Rev. Econ. Design 7, 233–255 (2002)

Review
of Economic
Design

Ambiguity in election games

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Abstract. We construct a model in which the ambiguity of candidates allows them to increase the number of voters to whom they appeal. We focus our analysis on two points that are central to obtain ambiguity in equilibrium: restrictions on the beliefs that candidates can induce in voters, and intensity of voters' preferences. The first is necessary for a pure strategy equilibrium to exist, while the second is necessary for ambiguity in equilibrium when there exists a Condorcet winner in the set of pure alternatives (e.g. the spatial model of electoral competition), and when candidates' only objective is to win the election. In this last case, an ambiguous candidate may offer voters with different preferences the hope that their most preferred alternative will be implemented. We also show that if there are sufficiently many candidates or parties, ambiguity will not be possible in equilibrium, but a larger set of possible policies increases the chance that at least one candidate will choose to be ambiguous in equilibrium.

> *Ambiguity thus increases the number of voters to whom a party may appeal. This fact encourages parties in a two party system to be as equivocal as possible about their stands on each controversial issue. And since both parties find it rational to be ambiguous, neither is forced by the other's clarity to take a more precise stand.*
>
> Anthony Downs, *An Economic Theory of Democracy* (1957)

We would like to thank Alberto Alesina, Antonio Cabrales, Steve Coate, Olivier Compte, Tim Feddersen, Itzhak Gilboa, Joe Harrington, Michel Le Breton, Alessandro Lizzeri, George Mailath, Steve Matthews, Steve Morris, Ignacio Ortuño, Tom Palfrey, Larry Samuelson, Murat Sertel, Fernando Vega, Eyal Winter and an anonymous referee for helpful comments. The first author acknowledges financial support from DGICYT-PB 95-0983. This work was done while the first author was visiting the Center in Political Economy at Washington University, and visiting the Center for Basic Research in the Social Sciences at Harvard University. Their hospitality is gratefully acknowledged. The support of the second author's research by the National Science Foundation is also gratefully acknowledged.

1 Introduction

Our aim in this paper is to analyze the strategic use of ambiguity by candidates in electoral competition. While there is general agreement that politicians sometimes choose to be ambiguous, it is not clear how such strategic ambiguity is to be reconciled with models based on rational voters with standard preferences. We will say that a candidate is ambiguous if at the end of the campaign voters are not sure what policy he will implement in case he wins. On the other hand, we say that a candidate is unambiguous if at the end of the campaign voters are certain which policy will be implemented in case he wins the election. Given a set of alternatives or policies, the ambiguity of candidates is represented by nondegenerate probability distributions or lotteries over the set of policies or pure alternatives. A lottery that assigns probability one to a certain policy (a degenerate lottery) reflects no ambiguity.

These lotteries are interpreted as the voters' beliefs about the policy preferences of the candidates. The relevance of the presence of ambiguity with respect to a candidate during an electoral campaign is reflected in the vote: voters' beliefs about the candidates' preferences will determine how they vote. Therefore, during the electoral campaign candidates make statements or promises that may affect voters' beliefs over their policy preferences or future performance, in order to maximize their expected utility. Even though the actual strategies of candidates during campaigns are rhetoric, endorsements, etc., in our analysis of ambiguity we will represent electoral campaigns as reduced form games in which the strategies of the candidates are the lotteries that represent voters' beliefs that are induced by the rhetoric.

We will characterize conditions under which a candidate decides to induce ambiguous beliefs in voters when he has the possibility of eliminating any uncertainty voters may have about what policy he will choose if elected. For this purpose we construct a model of two candidates competing to win an election, and we present conditions under which, in equilibrium, candidates choose to induce ambiguous beliefs. The presence of ambiguity in equilibrium in this framework allows us to obtain an explanation of how ambiguity may improve a candidate's chances of winning the election, as claimed by Downs in the quote above. An ambiguous candidate is able to convince voters who differ in their preferences that their (different) most preferred alternatives have some chance of being implemented, which will be attractive for voters who care intensely about these most preferred alternatives. In general, we can only have an equilibrium that exhibits ambiguous strategies if candidates can affect voters' beliefs, and when they can affect voters' beliefs, candidates choose ambiguous strategies.

Hence, a necessary condition for ambiguity is that some of a candidate's campaign strategies affect voters' beliefs. We assume candidates cannot guarantee voters that campaign promises will be honored; hence, voters will realize that when candidates care only about winning, the campaign strategies they choose are empty of informational content regarding the candidate's future performance. That is, since campaign statements are cheap talk, rational voters will not update their beliefs, and candidates cannot alter voters' beliefs, ambiguous or unambiguous.

Campaign statements may affect voters' beliefs only to the extent that they convey information about candidates' preferences and, hence, future policy choice. In Aragones and Postlewaite (2000) we provide conditions under which cheap talk statements can affect voters' beliefs when there are repeated elections. In that model,[1] a candidate is able to alter voters' initial beliefs, but is constrained in the beliefs he can induce about the policy he will implement if elected. The beliefs a candidate can induce depend on the voters' prior over his preferences over policies that will arise in both current and future periods. Some restriction on the beliefs that can be induced is necessary for the existence of equilibrium in ambiguous strategies: Fishburn (1972) shows that when candidates are allowed to induce any beliefs in voters, if an equilibrium exists only degenerate lotteries will be chosen.

For candidates who can alter voters' beliefs about what policy they will implement if elected, it must still be optimal for them to choose an ambiguous strategy. Alesina and Cukierman (1990) and Aragones and Neeman (2000) analyze models in which candidates care about more than just winning elections, and in which candidates may choose ambiguous strategies in equilibrium, even when voters' preferences over policies exhibit risk aversion. Candidates face a trade off between the probability of winning the election and the utility they derive if they are elected. In these cases, the probability of winning the election is lower with ambiguity and, therefore, the explanation that these papers offer for the presence of ambiguity is very different from the one we offer in this paper. If a candidate's only objective is to win the election, then ambiguity can only be optimal if, for at least some voters, ambiguity increases a candidate's appeal. This can be the case when voters' preferences are such that there is no Condorcet winner in the set of pure alternatives; in this case it is easy to see that for any unambiguous strategy we can find an ambiguous strategy that is preferred by a majority of voters. We will show that even when a Condorcet winner exists in the set of pure alternatives, as long as it is not the first choice of a majority of voters, an ambiguous strategy may defeat it in equilibrium. This will be the case if the voters' preferences over policies are "intense", that is, if the utility of a voter when his first choice is implemented is much larger than the utility he derives from the implementation of any other alternative. Ambiguous strategies arise as equilibrium outcomes because they provide voters with intense preferences the hope that their most preferred outcome might be chosen.

A simple example will illustrate this point. Suppose that the possibility of reducing taxes is the issue at the heart of the campaign. And suppose that any tax cut has to be compensated with a reduction of some public expenditure. We can imagine a situation in which a majority of voters prefer a tax cut rather than no tax cut at all, but their preferences differ about which kind of expenditure should be reduced. In this case a candidate who can convince voters that he will cut taxes, without specifying how, is inducing ambiguous beliefs on voters: voters are not sure which kind of expenditure he will cut.

For the preferences shown in Table 1, suppose that one of the candidates is able to convince the voters that he will not cut taxes, the Condorcet winner in the set of pure alternatives. Suppose further that voters' preferences are intense; that is,

[1] The model is described in the last section.

Table 1.

Voter 1	Voter 2	Voter 3
cut military	no tax cut	cut welfare
no tax cut	cut military	no tax cut
cut welfare	cut welfare	cut military

for voter 1, he would like very much to lower taxes by cutting military spending, but slightly prefers no tax cut to cutting welfare; similarly, voters 2 and 3 are close to indifferent between their second and third alternatives, strongly preferring their top-ranked alternative to either of them. A second candidate can obtain a majority of votes against the Condorcet winner with an ambiguous strategy – a promise to cut taxes with the precise programs to be cut to be determined in the future. Such an ambiguous strategy could induce in voters beliefs about the policy that will be implemented by this candidate that assign equal probabilities to the other two alternatives, cut military expenditures with probability one half, and cut welfare expenditures with probability one half, with voters 1 and 3 preferring the candidate that offers substantial probability that their first choice is implemented to a candidate that offers their second choice for sure. The outcome, of course, will be that the second candidate who refused to commit will be elected[2] In essence, the intensity of the voters' preferences for their first choice makes ambiguity strategically attractive to candidates.

This example is similar to that of Zeckhauser (1969) that showed the existence of a non-degenerate lottery defeating a Condorcet winner when preferences are intense. In this paper we analyze equilibria of an election model in which both candidates are allowed non-degenerate lotteries as strategies, and in this sense, we generalize Zeckhauser's analysis. Our paper is also related to Shepsle (1970), which deals with ambiguity in equilibrium in the case that there is a Condorcet cycle; our work differs in that we allow the case of a Condorcet winner. In addition to extending the results in these papers, we extend our analysis to the case that there is uncertainty about voters' intensities. Lastly, we show that ambiguity disappears as political competition increases.

2 A simple model of ambiguity

There are two candidates who compete for votes in an election. We assume that there are three alternatives: $Z = \{A, B, C\}$. We denote by $\triangle Z$ the set of probability distributions over the set of alternatives

$$\triangle Z = \{(p_A, p_B, p_C) : p_A, p_B, p_C \geq 0, p_A + p_B + p_C = 1\}.$$

[2] Given this framework, one can imagine a candidate promising a tax cut compensated by reducing part of the military expenditures and part of the welfare expenditures. Then we would have to define it as a new alternative, and redefine the voters' preferences accordingly. In the concluding remarks we discuss the introduction of such alternatives and the circumstances under which our conclusions are unchanged when the voters' preferences are intense.

We will abstract from the process by which politicians induce beliefs and model a politician's strategies as those beliefs that he induce in the voters. We discuss this reduced form aspect of our model and the type of processes that would justify it in the discussion section. We denote by $W_i \subseteq \Delta Z$ the set of beliefs that candidate i can induce in voters; $p^i \in W_i$ represents a strategy for candidate i. We will say that a strategy is *ambiguous* if it represents nondegenerate beliefs.

We will first assume that the strategy set for candidate i is given by $W_i = \{p \in \Delta Z : p_{Z_i} \geq a_i\}$ for some $a_i \in (0, 1]$, that is, a set that contains all probability distributions over the set of alternatives Z that assign a minimum probability $a_i > 0$ to some alternative Z_i. The alternative Z_i can be thought of as the outcome voters think is most likely to be candidate i's most preferred outcome. Such a strategy set implies that a candidate is able to convince voters that his true preferred policy is Z_i, but in addition, is able to induce in voters beliefs that to a limited extent his most preferred policy choice is one of the other alternatives. a_i can be interpreted as the scope of ambiguity available to which candidate i. It is important to note that Z_i is *not* necessarily candidate i's most preferred point, although that may be the case. Candidates' true preferences are irrelevant to voters' behavior; what matters is voters' *beliefs* about those preferences.

We assume that there are N voters, where N is a positive odd integer. Voters have preferences over alternatives that can be represented by von Neumann-Morgenstern utility functions. We normalize a voter's utility function so that it assigns one to the most preferred alternative and zero to the least preferred; we denote by $x \in [0, 1]$ the utility of the intermediate alternative. Hence, each voter is characterized by an ordering of the alternatives represented by $u : Z \to \{0, x, 1\}$ where $x \in (0, 1]$ represents the intensity of the voter's preferences. It is worthwhile commenting on the interpretation of x. If the alternatives have a natural linear order (such as varying amounts of money), "intense" preferences (i.e., $x < 0.5$) correspond to risk loving preferences. But as the example in the introduction illustrates, it may be the case that voters have intense preferences in problems unrelated to risk preferences.

We restrict attention to the case in which voters who have the same preference order have the same preference intensity as well for the sake of clarity. Hence, there are six types of voters, each with one of the six possible ordinal (strict) rankings of the three alternatives. We will use G_k both to denote the name of one of these groups of voters and to denote the number of voters in that group; no confusion should arise from this abuse of notation. The preferences for each of the groups are given in the following table.

G_1	A	$\succ$	B	$\succ$	C
G_2	A	$\succ$	C	$\succ$	B
G_3	B	$\succ$	A	$\succ$	C
G_4	B	$\succ$	C	$\succ$	A
G_5	C	$\succ$	A	$\succ$	B
G_6	C	$\succ$	B	$\succ$	A

Given a voter's beliefs, he or she votes for the candidate that maximizes his or her expected utility, and does not vote if indifferent.[3]

We analyze first the case in which candidates know the distribution of voters' preferences and their intensities. We provide conditions under which pure strategy Nash equilibria exist, and further, when a Nash equilibrium involves one or the other candidate being ambiguous.

3 Ambiguity as an equilibrium outcome

Our aim is to demonstrate conditions under which candidates will choose non-degenerate lotteries in equilibrium, that is, choose to induce beliefs that leave voters uncertain about the outcome that will be chosen if the candidate is elected. We first observe that if one of the pure alternatives is the first choice of a majority of voters, then no lottery can defeat it. In this case this alternative is the Condorcet winner (an alternative that defeats all other alternatives in pairwise elections using majority rule) in the set of pure alternatives Z as well as in the set of lotteries over pure alternatives, ΔZ. Thus, there cannot be ambiguity in this case.

3.1 With a Condorcet winner

A more interesting situation arises when the preferences of the voters are such that no pure alternative is the first choice of a majority of voters but there is a Condorcet winner among the pure alternatives, that is, a pure alternative that defeats all other alternatives in Z in pairwise elections using majority rule. In this case, it is still possible that a lottery might defeat this alternative under majority rule. Thus, even if a candidate can convince the voters that he will choose the alternative that is most preferred by a majority to any other alternative, this candidate may be defeated by an opponent who chooses to be ambiguous.[4] Without loss of generality assume that alternative B is the Condorcet winner in the set of pure alternatives, but is not the first choice of a majority of voters. Suppose also that this is candidate 1's most preferred outcome; we will refer to this candidate as the Condorcet candidate. This candidate has available a strategy that induces beliefs in voters that put probability 1 on this outcome by assumption. Suppose the other candidate's most preferred outcome is A. If this latter candidate has limited ability to be ambiguous, voters will believe with probability close to 1 that A will be the outcome if he is elected. Since B is a Condorcet winner that defeats A, it will also defeat lotteries that put very high probability on A. Consequently, if the second candidate has sufficiently limited ability to be ambiguous, the first candidate can be guarantee to voters that he prefers B and win the election. Hence, again, the winning strategy will be unambiguous.

[3] We assume that preferences over lotteries can be presented by expected utility for ease of exposition. Our results would still hold for a broader class of preferences satisfying some but not all of the axioms needed for expected utility.

[4] An example along these lines with three voters was constructed by Zeckhauser (1969).

But suppose that the second candidate has greater scope for being ambiguous. Specifically, suppose he can induce in voters beliefs that his most preferred outcome is either A or C, with the probability on each being close to 1/2. If each group of voters G_i has intense preferences (i.e., $x_i < .5$), all voters whose first choice is not B will prefer this lottery to the certainty of getting B. Hence, when voters have intense preferences and candidates have sufficient scope to be ambiguous, an ambiguous candidate will defeat one known to favor the Condorcet winner. Note that it may not be necessary for the ambiguous candidate to be able to induce a probability close to 1/2 on the outcome that is not his most preferred. While this degree of latitude to be ambiguous is sufficient to defeat the Condorcet winner when voters have intense preferences, the level of ambiguity necessary to be able to defeat the Condorcet winner is related to the intensity of preferences: the more intense the preferences (i.e., the smaller the x), the lower will be the required degree of ambiguity. If there is any scope at all for ambiguity, for sufficiently small x the Condorcet winner will be defeated.

The argument above does not demonstrate that the candidate who can guarantee voters that the Condorcet winner is his most preferred outcome will lose, only that sufficiently ambiguous candidates will defeat this candidate if he makes known to voters his most preferred outcome. If the candidate associated with the Condorcet winner has limited scope to be ambiguous himself, he will be defeated. But, if he has sufficient scope himself to be ambiguous, he may still have a winning (ambiguous) strategy. What this argument demonstrates is that if the non-Condorcet candidate has sufficient scope for ambiguity when voters have intense preferences, the winning strategy in any pure strategy equilibrium in the election will be ambiguous. Either the non-Condorcet candidate wins because the Condorcet candidate has insufficient flexibility to be ambiguous or the Condorcet candidate has sufficient flexibility to respond to any ambiguous strategy with a strategy that is itself ambiguous and appeals to a majority of the voters.

We will summarize this argument in the following proposition. Recall that candidate i's strategy set is $W_i = \{p \in \Delta Z : p_{Z_i} \geq a_i\}$

Proposition 1. *Consider a profile of preferences such that there is a Condorcet winner which is not the first choice of a majority of voters and voters have intense preferences (i.e., $x_i < 0.5, \quad for\ i = 1....6$). Suppose also that candidate 1 is a Condorcet candidate. Then there exist $a_1, a_2 > 0$ such that a pure strategy equilibrium exists in the voting game, and the winning strategy is ambiguous.*

The proof is left to the Appendix. Before going on, we will make several remarks about this result. The proof proceeds by showing that there will be a pure strategy equilibrium in which the winning strategy is ambiguous if and only if the $a_i's$ satisfy particular inequalities that depend on the given set of intensities $(x_1, ..., x_6)$. From the inequalities that determine the set of $a_i's$ for which the winning strategy is necessarily ambiguous (see Fig. 1), we can say several things.
First, suppose that for the levels of the ambiguity for the two candidates, a_1, a_2, there is a pure strategy equilibrium of the voting game in which the non-Condorcet candidate wins (necessarily with an ambiguous strategy). Then, if the voters' preferences become more intense (i.e., the $x_i's$ decrease), that candidate will continue

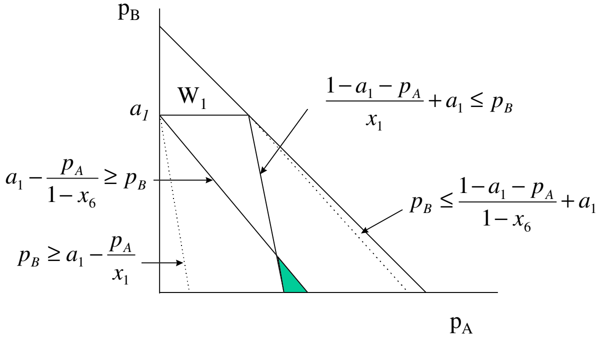

Fig. 1. Winning strategies for candidate 2

to have a winning strategy. If initially there is a pure strategy equilibrium with the Condorcet candidate winning with an ambiguous strategy, it may be the case that an increase in voters' intensities leads to nonexistence of pure strategy equilibria. However, if following an increase in voter's intensities there is a pure strategy equilibrium, it must be that the winning strategy is ambiguous. Roughly speaking, ambiguity is more likely when voters care more intensely about getting their most preferred outcome.

3.2 With a Condorcet cycle

We next consider the case of a Condorcet cycle in the set of pure alternatives Z, that is, every pure alternative is defeated by another pure alternative in a pairwise majority rule contest. Fishburn (1972) has shown that in this case there is no Condorcet winner in the set of lotteries over pure alternatives ΔZ either. Thus, if the strategy sets of the two candidates consist of all lotteries over pure alternatives, then there is no pure strategy equilibrium. In our framework, however, candidates are restricted in the set of beliefs that they can induce in voters, hence there is a possibility that pure strategy equilibria exist.

Suppose that the distribution of voters' preferences generates a Condorcet cycle: A defeats B, B defeats C, and C defeats A. Suppose further that candidate 1 and 2's strategy sets, W_1 and W_2, contain lotteries that put probability close to 1 on alternatives A and B respectively (each has very limited ability to be ambiguous). Since the Condorcet cycle is such that A defeats B, A will also defeat any lottery that puts probability sufficiently close to 1 on outcome B. Hence, it is straightforward that when there is sufficiently limited ability to be ambiguous, the candidate who can induce the beliefs in voters that his most preferred outcome is surely A will do so and, consequently, win the election. As in the case of a Condorcet winner discussed above, the winning strategy need not be ambiguous.

But suppose now that the candidate who prefers outcome A has available a greater scope to be ambiguous, that is, he is able to induce beliefs in the voters that place greater probability on outcomes other than A. If voters have intense preferences, the outcome will be similar to that in the case above with a Condorcet winner. If candidate 2 has sufficient scope to be ambiguous, he will have available a strategy that defeats A. In particular, if he is able to induce in voters beliefs that he will never choose A but is indifferent over B and C, he will be able to defeat a candidate who is nearly perfectly identified with outcome A. For any level of intensities x_i, one could compute in a manner similar to that in the proof of theorem 1 the levels of ambiguity a_1 and a_2 for which there is a pure strategy equilibrium to the voting game in which the winning strategy is ambiguous. It is worthwhile pointing out that in the case that voters do not have intense preferences ($x_i > 0.5$), there may be pure strategy equilibria in which the winning strategy is ambiguous. For example, if candidate 1 has minimal flexibility to be ambiguous while candidate 2 can induce beliefs in voters that his first choice is very likely C, candidate 2 will win, since in the voting cycle C defeats A, and consequently, any lottery putting probability close to 1 on A. However, the ability of candidate 2 to induce beliefs in voters that his most preferred outcome is almost certainly C when it is, in fact, B should probably be considered deception rather than ambiguity.

3.3 More than three alternatives

We restricted our model to the case in which there are three alternatives primarily for ease of exposition. Our aim was to show that intense preferences for voters could lead naturally to ambiguity in equilibrium. In the discussion and proof of the result, we took the utility of a voter's middle ranked alternative for his normalized von Neumann-Morgenstern utility function to be a measure of the intensity of his preferences. When there are three alternatives, this single number completely characterizes the cardinal information about the relative strength of the voter's preferences. When there are four or more alternatives, there isn't a single number that characterizes the relative strength of preferences for any pair of outcomes. But from the logic of our result it can be seen that it is the relative strength of the preference for the most preferred outcome over all other outcomes that matters, and we will continue to use the utility of the second ranked alternative as a measure of the intensity of preferences of voters. Suppose that in the example about cutting taxes offered in the introduction there had been an arbitrary finite number of types of expenditures that could be used to compensate the tax cut, and suppose that each voter had a most preferred alternative, but was close to indifferent over all other alternatives. The argument presented for the three alternative case would carry over to this more general case.

4 Uncertainty about voters' intensities

The model of electoral competition we presented in the last section was highly simplified in order to highlight the role that strategic ambiguity could play. A consequence of the assumption that candidates know precisely the intensity of voters' preferences is that after choosing strategies, one or the other of the candidates wins with probability 1. This made the expected payoffs to the candidates as a function of the joint choice of strategies highly discontinuous: any change in a candidate's strategy has either no effect whatsoever, or reverses the outcome completely. The difficulty in establishing the existence of pure strategy equilibria stemmed to a large extent from this discontinuity of the candidates' payoff functions. In this section we will extend the model above in a way that both makes the model more plausible from a positive point of view and increases the chance that pure strategy equilibria exist, thus enabling us to carry out several simple comparative statics exercises.

We maintain the assumption that voters with the same preference order have the same preference intensity, but assume that both candidates are uncertain about the intensities. We assume $x_i, i = 1, ..., 6$ are independently and identically distributed, and that the candidates have common beliefs about the distribution function F. We also assume that F is strictly concave,[5] has density function f and is twice continuously differentiable, with positive density everywhere. The restriction to concave F is consonant with our interest in the case that voters have intense preferences; with a concave F, there is higher probability on lower values of x_i than on higher values.

The probability with which a candidate wins the election, that is, the probability with which a candidate gets at least a majority of the votes, depends on the strategies chosen by the two candidates (p^1, p^2). Thus, the payoff function of candidate i (the probability that he wins) can be represented as a function $P_i(p^1, p^2)$ which takes values in $[0, 1]$.

The assumptions on F lead to a richer interaction between the candidates, with both candidates having positive probability of winning for typical joint choices. The conditions are not, however, sufficient to guarantee existence of a pure strategy equilibrium. If we add an assumption that the candidates' strategy sets are disjoint, the payoff functions P_i are continuous and there will be a pure strategy equilibrium. We will maintain our assumed structure of candidates' strategy sets, namely that they contain all lotteries that put some minimum probability on the candidate's most preferred alternative. We add the assumption that this minimum probability is at least 1/2 in order to guarantee that the candidates' strategy sets are disjoint.

Consider again the case in which the preference profile of the voters is such that there is a Condorcet winner in the set of pure alternatives but it is not the first choice of a majority of the voters. Suppose the Condorcet winner is the outcome that candidate 1 can choose with probability one. It is clear that the non-Condorcet candidate must choose an ambiguous strategy in equilibrium. If he were to play the unambiguous strategy that puts probability 1 on his most preferred outcome, the best response for the Condorcet candidate is to pick the degenerate lottery putting probability 1 on the Condorcet winner, which leads to the Condorcet candidate winning with probability 1. Trivially, this cannot be an equilibrium, since with any other strategy the non-Condorcet candidate will win with positive probability.

[5] A strictly concave probability distribution is sufficient for the results that follow, but it is not necessary. They can be obtained under weaker assumptions, for example: probability distributions that are symmetric or have monotone likelihood ratio will satisfy the first order condition. The second order condition can also be satisfied by nonconcave probability distributions.

Hence, in a pure strategy equilibrium, the non-Condorcet candidate must choose an ambiguous strategy. But it can be shown that the best response of the Condorcet candidate to any ambiguous strategy is to be ambiguous himself, that is, in any pure strategy equilibrium, both candidates will choose ambiguous strategies. Further, the strategic use of ambiguity is "escalating": in equilibrium both candidates will be maximally ambiguous, in the sense that had they not been constrained by the exogenously given strategy sets, a further reduction of the probability placed on their most preferred outcome would increase their probability of winning. We summarize these findings in the following proposition whose proof is left to the appendix.

Proposition 2. *Consider a profile of preferences such that there is a Condorcet winner which is not the first choice of a majority of voters and voters' intensities are given by a distribution function F satisfying the conditions above. Suppose that candidates' strategy sets are as described above with $a_i > \frac{1}{2}$ for $i = 1, 2$ and that candidate 1 is a Condorcet candidate. Then there is a pure strategy equilibrium. Further, in any pure strategy equilibrium, both candidates put the minimal possible probability on their most preferred outcome given their strategy sets.*[6]

This proposition demonstrates that for some political contests, the strategic use of ambiguity is a necessary component. The proof is similar to the proof of the first proposition in that it establishes that a best response to an opponent's strategy is the solution to a set of linear inequalities. From these inequalities, we can see how the probability that a particular strategy will lead to being elected is affected by the parameters of the problem. First, suppose we want to compare two problems with the voters having different intensity of preferences. One notion of an electorate having less intense preferences than a second would be that the distribution function of the first, say F, stochastically dominated the distribution function of the second, say G. In this case, G would put higher probability on lower intensities than F. The probability that any ambiguous strategy for the non-Condorcet candidate defeats the degenerate lottery that puts probability 1 on the Condorcet winner is higher for distribution function G than for F. Therefore an increase in the intensity of preferences as measured by stochastic dominance increases the benefit of any ambiguous strategy against the Condorcet winner.

We can also deduce from the inequalities determining the equilibrium how a change in the level of ambiguity available to a candidate, keeping his opponent's ambiguity level fixed, will affect the outcome (see Fig. 2). Specifically, the probability that a candidate will win increases if the level of ambiguity available to him increases.

5 Increasing political competition eliminates ambiguity

In this section we relax the condition that there are two parties and consider the case in which there are I parties, $I \geq 3$. We assume that the objective function of each party is to maximize the proportion of votes it obtains in the election. We

[6] The structure we place on the candidates' strategy sets is stronger than necessary for our results but eases exposition.

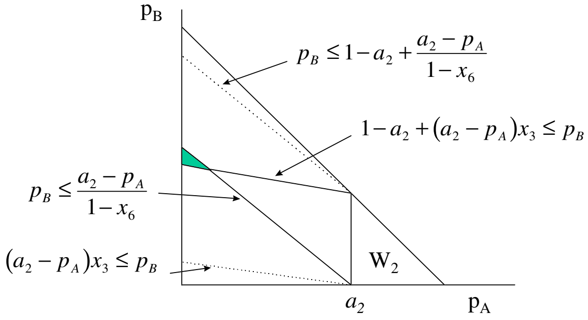

Fig. 2. Winning strategies for candidate 1

also relax the restriction on the set of alternatives and allow $|Z| \geq 3$. We assume that the set of strategies of each party is the set of all probability distributions over the set Z. We assume that voters have von Neumann-Morgenstern utility functions over Z and that they vote for the candidate who offers the highest expected utility. Lastly, we assume that if more than one party chooses the same lottery, the parties doing so get equal shares of the voters for whom this is the most preferred lottery on offer.

In the sections above, we provided conditions under which the election outcome was a pure strategy equilibrium in which the winning strategy was ambiguous. Roughly, the logic of the argument was that when voters had intense preferences, if the Condorcet outcome was not the most preferred outcome for a majority of the voters, it could be defeated by a lottery putting equal probability on the outcomes other than the Condorcet outcome. As discussed above, the logic is not restricted to the case of three alternatives. Consider the set of voters whose most preferred outcome is not the Condorcet outcome. A lottery that places equal probability on all the outcomes that are the most preferred outcome for some voter in this set will be preferred by all of these voters to the Condorcet outcome if preferences are sufficiently intense.[7]

We will show that when there are sufficiently many candidates or parties, there cannot be ambiguity in equilibrium. The idea can be illustrated with a simple example. Think of the voters being divided into groups, with each group having precisely those voters with an identical most preferred outcome. Suppose that of these groups, the smallest nonempty group contained 10% of the voters. Lastly, suppose that there are at least 11 parties and that in equilibrium, at least one chooses an ambiguous strategy. If the most preferred outcome of each of the groups is the unambiguous strategy of some party, then clearly an ambiguous strategy will get no votes. Hence, if an ambiguous strategy is chosen in equilibrium, it must put positive probability on at least one outcome that is the most preferred outcome of some group and that is not offered as an unambiguous strategy by some party. But

[7] Recall that the utility of the second ranked alternative is the measure of intensity and that preferences are said to be more intense when this number is lower.

if there are at least 11 parties, at least one must get less than 10% of the votes. If this party chooses instead an unambiguous strategy that is the most preferred outcome of some group that is not targeted by any other party (in the sense that some party has chosen this group's most preferred outcome as an unambiguous strategy, it will get the votes of this group. But this must increase the number of votes it gets, contradicting the supposition that an ambiguous strategy could be an equilibrium choice when there were 11 parties.

The logic of this example is formalized in the following theorem.

Proposition 3. *Fix the distribution of voters and their utility functions over Z. Let N' be the smallest number of voters with a common most preferred outcome. There is a nonempty set of pure strategy equilibria and every party chooses an unambiguous strategy in every pure strategy equilibrium if the number of parties is greater than N/N'.*

There is a natural intuition behind the result: when the number of candidates is sufficiently large, competition for votes will provide incentives for candidates to target any group of voters that is not already targeted by another party. On the margin, a candidate will do better by specializing in some subset of the untargeted voters than by competing with other ambiguous candidates for the entire pool of such voters.

The above result takes as given the number of alternatives. While this is a natural assumption to make, for many problems of interest, there is not a natural set of exogenously given alternatives; the alternatives are, in fact, determined within the political process. Consider reversing the order in which the number of alternatives and the number of parties are increased. For an initial set of alternatives, the fixed number of parties might be sufficiently large that the only pure strategy equilibria are in unambiguous strategies. But if the number of alternatives gets larger, there will be, in general, more groups of voters determined by most preferred outcomes. If the number of alternatives gets large enough, there won't be enough parties for each group to be targeted by at least one party. When the set of alternatives gets large enough, some party might find it advantageous to target the pool of disaffected voters with an ambiguous strategy that puts positive probability on precisely those alternatives that are some voter's most preferred outcome, but not the unambiguous strategy of any party. In the example of tax cuts, this would amount to a party choosing a strategy such as "We are not sure which type of expenditure should be cut, but it should definitely *not* be any of those the other parties propose." In summary, the logic of our model suggests that when the set of potential alternatives is large relative to the set of parties, voters are relatively dispersed with respect to their most preferred outcomes and preferences are intense, ambiguity is more likely to arise as an equilibrium strategy.

6 Concluding remarks

How do campaign statements affect voters? Campaign statements are cheap talk; that is, fixing all actions of all participants, no payoffs differ when messages alone are changed. Suppose that there is a single election in which candidates vie for office, and that are ideological, that is, that they have no utility for holding office, but only care about the policy chosen. In this case, any candidate who is elected will choose that alternative that he most prefers, regardless of any campaign promise that might have been made, and consequently no campaign promise can alter a rational voters' beliefs about the action will be taken by a candidate who is elected. Any statement that increased the probability of election for a candidate would be made by that candidate regardless of what he intended to do if elected. Hence, no campaign statement can convey information that alters the chance of election.[8]

When we move from a situation where there is a single election to one in which there is more than one election, campaign promises may be costly. Voters may vote differently in future elections if a candidate promises to do something if elected, but reneges on that promise after election. Simply put, voters may punish a politician for reneging on campaign promises by voting him out of office. Threats of such punishment can support an equilibrium in which campaign promises are kept. There is a credibility problem, however, with retrospective punishments that threaten to vote a candidate out of office if he reneges on a promise. In the election following a candidate's reneging on a campaign promise, it may be that the candidate is nevertheless more desirable than the alternative. At this point, voters would prefer the candidate despite his past behavior to his opponent. In game theoretic terms, the threat to vote a candidate out of office regardless of his opponent is a dominated strategy. In principle, we would like to restrict equilibrium to undominated strategies, a consequence of which is that voters must vote for candidates whose future choices are most preferred regardless of past behavior.

Hence it is not clear how, even for a candidate who will engage in multiple (but a finite number) of elections, reneging on campaign promises can be costly. If voters are restricted to voting for candidates solely on their prediction of future behavior, the only way that reneging on a campaign promise can alter future voting is if that reneging altered voters' prediction of the candidate's future behavior.

In an infinite period model, there is the possibility that campaign promises can be costly even if voters use undominated strategies. In Aragones and Postlewaite (2000) we analyze a dynamic model in which candidates make campaign promises, and voters use those promises to form beliefs about the policies the candidate will choose, if elected. They analyze equilibria of the model in which some promises will be kept, even when the promised policy differs from the elected candidate's ideal point, because of fear of voter reprisal. However, unlike the retrospective punishments described above, punishment are prospective. Voters discipline candidates by believing some promises a candidate makes as long as that candidate has never reneged on a promise in the past. Once he reneges, no future promises will be believed. Candidates only make promises they intend to keep, and keep those promises if elected.

For our purposes in this paper, there are several points that should be emphasized about the equilibria of a campaign rhetoric game. In general, candidates will *not* be able to induce all possible beliefs in voters. Some promised actions may be so

[8] See Harrington (1992) for an elaboration of this argument.

undesirable that a candidate may renege, understanding full well the consequences of voters not believing future promises. Such promises from candidates will not alter the beliefs or rational voters. In summary, a candidate will have available to him a subset of the set of possible beliefs he can induce in voters. It is important to note that the sets of beliefs that candidates can induce in voters are typically quite different, since they depend on voters' initial beliefs about the candidates.

The possibility of commitment. We have taken the approach in this paper that candidates cannot commit to post-election choices during an election. It is worthwhile discussing this modelling choice.

First, there are several disadvantages of assuming that commitment is possible. If candidates are assumed to be able to commit to their platform we often get cycles in pure strategies and we are forced to deal with mixed strategy equilibria (see for example, Myerson, 1993; Lizzeri and Persico, 1997). Mixed strategy equilibria are undesirable for several reasons. They do not offer clear qualitative predictions, and typically do not allow comparative statics. Additionally, in political models that assume commitment, it is not clear how commitment can be monitored if the platform committed to is represented by mixed strategy. Besides the difficulties that arise if commitment is assumed to be possible, there is a very real question of the degree to which politician, in fact, can commit. Commitment is problematic in most models simply because the physical strategies that are available to a candidate who has won an election are almost always unchanged by actions taken prior to the election. Hence, commitment cannot be taken to mean that a candidate has eliminated the possibility that he will do something other than what he has promised. An advantage of our approach is that without assuming commitment it yields pure strategy equilibria.

On the other hand our formal model of ambiguity can also be interpreted as if candidates can commit to certain lotteries. The same results would obtain, as long as candidates care only about winning the election. Thus we not only generalize Shepsle (1970) and Zeckhauser (1969) findings but we provide a way to endogenize their ad hoc assumptions.

Restrictions on the beliefs that can be induced. The restriction to subsets of beliefs is important both from a positive modelling point of view (it seems clear that candidates are limited in this way), and because of the consequences of the restriction on the analysis of our model (there are no pure strategy equilibria in ambiguous strategies in the absence of restrictions).

We took a particularly simple tack in restricting the beliefs available to candidates, namely that they had available all beliefs that put some minimal probability on a particular alternative. Some aspects of this particular restriction are important while other aspects could be relaxed without changing qualitatively our results. First, it is important that the candidates' strategy sets be disjoint; it is often the case that there will only be mixed strategy equilibria when they intersect.

The assumption that a candidate has available *all* beliefs that put minimal probability on one pure alternative is not strictly necessary. To guarantee that there is a pure strategy equilibrium in the election game, it is enough that the candidates' strategies be compact subsets of the sets we considered. To assure that the winning strategy is ambiguous, the candidates have to have available to them sufficiently ambiguous strategies in the sets we considered.

Repeated elections. Even though we have interpreted our formal model as though the two candidates made their choices of strategies simultaneously, our results would not change if we consider sequential choices. Suppose that instead of a campaign game we think of the policy choice of an incumbent who cares about reelection. His policy choice may affect voters' beliefs even before the electoral campaign has started. Thus the challenger has the possibility of choosing his strategy (induce beliefs in voters) after observing the incumbent's strategy. Our results allow this interpretation. Since the games we have presented are zero-sum games, and we find unique equilibria, the equilibrium choice of an incumbent as a "Stackelberg leader" will still be his maxmin strategy, that is the strategy chosen in the equilibrium of the simultaneous game. Thus, with our model we replicate not only the models of ambiguity in electoral campaigns but also the findings in Alesina and Cukierman (1990).

On the real line. We can adapt our findings to the spatial model of electoral competition. In this case, under the normal assumptions on voters' preferences (single peaked) there is a Condorcet winner in the set of pure alternatives, and in addition to the restrictions on the strategy sets, intensity of preferences is also needed for a nondegenerate lottery to defeat the Condorcet winner. Thus, our model replicates the results found by Shepsle (1972): intense preferences on the real line imply nonconcave utility functions, which in this set up are interpreted are risk tolerance. Even though it is standard to assume that individuals' utility functions over consumption exhibit risk aversion, it is not always a natural assumption when we consider utility over policies. Individuals can only derive utility from policies indirectly, that is, through the effects of policies on their possibilities of consumption. Thus, the indirect utility over policies need not inherit the same properties of the individuals' utility over consumption. Consider the following example where the policy choice is the level of gun control. The policy space may be represented by the interval [0,1] and a choice of a policy in the interval represents the extend to which the use of guns is restricted (0 means all is allowed, and 1 means no guns are allowed). Voters maximize their expected utility: the probability that they are alive, which increases with the policy, times the utility of their consumption, which decreases with the taxes they will have to pay to finance the implementation of the policy, and therefore with the level of policy chosen. It is reasonable to imagine that the probability of being alive as a function of the policy exhibits increasing returns to scale: the first ten percent of the guns that are eliminated may have a far smaller affect on one's chances of staying alive than the elimination of the last ten percent of the guns. Hence, even when the utility of voters over policy outcomes exhibits risk aversion, the utility function over policy may not be concave, and as a consequence convex combinations of extreme policies may attract more voters than a lottery that puts positive probability on each of the extreme policies.

Welfare consequences of ambiguity. We demonstrated conditions under which an ambiguous strategy could defeat a Condorcet winner. The ambiguous strategy that does this is a lottery that necessarily puts positive probability on a set of alternatives that by assumption a majority of people like less than the Condorcet outcome. It's worthwhile to note that this doesn't imply that ambiguity is necessarily welfare decreasing. Ex post, it's true that when the ambiguous candidate wins, there is an outcome that a majority of people would prefer to the outcome this candidate chose. But if we measure welfare ex ante, that is, before the uncertainty about candidates' true policy choices are known, welfare judgements are cloudier. Suppose a society could somehow ban ambiguity. When a Condorcet outcome is defeated by an ambiguous strategy, it's clear that a majority of voters would vote against outlawing ambiguity, since outlawing ambiguity is tantamount to voting for the Condorcet outcome in this case. If we were to make welfare judgements behind a veil of ignorance where one doesn't know if he will prefer the Condorcet outcome or the lottery, and not knowing whether one will win or lose if and when the ambiguous candidate chooses an outcome, it could be that in some cases, expected utility is higher when ambiguous strategies are allowed than when they are not.

Appendix

Proof of Proposition 1. First we find conditions for existence of a pure strategy equilibrium if the sets of strategies for the two candidates are given by $W_1 = \{p \in \Delta Z : p_B \geq a_1\}$ and $W_2 = \{p \in \Delta Z : p_A \geq a_2\}$, where alternative B is the Condorcet winner but it is not the first choice of a majority of voters. Notice that in equilibrium there must be one party that has a winning strategy, that is, a strategy that defeats any strategy available to the opponent. We will prove that a pure strategy equilibrium exists if and only if either one of the following two conditions is satisfied:

i) $a_1 > \max\left\{\dfrac{1}{1-x_1+1-x_6}, \dfrac{a_2}{(1-x_6)}\right\}$

ii) $a_2 > \max\left\{\dfrac{1-x_6}{1+(1-x_3)(1-x_6)}, a_1(1-x_6)\right\}$

If the first condition holds all winning strategies are nondegenerate lotteries. When the second condition holds all winning strategies are nondegenerate lotteries if and only if $a_2 < 1 - x_6$.

For a given set of lotteries $W \subseteq \Delta Z$, let $D(W)$ denote the set of lotteries that defeat all lotteries in W in pairwise contests. Candidate 2 has a winning strategy if and only if $W_2 \bigcap D(W_1) \neq \emptyset$, that is, his strategy set contains at least one lottery that defeats all strategies available to his opponent. A lottery that defeats all lotteries in W_1 must be $p' \in W_1^c$ such that $p' \in \bigcap_{p \in W_1} D(p)$. In particular, a winning strategy for candidate 2 must defeat the degenerate lottery that assigns probability one to the Condorcet winner. In order to defeat the lottery $(0,1,0)$ it is necessary and sufficient to find a lottery that is preferred by all voters in groups $G_1$and G_6 : that is all voters whose second choice is the Condorcet winner. Thus we need $p \in \Delta Z$ such that $E_p u_1 = p_A + p_B x_1 > x_1 = u_1(B)$ and $E_p u_6 = p_C + p_B x_6 > x_6 = u_6(B)$. Using these conditions we find

$$D(0,1,0) = \left\{(p_A, p_B, p_C) \in \Delta Z : 1 - \frac{p_A}{x_1} < p_B < 1 - \frac{P_A}{1-x_6}\right\}.$$

This set is non empty if and only if $x_1 + x_6 < 1$, which is satisfied if the voters in groups G_1 and G_6 have intense preferences.

With a similar argument, for any lottery $p \in W_1$ we can find the set of lotteries that defeat it:

$$D(p) = \left\{ p' \in \Delta Z : \frac{p_A}{x_1} + p_B - \frac{p_{A'}}{x_1} < p_{B'} < \frac{p_A}{1-x_6} + p_B - \frac{p_{A'}}{1-x_6} \right\} \\ \cup \{p' \in \Delta Z : p_A x_3 + p_B - p_{A'} x_3 < p_{B'}\} \\ \cup \{p' \in \Delta Z : p_C x_4 + p_B - p_{C'} x_4 < p_{B'}\}$$

The first subset is nonempty for all $p \in W_1$ as long as $x_1 + x_6 < 1$, and the other two subsets are empty for some $p \in W_1$ (for instance for p with $p_A = 0$ or $p_C = 0$). Since we are interested in the intersection of such sets for all $p \in W_1$ we should focus our attention in the intersection of the following sets:

$$\widetilde{D}(p) = \left\{ p' \in \Delta Z : \frac{p_A}{x_1} + p_B - \frac{p_{A'}}{x_1} < p_{B'} < \frac{p_A}{1-x_6} + p_B - \frac{p_{A'}}{1-x_6} \right\}.$$

Observe that $D(p) \subset D(p')$ for all p and p' such that

$$\frac{p_A}{1-x_3} + p_B - \frac{p_{A'}}{1-x_3} < p_{B'} < \frac{p_A}{x_1} + p_B - \frac{p_{A'}}{x_1}.$$

Therefore $D(W_1) = \bigcap_{p \in W_1} D(p) = \bigcap_{p \in \{p \in W_1 : p_B = a_1\}} D(p)$, that is, there exists a lottery $p \in \Delta Z$ that defeats all lotteries in W_1 if and only if $p \in \Delta Z$ defeats all lotteries in $\{p \in \Delta Z : p_B = a_1\}$. Finally, observe that $p \in \Delta Z$ will defeat all lotteries in $\{p \in \Delta Z : p_B = a_1\}$ if and only if it defeats $\{(0, a_1, 1-a_1), (1-a_1, a_1, 0)\}$. Thus, we have that

$$D(W_1) = D(0, a_1, 1-a_1) \cap D(1-a_1, a_1, 0) \\ = \left\{ p \in \Delta Z : \frac{1-a_1}{x_1} + a_1 - \frac{p_A}{x_1} < p_B < a_1 - \frac{p_A}{1-x_6} \right\}.$$

and this set is nonempty if and only if $x_1 + x_6 < 2 - \frac{1}{a_1}$. A condition similar to the one that guarantees existence of a lottery that defeats the Condorcet winner is also sufficient to guarantee that the set of lotteries that defeats any lottery in W_1 is not empty: intensities of preferences for voters in groups G_1 and G_6 need to be high enough (see Fig. 1).

Thus $W_2 \bigcap D(W_1) \neq \emptyset$ if and only if $a_1 > \frac{1}{1-x_1+1-x_6}$ and $a_1 > \frac{a_2}{(1-x_6)}$. The first condition guarantees that the set $D(W_1)$ is not empty and the second one guarantees that the two sets have a nonempty intersection. Observe that all lotteries in $D(W_1)$ are nondegenerate lotteries. Since the winning strategies must be contained in $W_2 \bigcap D(W_1)$, all of them must be nondegenerate lotteries.

Suppose that candidate 1's strategy set is S_1, a proper subset of W_1, then we have that $D(S_1) \supseteq D(W_1)$ thus the conditions that guarantee a winning strategy for candidate 2 are even weaker than the ones we find when the strategy set of candidate 1 is W_1. On the other hand, any proper subset of W_2 that intersects with $D(W_1)$ provides candidate 2 with a winning strategy.

Similarly, candidate 1 has a winning strategy if and only if $W_1 \bigcap D(W_2) \neq \emptyset$, that is, his strategy set contains at least one lottery that defeats all strategies available to his opponent. For any lottery $p \in W_2$ the set of lotteries that defeats it is given by:

$$D(p) = \left\{ p' \in \Delta Z : (p_A - p'_A)\, x_3 < p'_B - p_B < \frac{p_A - p'_A}{1 - x_6} \right\} \\ \cup \left\{ p' \in \Delta Z : p'_B - p_B > \max\left\{ \frac{p_A - p'_A}{x_1}, (p_A - p'_A)\frac{x_4}{1 - x_4} \right\} \right\}.$$

The first subset is nonempty for all $p \in W_2$, and the other subset is empty for some $p \in W_2$ (for instance for p with $p_C = 0$). Since we are interested in the intersection of such sets for all $p \in W_2$ we should focus our attention in the intersection of the following sets:

$$\widetilde{D}(p) = \left\{ p' \in \Delta Z : p_B + (p_A - p'_A)\, x_3 < p'_B < p_B + \frac{p_A - p'_A}{1 - x_6} \right\}$$

Notice that the lottery that assigns probability one to the Condorcet winner is contained in this set for all p with $\frac{p_A + p_C}{p_A} < \frac{1}{1 - x_6}$.

Using the argument developed for the previous case we can conclude that $D(W_2) = \bigcap_{p \in W_2} D(p) = \bigcap_{p \in \{p \in W_2 : p_B = a_2\}} D(p)$, that is, there exists a lottery $p \in \Delta Z$ that defeats all lotteries in W_2 if and only if $p \in \Delta Z$ defeats all lotteries in $\{p \in \Delta Z : p_A = a_2\}$, and $p \in \Delta Z$ will defeat all lotteries in $\{p \in \Delta Z : p_A = a_2\}$ if and only if it defeats $\{(a_2, 0, 1 - a_2), (a_2, 1 - a_2, 0)\}$. Thus, we have that

$$D(W_2) = D(a_2, 0, 1 - a_2) \cap D(a_2, 1 - a_2, 0) \\ = \left\{ p \in \Delta Z : \frac{1 - a_2 - p_B}{x_3} + a_2 < p_A < a_2 - p_B\,(1 - x_6) \right\}.$$

This set is not empty if and only if $a_2 > \frac{1 - x_6}{1 + (1 - x_3)(1 - x_6)}$ (see Figure 2). Thus $W_1 \bigcap D(W_2) \neq \emptyset$ if and only if $a_2 > \frac{1 - x_6}{1 + (1 - x_3)(1 - x_6)}$ and $a_2 > a_1(1 - x_6)$. The first condition guarantees that the set $D(W_2)$ is not empty and the second one guarantees that the two sets have a non empty intersection. Observe that all lotteries in $D(W_2)$ are nondegenerate lotteries as long as $a_2 < 1 - x_6$, that is, the set of strategies available to candidate 2 must be large enough. Otherwise $(0, 1, 0)$ is a winning strategy. Since the winning strategies must be contained in $W_1 \bigcap D(W_2)$, when $a_2 < 1 - x_6$ all winning strategies must be nondegenerate lotteries.

Suppose that candidate 2's strategy set is S_2, a proper subset of W_2, then we have that $D(S_2) \supseteq D(W_2)$ thus the conditions that guarantee a winning strategy for candidate 1 are even weaker than the ones we find when the strategy set of candidate 2 is W_2. On the other hand, any proper subset of W_1 that intersects with $D(W_2)$ provides candidate 1 with a winning strategy.

The conditions for existence of equilibrium involve only strict inequalities. If any of them is satisfied with equality the candidates tie in equilibrium (at least one group of voters is indifferent). Finally, when these conditions are not met, no candidate has a winning strategy, and therefore, there is no pure strategy equilibrium.[9] Thus, we have that there is no pure strategy equilibrium if and only if $a_1(1-x_6) < a_2 < \frac{1-x_6}{1+(1-x_3)(1-x_6)}$ or $\frac{a_2}{1-x_6} < a_1 < \frac{1}{1-x_1+1-x_6}$. These conditions characterize situations in which either both sets $D(W_1)$ and $D(W_2)$ are empty or, if one of them is not empty, its intersection with the opponents set is empty.

Proof of Proposition 2. Suppose that $W_1 = \{p \in \Delta Z : p_B \geq a_1\}$ and $W_2 = \{p \in \Delta Z : p_A \geq a_2\}$. We consider again the case in which the preference profiles of the voters is such that there is a Condorcet winner in the set of pure alternatives but it is not the first choice of a majority of the voters. As before, we assume that alternative B is the Condorcet winner.

First, notice that the probability with which a lottery p defeats $(0,1,0)$ is given by $F\left(\frac{p_A}{1-p_B}\right) F\left(1-\frac{p_A}{1-p_B}\right)$, that is, as discussed in the previous section, we need that the proposed lottery is preferred to alternative B by voters in groups G_1 and G_6. If we define $x(p_A, p_B) = \frac{p_A}{1-p_B}$ and maximize $F(x(p_A, p_B))F(1-x(p_A, p_B))$, we find the set of lotteries that defeat $(0,1,0)$ with maximal probability. The first and second order conditions only depend on the value of $x(p_A, p_B)$:

$$\frac{f(x)}{F(x)} = \frac{f(1-x)}{F(1-x)} \text{ and } \frac{f'(x)}{f(x)} + \frac{f'(1-x)}{f(1-x)} \leq \frac{f(x)}{F(x)} + \frac{f(1-x)}{F(1-x)}$$

Since we assume that $F(x)$ is a strictly concave function, we have that $\ln[F(x)]$ is also strictly concave, and thus its first derivative, $\frac{f(x)}{F(x)}$, is strictly decreasing. Therefore $\frac{f(x)}{F(x)} \neq \frac{f(y)}{F(y)}$ for all $x \neq y$ and $x = \frac{1}{2}$ is the only value that satisfies the first order condition. Furthermore, the strict concavity of $F(x)$ guarantees that $F''(x) = f'(x) \leq 0$; consequently, at $x = \frac{1}{2}$, $f'(x) < \frac{[f(x)]^2}{F(x)}$, and the second order conditions is satisfied at $x = \frac{1}{2}$. Thus, there is a local maximum at $x = \frac{1}{2}$. At the extreme points of the domain (0 and 1) the function $F(x)F(1-x)$ has value zero. Since $x = \frac{1}{2}$ is the only critical point, we have that $x = \frac{1}{2}$ is the only maximizer and the lotteries that satisfy $p_B = 1 - 2p_A$ defeat $(0,1,0)$ with maximal probability: $[F(1/2)]^2$.

Similarly, the probability with which a lottery p' defeats a lottery p in the set $W_1 = \{p \in \Delta Z : p_B \geq a_1\}$ is given by $F\left(\frac{p_{A'}-p_A}{p_B-p_{B'}}\right) F\left(1-\frac{p_{A'}-p_A}{p_B-p_{B'}}\right)$. Again, as before, this lottery must be preferred to all lotteries in W_1 by all voters in groups G_1 and G_6. We consider the problem of maximizing $F\left(\frac{p_{A'}-p_A}{p_B-p_{B'}}\right) F\left(1-\frac{p_{A'}-p_A}{p_B-p_{B'}}\right)$ and we find that the first and second order conditions of this problem are exactly as the previous ones. Thus, $\frac{p_{A'}-p_A}{p_B-p_{B'}} = \frac{1}{2}$ is the unique maximizer and the lotteries that satisfy $p_{B'} = p_B - 2(p_{A'} - p_A)$ defeat $p \in W_1 = \{p \in \Delta Z : p_B \geq a_1\}$ with maximal probability.

Observe that the maximal probability with which any lottery can defeat a strategy available to candidate 1, $F(x^*)F(1-x^*)$, depends only on the probability distribution of the intensity of the voters preferences. Given our assumption of concavity on the probability distribution this maximal probability is simply $m = [F(1/2)]^2$.

[9] We have not investigated the possibility of mixed strategy equilibria in which a candidate mixes, choosing to induce different beliefs with positive probability on more than one belief vector.

Now we will show that a pure strategy equilibrium exists if and only if $a_2 \geq \frac{1}{2}$ and $a_1 + a_2 > 1$, and that it is unique: in equilibrium candidate 1 chooses $(0, a_1, 1 - a_1)$ and candidate 2 chooses $(a_2, 0, 1 - a_2)$.

i) Suppose that $a_2 \geq \frac{1}{2}$ and $a_1 + a_2 > 1$. If $a_2 > 1 - \frac{a_1}{2}$, both candidates have dominant strategies: there is a unique equilibrium in which candidate 1 will choose $(0, a_1, 1 - a_1)$ and candidate 2 will choose $(a_2, 0, 1 - a_2)$. Candidate 2 cannot defeat any strategy of candidate 1 with probability m, and $(a_2, 0, 1 - a_2)$ is a best response for candidate 2 to any strategy available to candidate 1 since it is the closest lottery to a lottery that can defeat any strategy of candidate 1 with probability m. On the other hand, $(0, a_1, 1 - a_1)$ is a dominant strategy for candidate 1 because the distance between the lotteries that defeat it with probability m and the lotteries available to candidate 2 is maximal.

If $a_2 \leqslant 1 - \frac{a_1}{2}$, there is a subset of strategies of candidate 1 that candidate 2 cannot defeat with probability m (all lotteries $p \in W_1$ such that $p_B < 2(a_2 - p_A)$) and the best response of candidate 2 to any of these strategies is $(a_2, 0, 1 - a_2)$. If candidate 2 chooses $(a_2, 0, 1 - a_2)$, and $a_2 \geq \frac{1}{2}$ the best response for candidate 1 is $(0, a_1, 1 - a_1)$. Thus, there is a unique equilibrium in which candidate 1 chooses $(0, a_1, 1 - a_1)$ and candidate 2 chooses $(a_2, 0, 1 - a_2)$.

ii) If $a_1 + a_2 \leq 1$, that is, the strategy sets of the two candidates have a nonempty intersection, there is a subset of strategies for candidate 1 that candidate 2 cannot defeat with maximal probability: $p \in W_1$ such that $p_B < 2(a_2 - p_A)$. The best response of candidate 2 to any of these is $(a_2, 0, 1 - a_2)$. The best response of candidate 1 to $(a_2, 0, 1 - a_2)$ is now $(a_1, 1 - a_1, 0)$, but candidate 2 can choose a lottery that defeats it with probability m. Thus there is no pure strategy equilibrium in this case.

iii) Finally, suppose that $a_2 < \frac{1}{2}$ and $a_1 + a_2 > 1$. If $a_2 > \frac{a_1}{2}$ there is a subset of strategies of candidate 1 that candidate 2 cannot defeat with probability m: all lotteries $p \in W_1$ such that $p_B < 2(a_2 - p_A)$. The best response of candidate 2 to any of these strategies is $(a_2, 0, 1 - a_2)$. If candidate 2 chooses $(a_2, 0, 1 - a_2)$, the best response for candidate 1 is $(1 - a_1, a_1, 0)$. In this case there is no equilibrium, because candidate 2 will respond with $\left(1 - \frac{a_1}{2}, 0, \frac{a_1}{2}\right)$, the strategy that maximizes his probability of winning against $(1 - a_1, a_1, 0)$, and candidate 1 will want to move back to $(0, a_1, 1 - a_1)$. If $a_2 \leq \frac{a_1}{2}$, for every strategy available to candidate 1 there is a strategy available to candidate 2 that allows him to win with probability m, therefore, there is no pure strategy equilibrium. □

Proof of Proposition 3. Let I denote the number of candidates competing in the election and N' denote the size of the smallest group of voters that have the same alternative as their first choice. First notice that if each alternative is chosen with probability one by at least one candidate, then clearly a candidate that uses an ambiguous strategy will get no votes. Hence, if an ambiguous strategy is chosen in equilibrium, it must put positive probability on at least one alternative that is not chosen with probability one by any candidate. But if $N' \geqslant \frac{N}{I}$, there must be at least one candidate that gets less than $\frac{N}{I}$. If this candidate chooses instead an unambiguous strategy that is the most preferred outcome of some group that is not chosen by any other candidate, it will get the votes of this group. Since $N_z \geqslant N' \geqslant \frac{N}{I}$ for all z, this must increase his payoff.

We will show that for $I \geqslant \frac{N}{N'}$ there exists a set of pure strategy equilibria in which all candidates choose degenerate lotteries, and all alternatives are chosen with probability one by at least one candidate.[10]

If all candidates use only degenerate lotteries, that is, $P = (p_1, ..., p_I)$ with $p_i \in \Delta Z_d$ for all $i = 1, ..., I$, let k_z denote the number of candidates that assign probability one to alternative z. Then the payoff for candidate i will be given by: $\pi_i(P) = \frac{N_z}{k_z}$ if candidate i has decided to assign probability 1 to alternative z, where N_z represents the number of voters that have z as their first choice. We will show now that for each I we can find values for k_z, with $\sum_{z \in Z} k_z = I$ and $k_z \geqslant 1$ for all $z \in Z$ such that the corresponding strategies (degenerate lotteries) form a Nash equilibrium.

Observe that if $k_z \geqslant 1$ for all $z \in Z$, deviations to ambiguous strategies are not profitable since a voter will always vote for a candidate that chooses her most preferred alternative with probability one, a candidate that chooses an ambiguous strategy will receive zero votes.

Let $int\,[q]$ denote the integer part of q, where q is any positive number and let $t^* = I - \sum_{z \in Z} int\left[\frac{N_z I}{N}\right]$. Observe that $0 \leqslant t^* \leqslant |Z| - 1$.

First suppose that $t^* = 0$, and consider $\widetilde{k}_z = int\left[\frac{N_z I}{N}\right]$ for all $z \in Z$.

Notice that since $I \geqslant \frac{N}{N'}$ then we must have $\widetilde{k}_z \geqslant 1$ for all $z \in Z$.

Suppose that P represents the profile of strategies in which all candidates choose degenerate lotteries and such that for each alternative z there are exactly $\widetilde{k}_z$ candidates that assign probability one to it.

Since for each i we have that $\pi_i(P) = \frac{N_z}{int\left[\frac{N_z I}{N}\right]}$ for some $z \in Z$, we must have that $\pi_i(P) \geqslant \frac{N}{I}$, and this can only be true if $\pi_i(P) = \frac{N}{I}$ for all i. Any deviation of i from assigning probability one to z to assigning probability one to z' is not profitable since as a result the payoff corresponding to z increases and the one corresponding to z' decreases. Thus, the candidates' strategies corresponding to these values $\widetilde{k}$ form an equilibrium.

Now suppose that $t^* > 0$. Let Z_{t^*} be the set of alternatives corresponding to the most preferred ones by each one of the t^* largest groups of voters. Define $\widetilde{k}_z = int\left[\frac{N_z I}{N}\right] + 1$ for all $z \in Z_{t^*}$ and $\widetilde{k}_z = int\left[\frac{N_z I}{N}\right]$ for all $z \in Z \setminus Z_{t^*}$, and let P' represent the profile of strategies corresponding to these values. We will show that these strategies form an equilibrium.

Again we have that $\widetilde{k}_z \geqslant 1$ for all $z \in Z$, and therefore deviations to ambiguous strategies are not profitable in this case either.

Given P', for all $z \in Z_{t^*}$ we must have that $\pi_z(P') < \frac{N}{I - t^*}$ which is the payoff that all candidates that assign probability one to alternatives in $Z \setminus Z_{t^*}$ obtain. Thus, to prove that P' is an equilibrium we have to show that the candidates that obtain the smallest payoff (candidates that assign probability one to the alternative in Z_{t^*} which is most preferred by the smallest group of voters) do not want to deviate

[10] This condition implies that $I \geqslant |Z|$, since we always must have $N' \leqslant \frac{N}{|Z|}$

to assigning probability one to the alternative which guarantees the largest payoff (which is the alternative in $Z \setminus Z_{t^*}$ most preferred by the largest group of voters). That is, the following condition must hold:

$$\frac{N_z}{int\left[\frac{N_z I}{N}\right]+1} > \frac{N_{z'}}{int\left[\frac{N_{z'} I}{N}\right]+1}$$

where z is the alternative in Z_{t^*} which is most preferred by the smallest group of voters and z' is the alternative in $Z \setminus Z_{t^*}$ most preferred by the largest group of voters. And this condition always holds since $N_z > N_{z'}$. □

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