K > 0$.
4. Factor prices are competitive (ensuring the production firm optimizes): $r = \alpha(K/N)^{\alpha-1}$ and $w = (1 - \alpha)(K/N)^\alpha$.
5. The intermediary has zero net financial income: $C(l) + K(1 + r - \delta) + B = 0$.
6. The labor market clears: $N = \int e d\mu$.
7. The bond market clears: $B' = 0$.
8. Each contract market clears:

$$-l'(K, B, l)(a', s) = \int \mathbf{1}[a' = a'(a, e, s, h)]\mu(da, de, s, dh) \text{ for all } a', s.$$

9. The goods market clears.
10. Repayment probabilities are consistent:

$$p(a', s) = \mathbb{E}_s(1 - d(a', e', s', 0) + d(a', e', s', 0)\chi w e' / (-a')).$$

11. The model is in steady state: $K' = K, B' = B, l' = l$, and μ is consistent with household policies and stochastic transitions.

A.1.3 Equilibrium Characterization

I now characterize any equilibrium.⁵⁹ First consider the first order conditions from the firm's problem:

$$1 = \bar{q}_B(1 + r - \delta) + \theta \quad (42)$$

$$\bar{q}_B = \bar{q}_B \quad (43)$$

$$q(a', s)a' = \bar{q}_B \rho_s p(a', s)a' \quad (44)$$

where $\theta \geq 0$ is the Lagrange multiplier on the constraint $K' \geq 0$. For $a' \neq 0$, this gives the price schedule from the main text

$$q(a', s) = \bar{q}_B \rho_s p(a', s) \quad (45)$$

in the case of $K' > 0$ which is required in equilibrium. This price schedule can also be used for $a' = 0$ as a loan to repay nothing with a zero price satisfies the intermediary's first order condition.

Proposition 1. *Aggregate capital can found from the distribution as the unique K solving*

$$(1 + r - \delta)K = \int (a + d(a, e, s, h)(-a - \chi w e)) d\mu$$

where $r = \alpha(K/N)^{\alpha-1}$ and $w = (1 - \alpha)(K/N)^\alpha$ (with $N = \int e d\mu$).

Proof. Define

$$\tilde{p}(a', s') := \int_{e'} (1 - d(a', e', s', 0) + d(a', e', s', 0)\chi w e' / (-a')) f(e'|s') de'$$

so that $p(a', s) = \sum_{s'} F(s'|s)\tilde{p}(a', s')$.

The zero-profit condition of the firm requires $C(l) + (1 + r - \delta)K + B = 0$ and bond

⁵⁹Chatterjee et al. (2007) prove that an equilibrium exists for the environment with $\chi = 0$ and ρ_s age-invariant when the production function satisfies certain conditions.

market clearing requires $B = 0$. Consequently

$$(1 + r - \delta)K = \sum_{a',s} -l(a', s)\rho_s p(a', s)a' \quad (46)$$

$$= \sum_{a',s} \left(\sum_{a,h} \int \mathbf{1}_{a'=a'(a,e,s,h)} \mu(a, de, s, h) \right) \rho_s p(a', s)a' \quad (47)$$

$$= \sum_{a'} \sum_{a,s,h} p(a', s)a' \int \rho_s \mathbf{1}_{a'=a'(a,e,s,h)} \mu(a, de, s, h) \quad (48)$$

$$= \sum_{a'} \sum_{a,s,h} \left(\sum_{s'} F(s'|s) \tilde{p}(a', s') \right) a' \int \rho_s \mathbf{1}_{a'=a'(a,e,s,h)} \mu(a, de, s, h) \quad (49)$$

$$= \sum_{a',s'} \tilde{p}(a', s') \sum_{a,s,h} F(s'|s) a' \int \rho_s \mathbf{1}_{a'=a'(a,e,s,h)} \mu(a, de, s, h) \quad (50)$$

$$= \sum_{a',s'} \tilde{p}(a', s') a' \int \rho_s F(s'|s) \mathbf{1}_{a'=a'(a,e,s,h)} d\mu(a, e, s, h) \quad (51)$$

where (46) uses the definition of $C(\cdot)$, (47) uses contract market clearing, (48) just rearranges, (49) uses the definition of p and \tilde{p} , and (50)-(51) rearrange again. Now because households are born with zero assets, $a' \mu(a', s') = a' \int \rho_s F(s'|s) \mathbf{1}_{a'=a'(a,e,s,h)} d\mu$ for any a', s' . Using this fact,

$$(1 + r - \delta)K \quad (52)$$

$$= \sum_{a',s'} \tilde{p}(a', s') a' \mu(a', s') \quad (53)$$

$$= \sum_{a',s'} \tilde{p}(a', s') a' \sum_{h'} \mu(a', s', h') \quad (54)$$

$$= \sum_{a',s',h'} \tilde{p}(a', s') a' \mu(a', s', h') \quad (55)$$

$$= \sum_{a',s',h'} \left(\int_{e'} (1 - d(a', e', s', 0) + d(a', e', s', 0) \chi w e' / (-a')) f(e'|s') de' \right) a' \mu(a', s', h') \quad (56)$$

$$= \sum_{a',s',h'} \left(\int_{e'} ((1 - d(a', e', s', 0)) a' - d(a', e', s', 0) \chi w e') f(e'|s') \mu(a', s', h') de' \right) \quad (57)$$

$$= \int ((1 - d(a', e', s', 0)) a' - d(a', e', s', 0) \chi w e') d\mu(a', e', s', h') \quad (58)$$

where (53) substitutes, (54) uses a probability measure property, (55) rearranges, (56) uses the definition of \tilde{p} , (57) rearranges, and (58) uses the property $\mu(a, e, s, h) = f(e|s)\mu(a, s, h)$ (by a law of large numbers).

I also claim K is unique. This is clear once one notices w and $rK(= \alpha Y)$ are both increasing in K .

□

Using these characterizations, the simplified definition of equilibrium in the main text is readily obtained.

A.2 Business Cycle Model

A.2.1 The Intermediary's Problem

The financial intermediary's problem is to maximize the net present value of financial wealth using contracts, bonds, capital, and an aggregate-complete set of Arrow securities. He discounts future wealth using the state-contingent prices $\bar{q}_{z'}(\mathcal{S})$ of the Arrow securities. It is useful to think of a contract as a tuple (a', s, z') specifying an amount a' , the counterparty to the contract as summarized in s , and the contingency z' in which the debt (or savings) will be delivered. The intermediary's contract holdings can be thought of as a pair of portfolios $l_{z'} : A \times S \rightarrow \mathbb{R}$, one for each z' . It is useful to introduce two functions that aggregate the prices and yields of these portfolios. For any given portfolio of contract holdings $m : A \times S \rightarrow \mathbb{R}$, define

$$C(m; \mathcal{S}) := \sum_{a, s_{-1}} m(a, s_{-1}) \rho_s p(a, s_{-1}; \mathcal{S}) a \quad (59)$$

and

$$C'_{z'}(m; \mathcal{S}) := \frac{1}{\bar{q}_{z'}(\mathcal{S})} \sum_{a', s} m(a', s) q_{z'}(a', s; \mathcal{S}) a'. \quad (60)$$

$C(m; \mathcal{S})$ is the yield of a portfolio of contracts m and $C'_{z'}(m; \mathcal{S})$ is the price of those contracts normalized by $\bar{q}_{z'}(\mathcal{S})$. This normalization ensures $C'_{z'}(\cdot; \mathcal{S}) = C(\cdot; \mathcal{S}'_{z'})$ in equilibrium.

Using these definitions, the intermediary's problem can be written

$$P(A, B, K, l; \mathcal{S}) = \max_{A'_{z'}, B', K' \geq 0, l'_{z'}} D + \sum_{z'} \bar{q}_{z'}(\mathcal{S}) P(A'_{z'}, B', K', l'_{z'}; \mathcal{S}'_{z'})$$

$$D + \sum_{z'} \bar{q}_{z'}(\mathcal{S}) (C'_{z'}(l'_{z'}; \mathcal{S}) + A'_{z'}) + \bar{q}_B(\mathcal{S}) B' + K' = C(l; \mathcal{S}) + A + B + (1 + r(\mathcal{S}) - \delta) K.$$

Let the policy functions for the intermediary be denoted $A'_{z'}(A, B, K, l; \mathcal{S})$, $B'(A, B, K, l; \mathcal{S})$, $K'(A, B, K, l; \mathcal{S})$, and $l'_{z'}(A, B, K, l; \mathcal{S})$.

A.2.2 Equilibrium

For now, let \mathcal{S} be defined as z together with a distribution $\mu(a, e, s, h)$ of households and l, A, B, K, N satisfying $B = 0$, $K > 0$, $N > 0$, $A = 0$, and $C(l; \mathcal{S}) + A + B + (1 + \alpha(K/N)^{\alpha-1} - \delta)K = 0$. A recursive competitive equilibrium is a collection of price functions $r, w, \bar{q}_B, \bar{q}_g, \bar{q}_b, q_g, q_b$, repayment rates p , policy functions c, a'_g, a'_b, d , value functions V, P , and a law of motion Γ such that all of the following hold:

1. Household policies solve their problem.
2. The financial intermediary's policies solve his problem.

3. Factor prices are competitive (ensuring the production firm optimizes):

$$r(\mathcal{S}) = \alpha(K/N)^{\alpha-1} \quad (61)$$

$$w(\mathcal{S}) = (1 - \alpha)(K/N)^\alpha \quad (62)$$

4. Asset and factor markets clear:

$$N = \int e d\mu \quad (63)$$

$$K'(A, B, K, l; \mathcal{S}) > 0 \quad (64)$$

$$B'(A, B, K, l; \mathcal{S}) = 0 \quad (65)$$

$$A'_{z'}(A, B, K, l; \mathcal{S}) = 0 \quad (66)$$

where A, B, K, l and N are read off \mathcal{S} .

5. Each contract market clears:

$$-l'_{z'}(A, B, K, l; \mathcal{S})(a', s) = \int \mathbf{1}_{\{a'=a'_{z'}(a,e,s,h;\mathcal{S})\}} \mu(da, de, s, dh) \forall a', s, z' \quad (67)$$

6. The goods market clears.

7. Repayment probabilities are consistent:

$$p(a, s_{-1}; \mathcal{S}) = \sum_s \int \left(\frac{1 - d(a, e, s, 0; \mathcal{S}) + d(a, e, s, 0; \mathcal{S}) \chi w(\mathcal{S}) e / (-a)}{d(a, e, s, 0; \mathcal{S})} \right) f(e|s, z) de F(s|s_{-1}, z) \quad (68)$$

for all a', s , and \mathcal{S} (recall \mathcal{S} includes z).

8. The intermediary makes zero profits:

$$C(l'_{z'}; \mathcal{S}'_{z'}) + A'_{z'} + B' + (1 + r(\mathcal{S}'_{z'}) - \delta)K' = 0 \text{ for each } z' \quad (69)$$

$$\sum_{z'} \bar{q}_{z'}(\mathcal{S}) C'_{z'}(l'_{z'}; \mathcal{S}) + \sum_{z'} \bar{q}_{z'}(\mathcal{S}) A'_{z'} + \bar{q}_B(\mathcal{S}) B' + K' = 0 \quad (70)$$

9. The law of motion $\mathcal{S}'_{z'} = \Gamma(z', \mathcal{S})$ is consistent: μ' is generated by household policies and stochastic transitions, N' is given by stochastic transitions, and $l'_{z'}, A'_{z'}, B'$, and K' are given by their policy functions.

A.2.3 Equilibrium Characterization

In this section I give conditions on the allocations of households that greatly simplify the definition of equilibrium. In particular, if these conditions are met then there exist prices and optimal policies of the financial intermediary that satisfy market clearing and consistency conditions.

First, I discuss equilibrium pricing.

Lemma 1. *The intermediary is indifferent over all feasible policies if*

$$\begin{aligned} q_{z'}(a', s; \mathcal{S})a' &= \bar{q}_{z'}(\mathcal{S})\rho_s p(a', s; \mathcal{S}'_{z'})a' \\ \bar{q}_B(\mathcal{S}) &= \bar{q}_g(\mathcal{S}) + \bar{q}_b(\mathcal{S}) \\ 1 &= \bar{q}_g(\mathcal{S})(1 + r(\mathcal{G}) - \delta) + \bar{q}_b(\mathcal{S})(1 + r(\mathcal{B}) - \delta) \end{aligned}$$

Proof. If these hold, then the first order conditions for contract holdings, bonds, and capital are satisfied. The first order condition for Arrow securities is trivially satisfied: $\bar{q}_{z'}(\mathcal{S}) = \bar{q}_{z'}(\mathcal{S})$. Because the problem is linear, the second order conditions are satisfied. \square

I restrict attention to the prices satisfying the above.

Lemma 2. *Equilibrium prices imply $C'_{z'}(l'_{z'}; \mathcal{S}) = C(l'_{z'}; \mathcal{S}'_{z'})$ for any portfolio $l'_{z'}$. Further, $C'_{z'}(-l'_{z'}; \mathcal{S}) = -C'_{z'}(l'_{z'}; \mathcal{S})$.*

Proof. The first part is proved by

$$\begin{aligned} C'_{z'}(l'_{z'}; \mathcal{S}) &:= \frac{1}{\bar{q}_{z'}(\mathcal{S})} \sum_{a', s} l'_{z'}(a', s; \mathcal{S}) q_{z'}(a', s; \mathcal{S}) a' \\ &= \frac{1}{\bar{q}_{z'}(\mathcal{S})} \sum_{a', s} l'_{z'}(a', s; \mathcal{S}) \bar{q}_{z'}(\mathcal{S}) \rho_s p(a', s; \mathcal{S}'_{z'}) a' \\ &= \sum_{a', s} l'_{z'}(a', s; \mathcal{S}'_{z'}) \rho_s p(a', s; \mathcal{S}'_{z'}) a' \\ &=: C(l'_{z'}; \mathcal{S}'_{z'}). \end{aligned}$$

The second part is obvious. \square

Lemma 3. *Zero profits for the intermediary obtain if*

$$\begin{aligned} K' &= (-C'_g(l'_g; \mathcal{S}) + C'_b(l'_b; \mathcal{S})) / (r(\mathcal{G}) - r(\mathcal{B})), \\ B' &= (C'_g(l'_g; \mathcal{S})(1 + r(\mathcal{B}) - \delta) - C'_b(l'_b; \mathcal{S})(1 + r(\mathcal{G}) - \delta)) / (r(\mathcal{G}) - r(\mathcal{B})), \end{aligned}$$

and $A'_g = A'_b = 0$. This portfolio is feasible (and hence by Lemma 1 optimal) if $-C'_g(l'_g; \mathcal{S}) \geq -C'_b(l'_b; \mathcal{S})$.

Proof. For $A'_g = A'_b = 0$, zero profits obtain if

$$\sum_{z'} \bar{q}_{z'}(\mathcal{S}) C'_{z'}(l'_{z'}; \mathcal{S}) + \bar{q}_B(\mathcal{S}) B' + K' = 0 \quad (71)$$

$$0 = C(l'_{z'}; \mathcal{S}'_{z'}) + B' + (1 + r(\mathcal{S}'_{z'}) - \delta) K'. \quad (72)$$

In matrix notation, (71) is equivalent to

$$\begin{bmatrix} C'_g(l'_g; \mathcal{S}) & C'_b(l'_b; \mathcal{S}) \end{bmatrix} \begin{bmatrix} \bar{q}_g(\mathcal{S}) \\ \bar{q}_b(\mathcal{S}) \end{bmatrix} + \begin{bmatrix} K' & B' \end{bmatrix} \begin{bmatrix} 1 \\ \bar{q}_B(\mathcal{S}) \end{bmatrix} = 0. \quad (73)$$

Because prices satisfy

$$\begin{bmatrix} \bar{q}_g(\mathcal{S}) \\ \bar{q}_b(\mathcal{S}) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 + r(\mathcal{G}) - \delta & 1 + r(\mathcal{B}) - \delta \end{bmatrix}^{-1} \begin{bmatrix} \bar{q}_B(\mathcal{S}) \\ 1 \end{bmatrix}, \quad (74)$$

if

$$\begin{bmatrix} C'_g(l'_g; \mathcal{S}) & C'_b(l'_b; \mathcal{S}) \end{bmatrix} = - \begin{bmatrix} B' & K' \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 + r(\mathcal{G}) - \delta & 1 + r(\mathcal{B}) - \delta \end{bmatrix} \quad (75)$$

then (71) becomes

$$- \begin{bmatrix} B' & K' \end{bmatrix} \begin{bmatrix} \bar{q}_B(\mathcal{S}) \\ 1 \end{bmatrix} + \begin{bmatrix} K' & B' \end{bmatrix} \begin{bmatrix} 1 \\ \bar{q}_B(\mathcal{S}) \end{bmatrix} = 0. \quad (76)$$

which holds.

Now if (l'_g, l'_b, K', B') satisfying (75) imply (72), then zero-profits obtain. Note that (75) can equivalently be written

$$C'_g(l'_g; \mathcal{S}) + B' + K'(1 + r(\mathcal{G}) - \delta) = 0 \quad (77)$$

$$C'_b(l'_b; \mathcal{S}) + B' + K'(1 + r(\mathcal{B}) - \delta) = 0. \quad (78)$$

By Lemma 2, $C'_{z'}(l'_{z'}; \mathcal{S}) = C(l'_{z'}; \mathcal{S}'_{z'})$. Therefore (72) holds.

Finally, this is a feasible allocation so long as $K' \geq 0$. Inverting the relationship in (75) gives

$$K' = (-C'_g(l'_g; \mathcal{S}) + C'_b(l'_b; \mathcal{S})) / (r(\mathcal{G}) - r(\mathcal{B})) \quad (79)$$

$$B' = (C'_g(l'_g; \mathcal{S})(1 + r(\mathcal{B}) - \delta) - C'_b(l'_b; \mathcal{S})(1 + r(\mathcal{G}) - \delta)) / (r(\mathcal{G}) - r(\mathcal{B})). \quad (80)$$

Consequently, $K' \geq 0$ is equivalent to $-C'_g(l'_g; \mathcal{S}) \geq -C'_b(l'_b; \mathcal{S})$. □

Proposition 2. Define $\mu'_{z'}$ by

$$\mu'_{z'}(a', s; \mathcal{S}) := \int_{a,e,h} \mathbf{1}_{\{a' = a'_{z'}(a,e,s,h;\mathcal{S})\}} \mu(da, de, s, dh)$$

so that $\mu'_{z'}(a', s; \mathcal{S})$ is the measure of households choosing contract (a', s, z') . Also, define

$$K'(\mu'_g, \mu'_b; \mathcal{S}) := \frac{1}{r(\mathcal{G}) - r(\mathcal{B})} (C'_g(\mu'_g; \mathcal{S}) - C'_b(\mu'_b; \mathcal{S})) \text{ and}$$

$$B'(\mu'_g, \mu'_b; \mathcal{S}) := \frac{1}{r(\mathcal{G}) - r(\mathcal{B})} ((1 + r(\mathcal{G}) - \delta)C'_b(\mu'_b; \mathcal{S}) - (1 + r(\mathcal{B}) - \delta)C'_g(\mu'_g; \mathcal{S})).$$

If allocations satisfy

1. $K'(\mu'_g, \mu'_b; \mathcal{S}) > 0$ and

$$2. B'(\mu'_g, \mu'_b; \mathcal{S}) = 0,$$

and if prices satisfy

$$1. q_{z'}(a', s; \mathcal{S}) = \bar{q}_{z'}(\mathcal{S})\rho_s p(a', s; \mathcal{S}'_{z'}),$$

$$2. \bar{q}_g(\mathcal{S}) = (1 - \bar{q}_B(\mathcal{S})(1 + r(\mathcal{B}) - \delta))/(r(\mathcal{G}) - r(\mathcal{B})), \text{ and}$$

$$3. \bar{q}_b(\mathcal{S}) = (\bar{q}_B(\mathcal{S})(1 + r(\mathcal{G}) - \delta) - 1)/(r(\mathcal{G}) - r(\mathcal{B})),$$

then there exist optimal policies of the financial intermediary that satisfy zero profit conditions and clear Arrow security, capital, bond, and contract markets.

Proof. First note these prices satisfy the conditions in Lemma 1. Hence the intermediary is indifferent over all feasible allocations. Second, specifying $l'_{z'} = -\mu'_{z'}$ for the intermediary clears contract markets. Further, by Lemma 3, policies that specify $K' = (-C'_g(l'_g; \mathcal{S}) + C'_b(l'_b; \mathcal{S}))/r(\mathcal{G}) - r(\mathcal{B})$ and $B' = (C'_g(l'_g; \mathcal{S})(1 + r(\mathcal{B}) - \delta) - C'_b(l'_b; \mathcal{S})(1 + r(\mathcal{G}) - \delta))/r(\mathcal{G}) - r(\mathcal{B})$ and $A'_{z'} = 0$ result in zero profits (and clear Arrow security markets). By the second part of Lemma 2, the K', B' policies are then equivalent to $K' = (C'_g(\mu'_g; \mathcal{S}) - C'_b(\mu'_b; \mathcal{S}))/r(\mathcal{G}) - r(\mathcal{B})$ and $B' = (-C'_g(\mu'_g; \mathcal{S})(1 + r(\mathcal{B}) - \delta) + C'_b(\mu'_b; \mathcal{S})(1 + r(\mathcal{G}) - \delta))/r(\mathcal{G}) - r(\mathcal{B})$. This policy is feasible if $K'(\mu'_g, \mu'_b; \mathcal{S}) > 0$. Further, the bond market clears if $B'(\mu'_g, \mu'_b; \mathcal{S}) = 0$. Hence the specified policies are feasible, optimal, and clear all asset markets. \square

Proposition 3. *Today's capital stock K can be found in equilibrium from the joint distribution μ as the unique K solving*

$$(1 + r(\mathcal{S}) - \delta)K = \int (a + d(a, e, s, h; \mathcal{S})(-a - \chi w(\mathcal{S})e))d\mu$$

where $r(\mathcal{S}) = z\alpha(K/N)^{\alpha-1}$ and $w(\mathcal{S}) = z(1 - \alpha)(K/N)^\alpha$ (and $N = \int e d\mu$).

Proof. The proof is along the same lines as Proposition 1. First, define

$$\tilde{p}(a, s; \mathcal{S}) := \int (1 - d(a, e, s, 0; \mathcal{S}) + d(a, e, s, 0; \mathcal{S})\chi we/(-a))f(e|s, z)de$$

so that $p(a', s; \mathcal{S}'_{z'}) = \sum_{s'} F(s'|s, z')\tilde{p}(a', s'; \mathcal{S}'_{z'})$. Now, note that zero-profits, bond market clearing, and Arrow security market clearing imply $(1 + r(\mathcal{S}'_{z'}) - \delta)K' = -C'(l'_{z'}; \mathcal{S}'_{z'})$. From

this,

$$(1 + r(\mathcal{S}'_{z'}) - \delta)K' = \sum_{a', s} -l'_{z'}(a', s)\rho_s p(a', s; \mathcal{S}'_{z'})a' \quad (81)$$

$$= \sum_{a', s} \left(\sum_{a, h} \int \mathbf{1}_{a'=a'(a, e, s, h; \mathcal{S})} \mu(a, de, s, h) \right) \rho_s p(a', s; \mathcal{S}'_{z'})a' \quad (82)$$

$$= \sum_{a'} \sum_{a, s, h} p(a', s; \mathcal{S}'_{z'})a' \int \rho_s \mathbf{1}_{a'=a'(a, e, s, h; \mathcal{S})} \mu(a, de, s, h) \quad (83)$$

$$= \sum_{a'} \sum_{a, s, h} \left(\sum_{s'} F(s'|s, z') \tilde{p}(a', s'; \mathcal{S}'_{z'}) \right) a' \int \rho_s \mathbf{1}_{a'=a'(a, e, s, h; \mathcal{S})} \mu(a, de, s, h) \quad (84)$$

$$= \sum_{a', s'} \tilde{p}(a', s'; \mathcal{S}'_{z'}) \sum_{a, s, h} F(s'|s, z') a' \int \rho_s \mathbf{1}_{a'=a'(a, e, s, h; \mathcal{S})} \mu(a, de, s, h) \quad (85)$$

$$= \sum_{a', s'} \tilde{p}(a', s'; \mathcal{S}'_{z'}) a' \int \rho_s F(s'|s, z') \mathbf{1}_{a'=a'(a, e, s, h; \mathcal{S})} d\mu(a, e, s, h) \quad (86)$$

where (81) uses the definition of $C(\cdot)$, (82) uses contract market clearing, (83) just rearranges, (84) uses the definition of p and \tilde{p} , and (85)-(86) rearrange again. Now because households are born with zero assets, $a' \mu_{z'}(a', s') = a' \int \rho_s F(s'|s, z') \mathbf{1}_{a'=a'(a, e, s, h; \mathcal{S})} d\mu$ for any a', s', z' where I've used $\mu_{z'}$ to denote the distribution implicit in $\mathcal{S}'_{z'}$. Using this fact,

$$(1 + r(\mathcal{S}'_{z'}) - \delta)K' \quad (87)$$

$$= \sum_{a', s'} \tilde{p}(a', s'; \mathcal{S}'_{z'}) a' \mu_{z'}(a', s') \quad (88)$$

$$= \sum_{a', s'} \tilde{p}(a', s'; \mathcal{S}'_{z'}) a' \sum_{h'} \mu_{z'}(a', s', h') \quad (89)$$

$$= \sum_{a', s', h'} \tilde{p}(a', s'; \mathcal{S}'_{z'}) a' \mu_{z'}(a', s', h') \quad (90)$$

$$= \int ((1 - d(a', e', s', 0; \mathcal{S}'_{z'}))a' - d(a', e', s', 0; \mathcal{S}'_{z'}) \chi w(\mathcal{S}'_{z'}) e') d\mu_{z'}(a', e', s', h') \quad (91)$$

where (88) substitutes, (89) uses a probability measure property, (90) rearranges, and (91) uses the definition of \tilde{p} and the property $\mu_{z'}(a', e', s', h') = f(e'|s', z') \mu(a', s', h')$ (by a law of large numbers). □

B Data and Calibration

B.1 Data

The model was constructed to capture the salient features of Chapter 7 bankruptcy in the US. Figure 11 shows the annual percent of Chapter 7 filings per household from the period 1960 to 2010.⁶⁰ The number of households taking advantage of this bankruptcy provision has

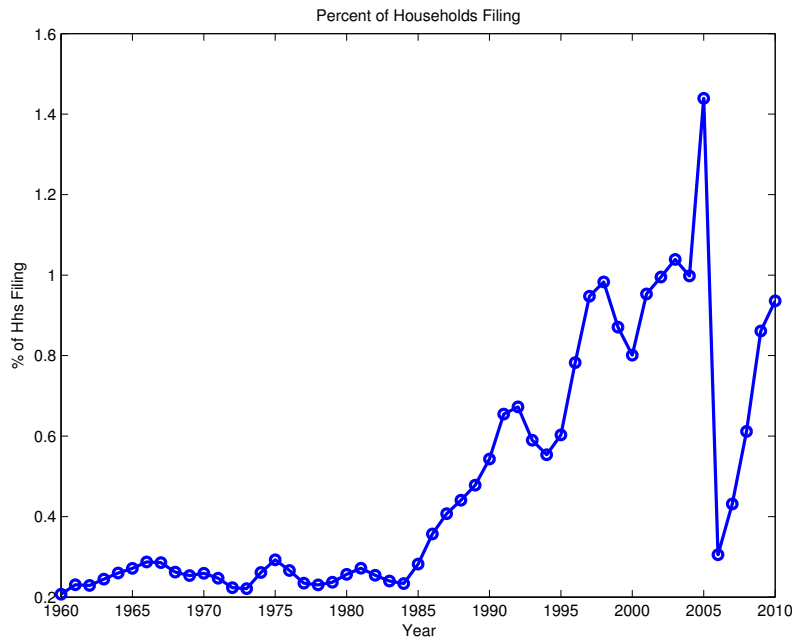


Figure 11: Chapter 7 Filings Per Household (1960-2010)

drastically increased since the 1984 and in 2005 experienced a sharp increase, then decrease, and subsequent recovery. The sharp increase is presumably due to anticipation of the 2005 reform (which mostly applied to filings made on or after October 17, 2005).⁶¹ Clearly, the sample period will drastically affect not only the level of bankruptcies but potentially their cyclicity and volatility. Because of this, I report statistics for each of the subperiods 1960-1984, 1984-2004, and 1960-2004. In addition to 1984 and 2005 being breakpoints visually,

⁶⁰The data that go back to 1960 are only available on a fiscal year basis ending in June (the series ending in December 1990). These are available at the Department of Justice website <http://www.uscourts.gov/Statistics/BankruptcyStatistics.aspx#calendar> in various pdf files. The data for 1960-1982 are at the end of the pdf file labeled “1983-2003 Bankruptcy Filings.” To recover the annual figure for year y prior to 1990, I use the average of y and $y + 1$. To test how well this works, I compare this method’s values with the known values for the period 1990-2004. This produces a good fit with an R^2 of .985. If 2005 is included however, this drops to .827 because of the very large rise at the end of 2005 presumably in anticipation of BAPCPA provisions about to begin (most provisions applied to cases filed on or after October 17, 2005).

⁶¹See <http://www.justice.gov/ust/eo/bapcpa/index.htm>.

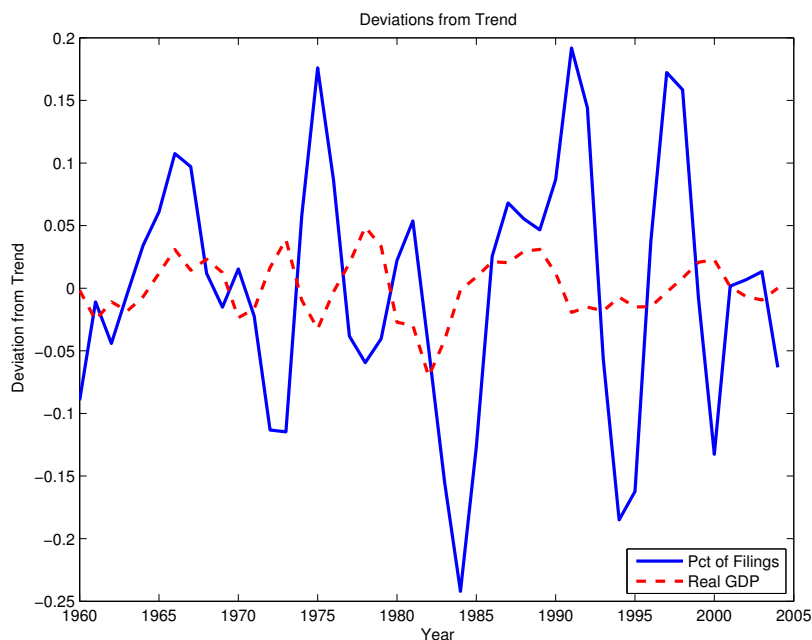


Figure 12: Cyclical Properties of Filings (1960-2004)

these were years of substantial bankruptcy reform.⁶²

Figure 12 plots log filings per household and log real GDP detrended using the HP filter with parameter 100 for the sample 1960-2004. Visually bankruptcy filings are much more volatile than output and appear to be countercyclical, but not strongly. This is borne out by the statistics reported in Table 11. Bankruptcy filings are between 2.8 and 7.0 times more volatile than output and are only mildly countercyclical with a correlation between $-.02$ and $-.45$ depending on the sample period.

The lack of strong countercyclicality in filings is at first counterintuitive: one would expect an increase in unemployment and hence a reduced ability to refinance debt to translate into a filings increase. But consider the case of an indebted household who has lost their job in a recession. The household has two choices. They can file for bankruptcy, lose access to their credit cards, and survive off unemployment insurance and other transfers. Alternatively,

⁶²There were essentially two rounds of substantial bankruptcy legislation. The first round began with the *Marquette* decision in 1979 which was subsequently amended by the Bankruptcy Amendment Act of 1984. Among other things, these made it easier for credit card companies to charge higher interest. The second round began with the “Bankruptcy Reform Act of 1999” which was passed by Congress but not signed into law. Subsequent revisions of this legislation resulted in the “Bankruptcy Abuse Prevention and Consumer Protection Act” (BAPCPA) which was signed into law in 2005. Several possible explanations for the rise in filings have recently been evaluated by Livshits et al. (2010). They find the rise was most likely due to changes in the credit market environment coming from decreased transaction costs of borrowing and the cost of filing for bankruptcy, not from legislative reform. However, they suggest the increase in information technology may have played a key role in driving down these costs which would also suggest a break in the early 1980s.

Statistic	1960-2004	1960-1984	1984-2004
Corr(Defaulting Pop,Output)	-.03	-.45	-.02
Stdev(Defaulting Pop) in %	10.25	7.53	11.07
Stdev(Output) in %	2.44	2.65	1.58
Stdev(Defaulting Pop)/Stdev(Output)	4.20	2.84	6.99
Mean(Defaulting Pop*) in %	.45	.25	.68

*In levels.

Table 11: Cyclical Properties of Chapter 7 Filings per Household

they can not file, keep their credit cards, and use these to supplement their income until the economy improves and they find a job.

In addition to a fraction of households filing, the model also features output, consumption, investment, and an aggregate labor supply, so I report the cyclical properties of these US data in Table 12.⁶³ Consumption, investment, and hours worked are all strongly procyclical. Investment and the fraction of households filing are both much more volatile than output while consumption and hours worked are somewhat less so.

Variable (x)	Stdev x in (%)	Stdev x/ Stdev y	Correlation of lagged x with y				
			x(-2)	x(-1)	x	x(+1)	x(+2)
Output (y)	2.44	1.00	0.01	0.58	1.00	0.57	-0.01
Consumption	1.69	0.69	0.02	0.59	0.92	0.59	0.10
Investment	7.19	2.94	0.04	0.53	0.89	0.38	-0.26
Hours Worked	1.60	0.66	-0.23	0.26	0.82	0.69	0.09
Defaulting Pop	10.25	4.20	0.17	0.00	-0.03	0.26	0.56

Table 12: US Business Cycle Properties (1960-2004)

B.2 Calibration

Table 13 gives the profiles of all variables omitted in the main text.

⁶³I follow [Ohanian and Raffo \(2011\)](#) and take annual hours worked from the Conference Board's Total Economy Database (TED) available at <http://www.conference-board.org/data/economydatabase/#files>. Output, consumption, and investment are taken from NIPA as real gross domestic product, real personal consumption expenditures, and real gross private domestic investment. The number of households (used to compute the fraction of households defaulting) is taken from the Census Bureau's historical tables available at <http://www.census.gov/population/www/socdemo/hh-fam.html#ht> (Table HH-1). The number of filings is taken from the Department of Justice website as already described. All variables have been logged and detrended using the HP filter with parameter 100.

Age	Parameters*						Age	Parameters*					
	ρ_s	θ_s	ϕ_h	γ_h	$\sigma_{\eta,h,1}^2$	$\sigma_{\varepsilon,h}^2$		ρ_s	θ_s	ϕ_h	γ_h	$\sigma_{\eta,h,1}^2$	$\sigma_{\varepsilon,h}^2$
20	.9994	1.18	.414	.693	.110	.059	53	.9947	1.34	.778	.893	.009	.078
21	.9994	1.18	.427	.693	.110	.059	54	.9942	1.32	.767	.978	.042	.070
22	.9994	1.19	.440	.693	.110	.059	55	.9937	1.30	.753	.982	.024	.070
23	.9994	1.25	.455	.693	.110	.059	56	.9931	1.28	.736	.894	.041	.062
24	.9994	1.33	.471	.693	.110	.059	57	.9925	1.26	.716	1.028	.036	.088
25	.9993	1.40	.487	.693	.080	.028	58	.9919	1.24	.693	.856	.004	.060
26	.9993	1.46	.504	.673	.062	.022	59	.9912	1.23	.667	1.014	.039	.093
27	.9993	1.51	.521	.884	.050	.043	60	.9905	1.22	.638	1.014	.039	.032
28	.9993	1.55	.539	.792	.053	.029	61	.9897	1.22	.605	1.014	.039	.032
29	.9993	1.59	.557	.828	.037	.043	62	.9888	1.22	.569	1.014	.039	.032
30	.9992	1.62	.575	.878	.025	.054	63	.9878	1.21	.530	1.014	.039	.032
31	.9992	1.64	.593	.908	.031	.067	64	.9867	1.21	.487	1.014	.039	.032
32	.9992	1.66	.611	.922	.015	.063	65	.9855	1.20	.487	1.014	.039	.032
33	.9991	1.67	.629	.904	.031	.051	66	.9841	1.20	.487	1.014	.039	.032
34	.9991	1.68	.647	.946	.020	.064	67	.9827	1.19	.487	1.014	.039	.032
35	.9990	1.68	.664	.963	.019	.061	68	.9813	1.19	.487	1.014	.039	.032
36	.9990	1.68	.681	.908	.004	.060	69	.9797	1.18	.487	1.014	.039	.032
37	.9989	1.68	.697	.921	.025	.077	70	.9780	1.18	.487	1.014	.039	.032
38	.9987	1.67	.712	1.048	.008	.074	71	.9760	1.17	.487	1.014	.039	.032
39	.9986	1.65	.727	.932	.019	.064	72	.9738	1.16	.487	1.014	.039	.032
40	.9984	1.64	.740	.973	.012	.064	73	.9714	1.15	.487	1.014	.039	.032
41	.9982	1.62	.753	.974	.013	.050	74	.9687	1.15	.487	1.014	.039	.032
42	.9980	1.60	.764	.949	.012	.051	75	.9657	1.14	.487	1.014	.039	.032
43	.9978	1.58	.774	.986	.022	.064	76	.9622	1.13	.487	1.014	.039	.032
44	.9976	1.56	.783	.954	.011	.074	77	.9583	1.12	.487	1.014	.039	.032
45	.9973	1.53	.790	.870	.018	.073	78	.9540	1.11	.487	1.014	.039	.032
46	.9971	1.51	.795	1.046	.011	.074	79	.9492	1.10	.487	1.014	.039	.032
47	.9968	1.49	.799	.990	.013	.072	80	.9438	1.09	.487	1.014	.039	.032
48	.9965	1.46	.801	1.050	.011	.076	81	.9379	1.08	.487	1.014	.039	.032
49	.9962	1.44	.801	1.000	.017	.073	82	.9312	1.06	.487	1.014	.039	.032
50	.9959	1.41	.799	.927	.006	.065	83	.9237	1.05	.487	1.014	.039	.032
51	.9955	1.39	.794	.983	.004	.061	84	.9154	1.04	.487	1.014	.039	.032
52	.9951	1.37	.788	1.048	.027	.080	85	.0000	1.02	.487	1.014	.039	.032

* ρ_s : the conditional probability of survival.

θ_s : adult-equivalent household size.

ϕ_h : deterministic earnings profile.

γ_h : income shock persistence.

$\sigma_{\eta,h,1}^2$: persistent shock variance.

$\sigma_{\varepsilon,h}^2$: transitory shock variance.

Table 13: Parameter Values for Profiles

C Computation

The steady state model is computed using a collection of standard techniques. The efficiency process is discretized using the method of [Tauchen \(1986\)](#). The persistent shock is discretized with 11 points with a coverage of ± 5 “average unconditional standard deviations in recessions,” $\bar{\sigma}_{\eta,b}/\sqrt{1-\bar{\rho}^2}$. The bar denotes the numerical average across ages. The transitory shock is discretized with 3 points with a coverage of ± 3 “average” standard deviation, $\bar{\sigma}_\varepsilon$. The “super-rich” efficiency process is discretized with only 3 points linearly spaced. The mass on each point is computed using, again, the method of [Tauchen \(1986\)](#) (e.g. the mass on the low point is the cdf evaluated at the midpoint of the lowest two points). The total number of efficiency states for each age is 36. The asset grid is comprised of 450 points, 70 strictly negative. These are very unevenly spaced between -5 and 4000 and concentrated close to zero.⁶⁴ The household problem is solved using backward induction with basic grid search assuming the asset policy conditional on repaying or being in bad standing is nearly monotonic (specifically I check 40 grid points below the previous asset choice in addition to all above). Cash-in-hand is not used, and there is no interpolation.

The business cycle model is computed using grid search, backward induction, and the method of [Krusell and Smith \(1998\)](#) (the KS method). The income process is discretized in exactly the same way and the asset grid is the same. The “moments” chosen for the KS method are aggregate wealth $A = (1+r-\delta)K$, aggregate labor N , and an equity premium W . The equity premium W is defined on the domain $[0, 1]$ and controls the risk-free bond price via

$$\frac{1}{\bar{q}_B(\mathcal{S})} = \frac{WF(b|z)}{WF(b|z) + (1-W)F(g|z)} \hat{R}(\mathcal{B}) + \frac{(1-W)F(g|z)}{WF(b|z) + (1-W)F(g|z)} \hat{R}(\mathcal{G}) \quad (92)$$

where \hat{R} is the forecasted gross return to capital (i.e. $1+r-\delta$). The probabilities ensure the equilibrium value of W is always close to, but slightly above, .5. The laws of motion take the following form:

$$\begin{bmatrix} K' \\ N' \\ W' \end{bmatrix} = \begin{bmatrix} \Gamma_{11,z} & \Gamma_{12,z} & \Gamma_{13,z} & 0 \\ \Gamma_{21,zz'} & \Gamma_{22,zz'} & 0 & 0 \\ \Gamma_{31,zz'} & \Gamma_{32,zz'} & \Gamma_{33,zz'} & 0 \end{bmatrix} \begin{bmatrix} 1 \\ N \\ A \\ W \end{bmatrix} \quad (93)$$

with A' calculated off K' , N' and z' using $A' = K'(1+z'\alpha(K'/N')^{\alpha-1} - \delta)$. Here a subscript of z means the coefficients are z -dependent and similarly for zz' . Importantly, W does *not* influence the one-step ahead forecast. This allows the risk-free bond price to be varied *holding fixed all other prices*.

Because the computational burden is extremely heavy, especially in terms of memory usage, the number of points used for the aggregate moments is kept to a minimum: 3 in the

⁶⁴Specifically, letting `linspace(a, b)` denote linear spacing between a and b , the $[-5, 0]$ has 71 points spaced according to $-(\exp(\text{linspace}(\log(\kappa), \log(5 + \kappa))) - \kappa)$ with $\kappa = .3$; the $[0, 100]$ range has 381 points spaced according to $\exp(\text{linspace}(\log(\kappa), \log(100 + \kappa))) - \kappa$ with $\kappa = .5$; and the $[100, 4000]$ range has 151 points spaced according to $\exp(\text{linspace}(\log(100 + \kappa), \log(4000 + \kappa))) - \kappa$ with $\kappa = -50$.

A direction linearly-spaced within $\pm 10\%$ of the steady-state value, 2 in the N direction placed at .963 and 1.048, and 2 in the W direction placed at .505 and .54. The coverage for N is the minimal possible without resorting to extrapolation. Because the natural borrowing limit and welfare depend on the worst possible scenario that can occur, and on how quickly it can be reached, the bounds and the number of gridpoints used influence the results. Somewhat surprisingly, this effect appears to be limited (see Table 15). At any rate, the bounds I've chosen I believe are reasonable especially given past US history where the capital-output ratio declined by some 20% in the Great Depression and even since 1960 hours worked per household has seen declines of 4% (relative to trend).⁶⁵

The model is simulated non-stochastically as in Young (2010). The economy is simulated for 600 periods and the first 200 periods are discarded. At each point in the simulation, the bond market must be cleared. Because the equity premium is included in the aggregate state space, this is a simple matter of linearly interpolating the asset policies to find a W such that (30) holds.

The approximate laws of motion are accurate. This is seen in Table 14 which records the maximum percent errors and the R^2 from 1-step and 50-step forecasts. 50-steps out, the errors do tend to accumulate with the maximum error getting close to .4% for some variables, but overall the fit is good.

Stat\Variable	Default			No Default		
	A'	N'	W'	A'	N'	W'
1-step						
R^2	.9996	.9995	.9973	.9997	.9995	.9974
Max % Error	.098	.127	.026	.097	.127	.023
50-step						
R^2	.9971	.9951	.9965	.9988	.9951	.9967
Max % Error	.352	.370	.028	.164	.370	.031

Table 14: Law of Motion Forecasting Accuracy

D Robustness

This section conducts robustness tests. The focus is on the consequences of eliminating default rather than restricting it.

⁶⁵Author's calculations. Hours worked per household are calculated as discussed in Appendix B. The capital-output ratio is constructed from NIPA data using the "current cost net stock of fixed assets" divided by nominal GDP.

D.1 Components of the Business Cycle

One of the key results is that business cycles reduce the welfare gain of eliminating default from a large amount 1.82% to a smaller amount .49%. However, business cycles in the model have three components: a TFP shock z , an aggregate labor supply shifter (LSS) ψ_z , and counter-cyclical earnings variance (CEV) of the persistent earnings shock $\sigma_{\eta,h,z}$. I now examine how each of these contribute to the total reduction in the welfare gain of eliminating default.

I proceed by starting with all three components and eliminating them successively until the model is equivalent to the steady state one. I try to hold fixed the effects of aggregate moment grids on the results by eliminating these in separate steps. The aggregate moments are A , W , and N where A is financial wealth, W is an equity premium, and N is aggregate labor supply; see Appendix C for more on these. I eliminate elements in the following order: CEV, LSS, the N grid, TFP, the W grid, and lastly the A grid. The last step results in the steady state model.⁶⁶

Table 15 reports the welfare gains of eliminating default after each of these steps. The most important business cycle components are CEV and the TFP shock. Each of these reduces the gain of eliminating default by around .55%. CEV likely matters because default is the only means of smoothing consumption intratemporally. The TFP shock likely matters because it drives capital accumulation or decumulation and consequently the risk due to fluctuating prices.

	Eliminating cumulatively from left to right							
	Nothing	CEV	LSS	N Grid	TFP	W Grid	A Grid	
Welfare Gain (%)	.49	1.04	1.08	1.03	1.70	1.70	1.82	
Population in Favor (%)	57.0	58.5	58.4	58.4	57.8	57.8	56.8	

Table 15: Contribution of Business Cycle Components to Welfare Results

It is worth noting what doesn't affect the welfare results much. The LSS matters very little and actually makes eliminating default look better by .04%. This is likely because the small earnings swings ($\pm 2.5\%$) have the same persistence as the business cycle and so are very easy to insure through borrowing and saving. What is really surprising is how little the grids matter. Apparently it is not just a positive probability of a deep recession occurring (which is what linear interpolation on a coarse grid for A) that matters but rather how likely

⁶⁶So that the model can be computed at every step, the grid coverage cannot be totally eliminated. In the third step the N grid is changed from $[\text{.963}, \text{1.048}]$ to $[1 - 10^{-8}, 1 + 10^{-8}]$. In the fourth step the z values are changed from $\{1 \pm .0224\}$ to $\{1 \pm 10^{-10}\}$ and the W grid is expanded from $[\text{.505}, \text{.54}]$ to $[\text{.497}, \text{.54}]$. The W grid is expanded because the equilibrium values of W are now below .505. In the fifth step the W grid is shrunk to $[\text{.497}, \text{.503}]$. For the last step I just report results from the steady state model (where z is always 1, N is always 1, and W is always .5). As measured by R^2 , the forecasts become successively worse (because the variation in the actual series goes to zero). Because of this I iterate until the coefficients don't change much and then stop the computation regardless of R^2 values.

it is.⁶⁷ This result is similar to when default is not eliminated but just made very costly. There the natural borrowing limit is not in effect but aggregate risk reduces the benefit of moving to a costly-default regime.

D.2 Guaranteed Earnings Prior to Retirement

It was argued that using a low minimum value for earnings during working life is reasonable. I now explore the robustness of the results to guaranteed earnings prior to retirement.

To provide guaranteed earnings, I do the following. Given the efficiency distribution of the benchmark economy, I replace any values less than a threshold τ with τ and re-normalize so that $N = 1$ in steady state. I consider 4 different thresholds $\tau \in \{.008, .058, .127, .233\}$ (the benchmark economy is any $\tau \leq .0043$). These represent a lower bound of roughly \$500, \$3500, \$7,600, and \$14,000 taking average household labor income to be \$60,000 and are the values considered in Athreya (2008).

Table 16 reports the results. In addition to the usual statistics, I report the worst R^2 values from the forecasts. The most glaring observation is that the gains from eliminating default are very large for $\tau \geq .058$. While the gains are high, in each case considered aggregate risk again reduces the benefit of eliminating default. This is especially true for $\tau = .058$ where the gain goes from 8.2% to 5.4%. While one might expect the results to be monotonic, they are not. In particular, for $\tau = .127$ the drop is only .7%. However, the forecast errors in the ND economies are high for $\tau \geq .058$ and so the results for high values of τ are likely inaccurate. Overall, these results suggest that, to the extent earnings are guaranteed in working life, it would be substantially welfare improving to eliminate default. However, the gain would be overstated if aggregate risk was not accounted for.

D.3 Retirement Schemes

In the benchmark calibration, labor income in retirement is comprised of a guaranteed fraction $\kappa_G = .15$ of average earnings and a fraction $\kappa_F = .35$ of earnings from the last period of working life. The average replacement rate is roughly 50% because $\kappa_G + \kappa_F = .5$. However, as already discussed, it is not the replacement rate that really matters but how much of it is guaranteed. I now examine the robustness of the results to alternative replacement schemes (κ_G, κ_F) subject to keeping $\kappa_G + \kappa_F = .5$.

Table 17 records the results. What is immediately striking is that when no earnings are guaranteed, eliminating default is a substantial loss, -2.5%, that slightly lessens after including aggregate risk. For the other schemes, the typical pattern of aggregate risk reducing the welfare benefit of eliminating default is observed.

⁶⁷The A grid is three linearly-spaced points $\{.9A_{SS}, A_{SS}, 1.1A_{SS}\}$ where A_{SS} is the steady state value. This grid with linear interpolation means that when A is slightly below its steady state value, a weight of almost 1 is placed on A_{SS} but a small weight is placed on $.9A_{SS}$. This can be interpreted as a small chance of moving to a deep recession. Apparently it is not so much whether this weight is positive but rather how positive it is.

$\tau =$.008	.058	.127	.233
Welfare Gain SS (%)	2.88	8.21	8.03	6.64
Welfare Gain BC (%)	1.25	5.41	7.21	5.10
Population in Favor SS (%)	56.4	48.4	47.5	42.4
Population in Favor BC (%)	56.1	43.8	48.4	41.9
CD				
K/Y SS	3.08	3.08	3.08	3.06
K/Y BC	3.08	3.08	3.08	3.06
Debt/Y SS	0.69	0.70	0.73	0.88
Debt/Y BC	0.53	0.53	0.53	0.70
Population in Debt SS	10.47	10.48	12.61	13.40
Population in Debt BC	9.01	9.04	9.27	12.28
Population Filing SS	0.18	0.18	0.18	0.19
Population Filing BC	0.26	0.25	0.30	0.27
Worst R^2 1-step Ahead	.9973	.9973	.9966	.9971
Worst R^2 50-step Ahead	.9951	.9951	.9910	.9844
ND				
K/Y SS	3.00	2.86	2.84	2.69
K/Y BC	3.03	2.95	2.89	2.78
Debt/Y SS	4.75	16.27	18.77	33.99
Debt/Y BC	2.89	9.86	13.80	25.09
Population in Debt SS	21.40	33.53	35.11	41.28
Population in Debt BC	17.85	27.38	31.23	37.53
Worst R^2 1-step Ahead	.9974	.9962	.9921	.9956
Worst R^2 50-step Ahead	.9951	.2121	.7478	.2639

Table 16: Robustness to Guaranteed Income

$(\kappa_G, \kappa_F) =$	(0, .5)	(.15, .35)	(.30, .20)	(.50, .0)
Welfare Gain SS (%)	-2.52	1.82	3.55	4.00
Welfare Gain BC (%)	-2.24	0.49	1.28	1.93
Population in Favor SS (%)	39.7	56.8	46.9	40.4
Population in Favor BC (%)	32.3	57.0	42.2	42.3
CD				
K/Y SS	3.19	3.08	2.99	2.88
K/Y BC	3.19	3.08	3.00	2.89
Debt/Y SS	1.27	0.69	0.69	0.71
Debt/Y BC	1.30	0.53	0.51	0.55
Population in Debt SS	10.19	10.47	11.27	12.65
Population in Debt BC	8.95	9.01	9.47	10.85
Population Filing SS	0.18	0.18	0.19	0.23
Population Filing BC	0.19	0.26	0.28	0.31
Worst R^2 1-step Ahead	.9986	.9973	.9964	.9949
Worst R^2 50-step Ahead	.9924	.9951	.8850	.9963
ND				
K/Y SS	3.21	3.00	2.81	2.65
K/Y BC	3.21	3.03	2.89	2.75
Debt/Y SS	0.29	4.37	10.81	16.03
Debt/Y BC	0.11	2.72	5.96	9.23
Population in Debt SS	6.16	20.76	29.70	35.42
Population in Debt BC	3.73	16.53	23.34	28.35
Worst R^2 1-step Ahead	.9984	.9974	.9967	.9913
Worst R^2 50-step Ahead	.9924	.9951	.8801	.9907

Table 17: Robustness of Results to Alternative Retirement Schemes

D.4 Profiles

Many of the parameters in the model are age-dependent. In particular, there are profiles for survival probabilities ρ_s , adult-equivalent household size θ_s , deterministic earnings ϕ_h , income persistence γ_h , persistent shock variance $\sigma_{\eta,h,z}^2$, and transitory shock variance $\sigma_{\varepsilon,h}^2$. While these bring the model closer in line with the data, they also make interpretation of the results more difficult. I examine the consequences of these profiles by separately setting ρ_s , θ_s , and ϕ_h to 1 and also by setting γ_h , $\sigma_{\eta,h,z}^2$, and $\sigma_{\varepsilon,h}^2$ to the age-independent estimates in Storesletten et al. (2004) (STY).⁶⁸

	$\rho_s = 1$	$\theta_s = 1$	$\phi_h = 1$	$\gamma_h, \sigma_{\eta,h,z}, \sigma_{\varepsilon,h}$ = STY est.
CD → ND				
Welfare Gain SS (%)	2.78	2.77	0.71	3.80
Welfare Gain BC (%)	1.00	0.73	0.14	2.35
Population in Favor SS (%)	49.1	55.8	47.6	61.4
Population in Favor BC (%)	48.8	54.7	50.0	64.2
CD				
K/Y SS	3.23	3.19	3.02	3.12
K/Y BC	3.25	3.19	3.02	3.11
Debt/Y SS	0.76	0.77	0.42	0.51
Debt/Y BC	0.60	0.54	0.35	0.38
Population in Debt SS	12.40	12.61	6.78	9.31
Population in Debt BC	9.82	9.65	5.82	6.96
Population Filing SS	0.16	0.18	0.29	0.08
Population Filing BC	0.18	0.25	0.35	0.11
ND				
K/Y SS	3.12	3.10	2.93	3.01
K/Y BC	3.17	3.14	2.96	3.04
Debt/Y SS	6.20	4.57	4.00	4.15
Debt/Y BC	3.81	2.69	2.66	2.41
Population in Debt SS	22.36	21.17	16.87	19.00
Population in Debt BC	18.89	17.58	13.71	15.23

Table 18: Robustness of Results to Flat Profiles

Table 18 reports the results. In each of these tests, aggregate risk reduces the benefit of eliminating default. For the STY estimates the gain is higher in steady state and drops slightly more than in the benchmark. The $\phi_h = 1$ case is worth pointing out because it is indicative of what results from an infinite-horizon economy might look like: because house-

⁶⁸For completeness, these are $\gamma_h = .952, \sigma_{\eta,h,g} = .125, \sigma_{\eta,h,b} = .211, \sigma_{\varepsilon,h} = .255$ for all h . As in the main text, I set $\sigma_{\eta,h,1} = .5\sigma_{\eta,h,g} + .5\sigma_{\eta,h,b} = .168$.

holds don't have life-cycle reasons to borrow, credit and default are less important. Overall, the results look about the same both in terms of welfare and allocations.

D.5 Flexible Portfolios

In the calibrated model, most households are restricted to use portfolios that just replicate a risk-free bond, $a'_g = a'_b$. I now relax this assumption, and, in particular, allow every household to choose any portfolio, i.e. to choose a'_g independently of a'_b .

	Flexible	Benchmark
Welfare Gain of SS (%)	1.82	1.82
Welfare Gain of BC (%)	0.88	0.49
Population in Favor SS (%)	56.8	56.8
Population in Favor BC (%)	58.9	57.0
CD		
K/Y SS	3.08	3.08
K/Y BC	3.07	3.08
Debt/Y SS	0.69	0.69
Debt/Y BC	0.58	0.53
Population in Debt SS	10.47	10.47
Population in Debt BC	9.97	9.01
Population Filing SS	0.18	0.18
Population Filing BC	0.27	0.26
Worst R^2 1-step Ahead	.9970	.9973
Worst R^2 50-step Ahead	.9890	.9951
ND		
K/Y SS	3.00	3.00
K/Y BC	3.00	3.03
Debt/Y SS	4.37	4.37
Debt/Y BC	3.16	2.72
Population in Debt SS	20.76	20.76
Population in Debt BC	17.42	16.53
Worst R^2 1-step Ahead	.9967	.9974
Worst R^2 50-step Ahead	.9910	.9951

Table 19: Robustness of Results to Flexible Portfolios

Table 19 reports some results. Under flexible portfolios, the welfare gain of eliminating default falls from 1.82% in steady state to .88% in the business cycle. This shows the result that aggregate risk reduces the welfare gain of eliminating default is robust to the assumption on portfolio restrictions. However, how aggregate risk affects each economy is different. One way this is seen is in how aggregates change. In particular, the capital-output ratio now

falls for the CD economy and stays the same for the ND economy. Additionally, while there is a contraction in debt use, the contraction is smaller than under the restricted portfolio assumption.