Microeconomic Theory II  
Preliminary Examination  
May 12, 2008

There are two parts, the first worth 20 points, and the second worth 100 points. The exam is worth 120 points in total.

Do all questions. Start each question on a new page, clearly labeled. Justify all answers and show all work. Label all diagrams clearly. Write legibly. If you need to make additional assumptions, state them clearly.

**Part A — Two questions, 10 points each.**

1. Consider the following two bidder auction for $10. All bids must be strictly positive integer dollar amounts and bidder 1 starts. The bidders take turns in either announcing a new bid, which must exceed the previous high bid by at least 1, or dropping out, in which case the other bidder is the highest bidder and wins the auction. Each bidder pays what he bid. Bidder 1 has a budget of $5 and bidder 2 has a budget of $3. Both bidders are risk neutral, and cannot bid more than their budget. Prove that this game has a unique stationary subgame perfect equilibrium, and describe it.  

2. Describe an economic environment and result that illustrates the claim that “private information necessarily leads to efficiencies.”

**Part B – Three questions, first two are worth 30 points each, and the last 40 points.**

1. Suppose the government (through its agency, the Internal Revenue Service—IRS) wants to ensure compliance with the tax code. A taxpayer, in filling out a tax return, has a choice, be truthful (T) or lie (L). The IRS can audit the return to determine whether the taxpayer was truthful on his return (A), or can decide not to audit (N). The taxpayer’s payoffs are given by $u_t(T, A) = 2$, $u_t(T, N) = 3$, $u_t(L, A) = -2$, and $u_t(L, N) = 4$. The government’s payoffs are given by $u_g(T, A) = -1$, $u_g(T, N) = 1$, $u_g(L, A) = 0$, and $u_g(L, N) = -2$. Suppose the IRS makes the audit decision after the taxpayer has submitted his return (for simplicity, suppose the IRS makes the decision before reading the return).

   (a) What is the normal form of this game? [5 points]
   (b) What are the Nash equilibria of this game? [5 points]
   (c) Suppose the game is infinitely repeated. Moreover, suppose that information becomes available at the end of the period that independently tells the IRS whether the taxpayer was truthful on his return (too late to punish the taxpayer in that period). That is, the IRS and the taxpayer repeatedly play the game described in the previous question, with the result of period $\tau$ play (including the truthfulness of the taxpayer) observed before the period $\tau + 1$’s play.

   Give bounds on the discount factor $\delta$ so that there is a subgame perfect equilibrium in pure strategies in which the taxpayer is truthful and the IRS does not audit. Be sure to fully describe the strategy profile and prove that it is subgame perfect (you can be informal as long as there is no ambiguity). [20 points]
2. The owner of a small firm is contemplating selling all or part of his firm to outside investors. The profits from the firm are risky and the investor is risk averse. The investor's preferences over $x$, the fraction of the firm the owner retains, and $p$, the price "per share" paid by the outside investors, are given by

$$u(x, \theta, p) = \theta x - x^2 + p(1 - x),$$

where $\theta$ is the value of the firm (i.e., expected profits). The quadratic term reflects the owner's risk aversion. The outside investors are risk neutral, and so the payoff to an outside investor of paying $p$ per share for $1 - x$ of the firm is then

$$\theta(1 - x) - p(1 - x).$$

There are at least two outside investors, and the price is determined by a first price sealed bid auction: The owner first chooses the fraction of the firm to sell, $1 - x$; the outside investors then bid, with the $1 - x$ fraction going to the highest bidder (ties are broken with a coin flip).

**Important convention:** The outside investors submit bids in "price per share" $p$, so the amount paid is $p(1 - x)$.

(a) Suppose $\theta$ is public information. What fraction of the firm will the owner sell, and how much will he receive for it? [5 points]

(b) Suppose now $\theta$ is privately known by the owner. The outside investors have common beliefs, assigning probability $\alpha \in (0, 1)$ to $\theta = \theta_1 > 0$ and probability $1 - \alpha$ to $\theta = \theta_2 > \theta_1$. Characterize the separating perfect Bayesian equilibria. Are there any other perfect Bayesian equilibria? [15 points]

(c) Maintaining the assumption that $\theta$ is privately known by the owner, suppose now that the outside investors' beliefs over $\theta$ have support $[\theta_1, \theta_2]$, so that there a continuum of possible values for $\theta$. What is the initial value problem (differential equation plus initial condition) characterizing separating perfect Bayesian equilibria? **DO NOT ATTEMPT TO SOLVE IT.** [10 points]

3. A public utility commission (the regulator) is charged with regulating a natural monopoly. The cost function of the natural monopoly is given by

$$C(q, \theta) = \begin{cases} 0, & \text{if } q = 0, \\ K + \theta q, & \text{if } q > 0, \end{cases}$$

where $q$ is the quantity produced, $K > 0$ is the publicly known fixed cost, and $\theta \in (0, 1)$ is marginal cost. The inverse demand curve for the good is

$$p(q) \equiv \max\{1 - 2q, 0\}.$$

Supposing there are no income effects for this good, consumer surplus is given by

$$V(q) = \int_0^q p(\tilde{q}) \, d\tilde{q} - p(q)q.$$

The regulator determines the firm's regulated price and subsidy, as well as whether the firm is allowed to operate at all. The firm cannot be forced to operate. As in the standard monopoly
problem, without loss of generality, the regulator chooses $q$ (with the regulated price given by $p(q)$).

The firm wishes to maximize expected profits,

$$\Pi(q) = p(q)q - C(q, \theta) + s,$$

where $s$ is the subsidy. The regulator maximizes the total of consumer surplus and firm profits net of the surplus,

$$V(q) + \Pi(q) - s = \int_{0}^{q} p(\tilde{q}) \, d\tilde{q} - C(q, \theta).$$

(a) Suppose the marginal cost $\theta > 0$ is publicly known. Solve the regulator’s problem. Interpret the resulting $q$, $p$ and $s$. [10 points]

(b) Suppose now the marginal cost $\theta$ is known only to the monopoly, with the regulator having some beliefs over $\theta$. The regulator’s problem is to determine the regulated price and subsidy as a function of some cost report from the firm. State the revelation principle for this problem and give a brief outline of its proof. [10 points]

(c) Suppose the regulator’s beliefs assign probability $\alpha_i \in (0, 1)$ to $\theta = \theta_i \in (0, 1)$, where $\alpha_1 + \alpha_2 = 1$ and $\theta_1 < \theta_2$.
   i. Describe the regulator’s problem, being explicit about the constraints. Which individual rationality constraint is implied by the other constraints, and why? Prove that the regulator’s optimal quantity schedule is monotonic in cost reports. [Assume that the regulator optimally allows both types to operate.] [10 points]
   ii. One of the incentive compatibility conditions is an implication of two other conditions holding with equality. Which one and why? Can you provide any intuition? Who, if anyone, earns information rents? [10 points]