Cole’s Problem

A. There is a risk-neutral principal, who can freely borrow and lend at rate $R$ who wants to efficiently insure a collection of (measure 1) agents each of whom lives for two periods. An agent’s first period income is stochastic, and given by $y \in \{y_1, y_2, ..., y_N\}$, where $y_i < y_{i+1}$; $y_1 > 0$ and $y_N < 1$. Assume that each of these draws is equally likely. An agent’s second period income is not stochastic and is equal to 1/2. Agent’s preferences are given by

$$E \left\{ \sum_{t=1}^{2} \beta^{t-1} u(c_t) \right\},$$

where $u' > 0$ and $u'' < 0$.

Since the revelation principal applies, a contract for an agent is a $2N$ set of consumptions $x_i^1$ in the first period and $x_i^2$ in the second period where $i = 1, ..., N$.

i) Assuming complete information and that the principal cares equally about all of the agent’s, form the representative contracting problem for the principal and characterize the efficient contract.

ii) Assuming that agents’ incomes are private information, form the contracting problem and try and characterize the efficient contract. In doing so, assume that the only incentive constraints that bind are for the agent with income $y_{i+1}$ misreporting that his income is $y_i$. Note that there is no incentive constraint on the agent with the lowest income. Contrast the efficient contract with private information to the one with complete information.

iii) What would happen to the efficient contract if the flow utility of consumption was $u(c) = c$? Use your results to discuss how the efficient contract is likely to change as agent’s become more or less risk averse.

B. Now assume that the agent’s can always store their income between the first and second period in a manner that cannot be detected by the planner. Assume that the agents earn the same rate of return, $R$, on their storage. How does the presence of this storage option change the contracting problem for the principal? Construct the new contracting problem and characterize the efficient contract. Contrast this contract to the efficient contract that you derived in part A.

C. How would one change the efficient contract if the agent’s could see each other’s incomes but the planner still could not? Would this help the planner come closer to achieving the complete information outcome?