Internet Appendix for Taxes and Chain Effects as Determinants of Informality

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1 Proofs.

Lemma 1 *If $\pi p_f < p_i < p_f$ then $\overline{\theta}_d(p_i, p_f)$ decreases with p_i and it increases with p_f. Further, if $\pi \leq \mu \leq 1$ then, $\overline{\theta}_d(p_i, \frac{p_i}{\mu})$ increases with p_i.*

Proof: If $\pi p_f < p_i \leq p_f$ formal firms prefer to buy the formal good. Hence

$$\frac{\partial \Pi_f^d(\theta_d)}{\partial p_f} = -\pi x_f(\theta_d, p_f) \tag{1}$$

Similarly, if $\pi p_f \leq p_i < p_f$, informal firms prefer to buy the informal good, and in an analogous fashion

$$\frac{\partial \Pi_i^d(\theta_d)}{\partial p_i} = -x_i(\theta_d, p_i) \tag{2}$$

This establishes the first part of the lemma, since increasing p_i reduces profits for informal firms and increasing p_f reduces profits for formal firms.

In order to sign the change in $\overline{\theta}_d(p_i, \frac{p_i}{\mu})$ we must establish the sign of:

$$\frac{1}{\mu}\frac{\partial \Pi_f^d(\theta_d)}{\partial p_f} - \frac{\partial \Pi_i^d(\theta_d)}{\partial p_i}. \tag{3}$$

for the marginal firm. If this is negative, the difference in profits in the formal and informal sectors for the marginal firm decreases and more firms will become informal. If $\pi p_i < p_f < p_i$,

$$\frac{1}{\mu}\frac{\partial \Pi_f^d(\theta_d)}{\partial p_f} - \frac{\partial \Pi_i^d(\theta_d)}{\partial p_i} = -\frac{\pi}{\mu} x_f(\theta_d, \frac{p_i}{\mu}) + x_i(\theta_d, p_i). \tag{4}$$

The marginal informal firm buys exactly $\overline{x}$. Hence, from Proposition 2

$$-\frac{\pi}{\mu} x_f(\overline{\theta}_d, \frac{p_i}{\mu}) + x_i(\overline{\theta}_d, p_i) \leq -\frac{\overline{x}}{\mu} + \overline{x} \leq 0$$

since we assume that $\mu \leq 1$ and the second part of the lemma follows.

The derivative $\frac{\partial \Pi_f^d(\theta_d)}{\partial p_f}$ $(\frac{\partial \Pi_i^d(\theta_d)}{\partial p_i})$ is not well defined when $p_i = \pi p_f$ (resp. $p_i = p_f$), but it is easy to see that, in this case, the change in profit difference between formality and informality for the marginal firm still equals $-\frac{\pi}{\mu} x_f(\overline{\theta}_d, \frac{p_i}{\mu}) + x_i(\overline{\theta}_d, p_i)$. ■

Comparative Statics

Market equilibrium yields:

$$I(p_i,p_f)=\underbrace{\int_0^{\overline{\theta}_u}\theta g_u(\theta)d\theta}_{\text{informal supply}}-\underbrace{\int_0^{\overline{\theta}_d(p_i,p_f)}x_i(\theta,p_i)g_d(\theta)d\theta}_{\text{informal demand}}=0 \tag{5}$$

$$F(p_i,p_f)=\underbrace{\int_{\overline{\theta}_u}^{\infty}\theta g_u(\theta)d\theta}_{\text{formal supply}}-\underbrace{\int_{\overline{\theta}_d(p_i,p_f)}^{\infty}x_f(\theta,p_f)g_d(\theta)d\theta}_{\text{formal demand}}=0 \tag{6}$$

The Jacobian matrix for the above functions is:

$$A = \begin{bmatrix} \frac{\partial I}{\partial p_i} & \frac{\partial I}{\partial p_f} \\ \frac{\partial F}{\partial p_i} & \frac{\partial F}{\partial p_f} \end{bmatrix} =$$

$$= \begin{bmatrix} \underbrace{\begin{matrix}\overline{\theta}_u g_u(\overline{\theta}_u)\frac{\partial\overline{\theta}_u}{\partial p_i}-\overline{x}g_d(\overline{\theta}_d)\frac{\partial\overline{\theta}_d}{\partial p_i} \\ -\int_0^{\overline{\theta}_d(p_i,p_f)}\frac{\partial x_i(\theta,p_i)}{\partial p_i}g_d(\theta)d\theta\end{matrix}}_{>0} & \underbrace{\overline{\theta}_u g_u(\overline{\theta}_u)\frac{\partial\overline{\theta}_u}{\partial p_f}-\overline{x}g_d(\overline{\theta}_d)\frac{\partial\overline{\theta}_d}{\partial p_f}}_{<0} \\ \underbrace{-\overline{\theta}_u g_u(\overline{\theta}_u)\frac{\partial\overline{\theta}_u}{\partial p_i}+x_f(\overline{\theta}_d)g_d(\overline{\theta}_d)\frac{\partial\overline{\theta}_d}{\partial p_i}}_{<0} & \underbrace{\begin{matrix}-\overline{\theta}_u g_u(\overline{\theta}_u)\frac{\partial\overline{\theta}_u}{\partial p_f}+x_f(\overline{\theta}_d)g_d(\overline{\theta}_d)\frac{\partial\overline{\theta}_d}{\partial p_f} \\ -\int_{\overline{\theta}_d(p_i,p_f)}^{\infty}\frac{\partial x_f(\theta,p_f)}{\partial p_f}g_d(\theta)d\theta\end{matrix}}_{>0} \end{bmatrix}$$

where the signs are obtained from noticing that

$$\frac{\partial\overline{\theta}_u}{\partial p_i}=\frac{\overline{y}}{\pi p_f}>0 \text{ and } \frac{\partial\overline{\theta}_u}{\partial p_f}=-\frac{p_i\overline{y}}{\pi p_f^2}<0.$$

For $\frac{\partial\overline{\theta}_d}{\partial p_f}$ and $\frac{\partial\overline{\theta}_d}{\partial p_i}$ notice that Π_i^d is maximized at the second argument in equation (4) in the article (because $\pi p_f < p_i < p_f$), which is independent of p_f and is negatively related to p_i. Likewise, Π_f^d is maximized at the first argument in equation (3) in the article, which is independent of p_f and negatively related to p_i. This leads to

$$\frac{\partial\overline{\theta}_d}{\partial p_i}<0 \text{ and } \frac{\partial\overline{\theta}_d}{\partial p_f}>0.$$

The determinant of the above matrix is:

$$|A| = \begin{pmatrix} \overline{\theta}_u g_u(\overline{\theta}_u)\frac{\partial \overline{\theta}_u}{\partial p_i} - \overline{x} g_d(\overline{\theta}_d)\frac{\partial \overline{\theta}_d}{\partial p_i} \\ -\int_0^{\overline{\theta}_d(p_i,p_f)} \frac{\partial x_i(\theta,p_i)}{\partial p_i} g_d(\theta) d\theta \end{pmatrix} \times \begin{pmatrix} -\overline{\theta}_u g_u(\overline{\theta}_u)\frac{\partial \overline{\theta}_u}{\partial p_f} + x_f(\overline{\theta}_d) g_d(\overline{\theta}_d)\frac{\partial \overline{\theta}_d}{\partial p_f} \\ -\int_{\overline{\theta}_d(p_i,p_f)}^{\infty} \frac{\partial x_f(\theta,p_f)}{\partial p_f} g_d(\theta) d\theta \end{pmatrix}$$
$$- \left(\overline{\theta}_u g_u(\overline{\theta}_u)\frac{\partial \overline{\theta}_u}{\partial p_f} - \overline{x} g_d(\overline{\theta}_d)\frac{\partial \overline{\theta}_d}{\partial p_f}\right) \times \left(-\overline{\theta}_u g_u(\overline{\theta}_u)\frac{\partial \overline{\theta}_u}{\partial p_i} + x_f(\overline{\theta}_d) g_d(\overline{\theta}_d)\frac{\partial \overline{\theta}_d}{\partial p_i}\right)$$

After cancelations and *ignoring the terms involving the integrals (which are positive)*, this expression becomes

$$\overline{\theta}_u g_u(\overline{\theta}_u) g_d(\overline{\theta}_d) \left[\underbrace{\left(x_f(\overline{\theta}_d)\frac{\partial \overline{\theta}_u}{\partial p_i}\frac{\partial \overline{\theta}_d}{\partial p_f} + \overline{x}\frac{\partial \overline{\theta}_u}{\partial p_f}\frac{\partial \overline{\theta}_d}{\partial p_i}\right)}_{\text{from } a_{11}\times a_{22}} - \underbrace{\left(x_f(\overline{\theta}_d)\frac{\partial \overline{\theta}_u}{\partial p_f}\frac{\partial \overline{\theta}_d}{\partial p_i} + \overline{x}\frac{\partial \overline{\theta}_u}{\partial p_i}\frac{\partial \overline{\theta}_d}{\partial p_f}\right)}_{\text{from } a_{12}\times a_{21}} \right] =$$

$$= \underbrace{\overline{\theta}_u g_u(\overline{\theta}_u) g_d(\overline{\theta}_d)}_{>0} \left[\underbrace{\left(x_f(\overline{\theta}_d) - \overline{x}\right)}_{>0} \left(\frac{\partial \overline{\theta}_u}{\partial p_i}\frac{\partial \overline{\theta}_d}{\partial p_f} - \frac{\partial \overline{\theta}_u}{\partial p_f}\frac{\partial \overline{\theta}_d}{\partial p_i}\right)\right]$$

Hence, as long as

$$\left(\frac{\partial \overline{\theta}_u}{\partial p_i}\frac{\partial \overline{\theta}_d}{\partial p_f} - \frac{\partial \overline{\theta}_u}{\partial p_f}\frac{\partial \overline{\theta}_d}{\partial p_i}\right) > 0$$

the determinant is positive. Because $\overline{\theta}_u$ is homogeneous of degree zero (or by direct inspection),

$$\frac{\partial \overline{\theta}_u}{\partial p_i} p_i + \frac{\partial \overline{\theta}_u}{\partial p_f} p_f = 0$$

and since $p_f > p_i$,

$$\left|\frac{\partial \overline{\theta}_u}{\partial p_i}\right| > \left|\frac{\partial \overline{\theta}_u}{\partial p_f}\right|.$$

So, if we can show that

$$\left|\frac{\partial \overline{\theta}_d}{\partial p_f}\right| > \left|\frac{\partial \overline{\theta}_d}{\partial p_i}\right|$$

we can conclude that the determinant is positive.

By definition,

$$\Delta \Pi^d = \Pi_i^d(\overline{\theta}_d, p_i, p_f) - \Pi_f^d(\overline{\theta}_d, p_i, p_f) = 0. \tag{7}$$

We can then apply the Implicit Function Theorem to get:

$$\frac{\partial \overline{\theta}_d/\partial p_i}{\partial \overline{\theta}_d/\partial p_f} = \frac{\partial \Pi_i/\partial p_i - \partial \Pi_f/\partial p_i}{\partial \Pi_i/\partial p_f - \partial \Pi_f/\partial p_f} = \frac{\partial \Pi_i/\partial p_i}{-\partial \Pi_f/\partial p_f}$$

because equation (3) in the article is active in the first argument and equation (4) in the article, in the second as previously mentioned. We can now appeal to the Envelope Theorem, which delivers

$$\frac{\partial \Pi_i^d}{\partial p_i} = -\overline{x}$$

and

$$\frac{\partial \Pi_f^d}{\partial p_f} = -x_f(\overline{\theta}_d)\pi.$$

By Proposition 4, $x_f(\overline{\theta}_d)\pi > \overline{x}$ and

$$\left|\frac{\partial \overline{\theta}_d}{\partial p_f}\right| > \left|\frac{\partial \overline{\theta}_d}{\partial p_i}\right|.$$

This allows us to compute the inverse of A as

$$A^{-1} = \frac{1}{|A|}\text{adj}(A)$$

where the adjoint matrix is given by

$$\begin{aligned}
\text{adj}(A) &= \begin{bmatrix} \frac{\partial F}{\partial p_f} & -\frac{\partial I}{\partial p_f} \\ -\frac{\partial F}{\partial p_i} & \frac{\partial I}{\partial p_i} \end{bmatrix} = \\
&= \begin{bmatrix} \underbrace{\begin{matrix} -\overline{\theta}_u g_u(\overline{\theta}_u)\frac{\partial \overline{\theta}_u}{\partial p_f} + x_f(\overline{\theta}_d) g_d(\overline{\theta}_d)\frac{\partial \overline{\theta}_d}{\partial p_f} \\ -\int_{\overline{\theta}_d(p_i,p_f)}^{\infty} \frac{\partial x_f(\theta,p_f)}{\partial p_f} g_d(\theta)d\theta \end{matrix}}_{>0} & \underbrace{-\overline{\theta}_u g_u(\overline{\theta}_u)\frac{\partial \overline{\theta}_u}{\partial p_f} + \overline{x} g_d(\overline{\theta}_d)\frac{\partial \overline{\theta}_d}{\partial p_f}}_{>0} \\ \underbrace{\overline{\theta}_u g_u(\overline{\theta}_u)\frac{\partial \overline{\theta}_u}{\partial p_i} - x_f(\overline{\theta}_d) g_d(\overline{\theta}_d)\frac{\partial \overline{\theta}_d}{\partial p_i}}_{>0} & \underbrace{\begin{matrix} \overline{\theta}_u g_u(\overline{\theta}_u)\frac{\partial \overline{\theta}_u}{\partial p_i} - \overline{x} g_d(\overline{\theta}_d)\frac{\partial \overline{\theta}_d}{\partial p_i} \\ -\int_0^{\overline{\theta}_d(p_i,p_f)} \frac{\partial x_i(\theta,p_i)}{\partial p_i} g_d(\theta)d\theta \end{matrix}}_{>0} \end{bmatrix}
\end{aligned}$$

Comparative Statics with respect to $\overline{y}$. To study the sensitivity of the equilibrium to $\overline{y}$, let $\overline{\theta}_u g_u(\overline{\theta}_u)\frac{\partial \overline{\theta}_u}{\partial \overline{y}} = c > 0$ and notice that

$$\begin{bmatrix} \frac{\partial I}{\partial \overline{y}} \\ \frac{\partial F}{\partial \overline{y}} \end{bmatrix} = \begin{bmatrix} c \\ -c \end{bmatrix}$$

Consequently,

$$\begin{bmatrix} \frac{dp_i}{d\overline{y}} \\ \frac{dp_f}{d\overline{y}} \end{bmatrix} = -\frac{1}{|A|}\text{adj}(A)\begin{bmatrix} c \\ -c \end{bmatrix} = \frac{1}{|A|}\text{adj}(A)\begin{bmatrix} -c \\ c \end{bmatrix}$$

So,

$$\frac{dp_i}{d\overline{y}} = -c\frac{(a_{22}+a_{12})}{|A|} = -\frac{c}{|A|}\left((x_f(\overline{\theta}_d)-\overline{x})g_d(\overline{\theta}_d)\frac{\partial\overline{\theta}_d}{\partial p_f} - \int_{\overline{\theta}_d(p_i,p_f)}^{\infty}\frac{\partial x_f(\theta,p_f)}{\partial p_f}g_d(\theta)d\theta\right) < 0$$

and

$$\frac{dp_f}{d\overline{y}} = c\frac{(a_{21}+a_{11})}{|A|} = \frac{c}{|A|}\left((x_f(\overline{\theta}_d)-\overline{x})g_d(\overline{\theta}_d)\frac{\partial\overline{\theta}_d}{\partial p_i} - \int_{0}^{\overline{\theta}_d(p_i,p_f)}\frac{\partial x_i(\theta,p_i)}{\partial p_i}g_d(\theta)d\theta\right)$$

Hence:

$$\begin{aligned}\frac{d\overline{\theta}_d}{d\overline{y}} &= \frac{\partial\overline{\theta}_d}{\partial p_i}\frac{dp_i}{d\overline{y}} + \frac{\partial\overline{\theta}_d}{\partial p_f}\frac{dp_f}{d\overline{y}} = \\ &= \frac{c}{|A|}\left(\frac{\partial\overline{\theta}_d}{\partial p_i}\int_{\overline{\theta}_d(p_i,p_f)}^{\infty}\frac{\partial x_f(\theta,p_f)}{\partial p_f}g_d(\theta)d\theta - \frac{\partial\overline{\theta}_d}{\partial p_f}\int_{0}^{\overline{\theta}_d(p_i,p_f)}\frac{\partial x_i(\theta,p_i)}{\partial p_i}g_d(\theta)d\theta\right) > 0.\end{aligned}$$

For the threshold in the upstream stage,

$$\frac{d\overline{\theta}_u}{d\overline{y}} = \underbrace{\underbrace{\frac{\partial\overline{\theta}_u}{\partial p_i}}_{>0}\underbrace{\frac{dp_i}{d\overline{y}}}_{<0} + \frac{\partial\overline{\theta}_u}{\partial p_f}\frac{dp_f}{d\overline{y}}}_{\substack{<0 \\ (indirect)}} + \underbrace{\frac{\partial\overline{\theta}_u}{\partial\overline{y}}}_{\substack{>0 \\ (direct)}}$$

where

$$\begin{aligned}\frac{\partial\overline{\theta}_u}{\partial p_i}\frac{dp_i}{d\overline{y}} + \frac{\partial\overline{\theta}_u}{\partial p_f}\frac{dp_f}{d\overline{y}} &= \frac{c}{|A|}\Bigg(\underbrace{\left(\frac{\partial\overline{\theta}_u}{\partial p_f}\frac{\partial\overline{\theta}_d}{\partial p_i} - \frac{\partial\overline{\theta}_u}{\partial p_i}\frac{\partial\overline{\theta}_d}{\partial p_f}\right)(x_f(\overline{\theta}_d)-\overline{x})g_d(\overline{\theta}_d)}_{<0} \\ &\quad \underbrace{+\frac{\partial\theta_u}{\partial p_i}\int_{\overline{\theta}_d(p_i,p_f)}^{\infty}\frac{\partial x_f(\theta,p_f)}{\partial p_f}g_d(\theta)d\theta}_{<0} - \underbrace{\frac{\partial\overline{\theta}_u}{\partial p_f}\int_{0}^{\overline{\theta}_d(p_i,p_f)}\frac{\partial x_i(\theta,p_i)}{\partial p_i}g_d(\theta)d\theta}_{>0}\Bigg) < 0\end{aligned}$$

with the first term above is negative since $\frac{\partial\overline{\theta}_u}{\partial p_f}\frac{\partial\overline{\theta}_d}{\partial p_i} = \left|\frac{\partial\overline{\theta}_u}{\partial p_f}\right|\left|\frac{\partial\overline{\theta}_d}{\partial p_i}\right| < \left|\frac{\partial\overline{\theta}_u}{\partial p_i}\right|\left|\frac{\partial\overline{\theta}_d}{\partial p_f}\right| = \frac{\partial\overline{\theta}_u}{\partial p_i}\frac{\partial\overline{\theta}_d}{\partial p_f}$. The expression can be further simplified using

$$\frac{\partial\overline{\theta}_d}{\partial p_i} = -\frac{\partial\Delta\Pi^d/\partial p_i}{\partial\Delta\Pi^d/\partial\overline{\theta}_d} = \frac{p_i\overline{x}}{p_i\overline{x} - p_f x_f(\overline{\theta}_d)} < 0$$

and

$$\frac{\partial\overline{\theta}_d}{\partial p_f} = -\frac{\partial\Delta\Pi^d/\partial p_i}{\partial\Delta\Pi^d/\partial\overline{\theta}_d} = \frac{-p_f x_f(\overline{\theta}_d)}{p_i\overline{x} - p_f x_f(\overline{\theta}_d)} > 0$$

where $\Delta\Pi^d$ is defined in equation (7). It is still unclear how this expression compares to $\frac{\partial\overline{\theta}_u}{\partial\overline{y}} = \frac{p_i}{\pi p_f}$ and what the sign of the expression is.

Comparative Statics with respect to $\overline{x}$. Let θ^* denote the θ at which an informal entrepreneur in the downstream sector becomes constrained by $\overline{x}$. Notice that

$$\begin{aligned} I(p_i,p_f) &= \underbrace{\int_0^{\overline{\theta}_u} \theta g_u(\theta)d\theta}_{\text{informal supply}} - \underbrace{\int_0^{\overline{\theta}_d(p_i,p_f)} x_i(\theta,p_i)g_d(\theta)d\theta}_{\text{informal demand}} = \\ &= \int_0^{\overline{\theta}_u} \theta g_u(\theta)d\theta - \int_0^{\theta^*} x_i(\theta,p_i)g_d(\theta)d\theta - \overline{x}(G(\overline{\theta}_d) - G(\theta^*)) \end{aligned}$$

Hence,

$$\begin{bmatrix} \frac{\partial I}{\partial \overline{x}} \\ \frac{\partial F}{\partial \overline{x}} \end{bmatrix} = \begin{bmatrix} -(G(\overline{\theta}_d) - G(\theta^*)) - \overline{x}g_d(\overline{\theta}_d)\frac{\partial\overline{\theta}_d}{\partial\overline{x}} \\ x_f(\overline{\theta}_d)g_d(\overline{\theta}_d)\frac{\partial\overline{\theta}_d}{\partial\overline{x}} \end{bmatrix}$$

As was the case before, we need to compute

$$\begin{aligned} \frac{dp_i}{d\overline{x}} &= \frac{1}{|A|}\Bigg[\underbrace{\big(G(\overline{\theta}_d) - G(\theta^*)\big)}_{>0}\underbrace{\left(-\overline{\theta}_u g_u(\overline{\theta}_u)\frac{\partial\overline{\theta}_u}{\partial p_f} + x_f(\overline{\theta}_d)g_d(\overline{\theta}_d)\frac{\partial\overline{\theta}_d}{\partial p_f} - \int_{\overline{\theta}_d(p_i,p_f)}^{\infty}\frac{\partial x_f(\theta,p_f)}{\partial p_f}g_d(\theta)d\theta\right)}_{>0} \\ &\quad \underbrace{+\, g_d(\overline{\theta}_d)\frac{\partial\overline{\theta}_d}{\partial\overline{x}}\frac{\partial\overline{\theta}_d}{\partial p_f}\overline{\theta}_u g_u(\overline{\theta}_u)(x_f(\overline{\theta}_d) - \overline{x}) - \overline{x}g_d(\overline{\theta}_d)\frac{\partial\overline{\theta}_d}{\partial\overline{x}}\int_{\overline{\theta}_d(p_i,p_f)}^{\infty}\frac{\partial x_f(\theta,p_f)}{\partial p_f}g_d(\theta)d\theta}_{>0}\Bigg] > 0 \end{aligned}$$

and

$$\begin{aligned} \frac{dp_f}{d\overline{x}} &= \frac{1}{|A|}\Bigg[\left(G(\overline{\theta}_d) - G(\theta^*) + \overline{x}g_d(\overline{\theta}_d)\frac{\partial\overline{\theta}_d}{\partial\overline{x}}\right)\left(\overline{\theta}_u g_u(\overline{\theta}_u)\frac{\partial\overline{\theta}_u}{\partial p_i} - x_f(\overline{\theta}_d)g_d(\overline{\theta}_d)\frac{\partial\overline{\theta}_d}{\partial p_i}\right) \\ &\quad - \left(\overline{\theta}_u g_u(\overline{\theta}_u)\frac{\partial\overline{\theta}_u}{\partial p_i} - \overline{x}g_d(\overline{\theta}_d)\frac{\partial\overline{\theta}_d}{\partial p_i} - \int_0^{\overline{\theta}_d(p_i,p_f)}\frac{\partial x_i(\theta,p_i)}{\partial p_i}g_d(\theta)d\theta\right)x_f(\overline{\theta}_d)g_d(\overline{\theta}_d)\frac{\partial\overline{\theta}_d}{\partial\overline{x}}\Bigg] \end{aligned}$$

Consequently,

$$
\begin{aligned}
\frac{d\overline{\theta}_u}{d\overline{x}} &= \frac{\partial\overline{\theta}_u}{\partial p_i}\frac{dp_i}{d\overline{x}} + \frac{\partial\overline{\theta}_u}{\partial p_f}\frac{dp_f}{d\overline{x}} = \\
&= \frac{1}{|A|}\Bigg[\left(G(\overline{\theta}_d) - G(\theta^*)\right) x_f(\overline{\theta}_d) g_d(\overline{\theta}_d) \left(\frac{\partial\overline{\theta}_d}{\partial p_f}\frac{\partial\overline{\theta}_u}{\partial p_i} - \frac{\partial\overline{\theta}_d}{\partial p_i}\frac{\partial\overline{\theta}_u}{\partial p_f}\right) \\
&\quad -\frac{\partial\overline{\theta}_u}{\partial p_i}\int_{\overline{\theta}_d(p_i,p_f)}^{\infty}\frac{\partial x_f(\theta,p_f)}{\partial p_f} g_d(\theta)d\theta \left(G(\overline{\theta}_d) - G(\theta^*) + \overline{x} g_d(\overline{\theta}_d)\right) \\
&\quad +\frac{\partial\overline{\theta}_u}{\partial p_f} x_f(\overline{\theta}_d) g_d(\overline{\theta}_d)\frac{\partial\overline{\theta}_d}{\partial\overline{x}}\int_0^{\overline{\theta}_d(p_i,p_f)}\frac{\partial x_i(\theta,p_i)}{\partial p_i} g_d(\theta)d\theta\Bigg] \\
&> 0
\end{aligned}
$$

<u>Comparative Statics with respect to π.</u> With respect to π, notice that

$$
\begin{bmatrix} \frac{\partial I}{\partial \pi} \\ \frac{\partial F}{\partial \pi} \end{bmatrix} = \begin{bmatrix} k \\ -k + \frac{\partial\overline{\theta}_d}{\partial\pi}(x_f(\overline{\theta}_d) - \overline{x}) g_d(\overline{\theta}_d) \end{bmatrix}
$$

where $k = \frac{\partial\overline{\theta}_u}{\partial\pi}\overline{\theta}_u g_u(\overline{\theta}_u) - \frac{\partial\overline{\theta}_d}{\partial\pi}\overline{x} g_d(\overline{\theta}_d)$. Then,

$$
\begin{aligned}
\begin{bmatrix} \frac{dp_i}{d\pi} \\ \frac{dp_f}{d\pi} \end{bmatrix} &= -\frac{1}{|A|}\mathrm{adj}(A)\begin{bmatrix} k \\ -k + \frac{\partial\overline{\theta}_d}{\partial\pi}(x_f(\overline{\theta}_d) - \overline{x}) g_d(\overline{\theta}_d) \end{bmatrix} = \\
&= \frac{1}{|A|}\mathrm{adj}(A)\begin{bmatrix} -k \\ k - \frac{\partial\overline{\theta}_d}{\partial\pi}(x_f(\overline{\theta}_d) - \overline{x}) g_d(\overline{\theta}_d) \end{bmatrix}
\end{aligned}
$$

and consequently,

$$
\frac{dp_i}{d\pi} = -k\frac{(a_{22} + a_{12})}{|A|} + \frac{a_{12}\frac{\partial\overline{\theta}_d}{\partial\pi}(x_f(\overline{\theta}_d) - \overline{x}) g_d(\overline{\theta}_d)}{|A|}
$$

and

$$
\frac{dp_f}{d\pi} = k\frac{(a_{21} + a_{11})}{|A|} - \frac{a_{11}\frac{\partial\overline{\theta}_d}{\partial\pi}(x_f(\overline{\theta}_d) - \overline{x}) g_d(\overline{\theta}_d)}{|A|}.
$$

Let $\Gamma_1 = \int_0^{\overline{\theta}_d(p_i,p_f)}\frac{\partial x_i(\theta,p_i)}{\partial p_i} g_d(\theta)d\theta$ and $\Gamma_2 = \int_{\overline{\theta}_d(p_i,p_f)}^{\infty}\frac{\partial x_f(\theta,p_f)}{\partial p_f} g_d(\theta)d\theta$. The total derivative (multiplied by $|A|$) is then

$$
\begin{aligned}
|A|\frac{d\overline{\theta}_d}{d\pi} &= |A|\left[\frac{\partial\overline{\theta}_d}{\partial p_i}\frac{dp_i}{d\pi} + \frac{\partial\overline{\theta}_d}{\partial p_f}\frac{dp_f}{d\pi} + \frac{\partial\overline{\theta}_d}{\partial\pi}\right] = \\
&= \left(\underbrace{\frac{\partial\overline{\theta}_d}{\partial p_i}\Gamma_2 - \frac{\partial\overline{\theta}_d}{\partial p_f}\Gamma_1}_{>0}\right)\underbrace{\frac{\partial\overline{\theta}_u}{\partial\pi}\overline{\theta}_u g_u(\overline{\theta}_u)}_{<0} \\
&\quad + \underbrace{\frac{\partial\overline{\theta}_d}{\partial\pi}}_{<0}\left(\underbrace{\Gamma_1\overline{\theta}_u g_u(\overline{\theta}_u)\frac{\partial\overline{\theta}_u}{\partial p_f} + \Gamma_1\Gamma_2 - \Gamma_2\overline{\theta}_u g_u(\overline{\theta}_u)\frac{\partial\overline{\theta}_u}{\partial p_i}}_{>0}\right) < 0
\end{aligned}
$$

Hence, both the direct and total effect of taxes on the cutoff are negative. Similarly

$$
\begin{aligned}
|A|\frac{d\overline{\theta}_u}{d\pi} &= |A|\left[\frac{\partial\overline{\theta}_u}{\partial p_i}\frac{dp_i}{d\pi} + \frac{\partial\overline{\theta}_u}{\partial p_f}\frac{dp_f}{d\pi} + \frac{\partial\overline{\theta}_u}{\partial\pi}\right] = \\
&= \underbrace{\Gamma_1 x_f(\overline{\theta}_d)g_d(\overline{\theta}_d)}_{<0}\left(\underbrace{\frac{\partial\overline{\theta}_d}{\partial\pi}\frac{\partial\overline{\theta}_u}{\partial p_f} - \frac{\partial\overline{\theta}_u}{\partial\pi}\frac{\partial\overline{\theta}_d}{\partial p_f}}_{>0}\right) \\
&\quad + \underbrace{\Gamma_2\overline{x}g_d(\overline{\theta}_d)}_{<0}\left(\underbrace{\frac{\partial\overline{\theta}_d}{\partial p_i}\frac{\partial\overline{\theta}_u}{\partial\pi} - \frac{\partial\overline{\theta}_d}{\partial\pi}\frac{\partial\overline{\theta}_u}{\partial p_i}}_{>0}\right) \\
&\quad + \underbrace{\frac{\partial\overline{\theta}_u}{\partial\pi}\Gamma_1\Gamma_2}_{<0} < 0
\end{aligned}
$$

and again both the direct and the total effect of taxes on the cutoff are negative.

2 Empirical Results

Correlation Matrix

	tax reg	tax sub	large cl	small cl	outsdhous	n_work	rev	otherjob	bkloan
tax sub	0.01	1.00							
large cl	0.12	-0.06	1.00						
small cl	0.06	-0.16	-0.10	1.00					
outside hh	0.09	-0.05	0.01	-0.03	1.00				
# employees	0.35	0.02	0.08	0.10	0.08	1.00			
revenue	0.29	0.05	0.17	0.03	0.09	0.30	1.00		
otherjob	-0.01	-0.06	-0.02	-0.06	0.01	0.04	-0.01	1.00	
bank loan	0.11	-0.02	0.01	0.00	-0.03	0.04	0.07	0.01	1.00
education	0.30	-0.16	0.08	0.10	0.06	0.17	0.13	0.24	0.06
age	0.03	0.02	0.01	0.01	0.00	0.04	0.05	0.01	0.00
gender	-0.07	0.05	0.06	0.08	0.10	-0.03	0.03	0.01	-0.06
ho×num	0.15	-0.06	0.05	0.03	0.05	0.08	0.10	0.06	-0.02
log inst	0.55	0.16	0.16	0.06	0.10	0.50	0.52	-0.05	0.15
log inv	0.38	0.01	0.12	0.09	0.07	0.29	0.27	0.06	0.16
profit	0.04	-0.08	0.07	0.04	0.05	0.06	0.37	-0.02	0.02
sup enf	-0.02	0.03	-0.04	-0.03	-0.11	0.03	0.01	0.02	0.04
cl enf	0.09	-0.06	0.03	0.16	-0.06	0.02	0.09	-0.01	0.04
log wage	0.33	-0.08	0.19	0.13	0.12	0.24	0.30	-0.08	0.05
cl form	0.31	-0.04	0.10	0.15	0.05	0.08	0.20	-0.07	0.05
sup form	0.04	-0.10	0.09	0.19	0.04	-0.01	0.03	-0.03	-0.05

Correlation Matrix (cont'd)

	education	age	gender	ho_num	loginst	loginv	profit	sup enf	cl enf
age	-0.12	1.00							
gender	-0.20	0.05	1.00						
ho_num	0.23	0.17	-0.06	1.00					
loginst	0.25	0.05	-0.02	0.12	1.00				
loginv	0.34	-0.02	-0.07	0.13	0.55	1.00			
profit	0.07	0.05	0.00	0.04	-0.04	0.07	1.00		
sup enf	-0.11	-0.04	-0.03	-0.01	0.05	0.01	-0.06	1.00	
cl enf	0.02	0.00	0.05	0.04	0.15	0.12	-0.02	0.63	1.00
log wage	0.21	0.09	0.08	0.09	0.55	0.37	0.05	-0.15	-0.03
cl form	0.11	0.03	0.07	0.08	0.35	0.21	0.01	0.10	0.47
sup form	0.10	0.01	0.13	0.07	0.01	0.04	0.05	-0.11	0.23

Correlation Matrix (cont'd)

	log wage	cl form
cl form	0.09	1.00
sup form	0.08	0.54

Probit Estimates (Chain Effects)

Dep. Var. = tax reg	**Coeff.** (Std. Err.) [Marg. Eff.]	**Coeff.** (Std. Err.) [Marg. Eff.]	**Coeff.** (Std. Err.) [Marg. Eff.]	**Coeff.** (Std. Err.) [Marg. Eff.]
large cl	0.373^{**}			0.331^{**}
	(0.049)			(0.049)
	[0.061]			[0.051]
small cl	0.168^{**}			0.107^{**}
	(0.035)			(0.036)
	[0.024]			[0.014]
supplier formal		2.803^{**}		0.115
		(0.294)		(0.329)
		[0.358]		[0.014]
client formal			4.976^{**}	4.745^{**}
			(0.296)	(0.330)
			[0.618]	[0.587]
outside hh	0.179^{**}	0.167^{**}	0.161^{**}	0.170^{**}
	(0.024)	(0.024)	(0.024)	(0.024)
	[0.022]	[0.021]	[0.02]	[0.020]
# employees	0.407^{**}	0.407^{**}	0.421^{**}	0.421^{**}
	(0.012)	(0.012)	(0.012)	(0.012)
	[0.052]	[0.052]	[0.052]	[0.052]
revenue	0.049^{**}	0.050^{**}	0.046^{**}	0.044^{**}
	(0.005)	(0.005)	(0.004)	(0.004)
	[0.006]	[0.006]	[0.006]	[0.005]
bank loan	0.381^{**}	0.382^{**}	0.361^{**}	0.364^{**}
	(0.033)	(0.034)	(0.034)	(0.034)
	[0.062]	[0.062]	[0.057]	[0.057]
otherjob	-0.229^{**}	-0.238^{**}	-0.234^{**}	-0.224^{**}
	(0.033)	(0.033)	(0.033)	(0.033)
	[-0.026]	[-0.026]	[-0.025]	[-0.024]
education	0.186^{**}	0.184^{**}	0.186^{**}	0.179^{**}
	(0.006)	(0.006)	(0.006)	(0.006)
	[0.024]	[0.024]	[0.023]	[0.022]

Probit Estimates (Chain Effects) (cont'd)

Dep. Var. = tax reg	**Coeff.** (Std. Err.) [Marg. Eff.]	**Coeff.** (Std. Err.) [Marg. Eff.]	**Coeff.** (Std. Err.) [Marg. Eff.]	**Coeff.** (Std. Err.) [Marg. Eff.]
age	0.035^{**}	0.035^{**}	0.037^{**}	0.037^{**}
	(0.005)	(0.005)	(0.005)	(0.005)
	[0.005]	[0.005]	[0.005]	[0.005]
age^2	0.000^{**}	0.000^{**}	0.000^{**}	0.000^{**}
	(0.000)	(0.000)	(0.000)	(0.000)
	[0.000]	[0.000]	[0.000]	[0.000]
gender	0.125^{**}	0.114^{**}	0.134^{**}	0.116^{**}
	(0.020)	(0.021)	(0.020)	(0.021)
	[0.015]	[0.014]	[0.017]	[0.014]
ho_num	0.030^{**}	0.029^{**}	0.029^{**}	0.029^{**}
	(0.003)	(0.003)	(0.003)	(0.003)
	[0.004]	[0.004]	[0.004]	[0.003]
N	47196	46654	46749	46744
Pseudo-R^2	0.3664	0.3657	0.3722	0.3743
χ^2	5491.36	5469.05	5597.23	5622.48

1. Significance levels : † : 10% $*$: 5% $**$: 1%
2. Standard errors clustered by census tract.
3. Controls include state and ECINF sector dummies

IV Probit Estimates (Chain Effects)

Dep. Var. = tax reg	Non-IV Coeff. (Std. Err.)	IV Coeff. (Std. Err.)	Dep. Var. = lscl	First Stage (IV) Coeff. (Std. Err.)
large cl	0.343**	4.220**	educurbsec	0.098**
	(0.048)	(0.371)		(0.017)
			nearest bank	-4×10^{-5}
				(4×10^{-5})
outside hh	0.177**	0.111**	outside hh	0.070
	(0.024)	(0.031)		(0.035)
# employees	0.407**	0.226**	# employees	0.043**
	(0.012)	(0.471)		(0.014)
bank loan	0.382**	0.195**	bank loan	0.044
	(0.033)	(0.051)		(0.051)
otherjob	-0.233**	-0.061	otherjob	-0.209**
	(0.033)	(0.040)		(0.049)
education	0.188**	0.074**	education	0.063**
	(0.006)	(0.022)		(0.006)
age	0.035**	0.020**	age	0.008
	(0.005)	(0.006)		(0.006)
age^2	-0.000**	-0.000**	age^2	0.000*
	(0.000)	0.000		(0.000)
gender	0.139**	-0.021*	gender	0.364**
	(0.020)	(0.029)		(0.031)
ho_num	0.029**	0.013**	ho_num	0.007†
	(0.003)	(0.004)		(0.004)
N	46,822	33,740		
Pseudo-R^2				0.14

1. Significance levels : † : 10% ∗ : 5% ∗∗ : 1%
2. The regressions also control for state and ECINF sector dummies.
3. The second regression uses the average level of education in the census tract as an instrument.
4. IV results obtained as bivariate probit.
5. Standard errors clustered by census tract.

Probit Estimates (Enforcement)

Dep. Var. = tax reg	Coeff. (Std. Err.)	Marg. Eff.	Coeff. (Std. Err.)	Marg. Eff.
sup enf	5.607** (1.463)	0.724		
cl enf			11.817** (1.294)	1.51
outside hh	0.178** (0.024)	0.022	0.177** (0.024)	0.022
# employees	0.407** (0.012)	0.053	0.412** (0.012)	0.053
revenue	0.051** (0.005)	0.006	0.049** (0.004)	0.006
bank loan	0.377** (0.033)	0.062	0.373** (0.033)	0.062
otherjob	-0.243** (0.033)	-0.027	-0.238** (0.033)	-0.027
education	0.192** (0.006)	0.025	0.186** (0.006)	0.024
age	0.035** (0.004)	0.005	0.035** (0.004)	0.004
age^2	0.000** (0.000)	0.000	0.000** (0.000)	0.000
gender	0.152** (0.020)	0.019	0.141** (0.020)	0.018
ho_num	0.030** (0.003)	0.004	0.029** (0.003)	0.004
N	46749		46749	
Pseudo-R^2	0.3628		0.3649	
$\chi^2_{(45)}$	5410.44		5482.02	

1. Significance levels : † : 10% ∗ : 5% ∗∗ : 1%
2. Standard errors clustered by census tract.
3. The regressions also control for state and ECINF sector dummies.

Probit Estimates (Tax Substitution)

Variable	Full Sample Coefficient (Std. Err.)	Tax Sub = 1 Coefficient (Std. Err.)
large cl	0.428**	0.059
	(0.049)	(0.208)
small cl	0.241**	-0.384**
	(0.036)	(0.128)
tax sub_large cl	-0.406†	
	(0.213)	
tax sub_small cl	-0.577**	
	(0.128)	
tax sub	0.348**	
	(0.030)	
outside hh	0.201**	0.201**
	(0.024)	(0.055)
n_employee	0.404**	0.371**
	(0.012)	(0.027)
revenue	0.047**	0.041**
	(0.004)	(0.009)
bank loan	0.381**	0.451**
	(0.033)	(0.074)
otherjob	-0.221**	-0.197*
	(0.033)	(0.081)
education	0.194**	0.198**
	(0.006)	(0.016)
age	0.034**	0.063**
	(0.005)	(0.013)
age2	0.000**	-0.001**
	(0.000)	(0.000)
gender	0.096**	0.004
	(0.021)	(0.053)
homeown_numroom	0.029**	0.051**
	(0.003)	(0.008)
N	46822	5732
Pseudo-R^2	0.3697	0.3285
$\chi^2_{(47)}$	5684.12	959.64

1. Significance levels : † : 10% $*$: 5% $**$: 1%
2. Standard errors clustered by census tract.
3. The regressions also control for state and ECINF sector dummies.