Discussion of

“Private Debt and Income Inequality: A Business Cycle Analysis”

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Motivation: Three Stylized Facts

- Increase in private debt: ratio of private debt to GDP has increased from roughly 90% to 144% between 1970 and 2004.

- Reduction in macro-volatility: standard deviation of growth rate of real GDP falls from 2.5% between 1960-1983 to 1.6% between 1984-2002. Volatility in debt (over GDP) increases between the two periods.

- Increase in micro-volatility: cross sectional variance in labor earnings has increased substantially in last 30 years.

- Sharp rise in wealth-, modest increase in consumption inequality.
Figure 1. The Evolution of Income and Consumption Inequality in the US

Gini Index

Standard Deviation of Logs

Note: Circles represent actual data points. All indexes are computed on cross sections of households in the CE survey. Income and Consumption are per adult equivalent. Standard deviations are based on the residuals from regressing, in each cross section, household consumption or income on controls for age and race of the reference person.
Figure 2. Decomposition of Income and Consumption Inequality: Data

(a) Decomposition of Std. Dev. of Log-Income

(b) Decomposition of Std. Dev. of Log-Consumption

Note: Circles represent actual data points.
This Paper

- The question: can we construct a coherent macro model that generates these facts, qualitatively and quantitatively?

- The answer: yes, although some of the facts are matched by brute force.
The Key Model Ingredients

Agent 1
\[
\beta = \beta_h \\
A_t k^\mu = \left( y_1 \frac{X_t}{W_t Z_t^\zeta} l \right)^{1-\mu}
\]

Agent 2
\[
\beta = \beta_h \\
A_t k^\mu = \left( y_2 \frac{X_t W_t^\theta}{Z_t^\xi} l \right)^{1-\mu}
\]

Agent 3
\[
\beta = \beta_l \\
A_t k^\mu = (y_3 X_t Z_t l)^{1-\mu} \\
b \leq \frac{m_t}{R} (h + k)
\]

• \( y_2 \gg y_3 > y_1 \)
The Key Model Ingredients

- Period utility function
  \[ \log(c) + j \log(h) - \tau \frac{1+\eta}{1+\eta} \]

- Depreciation of capital stocks \( \delta_k, \delta_h \) with \( \delta_k \gg \delta_h \)

- Constant relative prices of the consumption and housing, capital investment.

- AR(1) processes for \( A_t, m_t, W_t, Z_t \). The variable \( X_t \) captures deterministic growth.
The Thought Experiment

- Estimate time series processes for $A_t, m_t, W_t, Z_t, X_t$.

- Solve for policy functions using log-linearization

- Feed actual time series of shocks into the model and use policy functions to deduce time series predictions for debt, output, and inequality statistics.
Main Results

- Model successful in matching time series of output (on account of the $A_t$ shocks).

- Model successful in matching trends in between and within income inequality (on account of the $Z_t$ and $W_t$).

- Model successful in matching the trend (and somewhat less) and cyclical properties of debt-output ratio.

- Key for this: within-group income inequality.
FIGURE 5: Simulated time profiles of consumption, income and wealth inequality.
FIGURE 6: Simulated time series for debt

Notes: In the top right panel, constrained debt is debt held by impatient agent \((b'+b'')\), unconstrained debt is debt held by the patient agent \((d)\)
FIGURE 8: Counterfactual experiment: Simulated time series for debt over GDP (in percent)
The Key Mechanism

- Agent 1 and 2 are called one group. Distinguished by their fixed effect, i.e. agent 2 is a productivity-scaled up-version of agent 1 (in the data this is likely due to education, age or other observables which are commonly used to distinguish between groups -see e.g. Katz and Autor, 1999).

- In steady state high-productivity agent 2 is a creditor, low-productivity agent 1 is a debtor (this is arbitrary, from the perspective of the model).
Within-group shocks (mean reverting) make high-productivity agents even more productive. Reduce their lending in short run to invest in their productive capital to use higher productivity. In long run use proceeds to increase lending to finance higher permanent consumption. Reverse logic for low-productivity agents.

Thus long-run increase in debt and in wealth inequality. Also would expect substantial increase in within-group consumption inequality. Model? A bit hard to tell. Also: level of consumption inequality higher than that for income!
Comments: Housing vs. Durables

- Housing is modeled as capital stock that can be adjusted without adjustment costs and is perfectly divisible. If there were a rental market no distinction with any other asset. More reasonable to interpret $h$ as stock of total durables.

- But then $\delta_h = 0.01$ seems questionable.
Also, from the 2000 NIPA and *Fixed Assets and Consumer Durable Goods* of the Bureau of Economic Analysis I find

\[
\frac{I}{Y} = 13.5\%
\]

\[
\frac{I_h}{Y} = 12\%
\]

\[
\frac{K}{Y} = 1.2
\]

\[
\frac{H}{Y} = 1.45
\]

\(\frac{I_h}{Y}\) and \(\frac{H}{Y}\) used in the paper seems too low, unless \(H\) is interpreted strictly as housing.
An Endogenous Theory of the Expansion of Credit Lines and Private Debt

- Increase in variance of income shocks $\Rightarrow$ Higher income inequality

- Lower utility of being excluded from credit markets since $c_{i,t} = y_{i,t}$

- If consequence of default is exclusion from future credit markets, more private contracts are sustainable $\Rightarrow$ Better consumption insurance.

- Key: credit markets respond endogenously to income volatility change. Thus smaller increase in consumption inequality.
A Simple Model

- Two (types of) agents $i = 1, 2$

- Single, nonstorabile consumption good

- In each period one consumer has income $1 + \varepsilon$ and the other has income $1 - \varepsilon$

- Let $s_t \in \{1, 2\}$ denote the consumer that has endowment $1 + \varepsilon$
• Let \( \{s_t\}_{t=0}^{\infty} \) follows a Markov process with transition matrix
\[
\pi = \begin{bmatrix}
\delta & 1 - \delta \\
1 - \delta & \delta \\
\end{bmatrix}, \ \delta \in (0, 1)
\]

• Event history \( s^t = (s_0, \ldots, s_t) \) with \( \pi(s^t), \pi(s_0) = \frac{1}{2} \)

• Endowment process
\[
e^1(s^t) = \begin{cases}
1 + \varepsilon & \text{if } s_t = 1 \\
1 - \varepsilon & \text{if } s_t = 2
\end{cases}
\]
and similarly for agent 2

• Allocation \( c = (c^1, c^2) = \{c^1_t(s^t), c^2_t(s^t)\}_{t=0}^{\infty} \) maps event histories \( s^t \) into consumption.
• Preferences

\[ U(c^i) = (1 - \beta) \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) u(c^i_t(s^t)) \]

where \( u \) is continuous, twice differentiable, strictly increasing and strictly concave on \((0, \infty)\) and satisfies the Inada conditions and \( \beta < 1 \)

• Continuation utility of agent \( i \) from allocation \( c^i \)

\[ U(c^i, s^t) = (1 - \beta) \sum_{\tau=t}^{\infty} \sum_{s^\tau|s^t} \beta^{\tau-t} \pi(s^\tau|s^t) u(c^i_\tau(s^\tau)) \]

• Autarkic allocation: \( c^1 = e^1 \) and \( c^2 = e^2 \)
• Enforcement Constraints: for all \( i, t \) and \( s^t \)

\[
U(c^i, s^t) \geq U(e^i, s^t)
\]

\[
= (1 - \beta) \sum_{\tau = t}^{\infty} \sum_{s^t|s^t} \beta^{\tau-t} \pi(s^\tau|s^t) u(e^i_\tau(s^\tau))
\]

• Alvarez and Jermann (2000) show how to find constrained efficient allocations in this economy and how to decentralize them as a competitive equilibrium with state dependent borrowing constraints.
Analysis

- Value of Autarky $U(e^i, s^t) = U(e^i, s_t) = U(1 \pm \varepsilon)$

$$U(1 + \varepsilon) = \frac{1}{D} \left\{ (1 - \beta) u(1 + \varepsilon) + \beta(1 - \delta) [u(1 + \varepsilon) + u(1 - \varepsilon)] \right\}$$

with $D > 0$

- Similarly

$$U(1 - \varepsilon) = \frac{1}{D} \left\{ (1 - \beta) u(1 - \varepsilon) + \beta(1 - \delta) [u(1 + \varepsilon) + u(1 - \varepsilon)] \right\}$$

- $\varepsilon$ measures variance of the income process
The Value of Autarky and Income Dispersion

Value of Autarky

Income dispersion, $\varepsilon$

$U(1+\varepsilon)$

$U(1-\varepsilon)$

$U_{FB}$
The Efficient Consumption Allocation

<table>
<thead>
<tr>
<th>Time</th>
<th>Income</th>
<th>Consumption</th>
</tr>
</thead>
<tbody>
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<td>1−ε</td>
<td>1</td>
<td>1−εc</td>
</tr>
<tr>
<td>1+ε</td>
<td>1</td>
<td>1+εc</td>
</tr>
<tr>
<td>1ε</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1−ε</td>
<td>1</td>
<td>1−εc</td>
</tr>
</tbody>
</table>
Figure 3. Characterizing the Efficient Allocation

Value of Allocations

Income Dispersion, ε

0 ε₁ = ε_c(ε_h) = ε_c(ε₁) ε₁ ε₂

Autarky

Partial Risk Sharing

Full Risk Sharing

U(1+ε)

U(1-ε)

U(1+ε_c(ε_h)) = U(1+ε_h)

U^{FB}
Which Explanation is More Attractive?

- This paper: bigger shocks of the same stochastic process for within-group productivity

- Alternative theory: relies on the assumption that individual earnings process has become more volatile over time.

- Empirical evidence: Gottschalk and Moffitt (1994) document that in PSID the variance of transitory income shocks (on average in the population) has increased by 40% from the 1970’s to the 1980’s.
A very ambitious paper (too ambitious?).

Trends in debt and inequality could be generated by simpler models that do not rely on unobservable preference heterogeneity.

But if one wants to explain trends and cyclical properties of debt within the same model, maybe one really needs all the ingredients of the model.