There are 3 questions. Justify all answers and show all work. Write legibly. If you need to make additional assumptions, state them clearly. BE CONCISE.

1. Consider the following Envelope Game. There are two players and two envelopes. One of the envelopes is marked "player 1," and the other is marked "player 2." At the beginning of the game each envelope contains one dollar. Player 1 is given the choice between stopping the game and continuing. If he chooses to stop, then each player receives the money in his envelope and the game ends. If he chooses to continue, then a dollar is removed from his envelope and two dollars are added to player 2’s envelope. Then player 2 must choose between stopping the game and continuing. If he stops, then the game ends and each player keeps the money in his own envelope. If player 2 chooses to continue, then a dollar is removed from his envelope and two dollars are added to player 1’s envelope. Play continues like this, alternating between the players until either one of them decides to stop or k rounds of play have elapsed. (A round consists of one player’s decision.) If neither player elects to stop at the end of the kth round, then both players get zero. Assume the players want to maximize the amount they earn (that is, they do not discount).
   a. Draw the extensive form tree for the game for \( k = 5 \).
   b. What is the subgame perfect equilibrium for the game?
   c. Is there a Nash equilibrium for the game that is not subgame perfect?
   d. Suppose that the game is modified so that there is a chance that the game ends regardless of the players’ choices. In particular, suppose that at the beginning of stage \( t \), if the game has not yet ended the game will end with probability \( \frac{1}{4} \) before the amounts in the envelopes are altered. Players want to maximize the expected amount they earn. Is there a subgame perfect equilibrium in which players always choose to continue? Explain.

2. Suppose that an ambitious prosecutor (P) has accused a defendant (D) of committing a crime. Suppose that the trial involves evidence production by both parties. The probability that a side wins (that is, conviction for P or acquittal for D) increases the more money that side spends on the production of evidence. Specifically, suppose that the probability that D will acquitted is given by \( e_D \) and \( e_P \) are respectively the expenditures of D and P on evidence production. (Assume that the probability of acquittal is 1 if both sides choose 0 expenditure.) The defendant must pay 8 if he is convicted and 0 if he is acquitted. The prosecutor wins 8 if she wins and 0 if she loses the case.
   a. Write down the first order conditions and the best-response functions for the two parties.
   b. What is the Nash equilibrium for the game?
   c. Suppose now that the prosecutor must choose her expenditure \( e_P \) before the defendant chooses his expenditure, but that the defendant will not know the prosecutor’s choice before he chooses \( e_D \). How does your answer to part b change?
   d. Suppose as in part c that the prosecutor must choose her expenditure \( e_P \) before the defendant chooses his expenditure, but that now the defendant will learn with probability .5 that expenditure before he chooses \( e_D \). Describe carefully the strategy sets for the two players.
   e. For the problem described in part d, will this cause the expenditures \( e_D \) and \( e_P \) to increase or decrease relative to the equilibrium values for the problem in part c?
3. Consider a used car market model as in Akerlof’s Lemons paper, but with warranties. The value of the car to the seller is \( v_S = 10000P \), where \( P \) is the probability that the car will not break down during the next 20000 miles (the car will break down at most once). The seller knows \( P \). The buyer thinks \( P \) is uniformly distributed between .50 and 1.00. This implies that the value \( v_S \) is uniformly distributed between $5000 and $10000. The value to the buyer is \( v_B = 12000P \).

a. What is the range of prices, if any, at which some cars will be sold?

Now consider warranties of the form: If the car breaks down during the first 20000 miles the seller pays buyer $1000. Let \( p \) be the price for the car and warranty package.

b. Given a price \( p \), find the range of sellers (expressed in terms of \( P \)), who will accept the offer \( p \).

c. Given the answer in part b, what is the buyer’s expected value (excluding any warranty payment) of the cars that will be offered for sale at price \( p \)? What is the maximum price at which a buyer is willing to purchase?

d. Are there prices \( p \) at which some cars will be sold when all cars come with warranties as described but no cars can be sold when there are no warranties?