PART I (30%). Consider an economy where a representative consumer has utility function
\[ U = \sum_{t=0}^{\infty} \beta^t u(c_{mt}, c_{nt}, 1 - h_{nt} - h_{mt}), \]
where \( c_{jt} \) is consumption and \( h_{jt} \) is labor hours in sector \( j, j = m \) denoting market and \( j = n \) nonmarket activity. The individual budget constraint is
\[ c_t = w_t h_t + r_t k_{mt} + (1 - \delta_m) k_{mt} + (1 - \delta_n) k_{nt} - k_{m,t+1} - k_{n,t+1} - w_t h_t \tau_t + T_t, \]
where \( w_t \) is the real wage, \( r_t \) is the rental rate on market capital, \( k_{jt} \) is capital in sector \( j, \delta_j \) is the depreciation rate in sector \( j, \tau_t \) is a tax on labor income, and \( T_t \) is a lump sum transfer. Assume depreciation rates follow independent stochastic processes: \( \delta_{j,t+1} = \Delta_j(\delta_{j,t}, \varepsilon_j) \). \( R_t \) is set to balance the budget, \( g_t = w_t h_t \tau_t - T_t \), where \( g_t \) is government consumption and follows \( g_{t+1} = G(g_t, \eta_t) \). Nonmarket consumption satisfies \( c_{nt} = G(h_{nt}, k_{nt}) \), and there is a representative market firm with CRS technology \( y_t = F(h_{mt}, k_{mt}) \). Make other assumptions as you see fit.
1. Carefully define a recursive competitive equilibrium for this economy.
2. Derive the equations characterizing equilibrium.
3. Compare the equilibrium and the solution to the social planner’s problem.

PART II (35%). Lay out the basic textbook Pissarides model of search unemployment, with free entry by firms and generalized Nash bargaining, in as much detail as you can, and discuss how one finds equilibrium.

PART III (35%). Consider an overlapping generations economy where all agents born at \( t \) have utility \( U(c_{t1}, c_{t2}) = \log(c_{t1}) + \beta \log(c_{t2}) \) and endowment \( e_t = (e_1, e_2) \), with \( e_j > 0 \). The stock of fiat money is fixed at \( M \). Population grows at rate \( \gamma \).
1. Describe the set of nonmonetary equilibria.
2. Describe the set of monetary equilibria.
3. Give necessary and sufficient conditions for monetary equilibria to exist.