Economics 702
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Instructions: The exam is 2 hours long. Answer all three questions. Please answer each question in a separate book, write legibly and double space. Good Luck!

1 A Pure Exchange Model with Risk

Consider a pure exchange economy that lasts forever. There are two consumers that have endowments of the single consumption good that are stochastic. In each period there are two possible states of the world that can occur. Let by denote the state at time \( t \), \( s^t \) the history of states up to time \( t \) and \( S = \{1, 2\} \) the set of possible states of the world that can occur in each period. Also let \( S^t \) denote the \( t + 1 \)-fold Cartesian product of \( S \). The stochastic process \( \{s_t\}_{t=1}^{\infty} \) is a Markov process with transition matrix

\[
\pi(s_2|s_1) = \begin{pmatrix}
\gamma & 1 - \gamma \\
1 - \gamma & \gamma
\end{pmatrix}
\]

In period 0, the probability of state \( s_1 \) being drawn is \( \Pi_0(s_1) = 0.5 \) and the probability of state \( s_2 \) being drawn is given by \( \Pi_0(s_2) = 0.5 \). Denote by \( \Pi_t(s_1), \Pi_t(s_2) \) the corresponding unconditional probabilities at time \( t \). Finally, let \( \pi(s^t) \) denote the probability of event history \( s^t \).

Individuals \( i = 1, 2 \) have endowments \( e^t_i = \{e^t_i(s_t)\}_{s_t \in S} \) that depend on the state of the world and preferences over consumption allocations \( c^t = \{c^t(s)\}_{s \in S} \) given by

\[
U^t(c) = \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi(s^t) U(c^t(s^t))
\]

where the period utility function \( U : \mathbb{R}_+ \to \mathbb{R} \) is strictly increasing.

1. Let \( \gamma = 0 \) for the entire remaining question. Compute \( \pi(s^3 = (2, 1, 2, 1)) \) and \( \pi(s^3 = (2, 2, 2, 2)) \).

2. Let the utility function take the form

\[
U(c) = \begin{cases}
\frac{c^{1+\sigma}}{1+\sigma} & \text{if } \sigma > 0, \sigma \neq 1 \\
\log(c) & \text{if } \sigma = 1
\end{cases}
\]
Let endowments be given by

- \( e_t^1(s_t) = 1 = 2(1 + g)^t \)
- \( e_t^1(s_t) = 2 = 0 \)
- \( e_t^2(s_t) = 1 = 0 \)
- \( e_t^2(s_t) = 2 = 2(1 + g)^t \)

with \( g > 0 \) and \( \beta(1 + g)^{1-\sigma} < 1 \). Compute the unique Arrow Debreu equilibrium.

3. Suppose the realized state in period 0 is \( s_1 \). Compute the price of a two-period risk free bond that is traded in period zero (after the shock is realized) and that pays out one unit of consumption in period 2, independent of the realizations of the shocks in period 1 and 2.

4. Suppose the growth rate of the economy \( g \) increases. How does the long run (that is \( t \to \infty \)) equilibrium one-period risk free interest rate change? Briefly explain the economics behind this result.

2 The Stochastic Neoclassical Growth Model with Endogenous Labor Supply

The social planner in the stochastic neoclassical growth model chooses stochastic consumption, labor and capital allocations \( \{c_t, n_t, k_{t+1}\} \) to solve the following maximization problem

\[
\max E_0 \sum_{i=0}^{\infty} \beta^i (\ln(c_t) + \theta \ln(1 - n_t))
\]

s.t.

\[
c_t + k_{t+1} = z_t k_t^{\alpha} n_t^{1-\alpha}
\]

with \( \beta \in (0, 1) \) and \( \theta > 0 \) and \( \alpha \in (0, 1) \) being parameters. The initial endowment of capital \( k_0 \) is given. The technology shock \( z_t \) follows a \( N \)-state Markov chain with state space \( Z = \{z_1, z_2, \ldots, z_N\} \) be the state space of the Markov chain, and let \( \pi(z_{t+1}|z_t) \) denote the Markov transition matrix of the chain.

1. Write down the functional equation the solution to which solves the social planner’s problem. State explicitly what the state variables and control variables are.

2. Use the Benveniste-Scheinkman (envelope) theorem to derive an expression for the derivative of the value function with respect to the current capital stock \( k \).
3. Now suppose consumption goods and capital goods are produced in two different sectors. The production functions in both sectors are given by

\[ c_t = z_{1t}k_{1t}^{\alpha}n_{1t}^{1-\alpha} \]
\[ k_{t+1} = z_{2t}k_{2t}^{\beta}n_{2t}^{1-\gamma} \]

Capital and labor inputs in both sections have to satisfy

\[ k_{1t} + k_{2t} = k_t \]
\[ n_{1t} + n_{2t} = n_t. \]

The planner can allocate both capital and labor freely across the two sectors in the current period, that is, no time is required to relocate either capital or labor across sectors. The technology shocks both follow Markov chains with state space \( Z \) and transition matrices \( \pi_1(z_{1t+1}|z_{1t}) \) and \( \pi_2(z_{2t+1}|z_{2t}) \). Repeat question 1.

### 3 The Neoclassical Growth Model with Government Spending

Consider a world inhabited by an infinitely-lived representative agent. The agent’s momentary utility is given by

\[ U(c) - W(l), \]

where \( c \) is consumption, \( l \) is labor effort, \( U \) is a strictly increasing, strictly concave, \( C^2 \) function, and \( W \) is a strictly increasing, strictly convex, \( C^2 \) one. The agent discounts the future at rate \( \beta \), with \( 0 < \beta < 1 \). He produces output, \( y \), according to the following constant-returns-to-scale production function

\[ y = F(k,l), \]

where \( F \) is a strictly increasing, strictly concave, \( C^2 \) function in each of its arguments. Output in the economy can be used for three purposes, namely consumption, \( c \), for acquiring capital to use in production next period, \( k' \), and wasteful government spending, \( g \). Assume that the capital stock depreciates fully each period. The government finances its spending by a lump-sum tax each period in the amount, \( \tau \). It balances its budget on a period-by-period basis. Let \( g \) follow a simple deterministic difference equation of the form \( g' = G(g) \).

1. Formulate the representative agent’s dynamic programming problem. Can this setup be formulated as a simple planning problem?

2. Formulate a steady-state for the model. Write out the steady-state conditions in intensive form; i.e., in terms of \( k/l \) and \( l \). How are the steady-state levels of labor, \( l \), capital, \( k \), and the capital-labor ratio, \( k/l \), affected by the level of government spending? (Hint: Analyze the system in its intensive form representation.) What is the intuition underlying your finding? Can the government spending multiplier be greater than one?
3. Suppose that the economy is not in its steady state. Imagine that the level of government temporarily rises in the current period and then reverts back to its old time path from next period on. What will happen to \( l \) and \( k' \)? (Analyze the system in its standard form, as it doesn’t pay here to study the system in its intensive form representation.) What is the intuition underlying your finding? Can the government spending multiplier be greater than one? When answering this question, you may assume that the value function is a \( C^2 \) function. Other properties of the value function that you use should be justified.