Discussion of “Preferences and the Dynamic Representative Consumer”

by

Christos Koulovatianos

Dirk Krueger

University of Frankfurt, CEPR, CFS and NBER

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Motivation

- There is a lot of heterogeneity among households in the economy.

- Under what conditions can we do macroeconomics and asset pricing as if this heterogeneity did not exist?

- This paper: heterogeneity in preferences.

- Note: the paper assumes absence of uncertainty (or perfect insurance against this uncertainty).
Basic Setup

- types $i \in \mathcal{I}$, with measure $\mu(i)$.

- Initial asset holdings $a_0^i$, labor productivity $\theta^i(t)$, $t \geq 0$.

- Preferences: Time discount factor $\rho^i(t)$ and felicity function $u^i(., t)$.

- Budget constraint

$$\dot{a}^i(t) = r(t)a^i(t) + \theta^i(t)w(t) - c^i(t)$$
Main Question

- For which individual preference profiles \((\rho^i, u^i)\) does there exist a representative consumer with preferences \(\rho^{RC}(t)\) and \(v^{RC}(., t)\) such that, when faced with prices \((w(t), r(t))\) and with aggregate wealth \(a_0 = \int a_0^i d\mu\), the representative consumer would choose consumption allocations

\[
c^{RC}(t) = \int c^i(t) d\mu
\]
# The Theoretical Results

<table>
<thead>
<tr>
<th>Model</th>
<th>$\rho^i = \rho, u^i$ time-invar.</th>
<th>$\rho^i, u^i$ time-invar.</th>
<th>$\rho^i = \rho, u^i$ time-var.</th>
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</thead>
<tbody>
<tr>
<td>Sufficiency</td>
<td>$e^{\kappa_i} \frac{(\alpha c + \beta^i)^{1 - \frac{1}{\alpha}}}{\alpha (1 - \frac{1}{\alpha})} + \kappa$</td>
<td>$-e^{\kappa_i} \frac{c}{\beta^i} e^{-\beta^i} + \kappa$</td>
<td>$e^{\kappa_i} \frac{(\alpha c + \beta^i(t))^{1 - \frac{1}{\alpha}}}{\alpha (1 - \frac{1}{\alpha})} + \kappa$</td>
</tr>
<tr>
<td>Necessity</td>
<td>same as above</td>
<td>same as above</td>
<td>same as above</td>
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Table 1: Summary of Results
Implications of Results for Quasi-Aggregation

• Take $i = 1, 2$ (two groups), with relative sizes $p_i$ and cross-sectional distribution over assets $\Phi_i$ within the group. Decision rules
  
  $$a'(a, i, z) = \alpha_{iz} + \beta_{iz}a$$

• Despite heterogeneity in the population, under what conditions can the aggregate law of motion be written as
  
  $$K' = B_z + C_zK$$
  
  where
  
  $$K = \sum_i p_i \int a d\Phi_i(a)$$
  
  $$K' = \sum_i p_i \int a'(a, i, z) d\Phi_i(a)$$
When Quasi-Aggregation in Macroeconomics?

- **Case 1:** $\beta_{iz} = \beta_z$ for all $i$. This is the case the paper focuses on.

- **Case 2:** $\beta_{1z} \neq \beta_{2z}$, but $p_1 \approx 0$ or $\int a d\Phi_1(a) \approx 0$

- **Case 3:** $\beta_{1z} \neq \beta_{2z}$, and $p_i \gg 0$ and $\int a d\Phi_i(a) \gg 0$ but
  \[
  \frac{\int a d\Phi_i(a)}{K} \approx \theta_i, \text{ constant over time}
  \]
What does one need for Quasi-Aggregation to fail?

- Groups with different MPC’s out of current wealth, and

- These different groups all hold significant share of aggregate wealth, and

- Time variation (due to aggregate shocks, say) in the distribution of wealth across these different groups.

- Examples: OLG models with big aggregate shocks that affect wages and returns to capital differentially (Krueger and Kubler, 2004).
Conclusions

• Sufficient conditions are shown elsewhere. But important to know that these are the only cases that allow a representative consumer (i.e. to have necessary conditions).

• What do these results imply if the strong conditions of the theorems do not apply? Do they help to understand quasi-aggregation? Maybe not.