Crime and Poverty: A Search-Theoretic Approach

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Abstract: Numerous studies document, on the one hand, a positive correlation between crime and unemployment and, on the other hand, a negative correlation between the crime rate and educational attainment levels within given communities. We study this phenomenon in the context of a search-equilibrium model, in which agents choose between formal employment and pursuing crime related activities (theft). Prior to making this ‘occupational choice,’ agents first undertake costly schooling which raises their productivity upon formal employment. We show that crime acts, in essence, as a tax on human capital by affecting the probability that a worker’s earnings (possessions) are subsequently appropriated. There are multiple equilibria. High crime, low levels of educational attainment, long spells of unemployment, and poverty are correlated across them.

JEL Classification: I2, J2,J3,J4,J6.

Keywords: Crime, Search and Unemployment, Educational Choice and Human Capital

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1 Introduction

Criminal activity is characterized by several striking empirical features. It is subject to considerable temporal variation, it is geographically concentrated, and it exhibits wide dispersion across communities that possess ostensibly identical economic characteristics. Less affluent cities are disproportionately afflicted: in particular those characterized by chronic poverty, a poorly educated workforce and limited access to employment opportunities.

Thus, the U.S. witnessed a precipitous increase in the crime rate during the 1980s followed by an equally impressive decline during the 1990s. Indeed, at its peak in 1991, about 2% of the U.S. workforce – amounting to just over 2 million persons – was incarcerated in federal or State prisons and about 7% of the workforce was incarcerated, paroled or on probation at the reference time. Regarding geographic concentration, Scott Freeman, Jeffrey Grogger and Jon Sontselie (1996) note that in 1990 the median number of reported street robberies in Los Angeles equaled 4 per 1000 residents. Yet, 10% of neighborhoods had crime rates four times greater than the median. As for both temporal and spatial variability, Edward Glaeser, Bruce Sacerdote, and Jose Scheinkman (1996) document that the U.S homicide rate declined by 50% between 1933 and 1961 and that: “Ridgewood village reported 0.008 serious crimes per capita, whereas nearby Atlantic City reported 0.34.”

The changes in the U.S. crime rate were coterminous with two significant developments in the U.S. labor market: the sharp decrease in the earnings of young unskilled men in the 1980s and the rapid decline in the aggregate rate of unemployment in the 1990s. This immediately raises the question of whether or not the two events are related in some way. Of course, such a connection might be expected to hold on *a priori* grounds. The reason is that according to the canons of conventional microeconomic wisdom, the decision of whether or not to engage in criminal activities is a time allocation problem. As such, changes in the opportunities available to workers in the formal labor market will have a direct and ineluctable impact upon the crime rate, by affecting the opportunity cost of criminal behavior.

Yet, despite this compelling logic, few theoretical models have been constructed to date that can be used to address these connections more formally. This is particularly surpris-

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1 Similar disparities were observed between New York’s 1 precinct and the 123 precinct, with per-capita crime rates of 0.21 and 0.022 respectively.

2 See Grogger (1998) and the references cited therein. As noted by Imrohoroglu et al. (2000) and Merlo (2001), two other profound changes were also taking place at this time that could also account for the changes in the crime rate. First, there were significantly greater expenditures on policing as well as the adoption of new policing tactics in many locations. Second, there was a sizable demographic shift towards a more elderly population. (The vast majority of crimes are committed by those between the ages of 17 and 30).
ing, in view of the large body of empirical work that indicates that not only labor-market opportunities affect criminal behavior, but the crime rate itself also affects labor-market opportunities.\(^3\) Perhaps the most robust finding in this literature is the documented negative correlation between market wages and crime. While Grogger (1998) estimates that a 20% decline in the (youth) wage would lead to a 20% increase in the crime rate, Gould et al. (2002) document that changes in the wage can account for up to 50% of the trend in violent crimes and in property crimes. Recent studies by Raphael and Winter-Ebmer (2001) and by Gould et al. (2002) also indicate that there is a strong positive link between the unemployment and crime rates.\(^4\) Close empirical relationships are also observed between the rate of crime and human capital acquisition. For instance, Lawrence (1995) reports that in 1982: “In the general population, 85% of males 20-29 years of age have finished high school; only 40% of prisoners have done so. Six percent of prisoners have had no schooling at all.” Indeed, several studies including Witte and Tauchen (1994), Lochner (1999) and Locher and Moreti (2001) also indicate that completing high school significantly reduces criminal proclivities.

In this paper we propose a dynamic general (search) equilibrium framework that can be used to flesh out and better understand how the opportunities available in the formal labor market affect the crime rate and \textit{vice versa}.\(^5\) The model we construct captures, in a parsimonious manner, many of the salient stylized aspects of the empirical observations outlined above and illuminates several interesting feedback effects, presently not explored in the literature. Significantly, the crime rate, levels of educational attainment, interdiction rates, and both employment opportunities and wages are all determined endogenously. Perhaps somewhat surprisingly, the resultant framework is highly tractable and admits numerous comparative static experiments that may yield useful policy and empirically testable implications. These

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\(^3\)Evidence linking the crime rate to human capital accumulation is sparser. For instance, Grogger (1997) shows that propinquent violent crime has a significant and negative effect on educational attainment levels.

\(^4\)These results stand in sharp contrast with earlier work in the area (see for example Freeman (1995)), which indicates there is but a modest effect of unemployment on crime. In addition to more plentiful data, the hallmark of the more recent literature is carefully controlling for the endogeneity of the work/unemployment decision.

\(^5\)It is only recently that economists have begun to study the links between the socio-economic environment and the concomitant rate of crime. Indeed, the early literature on the economics of crime, pioneered by Becker (1968) and Ehrlich (1973), largely focuses on the determinants of malfeasant behavior and attempts to circumscribe the efficacy of punishment as a deterrent. Important recent exceptions include Imrohoroglu, Merlo and Rupert (2000a,b) who construct general (competitive) equilibrium models of crime. In his recent survey, Merlo (2001) emphasizes the importance of using a dynamic general-equilibrium framework to model criminal behavior.
include, to name but a few, the effects of an increase in policing expenditures, the effects of penal reform, as well as the effects of more traditional labor-market policies, such as subsidies intended to foster employment opportunities.

We consider a community that is populated by two types of homogeneous agents, corresponding to firms and to workers. Prior to entering the labor market, workers choose: (i) how much human capital to accumulate through (costly) schooling and (ii) the “occupation” (viz., work and crime) they intend to pursue. Workers are amoral in that – given identical pecuniary rewards – they are indifferent between committing crimes and pursuing formal employment. Human capital acquired through schooling raises each individual’s subsequent productivity upon employment. The interactions between all agents in the model are governed by random matching processes. Thus, in the formal labor market, firms with open vacancies and workers actively seeking jobs are brought together at random points in time through a stochastic matching technology. The flow rate of contacts depends upon the (endogenous) number of participants on each side of the market. Upon a successful match, the firm-worker pair negotiate a wage and production then takes place. Likewise, in the crime sector, consumers with goods held in inventory are randomly encountered with criminals who seek to appropriate them. In this setting, the outcome of a match is that either the criminal successfully steals his victim’s holdings of the consumption good or else is interdicted by the authorities, at a rate that depends upon, among other things, the resources devoted by the community to such efforts. In this latter event (at least in one variant of our model), the criminal is incarcerated and is subsequently released (paroled) at some random point in the future, depending primarily on government policies (including penal reform).

All agents are rational and forward looking. In making their entry decision, firms are influenced by average human capital in the community (which determines average worker productivity), wages, and the ease with which they can find suitable employees. Likewise, the level of human capital accumulated by each household (as well as its subsequent occupational choice) depends upon the aggregate circumstances that confront it in the community, including particularly criminal activities. The anticipated returns from formal employment depend upon the wage rate, the “tightness” of the labor market, and the crime rate, while the benefits of crime depend upon the ease of finding potential victims, the return that accrues from each theft (which, in turn, hinges on the average wage/income in the community), the likelihood of interdiction, and the sanctions imposed by the state in this latter event.

The model exhibits multiple steady-state equilibria, with high crime, low levels of educa-

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6 This latter decision is reversible ex ante, in the sense that at any point a worker can switch from formal employment to crime and vice versa (though this option is not exercised in the stationary steady-state).
tional attainment, long spells of unemployment, and poverty (low incomes) correlated across them. The multiplicity stems from three distinct positive feedback externalities that lead to greater levels of criminal activity increasing the marginal returns to crime at the individual level. First, for any given level of policing expenditures, an increase in the crime rate reduces the likelihood that any given criminal is interdicted by the authorities. In turn, this lowers the expected costs of committing crime (*the interdiction effect*).\(^7\) Secondly, an increase in the crime rate lowers the expected returns to formal employment, since workers anticipate that the fruits of their labor are more likely to be stolen by criminals (*the appropriation effect*).\(^8\) At the margin this encourages criminal behavior. Finally, crime acts, in essence, as an *indirect tax on human capital accumulation*, by reducing the value of any given level of schooling. As a consequence, low worker productivity discourages firms from entering the community, raising unemployment, and yielding more crime (*the human capital effect*).

One of the great merits of the search-equilibrium approach is the ease with which these various feedback channels can be identified separately and studied independently in a parsimonious manner. For instance, if police expenditures adjust (endogenously) to ensure a constant interdiction hazard rate and if the proceeds from crime are also subject to appropriation by criminals, neither the interdiction nor appropriation effects, described above, operate in the community. However, we show that despite this, the human capital effect alone is sufficient to generate multiple equilibria.

*Related Literature*

Some of the underlying strategic complementarities identified in this paper that stem from criminal activity have been studied elsewhere in the literature (although in quite different settings). Sah (1991) develops a model in which an increase in the number of criminals reduces the likelihood (for given enforcement expenditures) that any one criminal will be arrested. A similar externality is posited by Freeman et al. (1996). Our model not only admits a microfoundation that generates arrest probabilities *endogenously* but also allows us to perform experiments which maintain a constant per capita arrest hazard by endogenously adjusting the level of law enforcement expenditure. We demonstrate the possibility of multiple equilibria, even without the interdiction spillover effect identified by Sah.

Glaeser et al. (1996) consider a ‘random-graph’ framework, in which each agent in the community is conceived of as point on a lattice. Individual behavior is driven largely through peer group effects: certain agents imitate the behavior of their nearest neighbor (which is

\(^7\)A similar externality is studied by Ranj Sah (1991) and Freeman, Grogger, and Sonstelie (1996) under very different economic structures.

\(^8\)Murphy, Shleifer, and Vishny (1993) isolate a similar effect, but in a model of corruption.
either legitimate or malfeasant). They show a distribution of (Nash) equilibria may exist. This feature offers valuable insights into the variance of crime rates observed over both time and space. Although both random-graph and search-equilibrium theories may be connected at a somewhat deeper level, for many applications search theory has the decisive advantage of tractability. Moreover, by viewing search as taking place within a given community, the search-equilibrium approach offers a parsimonious framework for studying both local informational spillovers and *global* interactions (such as aggregate labor-market activity or the effects of changes in police expenditures). As Glaeser et al. note, this remains a desideratum, to their knowledge.

Murphy et al. (1993) explore a different type of externality. In their model, an increase in the level of corruption increases the relative returns to such activities, since the returns from legal activities are appropriated. This complimentarity generates multiple equilibria, with high income/low level of corruption and low income/high level of corruption configurations. Although our model also possesses a similar feature of multiple equilibria (by viewing corruption and crime as comparable), in addition we study the effects of criminal activity on human capital accumulation, wages, and the level of economic activity. Moreover, we study the possibility that the proceeds from crime may also be stolen – thieves are both *hunters* and *the hunted*. In this case, although the externality posited by Murphy et al. is absent, we still establish the possibility of multiple equilibria.

Our paper is also related to some recent and independent work by Lochner (1999), İmrohoroğlu, Merlo, and Rupert (2000a,b) and Burdett, Lagos, and Wright (2001). Lochner constructs a simple two-period life-cycle model of crime and educational choices. His main concern is exploring the effect of labor-market conditions upon crime and educational choices. Thus, in his model the crime rate neither affects the value of opting for formal employment nor engaging in nefarious criminal activities. Furthermore, an increase in the wage rate automatically increases the opportunity cost of crime. No allowance is made for the possibility that an increase in (formal) workers’ income levels might also increase the *value* of theft.

İmrohoroğlu et al. (2000a) construct a dynamic general competitive equilibrium model of crime and labor supply. They quantitatively evaluate the main determinants of the observed change in the crime rate between 1980 and 1996. However, in their baseline model, worker skills are assumed to be exogenous. It follows their model cannot be used to address the issues considered in this paper. İmrohoroğlu et al. (2000b) construct a political-economy model to study the effects of inequality, redistribution, and police expenditures on the crime rate. The search-equilibrium framework considered here can – and indeed is – extended to deal with such policy related issues. Yet, the main focus of this paper is explicating the links
that exist between criminal activity and investments in human capital, on which their papers are silent.

Perhaps the closest paper in spirit to this one is the search-equilibrium model due to Burdett, Lagos, and Wright. (2001). Just as in this paper, they consider an environment characterized by labor-market search and by the random interaction between criminals and formal workers. However, despite this similarity the focus of our two papers is quite different. In their model, all workers and firms are equally productive (there is no scope for workers to acquire human capital). An equilibrium wage distribution arises in their model, in which some firms use high wage payments to reduce costly labor turnover. \(^9\) Workers employed at high-wage firms prefer not committing (property) crimes, because they have too much to lose if they are apprehended, incarcerated, and as a result lose their jobs. It is, however, optimal for workers employed at low-wage firms to commit crimes when they have the opportunity to do so. \(^10\) Their paper examine the relationship between the distribution of wages and the level of crime. In our paper, an endogenous distribution of *accumulated human capital* arises in equilibrium, as opposed to an endogenous distribution of wages as in Burdett et al. The key property of our equilibrium is that those workers who *acquire skills* become formal workers (and do not commit crimes), while those who do not become criminals. Thus, our framework may be viewed as an “occupational choice” model of crime that focuses on *internal motivations* rather than *external incentives*, to use Rasmusen’s (1996) terminology.

2 The Model

We construct a continuous-time model of search and matching. The community is populated by two distinct risk-neutral economic units, corresponding to households and firms (or job vacancies), who discount the future at the rate \(r = \delta + r_1 > 0\), where \(\delta\) and \(r_1\) are the (common) death and time preference rates. Households are also born at the constant flow rate, \(\delta\), giving a constant steady-state population, of unity. There is a single perfectly divisible good with a normalized price of unity. The good is storable in unlimited quantity and at zero cost. Before providing a detailed description of the model, we first offer a general overview of its main features, focusing on three theaters of economic activity.

\(^{(i)}\) *The educational sector:* Prior to making their occupational choice households accumu-

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\(^9\) In equilibrium, both high- and low-wage firms expect to earn zero profits.

\(^{10}\) This formulation is similar in spirit to the employee crime model of Dickens et al. (1989). There firms pay “efficiency wages” to deter malfeasant behavior.
late human capital, increasing their productivity and wage income upon formal employment. Although all households are *ex ante* identical, their educational attainment levels may differ *ex post*. Crucially, educational choices depend upon the subsequent “career” (viz., formal employment or crime) households intend to pursue. Only formal workers choose more than the minimal level of education, as human capital is productive only in this sector.

(ii) *The primary labor market:* In this market job seekers and vacancies are brought together through a stochastic matching technology. All matches are transient and wages are determined in accordance with a (symmetric) Nash bargaining protocol. As soon as the wage is agreed, firms search for new trading partners and workers enter a consumption state, awaiting just the right moment to enjoy the fruits of their labor (this is governed by a Poisson process with parameter $\lambda$). Immediately after consumption, workers search for further job opportunities.

(iii) *The criminal sector:* This sector is populated by thieves who use their labor endowment to prey upon and to steal the goods currently held in inventory by other agents (including possibly other criminals). After a successful robbery, these agents too await the appropriate opportunity to consume and to derive utility from their illicit gains (also at the rate $\lambda$). After consuming their criminal proceeds, agents in this market then immediately search for new victims. We now formally describe the model.

2.1 **Firms**

Each firm possesses a fixed-coefficient’s technology that employs the labor services of one worker at a time. Firms maximize the present value of their expected net revenues. Through a suitable choice of units, the output (revenue) produced by a worker, with human capital $s$, is, $y = y_0 + s$, where $y_0 > 0$ captures the effects of exogenous productivity changes. We further assume that all worker-firm matches are transient and, upon matching, the wage, $w$, is agreed between the two parties according to a symmetric Nash bargain.\(^{11}\)

\(^{11}\)As we shall see these assumptions, in conjunction with those pertaining to worker preferences, lead to a simple (discrete) wealth distribution. We conjecture that extended employment with continuous flow wages and theft (at random intervals) could be modelled using the methods set out in Diamond and Yellin (1990). However, in this paper we are not interested in the properties of long-term employment contracts per se nor the effects of the distribution of wealth on crime.
The mass of firms in the economy is \( V \). There is unrestricted entry, in the sense that any number of firms can instantly enter the labor market and search for workers, after incurring a fixed-entry fee, \( v_0 > 0 \). This reflects unit capital costs as well as the costs of advertising the vacancy. Improvements in the organization of financial markets that reduce finance costs or alternatively tax incentives geared toward promoting investment lower \( v_0 \). Throughout we assume \( y_0 > r v_0 \), ensuring a non-trivial economic problem.

2.2 Households

Each household is endowed with an indivisible unit of labor that may be supplied inelastically without disutility from effort. Immediately after they are born, households can accumulate human capital \( s \) instantaneously by exerting schooling effort.\(^{12}\) The cost of acquiring human capital is \( g = g(s) \), where \( g' > 0 \), \( g'' > 0 \), and \( g(0) = 0 \). Defining the inverse elasticity \( \varepsilon(s) \equiv g/(sg') \), we assume \( 0 < \varepsilon(s) \leq \bar{\varepsilon} < 1 \) and \( \varepsilon' \geq 0 \) for all \( s \). The restriction on the inverse elasticity offers a simple sufficient condition for rendering certain comparative-static exercises, considered later, determinate. Obviously, the restriction is trivially satisfied in the constant elasticity case: \( g = s^{1/\bar{\varepsilon}} \) and \( \varepsilon \in (0,1) \). Further we assume that workers acquire human capital only through formal education, ruling out, for example, on-the-job learning.

The division of the \( \text{ex ante} \) identical (unit) population between formal employment and crime is an optimal \textit{occupational choice} (with populations determined in equilibrium and households free to switch between sectors).\(^{13}\) After acquiring human capital, households choose between formal employment and crime. Let \( N \) denote the mass of households in the primary labor market and let \( 1 - N \) denote the mass in the criminal sector. Each household (whether a legitimate worker or a criminal) is in either the consumption or acquisition state. Let, \( C \) and \( Q \) denote, respectively, the masses of households in the primary sector and in the criminal sector who are in the consumption state. Likewise, let \( E \) and \( R \) represent the masses in the acquisition state in the primary and criminal sectors. Thus, we have the

\(^{12}\)It is ready to include a time element by discounting each agent’s payoff according to the length of the schooling period.

\(^{13}\)In principle, it is possible to allow households to differ intrinsically in their abilities (both productive and stealing) and/or in their morality. In the present paper we prefer not to impose such \( \text{ex ante} \) heterogeneity. Instead, we seek to show that even with identical \( \text{ex ante} \) agents, we can obtain non-degenerate equilibria in which agents differ \( \text{ex post} \) (in particular their are sets of positive measure of criminals and formal workers).
following population identities:

\[ N \equiv C + E \quad (1) \]

\[ 1 - N \equiv Q + R \quad (2) \]

In order to ensure that, on the one hand, workers have something “saved” that criminals can actually steal and, on the other hand, to sidestep the difficult distributional issues (that generically arise in search equilibrium models that allow for storage) we adopt the following preference structure. At each point in time, households use their (indivisible) unit of labor for either “productive” or “felicity” purposes. In the former case we say that the household occupies the “acquisition state” (seeking goods to hold in inventory and to ultimately consume), while in the latter case it occupies the “consumption state” (seeking to derive utility by consuming goods currently held). In the acquisition state households use their unit labor endowment to augment their goods’ holdings through either formal employment or through theft. Households in the consumption state use their time to wait for a judicious moment to consume and to derive utility from goods they currently hold in inventory.

Instantaneous utility is given by, \( u = c \), where \( c \) is the level of consumption. Utility opportunities arise at random points in time, which are governed by a Poisson process with parameter \( \lambda \), where \( 0 < \lambda < r \). This formulation implies that if a household holds any goods in inventory it: \((i)\) optimally enters the consumption state and \((ii)\) consumes all of its inventory when the first opportunity to do so presents itself.\(^{14}\) Although households can, at any point in time, costlessly and instantaneously switch back and forth between the acquisition and consumption states they do so only if they successfully acquire goods, consume, or else become the victims of crime.

### 3 Matching, State-Transition and Asset Values

In this Section we describe the matching processes in the formal labor market and in the crime sector. These conditions are used to derive key equations governing population dynamics, as

\(^{14}\)Part \((i)\) is explained as follows. Suppose that a household unit holds some goods in inventory (if it does not it must enter the acquisition state). The household can either seek additional goods or else enter the consumption state. With linear utility and zero storage costs if the former choice were optimal, then the permanent deferral of consumption constitutes an optimal plan (since once additional goods are acquired the household faces the same decision problem). Yet, with \( r > \lambda > 0 \), the household can do better by entering the consumption state (where it almost surely attains positive utility in finite time). Part \((ii)\) is trivial, following from the assumption that utility is linear in consumption and from positive discounting, \( r > 0 \).
agents make the transition between the acquisition and consumption states. Then, we write down the asset values pertaining to households and firms. To facilitate the exposition, we assume, for the moment, that criminals are not subject to interdiction by the authorities. This extension is incorporated in Section 6 below.

3.1 Formal Labor Matching

Let \( \mu \) denote the flow rate at which workers locate jobs and \( \eta \) represent the corresponding rate at which firms locate suitable workers. Although the flow probabilities \( \mu \) and \( \eta \) are determined in equilibrium, each agent treats them as parametric in making his/her decisions. Since each job is filled by one and only one worker, it follows that:

\[
\mu E = \eta V = m_0 M(E, V)
\]

(3)

where \( m_0 > 0 \) and \( M(\cdot) \) is the matching technology governing the flow rate of contacts between the two sides of the labor market. We assume that \( M(\cdot) \) is a strictly increasing, concave, and constant-returns-to-scale function of \( E \) and \( V \), satisfying (i) the Inada conditions \( \lim_{j \to 0} M_j = \infty \) and \( \lim_{j \to \infty} M_j = 0 \), with \( j \in \{E, V\} \) and (ii) the boundary conditions \( M(0, V) = M(E, 0) = 0 \). The term \( m_0 \) parameterizes the matching efficacy in the labor market. For instance the establishment of a job placement center in the community or, alternatively, an improvement in communications or transportations infrastructure might be expected to increase \( m \) and to raise the flow contact rate for any given population of workers \( (E) \) and vacancies \( (V) \). The properties of the function \( M(\cdot) \) ensure the existence of a well-behaved hyperbolic Beveridge curve in \( (\mu, \eta) \) space, in which the absence of either side of the market results in zero matches. The constant-returns to scale assumption is made for simplicity (our results hold for any well-behaved quasi-concave function of \( E \) and \( V \)).

3.2 Criminal Activity

At any given moment a total of \( C+Q \) households occupy the consumption state. Let \( \alpha \) denote the flow probability with which members of \( C \) become crime victims and let \( \theta \alpha \) denote the corresponding rate for criminals \( (Q) \). Here, \( \theta \) captures the relative ease \( (\theta < 1) \) or difficulty \( (\theta > 1) \) with which criminals can protect their wealth holdings from appropriation by other criminals. We assume that criminals steal all of the goods held in their victim’s inventory, but derive utility at the discounted rate \( \phi \in (0, 1] \). If \( \phi = 1 \), they derive the full benefit from consumption, whereas \( \phi < 1 \) indicates strict discounting (for instance, criminals may incur a real resource cost in ‘fencing’ stolen property). Active criminals, \( R \), encounter victims at the
flow rate $\beta$. Agents treat the matching probabilities $\alpha$ and $\beta$ as parametric in formulating their optimal decisions.

Behavior in the criminal sector is governed by a simple matching technology in which the flow crime rate depends upon the mass of active criminals ($R$) as well as the (effective) mass of potential victims ($C + \theta Q$). Consider,

$$\alpha(C + \theta Q) = \beta R = \beta_0 B(C + \theta Q, R) = \beta_0 R$$

(4)

By definition, since each crime has a victim, the flow robbery rate ($\beta R$) must equal the flow crime rate ($\alpha(C + \theta Q)$), which explains the first equality in (4). The function $B(\cdot)$ is the matching technology, governing the contact rate between robbers and victims, while the parameter $\beta_0$ captures (exogenous) aspects of the urban environment, such as the quality of street lighting, policing, population density etc., that render crime more or less difficult to commit. As in the case of the labor-market matching technology, $M(\cdot)$, we assume that the function $B(\cdot)$ is homogeneous of degree one. Additionally, we assume, for simplicity, that the flow crime rate depends upon the measure of active robbers, $R$, alone.\footnote{As explained below, this feature implies that the steady-state equilibrium is amenable to a simple graphical analysis. Moreover, consider the matching technology: $B(\cdot) = b_0 B_1(R, C + \theta Q) + R$, where $B_1(\cdot)$ and hence $B(\cdot)$ satisfies all of the conditions imposed on $M(\cdot)$. Our analytic results hold generically for a set of values of $b_0 > 0$ with strictly positive measure.}

Note that, even though this formulation implies the contact rate is pinned down at $\beta_0$, the returns to crime, $\phi w$, are endogenous, depending upon, among other things, the conditions that prevail in the labor market.

### 3.3 Population Dynamics

In the primary labor market, workers acquire jobs and move into the consumption state at the flow rate $\mu$. Workers move out of this state either by successfully consuming the fruits of their labor ($\lambda$), death ($\delta$) or else after becoming the victims of crime ($\alpha$). Similarly, criminals enter the consumption state $Q$ after successfully committing a robbery ($\beta$). They exit the state $Q$ either by consuming ($\lambda$), death ($\delta$), or else by themselves becoming the victim of crime ($\theta \alpha$). These considerations yield,

$$\dot{C} = \mu E - (\lambda + \alpha + \delta)C$$

(5)

$$\dot{Q} = \beta R - (\lambda + \alpha \theta + \delta)Q$$

(6)

In the steady state, we must have: $\dot{C} = \dot{Q} = 0$.\footnote{As explained below, this feature implies that the steady-state equilibrium is amenable to a simple graphical analysis. Moreover, consider the matching technology: $B(\cdot) = b_0 B_1(R, C + \theta Q) + R$, where $B_1(\cdot)$ and hence $B(\cdot)$ satisfies all of the conditions imposed on $M(\cdot)$. Our analytic results hold generically for a set of values of $b_0 > 0$ with strictly positive measure.}
3.4 Asset Values

In order to study agents’ optimal choices and the wage agreement, it is first necessary describe the asset values of households, \( J \), and of firms, \( \Pi \). At any given point in time, each household occupies one of four states: \( C \), \( E \), \( R \) and \( Q \) and each vacancy is either filled (\( F \)) or unfilled (\( V \)). Denote the wage income accruing to households from employment by \( w \). Consider,

\[
egin{align*}
    r_{JE} &= \mu(J_C - J_E) \quad (7) \\
    r_{JC} &= \lambda w + (\lambda + \alpha)(J_E - J_C) \quad (8) \\
    r_{JR} &= \beta(J_Q - J_C) \quad (9) \\
    r_{JQ} &= \phi \lambda w + (\lambda + \theta \alpha)(J_R - J_Q) \quad (10) \\
    \Pi_F &= y_0 + s - w + \Pi_V \quad (11) \\
    r\Pi_V &= \eta(\Pi_F - \Pi_V) \quad (12)
\end{align*}
\]

These asset values are standard and intuitive. For instance (7) says that the flow value from job search, \( r_{JE} \), equals the flow probability of locating employment (\( \mu \)) times the capital gain the results from entering the consumption state (\( J_C - J_E \)). In equation (8) the flow value from occupying the consumption state, \( r_{JC} \), is made up of two terms: the flow utility derived from consumption, \( \lambda w \), and the flow probability of a change in state (\( \lambda + \alpha \)) times the capital loss from that event (\( J_E - J_C \)). Notice that households anticipate that they may re-enter the job-search state, \( J_E \), either because they successfully derive utility from their inventory (\( \lambda \)) or else because their goods are stolen (\( \alpha \)). Similar interpretations hold for (9) and (10), with the proviso that criminals’ payoffs are adjusted by \( \theta \) and \( \phi \), reflecting the possibility that they may be less or more subject to crime than other members of the community (\( \theta < \) or \( > 1 \)) and that they may incur a real resource cost in converting their spoils to goods suitable for consumption (\( \phi < 1 \)). In equation (11) the value of locating a worker is the value of instantaneous profits \( y_0 + s - w \) plus the value of the open vacancy, \( \Pi_V \). Finally, the flow value of opening a vacancy equals the flow probability of locating workers (\( \eta \)) times the capital gain from this event: \( \Pi_F - \Pi_V = y_0 + s - w \).

4 Bargaining, Schooling and Occupational Choice

In this Section we characterize the (partial-equilibrium) properties of wages, schooling levels, and occupational choices, as well as the steady-state matching and entry conditions. Throughout this section, we set: \( \theta = \phi = 1 \), indicating that criminals and workers are equally likely to be the victims of crime (\( \theta \)) and that the utility derived from stolen goods is not less than
that derived from legitimate ones ($\phi$). Toward the end of section 5, we analyze the effects of values of $\theta$ and $\phi$ that differ from unity.\footnote{Note that, at this point, neither the appropriation externality identified by Murphy, Schleifer and Vishny (1993) nor the interdiction effect studied by Sah (1991) operate, since, respectively, $\theta = 1$ (criminals are just as likely to be the victims of crime) and law enforcement is absent.}

### 4.1 Wage Determination

Upon matching, firms and workers bargain over the division of the surplus. We assume that negotiations are instantaneous and that the wage, $w$, is determined according to a symmetric Nash bargaining protocol.\footnote{Under a symmetric Nash bargain, the surplus (and hence the wage) depends upon the outside options $J_E$ and $J_C$ which, in turn, depend upon the conditions that prevail in the labor market.} The value to workers from accepting employment is: $J_C - J_E$ and the value to employers from (instantaneous) production is: $\Pi_F - \Pi_V (= y_0 + s - w)$. Under a symmetric Nash bargain, these two are equalized:

\[
J_C - J_E = y_0 + s - w > 0 \tag{13}
\]

Substituting the asset values (8), (7), (11), and (12) into (13) yields the wage offer agreed between workers and firms. Defining $\chi(\alpha) \equiv \lambda + \mu + \alpha$, we have:

**Lemma 1. (The Wage)** Under symmetric Nash bargaining between firms and workers, the wage offer is:

\[
w(s, \mu, \alpha; \lambda, y_0) = \frac{\chi(\alpha) + \mu}{\lambda + \chi(\alpha) + \mu}(y_0 + s) \tag{14}
\]

which satisfies: $\partial w/\partial s > 0$, $\partial w/\partial \mu > 0$, $\partial w/\partial \alpha > 0$, and $\partial w/\partial y_0 > 0$.

**Proof:** All proofs are in the Appendix.

An increase in $y_0$ or $s$ enlarges the ex post surplus accruing to any given match, which is shared between the two bargaining parties, thus leading to a higher wage. An increase in the worker contact rate, $\mu$, enhances workers’ bargaining power and hence raises $w$, by making alternative employment opportunities more readily available in the event of disagreement. An increase in the crime rate $\alpha$ raises $w$ by reducing the total surplus accruing to worker-firm matches: $(J_C + \Pi_F) - (J_E + \Pi_V)$. In bargaining the loss of surplus is shared by both parties, so that both $(J_C - J_E)$ and $(y_0 + s - w)$ fall. This latter event is accomplished through an increase in $w$. Although the gross wage $w$ rises with $\alpha$, the (expected) net-of-crime wage declines.

\[
\text{16 Note that, at this point, neither the appropriation externality identified by Murphy, Schleifer and Vishny (1993) nor the interdiction effect studied by Sah (1991) operate, since, respectively, } \theta = 1 \text{ (criminals are just as likely to be the victims of crime) and law enforcement is absent.}
\]

\[
\text{17 Under a symmetric Nash bargain, the surplus (and hence the wage) depends upon the outside options } J_E \text{ and } J_C \text{ which, in turn, depend upon the conditions that prevail in the labor market.}
\]
4.2 Educational Choices

Households that opt for formal employment recognize that education increases the output accruing to each match. Taking all contact rates as given, these agents solve: \( \max_S \{ J_E - g(s) \} \), where the wage is governed by (14). The first-order necessary (and sufficient) condition for a maximum is,

\[
\frac{\lambda \mu}{r[\lambda + \chi(\alpha) + \mu]} = g'(s)
\]

(15)

Straightforward manipulation of (15) yields:

Lemma 2. (Schooling Effort) The schooling effort function for primary-sector workers, \( s(\mu, \alpha) \), satisfies:

(i) \( \lim_{\mu \to 0} s = 0 \) and \( \lim_{\mu \to \infty} s = \frac{\lambda}{\alpha} \) where \( \frac{\lambda}{\alpha} \equiv g'(\lambda/r)^{-1} \);  
(ii) \( \lim_{\alpha \to 0} s < \infty \) and \( \lim_{\alpha \to \infty} s = 0 \);  
(iii) \( \partial s / \partial \mu > 0 \), \( \partial s / \partial \alpha < 0 \) and criminals optimally set \( s = 0 \).

The results in Lemma 2 are intuitive. If either \( \mu \to 0 \) or if \( \alpha \to \infty \), then costly education has no value, so that its level is optimally set to zero. With \( g(\cdot) \) strictly convex, \( s \) is bounded above regardless of how high (low) the value taken by \( \mu(\alpha) \). An increase in the arrival rate of job opportunities, \( \mu \), raises the marginal returns to education and hence the level chosen by workers. In essence, crime, \( \alpha \), acts as a tax on human capital accumulation by reducing the returns accruing to formal employment and subsequently the levels of educational attainment. As \( \alpha \to \infty \), the tax on human capital accumulation completely eliminates schooling incentive and hence \( s = g(s) = 0 \). Finally, criminals set \( s = 0 \), as education is costly and has no (normalized) direct benefit.

4.3 Steady-State Matching, Entry, and Populations

The two steady-state matching conditions (3) and (4) yield well-behaved Beveridge curves in contact-rate space in the labor market, \( SS^M \), and in the criminal sector, \( SS^B \):

Lemma 3. (Steady-State Matching) The Beveridge curves \( SS^M \) and \( SS^B \) take the following form,

\[
\eta \equiv \eta^{SS}(\mu; m_0)
\]

(16)

\[
\beta = \beta_0
\]

(17)

and satisfy, \( \partial \eta / \partial \mu < 0 \), \( \partial \eta / \partial m_0 > 0 \), and \( \partial \beta / \partial \beta_0 = 1 \).

The SS locus is downward sloping in \( (\mu, \eta) \) space and, under the Inada conditions, asymptotes at each axis. An improvement in the matching efficacy, \( m_0 \), shifts this locus out. As indicated
previously, the value of $\beta$ is uniquely pinned down by $\beta = \beta_0$, implying the $SS^B$ locus is horizontal (in $(\alpha, \beta)$ space) at this value.

The final steady-state condition determines the mass of vacancies, $V$. This must be consistent with the equilibrium firm-entry (EE) condition: $\Pi_V = (\eta/r)(y - w) = v_0$. Thus,

Lemma 4. (Equilibrium Entry) The equilibrium firm-entry locus EE is given by,

$$\eta = \frac{\eta_0}{\lambda} + \chi(\alpha) + \mu \frac{\lambda}{y_0 + s} \equiv \eta^{EE}(s, \mu, \alpha; y_0, v_0)$$

which possesses the following properties: $\partial \eta/\partial s < 0$, $\partial \eta/\partial \mu > 0$, $\partial \eta/\partial \alpha > 0$, $\partial \eta/\partial y_0 < 0$ and $\partial \eta/\partial v_0 > 0$.

Along the EE locus the expected discounted value of ex ante profits is exactly zero. An increase in the worker contact rate, $\mu$, raises the wage (lemma 1) and lowers ex post profits. Heuristically, zero ex ante profits are restored through an increase in the vacancy contact rate, $\eta$, making it easier (less time consuming) for firms to find workers. The other results are explained in a similar fashion.

The EE locus depends upon the three endogenous variables – the schooling level, $s$, and two contact rates, $\mu$ and $\alpha$. Although our ultimate goal is exploring the general-equilibrium properties of the system, it is helpful to first characterize the properties of the EE locus, admitting only endogenous changes in $s$:

$$d\eta^{EE}/d\mu = \{\partial \eta/\partial \mu\} + \{(\partial \eta/\partial s) \cdot (\partial s/\partial \mu)\}$$

Equation (19) decomposes the total effect of a change in $\mu$ upon $\eta^{EE}$ into the two indicated components. The direct effect of an increase in $\mu$ is to raise the wage and hence $\eta$, while the indirect human capital effect raises $s$ and thus lowers $\eta$. These conflicting effects suggest the EE locus may not be monotonic in $\mu$.

Lemma 5. (Non-Monotonic EE locus) For given $\alpha$, the EE locus satisfies limiting properties, $\lim_{\mu \to 0} \eta^{EE} = \eta_0 > 0$ and $\lim_{\mu \to \infty} \eta^{EE} = \infty$, and may possess decreasing segments in $(\mu, \eta)$ space.

For very small values of $\mu$ workers undertake little schooling, as the returns to education are low. As a consequence the work force is relatively low skilled, so that $\eta$ must be sufficiently large to make entry worthwhile for firms. The situation is quite different for very large values of $\mu$. Here education and worker productivity levels are high. Yet, with an extremely tight labor market, workers capture most of the rents from bargaining with firms (the wage, $w$, approaches $y$). In order to cover the ex ante entry cost the contact rate, $\eta$, must be
correspondingly large. An increase in $\mu$ has two separate effects: it raises the wage (Lemma 1) and the level of education and productivity of workers (Lemma 2). The former of these effects tends to raise $\eta$, by reducing the instantaneous profits from each successful match, whereas the latter effect tends to lower $\eta$ for analogous (but opposite) reasons. For values of $\mu$ not too large, the education effect may dominate and the EE locus may possess decreasing (in $\mu$) segments.\footnote{This claim is not simply “pie in the sky,” but rather can be demonstrated, by example, through the use of rather elementary simulations. Note also that as $\mu \to \infty$, $\eta \to (rv_0)(\lambda + \chi)/y_0$.} For sufficiently high values of $\mu$, an increase in $\mu$ has little effect on productivity (since $s$ approaches the upper bound $\bar{s}$, defined by $g'(\bar{s}) \equiv (\lambda/r) - \text{Lemma 2}$), but raises the wage. However, as $\mu \to \infty$, the EE locus converges asymptotically to a ray with a positive slope, $(rv_0)/(y_0 + \bar{s})$.

4.4 Occupational Choice

We now examine the occupational choices (viz. formal employment and crime) made by households. Since all households are ex ante identical the following no-arbitrage condition must hold if both the formal labor market and the criminal sectors are to coexist in the steady state:

$$\overline{J}_E \equiv J_E - g(s(\mu, \alpha)) = J_R \quad \text{(20)}$$

where $s(\mu, \alpha)$ is determined in accordance with Lemma 2. Condition (20) says that households are just indifferent between, on the one hand, acquiring the (optimal) education level and entering the primary labor market and, on the other, choosing the minimum level of education and engaging in crime related activities. Note that since formal employment entails costly ex ante education, equation (20) is satisfied only if $\mu \geq \beta_0$ ($\mu < \beta_0$ if robbers’ ex post utility levels exceed that of job searchers). Equation (20) gives the value of $\mu$ for which households are indifferent ex ante, between crime and formal employment. Define $\zeta(s) \equiv \varepsilon(s)s/y < \bar{s}$.

We can then use equations (7)-(10), (14)-(15) and (20) and the definition of $\varepsilon(s)$ to obtain:

$$\mu = \frac{\beta_0 \chi(\alpha)}{\chi(\alpha) - [\beta_0 + \chi(\alpha)]\zeta(s(\mu, \alpha))} \quad \text{(21)}$$

which implicitly defines an occupational choice (OC) locus, $\mu = \mu^{OC}(\alpha; \beta_0, y_0)$. Consider:\footnote{Recall that $\varepsilon_0 < 1$ is the upper bound on the inverse elasticity $g'()$.}

Condition 1. $(r + \lambda)(1 - \bar{s})/\bar{s} > \beta_0$.

Lemma 6 characterizes the OC locus,

Lemma 6. (Occupational Choice). Under Condition 1, the no-arbitrage condition (20) implicitly defines a function $\mu = \mu^{OC}(\alpha, \beta; y_0)$, for which: $\overline{J}_E \equiv J_R$, which satisfies:
(i) \( \lim_{\alpha \to 0} \mu^{OC} = \pi < \infty \) and \( \lim_{\alpha \to \infty} \mu^{OC} = \beta_0 \);
(ii) \( d\mu^{OC}/d\alpha < 0, \ d\mu^{OC}/d\beta_0 < 0, \ d\mu^{OC}/dy_0 < 0. \)

Condition 1 ensures that \( \mu^{OC} (\cdot) \) is well-defined and non-negative for all values of \( \alpha \geq 0. \) Intuitively, if \( \beta_0 \) is too high, then no (non-negative) value of \( \alpha \) exists that ensures equality in (20). The limiting properties are explained by noting that, as \( \alpha \to 0, \) then education approaches the upper bound \( \bar{\pi}(\mu) \) (Lemma 2). In this case, \( \mu \) is, by inspection, positive and finite. In contrast, as \( \alpha \to \infty \) then \( s = g(s) = 0 \) as education has no value. With \( g(\cdot) = 0, \) then \( \overline{J} = J_R \) only if \( \mu = \beta_0. \)

The comparative-static results may be understood as follows. Write the ex ante surplus from formal employment as, \( G(\cdot) \equiv \overline{J} - J_R = 0. \) Consider \( s = s(\alpha, \mu') \) where \( \mu' \) is harmlessly (from an expositional perspective) treated as given. Since the OC locus, \( \mu = \mu^{OC}(\alpha; \beta_0, y_0), \) is indeed defined by the fixed point \( \mu = \mu' \) according to equation (21) (such a point always exists under Condition 1). Totally differentiating \( G(\cdot) \) gives:
\[
G_\mu d\mu + G_\alpha d\alpha + G_s ds + G_{\beta_0} d\beta_0 + G_{y_0} dy_0 = 0
\]
where the subscripts on \( G \) connote partial derivatives. An increase in \( \mu \) raises the wage and hence both \( \overline{J} \) and \( J_R \) (the increase in \( w \) implies there is more available for criminals to steal from any given household). However, as \( \mu \) rises formal workers also contact jobs at a faster rate. This gives, \( G_\mu > 0. \) Since \( \mu > \beta_0, \) criminals are hurt more – all else equal – by an increase in the crime rate \( \alpha \) than are formal workers, implying: \( G_\alpha > 0. \) An increase in \( s \) has, via an application of the envelope Theorem, no first-order effect on \( \overline{J}; \) it does, however, raise output, wage income, and, consequently, \( J_R. \) It follows that \( G_s < 0. \) Now with \( \partial s / \partial \alpha < 0 \) (Lemma 2), then both the direct, \( G_\alpha > 0, \) and indirect, \( (G_s)(\partial s / \partial \alpha) > 0, \) effects of an increase in \( \alpha \) work in tandem, so that \( dG / d\alpha > 0, \) and, as a result, \( d\mu^{OC} / d\alpha < 0. \)

An increase in the predation rate, \( \beta_0, \) raises \( J_R \) with no first-order effect on \( \overline{J}; \) implying: \( G_{\beta_0} > 0. \) This explains why \( d\mu^{OC} / d\beta_0 > 0. \) An increase in productivity, \( y, \) benefits formal workers more than criminals \( (G_{y_0} > 0), \) which explains the last result.

In the remainder of the formal analysis it is convenient to work with the inverse function: \( \alpha = \alpha^{OC}(\mu; \beta_0, y_0), \) defined by \( \mu \equiv \mu^{OC}(\alpha^{OC} (\cdot); \beta_0, y_0), \) on the domain \( \mu \in (\beta_0, \overline{\pi}]; \) and by \( \alpha \equiv 0 \) for all \( \mu > \overline{\pi}. \) Applying Lemma 6 gives the limiting properties: \( \lim_{\mu \to \beta_0} \alpha^{OC} > 0 \) and \( \alpha^{OC}(\overline{\pi}, \cdot) = 0 \) as well as the derivatives: \( d\alpha / d\mu < 0, \ d\alpha / d\beta_0 > 0, \) and \( d\alpha / dy_0 < 0. \) Points above (below) \( \alpha^{OC} \) correspond to \( G > 0 \) (\( < 0 \)).

\(^{20}\)To see this consider the extreme, \( \mu \to \infty. \) Here, the employment state is transient and crime, \( \alpha, \) consequently has no effect on formal workers’ ex ante utility. Yet, with \( \beta \) finite, an increase in \( \alpha \) (given \( \theta = 1 \) would lower criminals’ ex ante utility. 17
5 Steady-State Equilibrium

From equations (1), (2), (4), (5) and (6), we can write the steady-state populations of $E$ and $Q$ (and $C$ and $R$) as functions of the contact rates: $\mu$, $\alpha$, and $\beta$ and the population $N$:

$$E = \frac{\beta(\delta + \lambda)}{\alpha \mu + \beta(\delta + \lambda + \mu)}$$ (22)

$$R = \frac{\alpha \mu}{\alpha \mu + \beta(\delta + \lambda + \mu)}$$ (23)

$$N = \frac{\beta \mu(\delta + \lambda)(\delta + \lambda + \mu + \alpha)}{(\delta + \lambda + \alpha)[\alpha \mu + \beta(\delta + \lambda + \mu)]}$$ (24)

where from the population identities, (1) and (2), $C$ and $Q$ are then determined. Define:

**Definition 1.** (Steady-State Equilibrium) A steady-state equilibrium is a tuple of population masses, a wage rate, an education level, and contact rates, $\nu \equiv \{(E^*, C^*, Q^*, R^*), (s^*, w^*), (\mu^*, \eta^*), (\alpha^*, \beta^*)\}$ satisfying:

(i) the population identities (1) and (2);

(ii) Lemmas 1 and 2 that determine the wage, $w$, and education, $s$;

(iii) the steady-state matching conditions (Lemma 3);

(iv) the equilibrium entry (EE) locus (Lemma 4);

(v) the occupational choice (OC) locus (Lemma 6).

The equilibrium is called non-degenerate if $s^* > 0$ and $0 < N^* < 1$ (i.e., if education is strictly positive and if there is a positive measure of agents in each occupation).

5.1 Existence and Characterization of Equilibrium

The model possesses a convenient recursive structure that helps delimit the conditions under which steady-state equilibrium exists and characterize its resultant properties. Substituting the inverse OC locus and the optimal education schedule into the EE locus (18) gives,

$$\eta = \eta^{EE}(s(\mu, \alpha^{OC}(\mu; \alpha^{OC}(\mu; \cdot)), \mu, \alpha^{OC}(\mu; \cdot)) \equiv \eta(\mu; \cdot)$$ (25)

which is solely a function of one endogenous variable, $\mu$, defined on the domain $\mu > \beta_0$ (noting that for $\mu \geq \bar{\mu}$, $\alpha = 0$). By construction, equation (25) embodies the behavior described in Lemmas 1, 2, 4, and 6 and equation (17) in Lemma 3. Equation (25) together with the SS locus (16), form a two-by-two system in $(\mu, \eta)$ space, whose solution – if it exists – yields steady-state values for $\mu^*$ and $\eta^*$. The other endogenous variables are recovered recursively. Specifically, $\mu^*$ gives: $\alpha^* = \alpha^{OC}(\mu^*; \beta_0; \cdot)$. The populations: $E^*, Q^*, C^*, R^*$ and $V^*$ are then
derived from $\mu^*, \eta^*, \alpha^*$ and $\beta^*$ (as are $w^*$ and $s^*$). Finally, simple substitution gives the asset values, $J^*$, for formal workers and for criminals. Consider,

**Condition 2.** $m_0 > \bar{m}_0$, where $\bar{m}_0$ solves $\eta^{SS}(\beta_0; \bar{m}_0) = \frac{\nu_0 2\lambda + r + \pi + \beta_0}{\theta_0 + s(\theta_0, \pi)}$.

**Proposition 1.** (*Existence*) Under Conditions 1 and 2, a non-degenerate steady-state equilibrium exists.

Condition 2 is sufficient to ensure the existence of a non-degenerate steady state but not necessary.

Figure 1 illustrates the EE and SS loci, together with steady-state equilibria as points A-C. The existence of non-degenerate steady-state equilibrium is, under the present circumstances, of some independent interest. Recall that we consider insofar a model of a “lawless” (i.e., no policing) society, with homogeneous ex ante agents, in which each household (whether criminal or non-criminal) faces the same crime rate ($\theta = 1$) and earnings opportunities ($\phi = 1$). Yet, even under these austere circumstances, some agents opt for formal employment. Intuitively, the market prices each activity in a manner that ensures both are active in equilibrium.

The parameter $m_0$ governs the matching-rate between job searchers and vacancies. If $m_0$ is “too low” each household engages in crime. Yet, with all agents opting for crime, $N^* = V^* = s^* = 0$ (which is the degenerate equilibrium). Conversely, with a sufficiently active labor market, $\mu > \bar{\mu}$, there are no criminals and hence no crime, $\alpha^* = 0$. The properties of $g(s)$ ensure $s$ is bounded above: $\lim_{\mu \to \infty} s(\mu, 0) = \bar{s} < \infty$ (where $g'(\bar{s}) = \lambda/r$ – see Lemma 2). Thus, the EE locus, equation (25), converges to a ray with a positive slope, $(rv_0)/(y_0 + \bar{s})$.

**Proposition 2.** (*Multiplicity*) Under Conditions 1 and 2, multiple non-degenerate steady-state equilibria may emerge.

Figure 1 illustrates equilibrium points A-C for $m_0 > \bar{m}_0$. In this case, Condition 2 is satisfied and the EE locus cuts the SS locus for exactly three times. In general, there may be more than the four non-degenerate equilibrium points when the EE locus exhibits more turning points, or there may be exactly two equilibria when Condition 2 does not hold and the EE locus cuts the SS locus from above.

In the remainder of this Section we examine the properties of the steady-state equilibrium and compare economic outcomes in the case of multiple equilibria. Proposition 3 describes the comparative-static properties of steady-state equilibrium points such as A and C in
Figure 1. Since firms reach \textit{ex ante} zero profit, the \textit{ex ante} welfare measure is simply:  
\[ \Omega^* = J^*_E - g(s^*), \]  
for any fixed value of \( \nu_0 \).\footnote{We do not solve for the local dynamics around steady-state equilibrium points. This is a difficult problem. However, points such as A and C, satisfy Samuelson’s Correspondence Principle. Moreover, for large enough values of \( m_0 \) such points always exists (Proposition 1).}

**Proposition 3. (Characterization of the Steady-state Equilibrium)** Around any steady-state equilibrium in which \((d\eta^{EE}/d\mu) - (d\eta^{SS}/d\mu) > 0\), we have:

(i) an improvement in labor-market efficacy (higher \( \eta^* \)) or productivity (higher \( \lambda^* \)) raises the level of education (higher \( s^* \)), encourages participation in the formal labor market and firm entry (higher \( N^* \) and \( V^* \)), and reduces the rates of crime and unemployment (lower \( \alpha^* \) and \( E^* \));

(ii) an increase in the predation rate (higher \( \beta_0 \)) or the entry cost (higher \( \nu_0 \)) lowers the level of education, discourages participation in the formal labor market and firm entry, and leads to higher crime and unemployment rates;

(iii) while an improvement in labor-market efficacy or productivity raises economic welfare (higher \( \Omega^* \)), an increase in the predation rate lowers it.

The ultimate source of utility derived by workers, firms, and criminals stems from the goods produced during job-seeker/vacancy matches. An improvement in labor-market efficacy (i.e., an increase in \( m_0 \)) raises the contact rates \( \mu^* \) and \( \eta^* \), which speeds up flow production. This encourages participation in the formal labor market by households (\( N^* \) rises), reduces the primary unemployment rate \( E^* \), and stimulates the entry of firms (\( V^* \) increases). One consequence of this population shift is that the crime rate, \( \alpha^* \), falls. Moreover, the reduction in crime and the increased ease of locating work, further fosters human capital accumulation (stimulating additional entry by firms). The rise in flow output translates into greater flow consumption levels for all agents, thus raising \textit{ex ante} welfare levels.

A productivity improvement, modelled as an increase in \( y_0 \), fosters human capital accumulation and the entry of firms (which further encourages education by raising \( \mu^* \)) and lowers unemployment in the formal sector. In turn, at the margin, households switch from crime to formal employment, which lowers the crime rate (again, further increasing education). An increase in the predation rate, \( \beta_0 \), encourages criminal activity and raises unemployment, lowering both educational attainment levels and \textit{ex ante} welfare. Finally, a reduction in the entry cost, \( \nu_0 \), leads to the entry of a greater number of firms. The increased ease of finding work reduces the unemployment rate, raises education levels and lowers the crime rate.

\footnote{Thus, it is not valid to conduct comparative-static exercises of this welfare measure with respect to changes in \( \nu_0 \).}
Given that the SS locus is strictly decreasing in the contact rate \( \mu \), multiple equilibria – if they exist – can be ordered by \( \mu^* \) (see Figure 1).

**Proposition 4. (Properties of Multiple Equilibria)** Consider an economy with \( n \geq 3 \) (non-degenerate) multiple equilibria indexed by \( i \) in which, after ordering: \( \mu^*_1 < \ldots < \mu^*_n \),

(i) the crime rate \( \alpha^*_i \) and the formal sector unemployment rate \( E^*_i \) decrease with \( i \);

(ii) primary sector activity, \( N^*_i \), educational attainment, \( s^*_i \), and vacancies, \( V^*_i \), all increase with \( i \);

(iii) the equilibria are Pareto rankable, with welfare levels strictly increasing in \( i \).

A high contact rate, \( \mu^* \), encourages formal employment (at the expense of criminal activity), stimulates human capital accumulation, and promotes entry by firms. Since all goods ultimately consumed are produced from matches between job seekers and vacancies, an increase in \( \mu^* \) leads to higher steady-state consumption and, hence household welfare levels.\(^{23}\) Equilibrium selection is driven by self-fulfilling prophecies regarding performance of the primary labor market. Thus, in Figure 1, the equilibrium at point \( C \) generates higher education and welfare levels, than the equilibrium points illustrated as \( A \) and \( B \). The expectation of a robust labor market (high \( \mu^* \)) fosters education by workers and entry by firms. In turn, this discourages criminal activity, further promoting education by workers and additional entry by firms. Point \( A \) is a low-level equilibrium for similar, but converse, reasons.

Our multiple equilibria results may be compared to the model of corruption developed by Murphy, Shleifer and Vishny (1993). They explore an appropriation externality in which an increase in the corruption rate increases its relative returns (since the returns from legal activities are appropriated). This complementarity generates multiple equilibria, with high income/low corruption and low income/high corruption configurations. Although our model also possesses this feature of multiplicity, we can establish the possibility of multiple equilibria even if the appropriation externality is absent, as the proceeds from crime may also be stolen (thieves are both the ‘hunters’ and ‘hunted’).\(^{24}\) In addition, we study the effects of criminal activity on human capital accumulation, wages, and the level of economic activity.

It is interesting that Propositions 3 and 4 have established a positive correlation between the unemployment rate and the crime rate as well as a negative correlation between the level of education and the crime rate. Then applying Lemma 1, we can conclude a negative

\(^{23}\)In equilibrium, firms earn exactly zero ex ante returns. It follows that welfare levels hinge solely on households’ ex ante utility levels: \( J^*_R - g(s^*) \) (which is equal to \( J^*_R \)) for any nondegenerate equilibria.

\(^{24}\)Of course, adding the appropriation effect will create another channel of multiplicity (see Proposition 5 below).
correlation between the wage rate in the primary labor market and the crime rate. These findings are consistent with empirical evidence summarized in the introduction.

5.2 The Case of $\phi < 1$ and $\theta \neq 1$

Up to this point, we have assumed that $\phi = \theta = 1$, indicating that there is no resource cost in fencing stolen property and that criminals are subject to the same rate of crime, $\alpha$, as are members of the formal labor market. For given $\mu$ the $OC$ locus is, $\left[ 1 - \frac{\beta_0 \phi}{\mu} \left( \frac{\chi(\alpha) + \mu}{\lambda + r + \theta \alpha + \beta_0} \right) \right] J_E = g(s)$, or, utilizing (15),

$$1 - \frac{\beta_0 \phi}{\mu} \left( \frac{\chi(\alpha) + \mu}{\lambda + r + \theta \alpha + \beta_0} \right) = g(s) \frac{s \varepsilon(s)}{y_0 + s}$$

(26)

The left hand side of (26) is the ex post return of formal employment relative to criminal activity. The right hand side is the ex ante cost-to-benefit ratio for undertaking education.

Understanding the effects of $\phi < 1$ is trivial. It lowers the ex post returns to crime for all values of $\alpha$ and $\mu$. The case of $\theta \neq 1$ is more interesting. Suppose as in Murphy et al. (1993) that criminals are in a better position, than are workers, to protect their holdings from appropriation (i.e., $\theta < 1$). From earlier arguments $\partial \{ \frac{s \varepsilon(s)}{y_0 + s} \} / \partial \alpha < 0$, as workers reduce their costly education ex ante as crime rises. Differentiating the left hand side of (26) with respect to $\alpha$ implies that its sign depends solely on $sgn \{ (\lambda + r + \mu) \theta - (\lambda + r + \beta_0) \}$. With $\mu > \beta_0$, this is positive at $\theta = 1$ and negative for $\theta = 0$. In the former case, the $OC$ locus can have at most one fixed point; while in the latter it may have more. Intuitively, with $\theta = 0$ an increase in the crime rate raises the relative returns to crime as only legal proceeds are stolen. Consider,

**Condition 3.** $\theta < \{ (\lambda + r + \beta_0) / (\lambda + r + \mu) \} < 1$.

**Proposition 5. (Appropriation Effect)** Under Condition 3, for each $\mu$ equation (26) may have more than one fixed point in $\alpha$ and the $OC$ locus is a correspondence in $(\alpha, \mu)$ space.

We have already established the possibility of multiple equilibria in the case wherein the $OC$ locus is a (single-valued) function. The possibility that the $OC$ locus is a (multi-valued) correspondence provides another source of multiplicity.

6 Crime and Punishment

Up to this point, criminals are assumed subject neither to interdiction by the authorities nor to punishment for the crimes they commit. Yet, an important issue in jurisprudence is the
appropriate setting of legal remedies for the purposes of deterring crime and for protecting
the innocent. Furthermore, communities, often with seriously limited resources, must deter-
mine appropriate levels of police expenditures. At a broader level, if society opts for a prison
system, choices must also be made regarding the treatment of the incarcerated. At one ex-
treme, there is the “short-sharp-shock” treatment, which emphasizes the punitive possibilities
associated with incarceration; at the other, the authorities may focus on the rehabilitation
opportunities afforded during an individual’s prison time. In this Section, we extend the
model set out above to examine these issues formally. In order to facilitate the exposition,
we de-emphasize the labor market details throughout this Section. Specifically, let \( \mu = \mu_0 \)
(exogenous) and let \( w = y_0 + s \).\(^{25}\)

6.1 Policing

The ranks of the police, \( P \), are filled by individuals from outside the community under study.\(^{26}\)
The supply of police is perfectly elastic at a flow wage normalized to unity, implying that \( P \)
also equals the flow expenditure on policing. We assume that police expenditures are
financed by a lump sum (flow) tax, \( \tau \), on each household (i.e., a per capita household tax). Budget
balance then gives, \( P = \tau \). The broad nature of the tax base is important. For instance, if
the tax is levied only on formal workers, then it distorts households’ \textit{ex ante} occupational
choices. We focus on studying and isolating the effects of policing efforts on crime, rather
than any effects that arise from distortionary taxes.

6.2 Punishment

Upon arrest and conviction, households are subject to legal penalty and their inventory
holdings are confiscated and destroyed.\(^{27}\) We admit two further types of legal sanction.

\(^{25}\)It is possible to derive this endogenously in the formal model by assuming a fixed supply of vacancies and
that workers have all the bargaining power with firms. Moreover, a generalization to allow for a fixed sharing
rule, \( w = \bar{\omega}(y_0 + s) \) with \( \bar{\omega} \in (0, 1) \), will not change the main conclusions.

\(^{26}\)On constrast, if the policies are recruited from the community the (lump-sum) taxes used to finance the
police wage bill simply represent transfer from one group (workers and firms) to another (the police). In this
case, the welfare cost of policing stem from the reduction in available productive manpower. The present
formulation is much simpler, since the police wage is exogenous to the community and the inflows into and
outflows from the police force are neglected.

\(^{27}\)Unnecessary distributional complications arise if property is returned to the original owner. The assumption
that the proceeds from crime are destroyed is made for simplicity, as it rules out such seizures as a source
of police financing.
First, there is an instantaneous utility penalty \( Z \) at the point of arrest. Second, convicted households are incarcerated, indexed by \( I \), during which time they receive a (flow) utility equal to zero (which is purely a normalization as any additional prison penalties/disutility can be incorporated in \( Z \)). Prisoners are released (paroled) at random points in time, according to a Poisson process with parameter \( \rho > 0 \). Thus, the legal environment is summarized by the tuple, \( (\rho, Z, P) \in R_+^3 \). In the limit as \( \rho \to \infty \), the incarceration state is transient and the model collapses, in essence, to a “fines only” system. The key opportunity afforded by incarceration is that undesirable elements are weeded out from the community \textit{ex post}, detained, and hence not in a position to commit further (property) crimes for the duration of their prison sentences. The case of \( \rho = 0 \) implies that convicted individuals remain in prison for the remainder of their lives.

6.3 Convictions

We assume that the legal process is imperfect, in that prosecutorial and forensic blunders imply all agents are subject to arrest and conviction at the rate \( q_0 \). However, expenditures on policing, \( P \), enable the community selectively to apprehend and to convict of members of the underground criminal fraternity, \( U = 1 - N = Q + R \). Let \( p \) denote the flow arrest rate and \( q \) denote the flow conviction rate of members of \( U \) resulting from police efforts.\(^{28}\) In order to reduce the notational burden, we assume that members of \( Q \) and \( R \) are subject to a common conviction rate. Thus, the conviction hazard rates for criminals and non-criminals are \( q + q_0 \) and \( q_0 \), respectively. Criminal convictions are governed by the matching process:

\[
pP = qU = \pi_0 \Pi(P, U),
\]

where \( \pi_0 > 0 \) and \( \Pi(\cdot) \) is a constant-returns-to-scale matching technology satisfying the same properties as \( M(\cdot) \).

Absent police expenditures, the judicial process is non-discriminating: both criminals and non-criminals face the same flow probability, \( q_0 \), of arrest and conviction. However, with \( \lim_{P \to \infty} q(P)/q_0 = \infty \), the community can – for sufficient expenditures – avoid type-II errors altogether (i.e., avoid convicting the innocent). The flow rate of convictions stemming from police activity, \( P \), must equal the rate with which members \( U \) are convicted, explaining the first equality in (27). The parameter \( \pi_0 \) is used to model (exogenous) improvements in detection/legal procedures that increase convictions for a given police presence \( P \). Since all agents are infinitesimal, they will treat \( p, q, \) and \( q_0 \) as parametrically given.

\(^{28}\)We emphasize conviction (rather than arrest), as this is the payoff-relevant event for the agents in the model.
6.4 Equilibrium Analysis

The possibility of prison leads to two new incarceration states: IE and IR for innocent workers and criminals, respectively. Our formal analysis of the model proceeds in two steps. First, we study the steady-state properties of the economy. This generates a key relationship - the steady-state locus, \( q^{SS} \) - in \((\alpha, q)\) space, along which all populations are unchanging through time. Second, we examine optimal occupational and educational choices and derive an occupational choice locus, \( q^{OC} \). This relationship gives, for each value of the crime rate \( \alpha \), the conviction rate \( q \) for which agents are just indifferent between working in the formal sector and committing a crime. As we explain below, an equilibrium is then simply a pair \((\alpha, q)\) consistent with both of these loci. Once this is done, we establish the possibility of multiple equilibria, discuss welfare, and consider the economics of rehabilitation.

6.4.1 The Steady-State Locus

The steady-state population of the underground criminal fraternity must satisfy,

\[
0 \leq U = U(\alpha, q; \mu_0, \rho) \leq 1
\]  

where \( U(\cdot) \) possesses the following properties: \( dU/d\alpha > 0, dU/dq < 0, dU/d\mu_0 > 0, dU/d\rho > 0, \partial^2 U/\partial^2 q > 0, 0 < \lim_{q \to 0} U(\cdot) < 1, \text{ and } \lim_{q \to \infty} U(\cdot) = 0. \) An increase in \( \mu \) raises the mass of active consumers, \( C \). In turn this increases the right hand side of the steady-state condition, \( \alpha(C + Q) = \beta_0 R \). The steady-state is restored by an increase in \( U \). Intuitively, an increase in the crime rate, \( \alpha \), or the parole rate, \( \rho \), leads to greater numbers of criminals. Conversely, an increase in the conviction rate, \( q \), lowers \( U \) by raising the population of criminals detained in prison. Finally, as \( q \to \infty \), criminals face instant arrest, in which case their number, \( U \), converges to zero. Equation (27) and the constant-returns-to-scale properties of \( \Pi(\cdot) \) yield the following steady-state conviction relationship,

\[
q = \pi_0 \Pi(P/U(\alpha, q; \mu_0, \rho); 1)
\]  

Since \( U(\cdot) \) is decreasing and convex in \( q \), the properties of \( \Pi(\cdot) \) ensure that the right hand side of (29) is strictly increasing and concave in \( q \). Consider,

Lemma 7. (The Modified SS Locus) The steady-state conviction relationship (29) implicitly defines a modified SS locus,

\[
q = q^{SS}(\alpha; P, \rho, \mu_0, \pi_0)
\]
along which all populations are invariant through time. The \(q^{\text{SS}}(\cdot)\) locus satisfies: \(dq/d\alpha < 0\), \(dq/P > 0\), \(dq/d\rho < 0\), \(dq/d\mu_0 < 0\) and \(dq/d\pi_0 > 0\) as well as the limiting property, \(\lim_{\alpha \to 0} q = \infty\).

An increase in the crime rate \(\alpha\) (or \(\rho\) or \(\mu\)) raises the population of active criminals \(m\), which, for given \(P\) reduces the likelihood with which any one of them will be convicted. Equation (30) may be inverted to yield the steady-state police expenditure function,

\[ P = P(\alpha, q; \rho, \mu_0, \pi_0) \] (31)

which gives, for each pair \((\alpha, q)\), the police expenditures necessary to ensure time-invariant populations. From the properties of (30), \(dP/d\alpha > 0\), \(dP/dq > 0\), \(dP/d\rho > 0\), \(dP/d\mu > 0\) and \(dP/d\pi_0 < 0\). While these properties are direct consequences of Lemma 7, it is useful to note that an increase in the crime rate \(\alpha\) lowers the probability that any given criminal is convicted and, in turn, police expenditures must rise to restore \(q\) to its given value.

### 6.4.2 The Occupational Choice Locus

With the possibility of arrest and incarceration the key asset values become,

\[ \Omega \equiv \max_{s \geq 0} \left\{ \frac{r + \rho}{r + \rho + q_0} \left[ \frac{\lambda \mu_0}{\lambda + r + \alpha + \mu_0 + q_0} \frac{y_0 + s}{r} - \frac{q_0 Z}{r} \right] - g(s) \right\} - \frac{P}{r} \] (32)

\[ J_R \equiv \frac{r + \rho}{r + \rho + q_0 + q} \left[ \frac{\lambda \beta_0}{\lambda + r + \alpha + \beta_0 + q_0 + q} \frac{y_0 + s}{r} - \frac{(q + q_0) Z}{r} \right] - \frac{P}{r} \] (33)

Equations (32) and (33) are structurally very similar. In addition to the absence of the education cost term, \(g(s)\), equation, (33) differs from (32) only in the replacement of: (i) of \(\mu_0\) by \(\beta_0\), which is the matching rate relevant to criminals, and (ii) \(q_0\) by \((q_0 + q)\), which is the arrest hazard rate pertinent for criminals. In (32) the term \((r + \rho)/(r + \rho + q_0)\) represents the loss of utility formal workers expect from (unwarranted) incarceration. The expression \((r + \rho)/(r + \rho + q_0 + q)\) possesses a similar interpretation in (33). Observe that as \(\rho \to \infty\), both these latter terms converge to unity, as prison becomes a transient state.

Equalizing (32) and (33) implicitly define the modified OC locus, which takes into account optimal schooling:

\[ q = q^{\text{OC}}(\alpha; \mu_0, \rho, Z) \geq 0 \]

(34)

This locus gives, for each \(\alpha\), the detection rate, \(q\), at which households are just indifferent \textit{ex ante} between formal work and crime (provided such a value exists). Consider,

**Lemma 8.** (The Modified OC Locus) For sufficiently large values of \(\mu_0 - \beta_0\) and sufficiently low values of \(y_0\), (i) \(q^{\text{OC}}(\alpha, \cdot) \geq 0\) for all \(\alpha \in [0, \bar{\alpha}]\), (ii) \(dq^{\text{OC}}/d\alpha < 0\) for all
\[ \alpha \in [0, \bar{\alpha}], \quad (iii) \quad q^{OC}(0; \cdot) > 0 \text{ and } q^{OC}(\bar{\alpha}; \cdot) \equiv 0 \quad \text{and} \quad (iv) \quad dq/d\mu < 0, \quad dq/d\rho > 0 \text{ and } dq/dZ < 0. \]

Condition 2 implies that if the crime rate is high, agents can be made indifferent between formal employment and crime only if the prosecution rate, \( q \), is sufficiently small.

**Definition 2. (Steady-State Equilibrium with Punishment)** A steady-state equilibrium with punishment is described by a pair \((\alpha^*, q^*)\) satisfying the modified SS and OC loci, (30) and (34).

Any tuple \((\alpha^*, q^*)\) on the OC locus is consistent with optimizing behavior on the part of households. Furthermore, if this tuple also lies on the SS locus, all populations are invariant through time. In summary, once a pair \((\alpha^*, q^*)\) satisfying both loci is found, the other variables, including \(s^*, C^*, E^*\) and \(U^*\), may be recovered recursively.

### 6.4.3 Multiple Equilibria

Although the labor market contact rate is assumed given as \(\mu_0\), multiple equilibria may arise from the positive feedback interactions between the crime rate, \(\alpha\), and the conviction hazard \(q\) alone. When police expenditure is nil \((P = 0)\), we have \(q = 0\) by construction. For any nondegenerate value of police expenditures \(P > 0\), an increase in the crime rate \(\alpha\) reduces the probability with which any given member of the criminal fraternity, \(U\), is subject to arrest. In turn, this raises the individual returns to crime. Consider,

**Proposition 6. (Steady-State Equilibrium with Punishment)**

(i) If \(P = 0\), then there is a unique equilibrium \((\alpha^*, q^*) \equiv (\bar{\alpha}, 0)\).

(ii) For any given value of \(P > 0\), multiple steady-state equilibria may exist. In particular, for any \(P, Z > 0\), the zero crime configuration, \(\alpha^* = 0\), is always an equilibrium.

In the absence of police expenditures, the SS locus is horizontal at \(q = 0\), giving rise to the equilibrium \((\bar{\alpha}, 0)\). If \(P > 0\) and \(\alpha = 0\), then \(q \to \infty\). Comparing (32) and (33), yields: \(\Omega > J_R\) at \(\alpha = 0\). In this case, all households opt for formal employment which gives \((\alpha^*, q^*) = (0, \infty)\) as an equilibrium. In Figure 2, we illustrate the results based on a simulation of the model, which indicates three equilibrium points \(A, B,\) and \(C\) (with \(C\) representing the low-crime equilibrium). Welfare levels across the equilibria are Pareto rankable and decrease with the crime rate \(\alpha\). Of course, it is worth bearing in mind that throughout this section we have assumed that \(\mu_0\) is given. Allowing \(\mu_0\) to adjust endogenously would give rise to an additional source of multiple equilibria, for the reasons already described in Section 5. Finally, the equilibrium depicted at \(C\) possesses natural comparative-static properties. For
instance, here an increase in police expenditures $P$ or the labor-market matching rate $\mu_0$ lowers the crime rate $\alpha$ (Lemma 7).

### 6.5 Welfare Analysis

In steady-state equilibrium firms earn zero expected returns and for points along the $OC$ locus, $\Omega \equiv J_R$. In view of this, we can again identify social welfare with formal workers’ ex ante utility $\Omega$. Although ascertaining the social welfare objective is relatively straightforward, determining the optimal legal environment, $(\rho, Z, P)$, is complicated by the existence of multiple steady-state equilibria. The reason is, of course, that each choice of $(\rho, Z, P)$ is potentially associated with different equilibria and hence different welfare levels.\(^{29}\)

In view of this difficulty, we adopt the following approach in analyzing the welfare properties of the model. First, we write ex ante worker welfare (32) as the function $\Omega(\alpha; \mu_0, \rho, Z, P)$. Second, the $OC$ locus, Lemma 8, gives: $q = q^{OC}(\alpha; \mu_0, \rho, Z)$, which is defined for $\alpha \in [0, \bar{\alpha}]$. Third, the police expenditure required to sustain any pair $(\alpha, q)$ as a steady-state is, according to equation (31), $P = P(\alpha, q; \mu_0, \rho, Z)$. Since any steady-state equilibrium lies on the $OC$ locus, we can use $q^{OC}(\cdot)$ in $P(\cdot)$ to derive, $P = P^{SS}(\alpha; \mu_0, \pi_0, \rho, Z)$ for $\alpha \in [0, \bar{\alpha}]$. This determines for each $\alpha \in [0, \bar{\alpha}]$ (and, of course, the other parameters) the (unique) level of law enforcement $P$ that ensures $\alpha$ is a steady-state equilibrium.

Using the expression $P^{SS}(\cdot)$ in $\Omega$ yields the welfare level derived in steady-state equilibrium at $\alpha$. By selecting $(\rho, Z)$ one can analyze the maximum social welfare attainable at a steady-state equilibrium with a crime rate $\alpha$. Although analytic results are difficult to obtain, the model can be simulated, which gives maximized equilibrium welfare for each $\alpha$ along with the optimal choice of policy instruments.\(^{30}\) From this,

**Proposition 7.** (Welfare)

(i) If $q_0 = 0$, there is a unique zero-crime equilibrium with $\alpha^* = 0$, $Z^* = \infty$ and $\rho^* = 0$.

(ii) If $q_0 > 0$, then under proper parameter values, $Z(\alpha^*) = 0$ and $0 < \rho(\alpha^*) < \infty$ for steady-state equilibrium value of $\alpha^* > 0$, whereas maximized equilibrium welfare $\Omega^*$ need not decrease with the crime rate $\alpha^*$.

Part (i) is easily explained. In the absence of judicial error, $q_0 = 0$, legal sanctions are imposed only on criminals. For any $P > 0$, sufficiently draconian penalties deter any agent from opting

\(^{29}\)For instance, we have already seen in Proposition 6(ii) that for any legal environment with $P, Z > 0$ there is always a zero-crime equilibrium.

\(^{30}\)Our numerical analysis considers: $g(s) = s^2$ and $\Pi \equiv (P)^{1/2}(U)^{1/2}$, with $y_0 = \beta_0 = 1$, $\mu_0 = 1.3$ and $\delta = \lambda = q_0 = r = .1$. 

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for crime, so \( \alpha^* = 0 \) is the only equilibrium. However, with \( q_0 > 0 \) even innocent members of the community run some risk of conviction and punishment. This naturally bounds the severity of any legal remedies, leading to the optimal tolerance of positive amounts of crime. Here, detention is preferable to an instantaneous fine \( (Z^* = 0) \), since incarceration improves the \textit{ex post} composition of agents in the community, by weeding out criminal elements. Moreover, the possibility of a miscarriage of justice (arresting a member of \( C \) or \( E \)) gives \( \rho > 0 \), so that ultimately all agents are eligible, their mortality notwithstanding, for release. Our numerical exercises illustrate regions over which welfare increases with the equilibrium crime rate \( \alpha \) – police expenditures fall sufficiently rapidly that the deleterious effect of the increase in crime is outweighed by the benefits of lower taxes. Finally, with \( P = q^* = 0 \), the judicial process is non-discriminating and, in effect, acts simply as a dissipative tax on all agents without affecting their \textit{ex ante} occupational choices.

### 6.6 Rehabilitation and Recidivism

One of the great opportunities afforded by prison is the possibility of rehabilitation through, for example, education. We now extend the model developed in this section to study this possibility. Suppose that an individual enters prison with an education level \( s^0 \). Consider that through fiat, the authorities can compel prisoners to undertake schooling, \( s^P \), with the cost of education borne by the inmate given by: \( \max\{g(s^P) - g(s^0), 0\} \). Further assume that the human capital acquired by the inmate is \( \max\{s^0, s^P\} \). Once again, we assume that education varies across the intensive margin. The assumption above captures the intuitive notion that teaching basic arithmetic is costly but useful to an individual only if the individual in question does not know basic arithmetic. Despite the admitted simplicity of this setup, we are able to offer two key findings.\(^{31}\) First, we show the possibility of recidivism and reform as individual best responses to prison education. Second, our findings point to interesting role for using \textit{in-prison education as a punishment}!

Our goal is simple and somewhat limited. It is to consider only the partial-equilibrium properties of the model, in which we derive agents’ best responses, upon release from prison, after acquiring education \( s^P \). Let \( (\rho, Z, P) \) be given and consider a steady-state equilibrium configuration \( (\alpha^*, q^*) \), with a positive crime rate: \( \alpha^* > 0 \). We perform a local analysis in

\(^{31}\) Naturally, a richer framework would: \((i)\) account for resources devoted by the community to education, \((ii)\) allow ‘time-to-build’ human capital, \((iii)\) admit learning heterogeneity, since acquiring given \( s^P \) may be very costly indeed for some individuals, and \((iv)\) allow for the loss of skills during extended periods of incarceration. Yet, these extensions would take us to far afield from our present study.
that the authorities only educate, at the margin, convicted individuals with zero measure (this leaves the steady-state unaltered). Let $J_{IR}$ and $J_{IE}$ denote the \emph{ex ante} utility levels for (incarcerated) criminals and non-criminals respectively. Consider,

**Proposition 8. (Rehabilitation and Recidivism)**

(i) (\textbf{Formal workers}) For any $s^P \geq 0$, formal workers re-enter the formal labor market, upon release from prison.

(ii) (\textbf{Criminals}) There exists a $\tilde{s} \in (0, s^*)$ with $s^* = \arg\max\{J(s)_E - g(s)\}$ such that:
    
    (a) (\textbf{Recidivism}) criminals return to crime upon release for all $s^P < \tilde{s}$;
    
    (b) (\textbf{Reform}) criminals enter the formal labor market for all $s^P \geq \tilde{s}$.

(iii) (\textbf{Education as a punishment})

    (a) If $s^P \in [\tilde{s}, s^*)$, then $J_{IE} = \Omega$ and $J_{IR} < J_R$;
    
    (b) if $s^P \geq s^*$, then $J_{IE} < \Omega$ and $J_{IR} < J_R$.

Results (i) and (ii) are a direct consequence of the fact that \emph{ex post} welfare in the formal labor market monotonically increases with human capital, $s$. It follows that workers optimally re-enter the formal labor market. However, criminals opt for formal employment only if they acquire sufficient human capital in prison to make it worthwhile to switch occupations. This result may help the exceedingly high recidivism rates among released (ex) felons, as perhaps the convicted are provided with insufficient education to make a career change worthwhile.\footnote{An alternative explanation is a stigma effect (c.f. Rasmusen (1996)) associated with crime and incarceration. This could be modelled in our framework as a reduction in the job contact rate for released prisoners.}

Finally, part (iii) points to education as a tool of rehabilitation and punishment. Education at the level $s^P \in [\tilde{s}, s^*)$ acts as a selective penalty for members of the criminal fraternity alone. In a general-equilibrium model it would act as an \emph{ex ante deterrent}. Here, it also acts as an instrument of reform, as it provides sufficient education to ensure criminals optimally enter the formal labor market upon release. Nevertheless, education levels in excess of the optimal level schooling $s^*$ would impose costs on both criminals and formal workers.

7 Concluding Remarks

We study the accumulation of human capital, crime, and unemployment in the context of a simple search-equilibrium model. In determining the level of human capital they wish to accumulate, agents take into account not only the cost of acquiring human capital but also the (endogenously determined) crime rate. Crime acts, in effect, as a tax on the returns to human capital. There are multiple equilibria. High crime, low levels of educational attain-
ment, unemployment, and poverty are correlated across them. The selection of a particular equilibrium outcome is based primarily on self-fulfilling prophecies.

One of the main findings is that a finite prison sentence may dominate fines as a means of deterring crime. This stands in contrast to much of the previous literature (cf. Becker (1968) and the follow-up research), wherein fines/compensatory damages are optimal. Our result stems from the fact that agents make an *ex ante* occupational choice and that prison: (i) acts as a penalty that favorably affects the *ex ante* composition of agents and (ii) improves the *ex post* composition of the population by ‘weeding out’ members of the criminal fraternity. In additional our model may help explain the alarmingly low levels of education undertaken by those who choose a life of crime. Our framework is one in which all agents are *ex ante* identical, but differ according to their *ex post* career choices and educational attainment levels. In view of this, our model points to education as an obvious tool for rehabilitation. In the absence of ‘stigma’ effects, providing inmates with sufficient levels of education, leads them to choose formal employment upon release. Moreover, education can, in itself, serve as a tool of rehabilitation and as a punishment.

Our model admits many potential extensions, and we mention just two of them. First, we assume that individuals make a dichotomous choice between work and crime. In practice, most criminals also secure formal employment at various points, often committing crime during their spare time. We believe we could model this phenomenon by introducing leisure as an argument in the utility function and allowing *ex ante* productive heterogeneity. A second set of extensions deals with the consequences of extended periods of incarceration on human capital levels.
References


Appendix

This appendix provides proofs of lemmas and propositions presented in the main text.

Lemma 1. (The Wage)

The Bellman Equations (7)-(12) give,

\[
\begin{align*}
J_E &= \left[ \frac{\lambda \mu}{\lambda + \mu + \chi} \right] (w/r) \\
J_C &= \left[ \frac{\lambda(\mu + r)}{\mu + \lambda + \chi} \right] (w/r) \\
J_R &= \left[ \frac{\lambda \beta}{\beta + \lambda + \chi + (\theta - 1)\alpha} \right] (\phi w/r) \\
J_Q &= \left[ \frac{\lambda(\beta + r)}{\beta + \lambda + \chi + (\theta - 1)\alpha} \right] (\phi w/r)
\end{align*}
\]

where \( \chi \equiv \lambda + r + \alpha \). Using (A1) in (13) yields: \( \lambda w/(\chi + \mu) = y - w \). This is solved as (14) in the text. The partial derivatives are obtained through standard methods. ||

Lemma 2. (Education)

By substituting the wage (14) into the asset value (A1),

\[
\max_{s \geq 0} \{J_E - c(s)\} = \max_{s \geq 0} \left\{ \left[ \frac{\lambda \mu}{\lambda + \mu + \chi} \right] \left[ (y_0 + s)/r - g(s) \right] \right\}
\]

The program is convex, so, under Conditions 1 and 2, the first-order conditions are necessary and sufficient for a maximum. Thus,

\[
\left[ \frac{\lambda \mu}{\lambda + \mu + \chi} \right] (1/r) \equiv g'(s)
\]

which implicitly defines the optimal schooling level, \( s(\mu, \alpha) \). The derivatives are obtained by totally differentiating equation (A3). By l’Hospital’s rule: \( \lim_{\mu \to \infty} [\lambda \mu/(\lambda + \mu + \chi)](1/r) = (\lambda/r) \). This implicitly defines the educational upper bound, \( \bar{s} \), by: \( (\lambda/r) \equiv g'(\bar{s}) \). Similarly, \( \lim_{\alpha \to \infty} [\lambda \mu/(\lambda + \mu + \chi)](1/r) = 0 \), which gives \( s = 0 \). For \( \alpha = 0 \), then \( \chi = \lambda + r \). Direct substitution then gives, \( \lambda \mu/(\mu + 2\lambda + r)](1/r) \equiv g'(s(\mu)) \). ||

Lemma 3. (Steady-State Matching)

Under constant returns to scale,

\[
\mu E = \eta V = Vm_0 M(E/V, 1) = Vm_0 M(\eta/\mu, 1)
\]

Equation (A4) implicitly defines the SS locus, \( \eta = \eta^{SS}(\mu; m_0) \). Straightforward differentiation establishes the properties. ||

Lemma 4. (The EE locus)

With unrestricted entry, the \textit{ex ante} return from entering the market, \( \Pi_V - \nu_0 \), must be zero. Hence, \( (y - w)\eta = rv_0 \). Using the wage equation (14) yields,

\[
\eta = (rv_0/\lambda)[(\lambda + \chi + \mu)/(y_0 + s)]
\]
Routine manipulation yield, with Lemma 2, the partial derivative properties. ||

Lemma 5. (Non-monotonicity)
As \( \mu \to 0 \), then \( s(\cdot) = 0 \), since costly education has no value. Equation (4) gives: \( \eta_0 = \left( \frac{\nu_0}{\lambda y_0} \right)(\lambda + \chi) > 0 \). As \( \mu \to \infty \), then \( s \to \bar{s} \) (Lemma 2). From (A5), \( \lim_{\mu \to \infty} \eta = \infty \).
This establishes the limiting properties of the EE locus. Using parameter values similar to those in Laing, Palivos, and Wang (1995) gives an EE locus that possesses decreasing segments. ||

Lemma 6. (The OC locus).
Let \( \mu_0 \) be given together with the educational choice: \( s(\alpha, \mu_0) \). Write equation (19) as:
\[
\mu = \frac{\beta_0 \chi}{\chi - (\beta_0 + \chi) \zeta(s(\alpha, \mu'))}
\]  
(A6)
Under Condition 1, \( \chi - (\beta_0 + \chi) \zeta(s(\alpha, \mu')) > 0 \) for all \( \mu' \) ensuring (A6) non-negative and well-defined for all \( \mu' \). Differentiating (A6) with respect to \( \mu' \) yields:
\[
\text{sgn} \left\{ \frac{\partial \mu}{\partial \mu'} \right\} = \text{sgn} \left\{ \frac{\partial x}{\partial s} \left( \frac{\partial s}{\partial \mu'} \right) \right\} > 0
\]
The limiting properties are,
\[
\lim_{\mu' \to 0} \mu = \beta_0 \quad (A7a)
\]
\[
\lim_{\mu' \to \infty} \mu = \frac{\beta_0 \chi}{\chi - (\beta_0 + \chi) \zeta(s)} \leq \frac{\beta_0}{1 - \zeta(s)} < \infty \quad (A7b)
\]
It follows that, for each \( \alpha \), there is a unique fixed point: \( \mu = \mu' \equiv \mu^{OC}(\alpha; \beta_0, y_0) \). Around this point, \( d\mu/d\mu' < 1 \) (as the function (A6) is increasing and bounded above, it can have at most one fixed point). Totally differentiating equation (A6), using this latter condition, yields the partial derivatives reported in Lemma 6. The limiting properties follow directly from (A6).||

Proposition 1. (Existence).
From Lemma 4, the EE locus is continuous with \( \lim_{\mu \to -\beta_0} \eta^{EE} = \frac{\nu_0}{\lambda y_0 + s(\beta_0, \bar{a})} \equiv \eta_0 > 0 \) and \( \lim_{\mu \to \infty} \eta^{EE} = \infty \), for given \( \alpha \). The properties of \( g(s) \) ensure \( s \) is bounded above: \( \lim_{\mu \to \infty} s(\mu, 0) = \bar{s} < \infty \) (where \( g'(\bar{s}) \equiv \lambda/\tau \)). It follows that for large enough \( \mu \) the EE locus, equation (25), converges asymptotically to a ray with a positive slope \( r\nu_0/(y_0 + \bar{s}) \), which is linear and increasing in \( \mu \). Also, the SS locus (16) is strictly decreasing in \( \mu \), asymptotes at each axis, and increases in \( m_0 \). Define \( m_0 \) as a lower bound of \( m_0 \) to satisfy: \( \eta^{SS}(\beta_0; m_0) = \eta_0 \). For any \( \eta = \eta^{SS}(\beta_0; m_0) > \eta_0 \), which is true for any \( m_0 > m_0 \), the strictly decreasing SS locus must intersect with the EE locus. This, together with the recursive property of the system of well-defined functions, proves the Proposition. ||

Proposition 2. (Multiplicity).
This is trivial. For small value of \( m_0 \) only the degenerate \( s^* = N^* = 0 \) equilibrium exists. As shown in Proposition 1, for any given matching technology, \( M(\cdot) \), there is a unique \( m_0 \).
for which the SS and EE loci coincide only at a point of tangency. For values of $m_0 > m_0$, the SS locus may cut the EE locus three times or more.

**Proposition 3. (Comparative Statics).**
The derivatives are obtained, through routine methods, by totally differentiating the EE locus and the SS locus around a steady-state equilibrium point (using Lemmas 2 and 6). The condition, $(dη^{EE}/dμ) - (dη^{SS}/dμ) > 0$, yields the results.

**Proposition 4. (Multiple Equilibria)**
The inverse OC locus implies that the crime rate decreases with $\mu$ (Lemma 6). The schooling effort function $s(\mu, \alpha)$ increases with $\mu$ and decreases with $\alpha$. It follows that higher values of $\mu^*$ lead, unambiguously, to higher values of $s^*$ as both the direct ($\mu$) and indirect ($\alpha$) effects work in tandem. Equation (24), determining the primary sector labor force, $N$, increases with $\mu$ and falls with $\alpha$. It follows that higher values of $\mu^*$ lead to higher values of $N^*$. Finally, a worker’s *ex ante* welfare is given by (20), which increases (decreases) with $\mu$ and $\alpha$. The welfare results are then immediate.

**Proposition 5. (Appropriation Effect)**
Recall that the right hand side of (26) is decreasing in $\alpha$. Given $\theta < \{(\lambda + r + \beta_0)/(\lambda + r + \mu)$, the left hand side of (26) decreases with $\alpha$. Thus, multiple solutions may emerge.

**Lemma 7. (The Steady-state Locus).**
Tedious but straightforward manipulations give, $U = B_1/B_2$, where

$$B_1 \equiv \alpha(q + q_0 + \alpha + \beta_0 + \delta + \lambda)$$
$$B_2 \equiv (\alpha q_0^2 \mu_0 + B_3)(q_0 + \delta + \rho) + q\{q_0^2 \beta_0 + \alpha^2 \mu_0 + \beta_0 (\delta + \mu_0 + \lambda) (\delta + \rho) + \alpha[\mu_0 (2\delta + \lambda + \rho) + \beta_0 (\delta + \mu_0 + \rho)] + q_0[\alpha (\beta_0 + 2\mu_0) + \beta_0 (2\delta + \lambda \mu_0 \rho)]\}$$
$$B_3 \equiv (q_0 + \alpha + \delta + \lambda)[\alpha \mu_0 + \beta_0 (q_0 + \delta + \lambda + \mu_0)]$$

Even more tedious manipulations yield the reported derivatives on $U(\cdot)$. The function $\Pi(P/U(\cdot); 1)$ is increasing and concave in $q$, satisfying $\Pi(\cdot) > 0$ at $q = 0$. It follows there is a unique fixed point $q$ at which, $q \equiv \Pi(P/U(q; \cdot); 1)$. This fixed point implicitly defines the $q$ locus. The comparative static properties of $q^{SS}$ follow directly from total differentiation and the properties of $U$.||

**Lemma 8. (The OC locus)**
For ease of notation, define: $R \equiv r + \rho + q_0$ and $A(\alpha) \equiv \lambda + r + \alpha + q_0$. The first-order condition of educational choice gives:

$$\frac{r + \rho}{rR} \frac{\lambda \mu_0}{A(\alpha) + \mu_0} = g'(s) = \frac{g(s)}{\varepsilon s}$$
implying that \( \lim_{\alpha \to 0} s < \infty \) and \( \lim_{\alpha \to \infty} s = 0 \). Using the above expression, the \( OC \) locus can be written as:

\[
\Psi(\alpha, q) \equiv \left[ 1 - \frac{\beta_0}{\mu_0} \frac{R}{R + q} A(\alpha) + \beta_0 + q \right] - \zeta(s) \left[ 1 - \frac{\hat{Z}}{g(s)} \right] = 0 \quad (A8)
\]

where, \( \hat{Z} \equiv (Z/r)[q_0/(R + q_0)][(r + \rho)/R] < Z/r \). Consider the case where \( q = 0 \),

\[
\Psi(\alpha, 0) \equiv \left[ 1 - \frac{\beta_0}{\mu_0} A(\alpha) + \beta_0 \right] - \frac{\varepsilon s}{y_0 + s} = 0
\]

It is easily seen that \( \lim_{\alpha \to \infty} \Psi(\alpha, 0) = 1 - \frac{\beta_0}{\mu_0} > 0 \) and that \( \lim_{\alpha \to 0} \Psi(\alpha, 0) = 1 - \frac{\beta_0}{\mu_0} \frac{\lambda + r + q_0 + \mu_0}{\delta = r + q_0 + \beta_0} \), which is negative for sufficiently low values of \( y_0 \). Moreover, \( \partial\Psi(\alpha, 0)/\partial\alpha > 0 \). This system possesses a unique fixed point at \( \overline{\alpha} > 0 \) and \( q^{OC}(\overline{\alpha}, \cdot) = 0 \). Total differentiation of (A8) with respect to \( \alpha \) and \( q \) gives:

\[
K_1 d\alpha + K_2 dq = 0
\]

where, \( \text{sgn.}\{K_1\} < 0 \) for sufficiently large values of \( (\mu_0 - \beta_0) \) and sufficiently low values of \( y_0 \) and \( \text{sgn}\{K_2\} < 0 \) unambiguously. This yields, \( \partial q^{OC}/\partial\alpha < 0 \) and \( q^{OC}(0, \cdot) > 0 \). \|

**Proposition 6. (Steady-state Equilibrium with Punishment)**

(i) If \( P = 0 \), then the steady-state value of \( q \) is \( q = 0 \). This gives \( (\alpha^*, q^*) = (\overline{\alpha}, 0) \) as the equilibrium. (ii) Numerical examples demonstrate the possibility of multiple equilibria. For \( P, Z > 0 \) and \( \alpha = 0 \), then \( q \to \infty \). In this case, \( \Omega > J_R \), implying that no agent chooses crime (i.e., \( \alpha^* = 0 \)) as the equilibrium.\|

**Proposition 7. (Welfare)**

(i) Let \( q_0 = 0 \) and \( P > 0 \). Then, for some \( Z > 0 \), \( J_R < 0 \). In this case, \( \Omega > 0 > J_R \), implying all agents choose formal employment, so that \( \alpha^* = 0 \).

(ii) Consider: \( g(s) = s^2 \) and \( \Pi \equiv (P)^{1/2}(U)^{1/2} \). Let, \( y_0 = \beta_0 = 1, \mu_0 = 1.3 \) and \( \delta = \lambda = q_0 = r = .1 \). The results of this simulation are as presented in the text. \|

**Proposition 8. (Rehabilitation and Recidivism)**

(i)/(ii) Criminal activity provides a level of utility \( J_R \). The \textit{ex post} utility of formal employment with education \( s^p \) is, \( J(s^p)_E \equiv [\lambda_0(y_0 + s^p)/r] / [\chi(\alpha) + \mu_0] \) which is monotone increasing in \( s^p \). From previous arguments, if \( s^p = 0 \), then \( J(s^p)_E < J_R \), whereas if \( s^p = s^* = \arg\max\{J(s)_E - g(s)\} \), then \( J(s^p)_E = J_E > J_R \). It follows that there is a unique, \( \overline{s} \in (0, s) \), such that \( J(\overline{s}) = J_R \). (iii) Since \( s^* = \arg\max\{J(s)_E - g(s)\} \), it follows that any \( s^p \neq s^* \) gives \textit{ex ante} utility \( J(s^p) - g(s^p) < \Omega(s^*) = J_R \). \|
Figure 1: Steady-State Equilibrium

Figure 2: Steady-State Equilibrium with Punishment