Cole’s Problem for the Comp 2009

Assume that there is a single nonstorable consumption good each period. Assume that there is a unit measure of households each of whom has preferences given by

\[ E \left\{ \sum_{t=0}^{\infty} \beta^t V(c_t) \theta_t \right\}, \]

where \( c_t \) is consumption in period \( t \) and \( \theta_t \) is a preference shock in period \( t \) which affects the flow utility from consumption. Assume that \( V'' > 0 \) and \( V'' < 0 \). Assume that \( \theta_t \) is idiosyncratic, serially independent and drawn from a distribution \( \mu \). We will assume that the taste shocks take on a finite set of possible values \( \theta_t \in \Theta = \{\theta^1, ..., \theta^n\} \), where \( \theta^j > \theta^{j+1} \) and \( \theta^n > 0 \). (Note that a lower report means you’re hungrier.)

A) Consider the social planning problem in which the planner tries to maximize the ex ante welfare of his subjects given that he has a fixed amount of output \( y \) to hand out in each period. Formally, the planner is picking a consumption arrangement \( c_t(\theta^t) \), where \( \theta^t \in \Theta^t \), so as to maximize social welfare subject to a resource constraint. (i) Write down the objective function of the planner and the resource constraint for his choice variable \( c_t(\theta^t) \). (ii) Derive the optimality conditions for his choice of \( c_t(\theta^t) \). (iii) Use them to characterize his optimal choice for the special case in which \( V(c) = \exp(c) \). Show that the optimal allocation is stationary and increasing in \( \theta \).

B) Assume that the individual’s preference shocks are private information. Assume that the revelation principal applies (which it does) and that we can characterize the efficient arrangement using the truth-telling equilibrium of a direct mechanism. Define the continuation payoff to the agent under the plan as

\[ \omega_k(\theta^{k-1}) = E_{k-1} \left\{ \sum_{t=k}^{\infty} \beta^{t-k} V(c_t(\theta^t)) | \theta_{t+1}^{k-1} \right\}, \]

where \( k \) denotes the time period. (i) Write down the additional constraint that we must now add to the social planner’s problem in (A) to insure that the agent’s reports are incentive compatible. (ii) Argue that temporary incentive compatibility implies this constraint if \( V \) is bounded (both above and below). Be sure to define temporary incentive compatibility and to state how many temporary incentive compatibility constraints there are. Be brief as I only need to see the outline of the proof.

C) Assume that the planner is trying to minimize the discounted cost of delivering some
promised utility to his subjects. Assume that the planner discounts future expenditures at rate $\beta^t$ is period $t$. Assume that there are a set of possible continuation payoffs to the agent $\Omega$ which we can deliver. and that the cost of these payoffs to the planner is $P(\omega)$. (i) Taking $v$ as his current distribution of utility promises, and write down the recursive optimization problem one could use to implicitly characterize $P$. To formulate the planner’s problem, it will be useful to think of him as picking the value $u(\hat{\theta}, v) = V(c(\hat{\theta}, v))$ and his continuation utility $\omega(\hat{\theta}, v)$ as functions of the agent’s report $\hat{\theta}$ and promised utility $v$. It will also be useful to define the inverse of $V$ as $C(u) = V^{-1}(u)$. (ii) Given $P(v)$, and the distribution of utility promises $h(v)$, what is the planner’s cost of honoring his promises.

D) Assume that only the local downward misreporting constraint binds in your recursive program to characterize $P(v)$. Construct the first-order conditions for the optimal choices $u(\hat{\theta}, v)$ and $\omega(\hat{\theta}, v)$. Argue that $P'$ is a Martingale.