Growth Models (A two class of labor economy with something else)

There is an economy with many consumers and infinite time. There are two types of households, those that are strong called Herreños and those that are smart, called Gomeros. Each type has its own type of labor that they provide inelastically. Herreños cannot work as smart and Gomeros cannot work as strong. They have identical preferences over expected discounted streams of consumption. Consumers within types are identical and their measure is the same.

Firms can produce output according to a production function given by

\[ z_t F(k_t, n^H_t, n^g_t, N_{t-1}) \]

where \( z_t \) is an aggregate Markovian shock with transition matrix \( \Gamma \), \( (k_t, n^H_t, n^g_t) \) are own quantities of inputs: capital, strong labor and smart labor and \( N^g_t \) is the aggregate stock of smart labor of the previous period.

Capital depreciates at rate \( \delta \). Output can be used either for consumption or for investment purposes.

1. (5 points) Does the First Welfare Theorem apply?

2. (10 points) Make a small change in this economy that results in the opposite answer to the previous question.

Suppose now that the household owns capital and rents it to firms.

3. (5 points) Define a recursive competitive equilibrium. Make sure that you list the state variables.

Now assume that the government taxes/subsidizes strong labor at rate \( \tau \) and returns the proceeds in a lump sum manner.

4. (5 points) Write a formula that links the equilibrium transfer as a function of the \( \tau \) and the state variables.

5. (10 points) Can you think of a tax/transfer scheme that uses only labor taxes, perhaps different across types, in a way that achieves optimality in the version of the economy that is not optimal? (Notice that your answer depends on what you answered in the first two questions)?
Lucas Trees

Assume there is a representative agent economy. There is a are two trees in the economy, one produces apricots and the other bananas.

Preferences are given by

\[ E \left\{ \sum_{t=1}^{\infty} \beta^t \left( a_t^\rho b_t^{1-\rho} \right)^{1-\sigma} \right\} \]

where \( a_t \) and \( b_t \) stand for apricots and bananas respectively. Apricot production follows a Markov process with transition \( \Gamma^a \) and bananas are iid with finit support \( b \in B \).

6. (10 points) Price an asset that entitles the holder to 10% of an apricot tree fruit every odd year and 50% of a banana tree every even year as a function of the state variables.

7. (5 points) Price an option to buy this asset at price \( q \) two periods from now.

Industry Equilibria

Imagine that the shock that affects a firm’s productivity can take 5 values \( \{ s^1, \ldots, s^5 \} \) with transition matrix \( \Gamma \). Assume that \( \Gamma \) is such that if \( s_i > s_j \) then \( E\{s|s_i\} > E\{s|s_j\} \). Adjusting the size of the labor force entails costs given by increasing function \( \phi(n - n^-) \) with \( \phi(0) = 0 \) where \( n \) is today’s labor today and \( n^- \) labor yesterday of the firm. There are fixed costs to entry of \( \xi \).

8. (5 points) Pose the problem of the firm, and describe its objects.

9. (5 points) Write the transition function that characterizes the evolution of firms. Verify that what you construct is indeed a transition function.

10. (5 points) Can you say something about how this economy differs from one where there are no adjustment costs? Answer with respect to price quantity and properties of the size distribution of firms.

11. (5 points) Imagine now that the demand function is \( y_t = \gamma(t - p_t), \gamma > 1 \). Define a stationary equilibria. How would you transform the economy to use the tools that we have learned to deal with economies without growth.

Health is Capital

Imagine a finitely lived agent with preferences given by

\[ E \left\{ \sum_{i=1}^{t} \beta^i \psi^i(h_i) u(c_i, e_i) \right\} \]
where \( i \) is age, \( h \) is health, \( e \) is effort in self care. Where \( 0 < \psi^i(h) < 1 \), with \( \psi \) monotonically increasing, is the survival probability. \( U(c,e) > 0 \), and \( u_e > 0 \) and \( u_e < 0 \). The agent’s health evolves according to

\[
    h^{i+1} = \phi(h^i, e) + (1 - \delta)h^i + x
\]

where \( \phi_h > 0 \) and \( \phi_e > 0 \), and \( x \) are expenditures in health care. Agents can save at rate \( r \).

12. (5 points) Write the problem of this agent recursively.

13. (10 points) Derive and comment the first order condition associated to the allocation of goods.

14. (5 points) If \( \psi^i(h_i) = \psi(h_i) \), how is the implied survival rate by age?

**Contracting Frictions in Island Economies**

Consider an economy where agents have an endowment of 1 unit of time each period. They have access, in every period, to a farming technology that when working \( y \) units of time they get \( s_t \) units of the good. \( (s_t \) is Markov with transition \( \Gamma \) and is identically independently distributed across all agents). They also have access to a storage technology (the good today can be transformed into the same amount of good tomorrow). Preferences are given by

\[
    E\left\{ \sum_{t=0}^{\infty} \beta^t \frac{c^{1-\sigma}}{1-\sigma} \right\}
\]

where \( \beta < 1 \).

Assume first that there are no credit markets.

15. (5 points) Write the problem of the agent.

16. (5 points) Is there a stationary distribution? Make any necessary additional assumption that you may need and explain why you need it.

Imagine now that there are credit markets but that every unit exchanged loses 2% when changing hands.

17. (5 points) Define recursive equilibria in this case.

18. (5 points) Describe a necessary condition for credit to exist when there is also a storage technology.