Microeconomic Theory I
Preliminary Examination
University of Pennsylvania

May 11, 2009

Instructions

You have two hours to complete this exam.

Each question is of equal value. Answer all parts.

If you need to make any additional assumptions, state them clearly.

Be concise.

Good luck!
1. Consider a two-period exchange economy with two agents, $i = 1, 2$. Agent $i$’s utility function over consumption plans is $U_i(x_1, x_2) = u_i(x_1) + \delta_i u_i(x_2)$, where $u_i$ is strictly increasing, strictly concave and differentiable, and $\delta_i > 0$. The agents’ initial endowments are $\omega_i = (1, 0)$ and $\omega^2 = (0, 1)$.

(a) Show that if $\delta_i > \delta^2$, then any interior Pareto efficient allocation $(x^1, x^2)$ satisfies $x_1^1 < x_1^2$, that is, agent 1 consumes less in period 1 than in period 2.

For the remaining parts, assume $\delta^1 = \delta^2$.

(b) What is the Walrasian equilibrium allocation for this economy?

(c) What can you say about the Walrasian equilibrium price if the initial endowments are changed to $\omega^1 = (1, 0)$ and $\omega^2 = (0, 2)$? Explain.

(d) Now assume $u_i(x_t) = \ln x_t$ for both agents and dates, and $\delta^1 = \delta^2 = 1$. Let the endowments be $\omega^1 = (1, 0)$ and $\omega^2 = (0, k)$, where $k > 0$. Show that when agent 2’s second-period endowment increases, the resulting Walrasian equilibrium price change is detrimental to agent 2, but the increase in his endowment more than compensates. (That is, agent 2’s equilibrium utility increases in $k$.)

2. Consider a two-period GEI model of an exchange economy with a single commodity per state. Suppose there are 5 states and 3 assets. The assets pay

\[
A = \begin{pmatrix}
1 & 1 & 1 \\
2 & 0 & 3 \\
1 & 3 & 0 \\
0 & 0 & 1 \\
2 & 1 & 2 \\
\end{pmatrix}
\]

where the $j^{th}$ element of the $i^{th}$ row is the amount the $j^{th}$ asset pays in state $i$. Which of the following asset prices preclude arbitrage?

(a) $q = (1, 2, 1)$

(b) $q = (2, 4, 1)$

(c) $q = (1, 1, 1)$

(d) $q = (6, 5, 7)$

(e) $q = (2.1, 1.2, 2.2)$
3. Consider a public goods economy with \( n \) consumers, one public good \( x \), and one private good \( y \). The utility function of consumer \( i \) is \( u^i(x, y_i) \), and her initial endowment of the private good is \( \omega_i > 0 \). The production function for transforming private good into public good is \( f : \mathbb{R}_+ \to \mathbb{R}_+ \), and it satisfies \( f(0) = 0 \). Each \( u^i \) and \( f \) is continuous, strictly increasing, and concave, but not necessarily differentiable.

(a) Prove or disprove: the Utility Possibility Set is convex.

(b) Prove or disprove: an allocation is Pareto optimal if \( \lambda \in \mathbb{R}^n_+ \setminus \{0\} \) exists such that the allocation maximizes \( \sum_i \lambda_i u_i(x, y_i) \) subject to the feasibility constraints.

(c) Prove or disprove the converse of the claim in (b).

4. Consider the asymmetric information bilateral trading environment in which a seller can produce one unit of a discrete good for a buyer. Both parties are risk neutral. The seller’s production cost \( \tilde{c} \) and the buyer’s value \( \tilde{v} \) for the good are \textit{ex ante} independently distributed, each with a positive density function on \([0, 1]\). Given a number \( a \), define a trading revelation mechanism \( \langle q, T_s, T_b \rangle \) in the following way:

\[
q(v, c) = \begin{cases} 
1 & \text{if } v \geq c \\
0 & \text{if } v < c,
\end{cases}
\]

\[
T_s(v, c) = E\tilde{v}q(\tilde{v}, c) + E\tilde{c}q(\tilde{c}, v) - a,
\]

\[
T_b(v, c) = -E\tilde{c}q(v, \tilde{c}) - E\tilde{v}q(\tilde{v}, c) + a.
\]

(The \( T_i \) determines the monetary transfer to player \( i \), and \( q \) the probability of trade.) What are the merits and demerits of this mechanism, as a function of \( a \)? Prove your claims.