Discussion of Rui Zhao
“Repeated Two-Sided Moral Hazard”

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MAIN RESULTS

1. Under fairly general conditions Pareto optimal contracts under repeated two-sided moral hazard are recursive

2. Partial characterization of optimal contracts

3. Obvious next step: explore further properties of optimal contracts using numerical “examples”
MAIN ASSUMPTIONS

1. For all $\theta_i \in \Theta_i$, all $a_i \in A_i$
   
   \[
   \pi(\theta_i | a_i) > 0
   \]

2. $u_i : (c, \infty) \rightarrow \mathbb{R}$ is strictly concave and

   \[
   \lim_{c \rightarrow \infty} (c) = -\infty
   \]

3. There exist $a_i, a'_i \in A_i$ such that $g_i(a_i) \neq g_i(a'_i)$

4. (Infinite horizon)
RECURSIVE FORMULATION

• State variable

\[ U : \text{promise of expected discounted lifetime utility for agent 1} \]

• Control Variables:

\[ \alpha_i : \text{probability distribution over effort } a_i \in A_i \]
\[ c_i(\theta) : \text{consumption, conditional on public signal } \theta \in \Theta_1 \times \Theta_2 \]
\[ U(\theta) : \text{continuation utility of agent 1, conditional on } \theta \]
Timing

Today                                               Tomorrow        Time

Come in with U

Pick efforts

\( \frac{?}{i} \)

Output \( x(\text{?}) \) realized

Draw \( \frac{?=(\text{?}, \text{?})}{i} \)

with \( \frac{? \sim ?}{i} \) \( \frac{(\text{?}, \text{?})}{i} \)

Pick cons. \( c(\text{?}), U(\text{?}) \)

Come in with \( U'=U(\text{?}) \)

Today                                               Tomorrow        Time

Figure 1:
- Bellman equation

\[
V(U) = \max_{\alpha_i,c_i(\theta),U(\theta)} \sum_{\theta_1 \in \Theta_1} \sum_{\theta_2 \in \Theta_2} \pi^1(\theta_1|\alpha_1)\pi^2(\theta_2|\alpha_2) \star \{u_2(c_2(\theta)) + \delta V(U(\theta)) - g_2(\alpha_2)\}
\]

subject to

- Promise Keeping

\[
U = \sum_{\theta_1 \in \Theta_1} \sum_{\theta_2 \in \Theta_2} \pi^1(\theta_1|\alpha_1)\pi^2(\theta_2|\alpha_2) \star \{u_1(c_1(\theta)) + \delta U(\theta) - g_1(\alpha_1)\}
\]
\(-\) IC, agent 1: for all \(a_1 \in A_1\)

\[
\sum_{\theta_1 \in \Theta_1} \sum_{\theta_2 \in \Theta_2} \pi^1(\theta_1 | \alpha_1) \pi^2(\theta_2 | \alpha_2) \ast \\
\{ u_1(c_1(\theta)) + \delta U(\theta) - g_1(\alpha_1) \} \\
\geq \sum_{\theta_1 \in \Theta_1} \sum_{\theta_2 \in \Theta_2} \pi^1(\theta_1 | a_1) \pi^2(\theta_2 | \alpha_2) \ast \\
\{ u_1(c_1(\theta)) + \delta U(\theta) - g_1(a_1) \}
\]

\(-\) IC for agent 2: for all \(a_2 \in A_2\)

\[
\sum_{\theta_1 \in \Theta_1} \sum_{\theta_2 \in \Theta_2} \pi^1(\theta_1 | \alpha_1) \pi^2(\theta_2 | \alpha_2) \ast \\
\{ u_2(c_2(\theta)) + \delta V(U(\theta)) - g_2(\alpha_2) \} \\
\geq \sum_{\theta_1 \in \Theta_1} \sum_{\theta_2 \in \Theta_2} \pi^1(\theta_1 | \alpha_1) \pi^2(\theta_2 | a_2) \ast \\
\{ u_2(c_2(\theta)) + \delta V(U(\theta)) \} - g_2(a_2)
\]

\(-\) Resource Feasibility: for all \(\theta \in \Theta_1 \times \Theta_2\)

\[c_1(\theta) + c_2(\theta) = x(\theta)\]
MAIN CHARACTERIZATION

• Two-Sided Moral Hazard: Sub-martingale result

\[
\frac{u'_1(c_1(\theta))}{u'_2(c_2(\theta))} = \sum_{\theta'_2} \pi(\theta'_2|\alpha'_2) \left[ \sum_{\theta'_1} \pi(\theta'_1|\alpha'_1) \frac{u'_2(c_2(\theta'))}{u'_1(c_1(\theta'))} \right]^{-1}
\]

\[
\leq \sum_{\theta'_1} \sum_{\theta'_2} \pi(\theta'_2|\alpha'_2) \pi(\theta'_1|\alpha'_1) \frac{u'_1(c_1(\theta'))}{u'_2(c_2(\theta'))}
\]

where

\[
c_i(\theta) = c_i(U; \theta)
\]

\[
\alpha'_i = \alpha'_i(U') = \alpha'_i(U(\theta))
\]

\[
c_i(\theta') = c_i(U'; \theta') = c_i(U(\theta); \theta')
\]
- One-Sided Moral Hazard (Rogerson 1985)): Agent 1 is risk neutral principal; agent 2 has moral hazard problem; equal discount factors: Martingale result

\[
\frac{1}{u_2'(c_2(\theta))} = \sum_{\theta'} \pi(\theta'|\alpha_2') \left[ \frac{u_2'(c_2(\theta'))}{1} \right]^{-1}
\]

\[
= \sum_{\theta'} \pi(\theta'|\alpha_2) \frac{1}{u_2'(c_2(\theta'))}
\]

or \[
\frac{1}{u_2'(c_2(\theta))} = E_{\theta'|\alpha_2} \left\{ \frac{1}{u_2'(c_2(\theta'))} \right\}
\]
MAIN CHARACTERIZATION

- Two-Sided Moral Hazard

\[
\frac{u'_1(c_1(\theta))}{u'_2(c_2(\theta))} = \sum_{\theta'_2} \pi(\theta'_2|\alpha'_2) \left[ \sum_{\theta'_1} \pi(\theta'_1|\alpha'_1) \frac{u'_2(c_2(\theta'))}{u'_1(c_1(\theta'))} \right]^{-1}
\]

where

\[
c_i(\theta) = c_i(U; \theta) \\
\alpha'_i = \alpha'_i(U') = \alpha'_i(U(\theta)) \\
c_i(\theta') = c_i(U'; \theta') = c_i(U(\theta); \theta')
\]

- One-Sided Moral Hazard (Rogerson 1985)): Agent 1 is risk neutral principal; agent 2 has moral hazard problem; equal discount factors

\[
\frac{1}{u'_2(c_2(\theta))} = \sum_{\theta'} \pi(\theta'|\alpha'_2) \left[ \frac{u'_2(c_2(\theta'))}{1} \right]^{-1}
\]

or

\[
\frac{1}{u'_2(c_2(\theta))} = E_{\theta'|\alpha'_2} \left\{ \frac{1}{u'_2(c_2(\theta'))} \right\}
\]
SMALL REMARKS

• What restrictions does the use of public actions impose?

• Implementation of optimal contracts?

• Connection with observables? Characterization of consumption (compensation) process? Note that the results about long-run behavior still involve endogenous choices.
Numerical Examples

- Suppose $A_i = \{a_l, a_h\}$ and $\Theta_i = \{\theta_1, \theta_2\}$

- Only one (continuous) state variable $U$

- 14 control variables

- Potential problem: unbounded domain of $U$, $U(\theta)$

- Potential resolution: lower bounds on continuation utilities (because of limited commitment)

\[
U(\theta) \geq u_1 \\
V(U(\theta)) \geq u_2
\]

- But: this may change implications of the model drastically: see Atkeson and Lucas (1992) vs. (1995)