This question will be graded both on the analytical ability that you demonstrate and on the economic intuition that you display. Please write legibly and double space. Good Luck!

(Malthus) Consider the following overlapping generations model. There are two generations alive at any period in time, namely the young and old. Young people supply one unit of time on the labor market. A unit of labor earns the wage rate $w$. A young person uses his labor income either to consume in the current period or to save for the future. All savings is in the form of land holdings. A young person can purchase a unit of land for the price $q$ in terms of consumption. Next period they collect the rentals on the land and then can resell it. Old people don’t work and must live off of their retirement savings, which earns a gross rate of return represented by $i$. Let a person’s lifetime utility function be given by

$$\ln c_1 + \beta \ln c_2',$$

where $c_1$ is the person’s consumption when young and $c_2'$ denotes his consumption when old.

Output, $y$, in the economy is produced in line with the constant-returns-to-scale production function

$$y = n^{1-\theta} l^\theta,$$

where $n$ is the amount of labor hired and $l$ denotes the amount of land utilized. Let the rental rate on land be represented by $r$. There is one unit of land available in the economy.

Last, the number of children that a young adult will have, or his fertility, $f$, is exogenously specified by the invertible function

$$f = F(c_1 - 1 + \theta).$$

Here $F : [-1+\theta, \infty] \to R_+$, with $F_1 > 0$ and $F(0) = 1$. A young person does not take into account the cost and benefits of having children in his maximization problem. Answer the following:

1. Formulate the Malthusian steady state. What is the equilibrium size of the population, $n$? What is the equilibrium interest rate, $i$? How is the interest rate related to labor’s share of income? What is the intuition for this result? Suppose that an economic historian gives an observation on $i$. Is it possible that a discount factor greater than one is needed to be consistent with this observation? Explain the economics underlying your finding.

2. Suppose that the production function is given by

$$y = zn^{1-\theta} l^\theta,$$

where $z$ grows at gross rate $\gamma^\theta > 1$. Reformulate the Malthusian steady state. What is the equilibrium size of the population? What is the equilibrium interest rate?