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Low Reserve Prices in Auctions

by

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Abstract. Received auction theory prescribes that a reserve price which maximizes expected profit should be no less than the seller's own value for the auctioned object. In contrast, a common empirical observation is that many auctions have reserve prices set below sellers' values, even at zero. This paper revisits the theory to find a potential resolution of the puzzle for second-price auctions. The main result is that an optimal reserve price may be less than the seller's value if bidders are risk averse *and* have interdependent values. Moreover, the resulting outcome may be arbitrarily close to that of an auction that has no reserve price, an absolute auction.

Keywords. *reserve price, risk aversion, interdependent values, second-price auction*
JEL Classification. D44, D82

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1 Introduction

Reserve prices in real-world auctions are often very low. This is most obviously true of absolute auctions, which have no reserve price and are in widespread use.[1] Online auctions regularly have reserve prices so low that they must be less than the sellers' values.[2] In contrast, received theory predicts that a profit-maximizing monopoly seller should exclude low-valued buyers by setting a reserve price higher than his own value. Even in settings with competing auctions or endogenous entry, theory prescribes that the reserve should be set no lower than the seller's value.[3] Reinforcing the discrepancy between observed and theoretical reserve prices is empirical evidence that raising reserve prices above a seller's value can lower profit.[4]

In this paper, for second-price auctions, we provide a potential resolution to this puzzle by generalizing the standard theoretical model in two ways. Both ways are plausible for many settings in which empirical reserve prices seem too low, such as art, antique, collectibles and online auctions. First, we allow the bidders to have positively interdependent values, so that a bidder's value for the good depends on the private information of other bidders. Price discovery is one reason this is a natural assumption, even in the supposedly "private value" setting of an art auction. For example, a bidder's value for a rare painting may be determined not only by his own aesthetic taste, but also by the painting's resale price or the prestige of owning it, which in turn are positively related to the other bidders' private tastes. Second, we allow the bidders to be risk averse, which is plausible when they are consumers or small businesses. Our model is otherwise entirely standard, with fully rational

[1] See, e.g., Cassady (1967), McAfee and McMillan (1987), Ashenfelter (1989), Ashenfelter and Graddy (2003), and Jehiel and Lamy (2015).

[2] See Hasker and Sickles (2010) for a recent survey and the references therein.

[3] See, e.g., Riley and Samuelson (1981), Myerson (1981); Krishna (2010), Hu, Matthews and Zou (2010), Engelbrecht-Wiggans (1987), McAfee and McMillan (1988), Levin and Smith (1994, 1996), and Moreno and Wooders (2016).

[4] Reiley (2006), Walley and Fortin (2005).

bidders and independent types. The main result is that profit-maximizing reserve prices can be lower than the seller's value and arbitrarily close to zero.

Low optimal reserve prices arise in our model due to an interaction between the bidders' interdependent values and their risk aversion. In a second-price auction with a reserve price large enough to exclude some types of bidders from bidding, in equilibrium the lowest bid a bidder will ever make is larger than the reserve price if their values are positively interdependent. This is true even if the bidders are risk neutral, but the "gap" between the smallest equilibrium bid and the reserve price is larger the more risk averse they are.[5] When the gap is large enough, *lowering* the reserve price paradoxically *increases* each bidder's ex ante expected payment. This is because lowering the reserve induces marginal types of bidder to bid, which increases the likelihood that the winning bidder will pay the second-highest bid rather than the lower reserve price. Provided the bidders exhibit both value interdependence and risk aversion, increasing either one enough causes the gap to be so large that any profit-maximizing reserve price must be lower than the seller's value for the good, so long as the seller's value exceeds the lowest possible bidder value. In this case there is positive probability that the good will be sold for a price less than the seller's value.

Our final result bears on the use of absolute auctions. We establish the existence of an interval of possible seller values for which the seller's optimal reserve price is arbitrarily close to the lowest possible bidder value if the bidders are sufficiently risk averse, and the interdependency of their values exceeds a certain bound. In this case the outcome is nearly the same as if there were no reserve price, i.e., an absolute auction is nearly profit maximizing.

[5] Milgrom and Weber (1982) (fn. 26) note the existence of this gap in some second-price auctions with interdependent values. The observation that the gap increases with bidder risk aversion is new to this study, to the best of our knowledge.

1.1 Related Literature

An early theoretical study of low reserve prices is by Levin and Smith (1996), who consider profit-maximizing reserve prices in a symmetric setting in which the bidders' types are affiliated. Their main result is that as the number of bidders becomes large, the profit-maximizing reserve price converges to the seller's value from above in both first- and second-price auctions. They do not show that the optimal reserve price is ever below the seller's value – because they assume the latter is equal to the lowest possible bidder value, there is no benefit to the seller from setting a reserve price less than this amount. More importantly, the logic of Levin and Smith (1996) depends on the types being positively affiliated. The more these are affiliated, the lower is the probability of the event in which one bidder's type is high enough for him to bid and the other bidders' types are so low that they do not bid – this is the event in which raising the reserve price increases the payment to the seller.[6] In contrast, in the present paper the bidders' types are independently distributed; reserve prices are low here solely because of the interaction of risk aversion and value interdependence in second-price auctions.

Vincent (1995) provides an example in which a risk neutral seller prefers to use a secret reserve price rather than a posted one in a second-price auction. While his focus is on secret reserve prices rather than low posted reserve prices, like ours his argument also centers on the gap between the lowest bid and posted reserve price in second-price auctions that exists because of positively interdependent values. In his example, the reduced bidder participation caused by this gap is alleviated by moving to a secret reserve price format. (This result requires the seller to privately know his value.)

In a recent paper, Jehiel and Lamy (2015) present models and conditions under which very low reserve prices, absolute auctions, and secret reserve prices are optimal for risk neutral sellers. They focus on the limiting case of large numbers

[6] Krishna (2010, section 8.4) gives a textbook discussion of this logic.

of bidders sorting themselves among competing sellers. A major innovation is their assumption that some bidders do not fully understand how participation rates vary with the auction format, and how reserve prices are distributed when secret. Our analysis differs in that our bidders are fully rational and do not choose between competing auctions.

1.2 Organization of the Paper

In Section 2 we examine the incentives of a seller to lower the reserve price in a simplified "reduced form" model of a second-price auction with value interdependent, risk averse bidders. This reduced form model is obtained from a complete model that we present in Section 3 and study in the remainder of the paper. In Section 4 we derive the effects of increasing the bidders' risk aversion, and also show that if they are risk neutral a profit-maximizing reserve price must exceed the seller's value. In Section 5 we obtain conditions under which an optimal reserve is lower than the seller's value, and also conditions under which it is arbitrarily close to the lowest possible bidder value. The Appendix contains proofs missing from the text.

2 An Incentive to Lower the Reserve

Consider a second-price auction for a good (object, asset) owned by a seller who has value v_0 for it. The seller publicly announces a reserve price, r, which he chooses to maximize expected profit. Each of $n \geq 2$ bidders then either submits a sealed bid no less than r, or does not bid. The good is sold if and only if at least one bid is submitted, in which case it is sold to a bidder chosen (randomly) among those who bid the highest, for a price equal to the maximum of the reserve price and the other submitted bids.

The information structure is characterized by symmetry and independence. The bidders' *types*, $t_1, \ldots, t_n$, are independently drawn from the same distribution F on the interval $[0, 1]$, upon which it has a continuous and positive density, f. Each bidder privately learns his type before the auction is held. For an arbitrarily chosen bidder, we denote his type and the maximum type of the other $n-1$ bidders as X and Y_1, and the realizations of these independent random variables as x and y, respectively.

The payoff structure is also symmetric, with either private or interdependent values. We let $U(-p, x, y)$ denote the conditional expected utility of a bidder when he purchases the object for price p, his type is x, and the maximum of the other bidders' types is y. The function U is continuously differentiable, with partial derivatives satisfying $U_1 > 0$, $U_2 > 0$, and either $U_3 = 0$, the case of *private values,* or $U_3 > 0$, the case of *(positively) interdependent values.* Familiar examples are $U(-p, x, y) = u(x-p)$ (private values, possibly risk averse) and $U(-p, x, y) = v(x, y) - p$ (interdependent values, risk neutral). We give a more general specification in the next section.

The expected utility of a losing bidder is normalized to equal zero. A bidder is assumed to want the good if its price is low enough, but not if its price is too high: $\underline{p} > 0$ and $\bar{p} > \underline{p}$ exist such that $U(-\underline{p}, x, y) > 0$ and $U(-\bar{p}, x, y) < 0$ for all $x, y \in [0, 1]$.

A *strategy* for bidder i is a function β_i mapping each of his types into the set $\{No\} \cup [r, \infty)$. When his type is x, he does not bid if $\beta_i(x) = No$, and otherwise he submits the bid $\beta_i(x) \in [r, \infty)$. We focus on a symmetric equilibrium,[7] characterized by a *screening level* $s \in [0, 1]$ and a continuous strictly increasing *bid function* $b : [0, 1] \rightarrow \mathbb{R}$. Given s and $b(\cdot)$, each bidder's equilibrium strategy, β^*, is defined by

$$\beta^*(x) := \begin{cases} No & \text{for } x < s \\ b(x) & \text{for } s \leq x \leq 1. \end{cases} \tag{1}$$

Thus, a bidder submits a bid if and only if his type exceeds or equals the screening level, in which case the bid he submits is determined by the bid function.

The equilibrium screening level is determined by the reserve price. Assuming for the moment that $s \in (0, 1)$, a type s bidder must be indifferent between bidding

[7] However, it should be noted that second-price auctions in general have asymmetric equilibria, even ones in undominated strategies. See, e.g., Milgrom (1981) and Krishna (2010).

and not bidding, since otherwise a somewhat lower type would want to bid, or a somewhat higher type would not want to bid. Neglecting the zero-probability event of a tie, a type s bidder wins if and only if the other bidders' types are less than s, and he then pays the reserve price. Accordingly, for $s \in (0,1)$ the reserve price and screening level are related by

$$\mathbb{E}[U(-r, s, Y_1) \,|\, Y_1 \leq s] = 0. \tag{2}$$

In contrast, the bid function does not depend on the reserve price. It is instead determined as usual in a second-price auction: for any $x \in [0,1]$, the bid $b(x)$ is equal to the most a type x bidder would be willing to pay for the good given that the maximum of the other bidders' types is also equal to x. That is, $b(x)$ is given by

$$U(-b(x), x, x) = 0. \tag{3}$$

Our assumptions on U imply $b(\cdot)$ is differentiable, with $b'(s) > 0$ for $s \in (0,1)$.

Despite the generality of the present setting, standard arguments (e.g., Milgrom, 1981; Milgrom and Weber, 1982) show that for the s and $b(\cdot)$ determined by (2) and (3), the β defined by (1) is a symmetric equilibrium strategy.[8]

For each $s \in [0,1]$, precisely one r, say $r(s)$, satisfies (2). We thus obtain a *reserve price function*, $r : [0,1] \to \mathbb{R}$. Our assumptions on U imply $r(\cdot)$ is differentiable, with $r'(s) > 0$ for $s \in (0,1)$. When choosing the reserve, the seller need not consider any $r < r(0)$ because it yields the same outcome as does $r(0)$ (all types bid). Similarly, he need not consider any $r > r(1)$ because (with probability one) it yields the same outcome as does $r(1)$ (no type bids). We can thus assume the seller chooses a reserve price from the interval $[r(0), r(1)]$. This allows a convenient change of variable from r to s; we can view the seller as choosing s by setting the reserve price $r = r(s)$.

[8] Other symmetric equilibria exist, differing only in what a type s bidder does: this type may submit any bid in $[r(s), b(s)]$, or not bid at all. All our results hold for any of these equilibria.

The seller's incentives for choosing s are determined in part, as we shall see, by the magnitude of the *gap* between the smallest possible equilibrium bid and the corresponding reserve price:

$$\gamma(s) := b(s) - r(s). \tag{4}$$

It is immediate from (2) and (3) that $\gamma(0) = 0$, that $\gamma(s) \equiv 0$ when values are private, and that $\gamma(s) > 0$ for $s > 0$ when values are interdependent. It is intuitive that value interdependency implies $b(s) > r(s)$: $b(s)$ is the willingness-to-pay of type s conditional on the event that the largest of the other bidders' types is also equal to s, but $r(s)$ is his willingness-to-pay conditional on the less favorable event that the largest of the other bidders' types is less than or equal to s.

Consider now an arbitrary $s \in (0,1)$ and a type $x > s$ bidder. Let $p(y,x,s)$ be this bidder's payment to the seller if the maximum of the other bidders' types is y (assuming for concreteness that the bidder wins in case of a tie):

$$p(y,x,s) = \begin{cases} r(s) & \text{for } y \in [0,s) \\ b(y) & \text{for } y \in [s,x] \\ 0 & \text{for } y \in (x,1] \end{cases}.$$

Raising s to $s' \in (s,x)$ changes this payment only for $y < s'$, and then

$$p(y,x,s') - p(y,x,s) = \begin{cases} r(s') - r(s) & \text{for } y \in [0,s) \\ r(s') - b(y) & \text{for } y \in [s,s') \\ 0 & \text{for } y \in [s',1] \end{cases}.$$

This difference is positive if $y \in [0,s)$, since $r(\cdot)$ is increasing. It is also positive for $y \in [s,s')$ in the private values case, because then $r(s') = b(s') > b(y)$ (zero gap). This gives us the well-known result that with private values, the bidder's expected payment to the seller,

$$\pi(x,s) := \mathbb{E}\left[p(Y_1,x,s)\right],$$

increases in s (and hence the reserve price).

In contrast, the fact that $r(s) < b(s)$ when values are interdependent implies that we can find $s' > s$ such that $r(s') < b(s)$ (see Figure 1 below). Then, for $y \in [s, s')$ the change in the payment of a type x bidder is negative:

$$r(s') - b(y) < b(s) - b(y) \leq 0.$$

Raising the reserve price from $r(s)$ to $r(s')$ causes the bidder's payment to fall from $b(y)$ to $r(s')$ because the type y bidder bids when the reserve price is $r(s)$, but not when it is $r(s')$. The resulting change in the bidder's expected payment, $\pi(x, s') - \pi(x, s)$, is thus the hatched area less the grey area in Figure 1 (both areas probability weighted). If the grey area is larger, the bidder's expected payment falls when the reserve price is raised from $r(s)$ to $r(s')$.

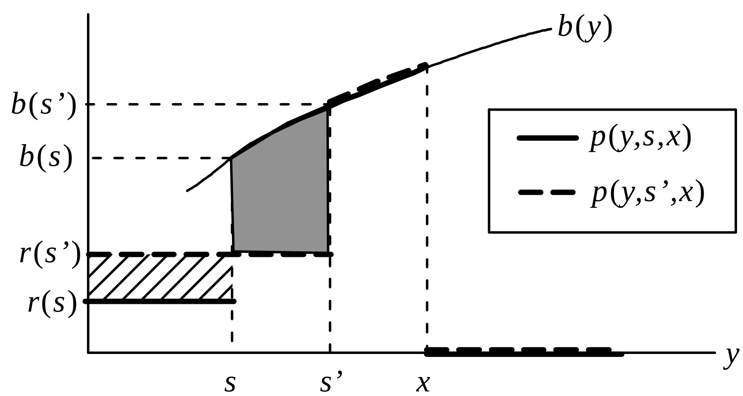

Figure 1

More formally, let $G(x) := F(x)^{n-1}$ denote the distribution of Y_1, and let $g(x) = G'(x)$. The expected payment to the seller of a type $x \geq s$ bidder is then

$$\pi(x, s) = r(s)G(s) + \int_s^x b(y)g(y)dy. \tag{5}$$

We write the partial derivative of $\pi(x, s)$ with respect to s as $\pi_s(s)$ because it does not depend on x :

$$\pi_s(s) := \pi_s(x, s) = G(s)r'(s) - g(s)\gamma(s). \tag{6}$$

This expression suggests that $\pi(x, s)$ decreases in s if the gap is large, in which case the bidder's expected payment paradoxically *increases* if the reserve price is *lowered*.

In order to relate the sign of π_s to the seller's incentives for choosing a screening level, note that his expected profit as a function of s is v_0 times the probability of no sale plus the sum of the bidders' expected payments:

$$\Pi(s) := v_0 F(s)^n + n \int_s^1 \pi(x, s) f(x) dx. \tag{7}$$

Differentiation yields, after some algebra using (5) and (6),

$$\Pi'(s) = nG(s)f(s)\left[v_0 - r(s)\right] + n(1 - F(s))\pi_s(s). \tag{8}$$

The first term in (8) is positive if $r(s) < v_0$, since then raising the reserve price raises the sale price in the event that only one bidder bids. But the second term is negative if $\pi_s(s) < 0$, since in this case raising the reserve price lowers the bidders' expected payments. If s^* is an interior maximizer of the seller's expected profit, then $\Pi'(s^*) = 0$ holds, which implies that the optimal reserve price is less than the seller's value if $\pi_s(s^*) < 0$.

3 A Specific Model

In order to obtain sharper results, we now consider a specific but relatively general model that gives rise to a reduced form utility function satisfying the assumptions of the previous section. The model is along the lines of Milgrom and Weber (1982).

Accordingly, we now assume that if bidder i obtains the object when his type is t_i and the vector of the other bidders' types is t_{-i}, his value for the object (maximal amount he would be willing to pay for it) is $\hat{v}(t_i, t_{-i})$. The function $\hat{v}$ is assumed to be invariant with respect to permutations of its last $n-1$ arguments, to be continuously differentiable with partial derivatives $\hat{v}_1 > 0$ and $\hat{v}_j \geq 0$ for $j > 1$, and to satisfy $\hat{v}(\mathbf{0}) = 0$.

Also of interest will be the expected value of a bidder with type x conditional on the maximum of the other bidders' types being equal to y,

$$v(x, y) := \mathbb{E}\left[\hat{v}(x, Y) \middle| \, Y_1 = y\right],$$

where $Y_1 > Y_2 > ... > Y_{n-1}$ are the order statistics of the other bidders' types, and $Y := (Y_1, ..., Y_{n-1})$. Noting that $v_x > 0$ because $\hat{v}_1 > 0$,[9] we henceforth assume a normalization,

$$\min_{(x,y)\in[0,1]^2} v_x(x, y) = 1, \tag{9}$$

accomplished by rescaling $\hat{v}$ if necessary.[10] We use the bidder's conditional expected value function to define, at any $(x, y) \in [0, 1]^2$, a local *interdependence measure:*

$$\rho(x, y) := \frac{v_y(x, y)}{v_x(x, y)}.$$

This function measures the sensitivity of a bidder's conditional expected value to increases in the maximum of the other bidders' types relative to its sensitivity to increases in his own type. Note that ρ is bounded below by 0, and $\rho \equiv 0$ in the private values case.

Linear Example. *A linear value function satisfying our assumptions is given by*

$$\hat{v}(t_i, t_{-i}) = t_i + \theta \sum_{j\neq i} t_j,$$

for some $\theta \geq 0$. Letting the type distribution be uniform on $[0, 1]$, a straightforward calculation shows that the conditional expected value function is

$$v(x, y) = x + \tfrac{1}{2}\theta n y.$$

From this one obtains the interdependence measure $\rho(x, y) = \frac{1}{2}\theta n$. This ρ is constant in (x, y) but, as is suggested by the nature of $\hat{v}$, increasing in the parameter θ and the number of bidders n.

The last component of the model to be specified is the risk attitude of the bidders. Each bidder is assumed to have the same Bernoulli utility function u for

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[9] For clarity, we write $v_x(x, y)$ and $v_y(x, y)$ for partial derivatives of v rather than $v_1(x, y)$ and $v_2(x, y)$.

[10] Specifically, dividing $\hat{v}$ by $\min_{t\in[0,1]^n} \hat{v}_1(t)$ yields a value function satisfying (9).

money, which is twice differentiable and satisfies $u' > 0$ and $u(0) = 0$. We now have an explicit expression for $U(-p, x, y)$, the conditional expected utility of a bidder when he purchases the good for price p, his type is x, and the maximum of the other bidders' types is y:

$$U(-p, x, y) := \mathbb{E}\left[u(\hat{v}(x, Y) - p) | Y_1 = y\right]. \tag{10}$$

This U satisfies the assumptions made in the previous section. If $\rho > 0$, then values are interdependent in the sense of the previous section: $U_3 > 0$.

4 The Role of Risk Aversion

Our first proposition derives useful comparative statics results about the effect of increasing the bidders' risk aversion on the equilibrium bid, reserve, and gap functions. Let $A := -u''/u'$ denote the bidders' Arrow-Pratt absolute risk aversion function.

Proposition 1 *For $k = 1, 2$, let $u_k : \mathbb{R} \to \mathbb{R}$ be twice differentiable with $u_k' > 0$ and $u_k(0) = 0$, and let A_k be its absolute risk aversion measure. Let r_k, b_k, and γ_k be the corresponding reserve, bid, and gap functions defined in* (2)-(4) *using* (10). *Then, for all $s \in (0, 1]$,*

(i) $b_1(s) \leq b_2(s)$ *if* $A_1 \geq A_2$;

(ii) $r_1(s) \leq r_2(s)$ *if* $A_1 \geq A_2$; *and*

(iii) $\gamma_1(s) \geq \gamma_2(s)$ *if* $A_1 \geq \hat{\lambda} \geq A_2$ *for some* $\hat{\lambda} \in \mathbb{R}$.

If the inequalities in these implications are made strict, then (i) *remains true if $\rho > 0$ and $n > 2$; and* (ii)-(iii) *remain true if $\rho > 0$.*

Parts (i) and (ii) of Proposition 1 hold because, when bidder i's risk aversion increases, his willingness-to-pay conditional on any event on which his value $\hat{v}(t_i, \tilde{t}_{-i})$ is random must decrease. His bid $b(s)$ is his willingness-to-pay conditional on the event $\{X = s, Y_1 = s\}$, and on this event his value is random if $n > 2$ and $\rho > 0$ (so that the values are interdependent). The reserve price $r(s)$ is his willingness to-pay conditional on the event $\{X = s, Y_1 \leq s\}$, and his value on this event is random if $\rho > 0$ (since $s > 0$ and $n \geq 2$).

More surprising is part (iii) of Proposition 1, which states that the gap $\gamma = b - r$ increases in bidder risk aversion. It does so because an increase in risk aversion decreases $b(s)$ less than it decreases $r(s)$. This is true because the event upon which $b(s)$ is determined, $\{X = s, Y = s\}$, has less residual uncertainty than does the event upon which $r(s)$ is determined, $\{X = s, Y \leq s\}$, which causes $b(s)$ to be less sensitive than $r(s)$ to changes in risk aversion.

The fact that the gap increases in bidder risk aversion if values are interdependent is why risk aversion and value interdependence are both required in order for the optimal reserve to be less than the seller's value. Recall from the discussion of (8) that the optimal reserve is less than the seller's value if an active bidder's expected payment to the seller decreases in the screening level, and from (6) that the gap must be sufficiently large for this to be true. The gap, it turns out, is not large enough if the bidders are risk neutral, regardless of the amount of value interdependence. This is shown in the proof of Proposition 2 below, which establishes that $r(s^*) > v_0$ if the bidders are risk neutral.

We first derive the bid, reserve, and gap functions for risk neutral bidders, b_0, r_0, and γ_0. For $u(z) \equiv z$ we have $U(-p, x, y) = v(x, y) - p$. Hence, (3) yields $b_0(s) = v(s, s)$, and (2) implies

$$r_0(s) = \frac{1}{G(s)} \int_0^s v(s, y) dG(y). \tag{11}$$

Integrating (11) by parts and using the definition $\gamma_0 = b_0 - r_0$ yields

$$\gamma_0(s) = \int_0^s \frac{G(y)}{G(s)} v_y(s, y) dy. \tag{12}$$

Proposition 2 *For any $v_0 \in [0, r_0(1))$, the profit-maximizing reserve price is strictly larger than v_0 if the bidders are risk neutral.*

Proof. For $0 < s < x \leq 1$, let $\pi^0(x, s)$ denote the expected payment of a bidder with type x given screening level s. From (6) and the definition of γ_0, the derivative of $\pi^0(x,s)$ with respect to s does not depend on x and can be written as

$$\pi^0_s(s) = G(s)r_0'(s) - g(s)\left(b_0(s) - r_0(s)\right).$$

Using (11), differentiating $G(s)r_0(s)$ yields

$$G(s)r_0'(s) + g(s)r_0(s) = g(s)b_0(s) + \int_0^s v_s(s,y)dG(y).$$

These two equalities imply the following equality, and (9) the inequality:

$$\pi^0_s(s) = \int_0^s v_s(s,y)dG(y) \geq G(s).$$

From this and (8) we obtain

$$\Pi'(s) \geq nG(s)\left[\left(v_0 - r_0(s)\right)f(s) + 1 - F(s)\right]. \tag{13}$$

Note that if $v_0 > 0$, then $\Pi'(s) > 0$ at any s in the nonempty interval $(0, r^{-1}(v_0)]$, since for such s we have $r(s) \leq v_0$. This implies that any maximizer s^* of Π on $[0,1]$ satisfies $r(s^*) > v_0$. If instead $v_0 = 0$, then because $r_0(s) = 0$, the square-bracketed term in (13) converges to 1 as $s \to 0$. Hence, $\bar{s} > 0$ exists such that $\Pi'(s) > 0$ on the interval $(0, \bar{s}]$, implying that $s^* > \bar{s} > v_0$. ■

Consequently, a necessary condition for the optimal reserve price to be less than v_0 is that the bidders have interdependent values *and* risk averse utility.

5 Low Reserve Prices

We now find conditions under which a profit-maximizing reserve price $r(s^*)$ is less than the seller's value. Since $r(s^*) \geq r(0) = 0$, the optimal reserve cannot be less than v_0 if $v_0 \leq 0$. If instead $v_0 \geq r(1)$, the outcome is uninteresting because either $s^* = 1$ and the object is surely not sold, or $s^* < 1$ and $r(s^*) < v_0$ holds trivially. The interesting case, and the one we focus on in this section, is therefore $v_0 \in (0, r(1))$.

Remark. *We note that while the interval* $(0, r(1))$ *of interesting seller values does shrink as the bidders become more risk averse (by Proposition* 1*), under our assumptions it always contains the unit interval* $(0, 1)$*, since*

$$r(s) \geq s \text{ for all } s \in [0, 1]. \tag{14}$$

To see why (14) *is true, note that* (10) *and* (2) *imply that for any* $s \in [0, 1]$,

$$\begin{aligned} u(0) = 0 &= \mathbb{E}[u(\hat{v}(s, Y) - r(s)) | \, Y_1 \leq s] \\ &\geq \mathbb{E}[u(\hat{v}(s, Y) - r(s)) | \, Y_1 = 0] \\ &= u(v(s, 0) - r(s)), \end{aligned}$$

and so $r(s) \geq v(s, 0)$*. This implies* (14)*, as* (9) *and* $v(0, 0) = 0$ *imply* $v(s, 0) \geq s$*.*

Now we rewrite the seller's marginal profit in (8) as

$$\Pi'(s) = nf(s) \left((v_0 - r(s)) \, G(s) + \frac{(1 - F(s))}{f(s)} \pi_s(s) \right). \tag{15}$$

For any v_0, denote the set of profit-maximizing screening levels as

$$S^*(v_0) := \arg \max_{s \in [0,1]} \Pi(s).$$

This set is nonempty because Π is continuous. Consider some $v_0 \in (0, r(1))$ and $s^* \in S^*(v_0)$. From (15) we see that $\Pi'(1) = nf(1)[v_0 - r(1)] < 0$, and hence $s^* < 1$. If $s^* = 0$ then $r(s^*) = 0 < v_0$ is immediate. Otherwise, $s^* \in (0, 1)$ and the first-order condition $\Pi'(s^*) = 0$ implies

$$(v_0 - r(s^*)) \, G(s^*) + \frac{(1 - F(s^*))}{f(s^*)} \pi_s(s^*) = 0, \tag{16}$$

reaffirming that $r(s^*) < v_0$ when $\pi_s(s^*) < 0$. We now seek conditions under which the latter inequality holds. In order to obtain easily verified conditions, we make use of the following assumptions.

A0. F is the uniform distribution on $[0, 1]$;

A1. A is a nonincreasing function; and

A2. $\hat{v}$ is a supermodular function.

Assumption A0 is made for the sake of simplicity and without loss of generality: given any continuous type distribution F, the familiar normalization of letting $F(t_i)$ be the type of player i yields an equivalent model with uniformly distributed types.[11]

Assumption A1 requires the bidders to exhibit nonincreasing absolute risk aversion, a commonly assumed property. As it is preserved under integration (e.g., Jewitt, 1987), A1 implies that $-U_{11}/U_1$ is nonincreasing in $-p$.

Assumption A2 implies positive *marginal* interdependence: a bidder's marginal value with respect to his own type is nondecreasing in the other bidders' types. Because supermodularity is preserved under integration (e.g., Vives, 1990), A2 implies that $v_x(x, y)$ is nondecreasing in y.

Now we define a function M on $\mathbb{R}^3_+$ by

$$M(s, \lambda, m) := \frac{(n-1)m}{n} H\left(\frac{\lambda m s}{n}\right) - 1, \tag{17}$$

where the function H is defined by

$$H(x) := 1 + \frac{e^{-x} - 1}{x}. \tag{18}$$

(We suppress the argument n of M for the sake of clarity.) An important property of M is that it strictly increases in s, λ, and m at any $(s, \lambda, m) \gg 0$.[12] Note also that $M(s, \lambda, m) = -1$ unless $(s, \lambda, m) \gg 0$.

Henceforth, we let λ and m denote parameters of the bidders' utility and value functions, respectively. First, we define

$$\lambda := \inf_{z \in \mathbb{R}} A(z) \tag{19}$$

[11] Technically, this rescaling of types should be done before the rescaling of $\hat{v}$ that yields the normalization (9).

[12] This is because H strictly increases on $\mathbb{R}_+$, with $\lim_{x \to 0} H(x) = 0$ and $\lim_{x \to \infty} H(x) = 1$.

to be the lower bound of the Arrow-Pratt measure of u.[13] Second, we define

$$m := \min_{(x,y)\in[0,1]^2} \rho(x,y) \tag{20}$$

to be the minimal amount of value interdependence. In light of Proposition 2, in the remainder of the paper we assume both λ and m are positive. Note that these parameters, and s, are positive if $M(s,\lambda,m) \geq 0$. The following lemma establishes that this inequality implies $\pi_s(s) < 0$, given A0-A2.

Lemma 1 *Under* A0-A2, *if* $M(s,\lambda,m) \geq 0$, *then*

$$\pi_s(s) < -M(s,\lambda,m)G(s). \tag{21}$$

Given its assumptions, Lemma 1 implies that if $M(1,\lambda,m) \geq 0$, then $\pi_s(s) < 0$ for sufficiently large $s \in (0,1]$. This suggests that an optimal reserve price may be less than the seller's value if (λ, m) is in the set

$$\Omega := \{(\hat{\lambda},\hat{m}) \in \mathbb{R}^2_+ : M(1,\hat{\lambda},\hat{m}) > 0\}.$$

Four properties of Ω are noteworthy: (a) it does not depend on the model primitives u, $\hat{v}$, or v_0; (b) it is a subset of $\mathbb{R}^2_{++}$; (c) it is comprehensive from above;[14] and (d) it is nonempty.[15] The following proposition implies that if $(\lambda,m) \in \Omega$, then seller values v_0 exist for which any profit-maximizing reserve price is less than v_0.

Proposition 3 *Under* A0-A2, *a strictly decreasing function* $\underline{v} : \Omega \to (0, r(1))$ *exists such that if* $(\lambda,m) \in \Omega$ *and* $v_0 \in [\underline{v}(\lambda,m), r(1))$, *then* $r(s^*) < v_0$ *for all* $s^* \in S^*(v_0)$.

[13] Note that U inherits this lower bound, i.e., $-U_{11}/U_1 \geq \lambda$ holds, which follows from integrating both sides of $-u'' \geq \lambda u'$.

[14] That is, $(\hat{\lambda},\hat{m}) \in \Omega$ implies $(\lambda',m') \in \Omega$ for all $(\lambda',m') \geq (\hat{\lambda},\hat{m})$. This is true because M increases in $(\hat{\lambda},\hat{m})$.

[15] Two easy ways to see that $\Omega \neq \varnothing$ are to observe that (i) $M(1,\hat{\lambda},\hat{m}) \to \infty$ as $m \to \infty$ if $\hat{\lambda} > 0$, or (ii) $M(1,\hat{\lambda},\hat{m}) \to m(n-1)/n - 1$ as $\hat{\lambda} \to \infty$, and for any $n > 1$ this limit is positive if m is sufficiently large.

Proof. The function M is continuous, and it is strictly increasing on $(0,1] \times \Omega$. Hence, since $M(0,\hat{\lambda},\hat{m}) = -1 < 0 < M(1,\hat{\lambda},\hat{m})$ for $(\hat{\lambda},\hat{m}) \in \Omega$, the equation $M(\underline{s},\hat{\lambda},\hat{m}) = 0$ defines a strictly decreasing function $\underline{s} : \Omega \to (0,1)$. Define $\underline{v}$ on Ω by $\underline{v}(\hat{\lambda},\hat{m}) := r(\underline{s}(\hat{\lambda},\hat{m}))$. Since $r : [0,1] \to \mathbb{R}_+$ is strictly increasing and $r(0) = 0$, the function $\underline{v}$ is strictly decreasing and satisfies $0 < \underline{v} < r(1)$.

Now, for the (λ, m) defined in (19)-(20), simplify notation by letting $\underline{s} = \underline{s}(\lambda, m)$ and $\underline{v} = \underline{v}(\lambda, m)$. If $r(s^*) < \underline{v}$, then $r(s^*) < v_0$ because $\underline{v} \leq v_0$. So suppose $r(s^*) \geq \underline{v}$. Then $r(s^*) \geq r(\underline{s})$. This implies $s^* \geq \underline{s}$, and so $M(s^*, \lambda, m) \geq M(\underline{s}, \lambda, m) = 0$. Hence, Lemma 1 implies $\pi_s(s^*) < 0$. Now, note that $s^* < 1$, because $s^* = 1$ would imply the first-order condition $\Pi'(1) \geq 0$, which by (15) would imply the contradiction $v_0 \geq r(1)$. Thus, $s^* \in (0,1)$, and so the first-order condition (16) holds. This, $\pi_s(s^*) < 0$, and $s^* < 1$ yield $r(s^*) < v_0$. ■

Recall that Ω is comprehensive from above. Hence, Proposition 3 implies that if $(\lambda, m) \in \Omega$, then replacing u by a more risk averse utility function and $\hat{v}$ by a value function with a larger interdependence measure, both of which satisfy the maintained assumptions and A1-A2, yields another bidder environment in which there is an interval of seller values for which any optimal reserve must be less than the seller's value. The following corollary sharpens this observation by showing that even for arbitrarily small positive seller values, an optimal reserve price will be less than the seller's value if the interdependence lower bound m exceeds $n/(n-1)$ and the bidder's risk aversion lower bound λ is sufficiently large.

Corollary 1 *Given* A0-A2 *and any $v_0 \in (0, r(1))$, there exists $\lambda_0 > 0$ such that if $\lambda > \lambda_0$ and $m > n/(n-1)$, then $r(s^*) < v_0$ for all $s^* \in S^*(v_0)$.*

Proof. Since r is continuous and $r(0) = 0 < v_0$, there exists $s_0 \in (0,1)$ such that $r(s_0) < v_0$. Since $H(\infty) = 1$, we see from (17) that $M(s_0, \hat{\lambda}, m) \to m(n-1)/n-1 > 0$ as $\hat{\lambda} \to \infty$. This implies the existence of $\lambda_0 > 0$ such that $M(s_0, \lambda_0, m) = 0$. Let $\lambda > \lambda_0$. Since $(s_0, \lambda_0, m) \gg 0$, $M(s_0, \lambda, m) > M(s_0, \lambda_0, m)$, and so $(\lambda, m) \in \Omega$. Let

$\underline{s}$ and $\underline{v}$ be the functions on Ω defined in the proof of Proposition 3. Since $\underline{s}$ is a decreasing function, we have $\underline{s}(\lambda, m) \leq s_0$. This, as r is an increasing function, yields

$$\underline{v}(\lambda, m) = r(\underline{s}(\lambda, m)) \leq r(s_0) < v_0.$$

Proposition 3 now implies $r(s^*) < v_0$ for all $s^* \in S^*(v_0)$. ■

Our last proposition bears on the question of why absolute auctions are ever used. It gives conditions under which an optimal reserve price is arbitrarily close to the lowest possible value, $\hat{v}(\mathbf{0}) = 0$, if the risk aversion and interdependence lower bounds are sufficiently large. This result is consistent with the use of absolute auctions, since the equilibrium outcome of an auction with a reserve $r \approx 0$ is approximately the same as that of an auction with no reserve at all.

Proposition 4 *Given* A0-A2*, any* $v_0 \in (0, 1]$*, and* $\varepsilon \in (0, r(1))$*, there exists* $\lambda_\varepsilon > 0$ *such that if* $\lambda > \lambda_\varepsilon$ *and* $m > 2n/(n-1)$*, then* $r(s^*) < \varepsilon$ *for all* $s^* \in S^*(v_0)$*.*

Proof. Define $s_\varepsilon := r^{-1}(\varepsilon)$, and note that $s_\varepsilon \in (0, 1)$ because $\varepsilon \in (0, r(1)) = (r(0), r(1))$ and r is strictly increasing. Since $m(n-1)/n > 2$, we have

$$\lim_{\lambda \to \infty} M(s_\varepsilon, \lambda, m) = m(n-1)/n - 1 > 1.$$

Thus, $\lambda_\varepsilon > 0$ exists such that $M(s_\varepsilon, \lambda_\varepsilon, m) = 1$. Let $s \in [s_\varepsilon, 1)$ and $\lambda > \lambda_\varepsilon$. Then, since M is an increasing function, $M(s, \lambda, m) > 1$. Lemma 1 thus implies inequality (21). It, together with (15) and A0, yield

$$\begin{aligned} \Pi'(s) &= ns^{n-1}(1-s)\left(\frac{v_0 - r(s)}{1-s} + \frac{\pi_s(s)}{s^{n-1}}\right) \\ &< ns^{n-1}(1-s)\left(\frac{v_0 - r(s)}{1-s} - M(s, \lambda, m)\right). \end{aligned} \tag{22}$$

From (14) we have $r(s) \geq s$, which with $v_0 \leq 1$ implies $v_0 - r(s) \leq 1 - s$. Hence, from (22) and $M(s, \lambda, m) > 1$ we obtain

$$\Pi'(s) < ns^{n-1}(1-s)\,(1 - M(s, \lambda, m)) < 0.$$

As this is true for any $s \in [s_\varepsilon, 1)$, we conclude that $s^* < s_\varepsilon$, and hence $r(s^*) < \varepsilon$, for any $s^* \in S^*(v_0)$. ■

Lastly, we note that the lower bounds on value interdependence required by the hypotheses of Corollary 1 and Proposition 4 are not implausibly large. In the Linear Example of Section 3 we have $m = \frac{1}{2}\theta n$, and so the two bounds become $\theta(n-1) > 2$ and $\theta(n-1) > 4$, respectively.

6 Conclusion

When applied to auctions, the "Monopoly Exclusion Principle" stipulates that a profit-maximizing reserve price should be larger than the seller's value for the object being sold. This well-known theoretical proposition fails to hold empirically in those auctions in which the seller is observed to set no reserve price, or only a very low one. The contribution of this paper is to show that in second price auctions, even in theory the Exclusion Principle does not hold if bidders are sufficiently risk averse *and* the interdependence of their values exceeds a certain bound: the profit-maximizing reserve price is then less than the seller's value. As bidder risk aversion increases, the optimal reserve converges to the lowest possible bidder value, so that the equilibrium outcome is approximately that of an absolute auction.

Appendix

Proof of Proposition 1. For $k = 1, 2$, let U^k be the reduced form utility function defined from u_k by (10).

(i) From (3) and (10) we obtain

$$\mathbb{E}\left[u_1(\hat{v}(s, s, Y_{-1}) - b_1(s))|Y_1 = s\right] = 0 = u_1(0).$$

Thus, since u_1 is more risk averse than u_2,

$$\mathbb{E}\left[u_2(\hat{v}(s, s, Y_{-1}) - b_1(s))|Y_1 = s\right] \geq u_2(0),$$

and this inequality is strict if $A_1 > A_2$ when $n > 2$ and $\rho > 0$ (so the values are interdependent). From (3) and (10) we also have

$$\mathbb{E}\left[u_2(\hat{v}(s,s,Y_{-1}) - b_2(s))|Y_1 = s\right] = 0 = u_2(0).$$

This proves that $b_1(s) \leq (<) b_2(s)$, since $u_2' > 0$.

(ii) This is proved similarly to (i), using (2) rather than (3).

(iii) Let $\hat{u}(w) := \left(1 - e^{-\hat{\lambda} w}\right)/\hat{\lambda}$, let $\hat{U}$ be the reduced form utility function defined from this CARA $\hat{u}$ by (10), and let $\hat{r}$, $\hat{b}$, and $\hat{\gamma}$ be defined from $\hat{U}$ by (2)-(4). Note that (iii) together with its strict version consists of four implications:

$$\begin{aligned}
&\text{(a)} \quad A_1 \geq \hat{\lambda} \;\Rightarrow\; \gamma_1 \geq \hat{\gamma}, \\
&\text{(b)} \quad A_1 > \hat{\lambda} \text{ and } \rho > 0 \;\Rightarrow\; \gamma_1 > \hat{\gamma}, \\
&\text{(c)} \quad \hat{\lambda} \geq A_2 \;\Rightarrow\; \hat{\gamma} \geq \gamma_2, \\
&\text{(d)} \quad \hat{\lambda} > A_2 \text{ and } \rho > 0 \;\Rightarrow\; \hat{\gamma} > \gamma_2.
\end{aligned}$$

It suffices to prove (a) and (b), since the proofs of (c) and (d) are very similar.

To prove (a), suppose by way of contradiction that $s \in (0,1]$ exists such that $\gamma_1(s) < \hat{\gamma}(s)$. This and (4) yield

$$-r_1(s) + b_1(s) - \hat{b}(s) < -\hat{r}(s). \tag{23}$$

From (2) and (3) we obtain

$$U^1(-b_1(s), s, s) = \mathbb{E}[U(-r_1(s), s, Y_1) \,|\, Y_1 \leq s]. \tag{24}$$

Since $A_1 \geq \hat{\lambda}$, $U^1(\cdot, s, y)$ is weakly more risk averse than $\hat{U}(\cdot, s, y)$ for any y, and so the above equality implies

$$\hat{U}(-b_1(s), s, s) \leq \mathbb{E}[\hat{U}(-r_1(s), s, Y_1) \,|\, Y_1 \leq s]. \tag{25}$$

Now, because $\hat{U}(\cdot, s, y)$ is a CARA function for any y, adding $b_1(s) - \hat{b}(s)$ to the first argument of $\hat{U}$ in the above inequality maintains the inequality:

$$\hat{U}(-\hat{b}(s), s, s) \leq \mathbb{E}[\hat{U}(-r_1(s) + b_1(s) - \hat{b}(s), s, Y_1) \,|\, Y_1 \leq s]. \tag{26}$$

But this, together with (3) and (23), imply an impossibility:

$$0 = \hat{U}(-\hat{b}(s), s, s) < \mathbb{E}[\hat{U}(-\hat{r}(s), s, Y_1) \,|\, Y_1 \leq s] = 0. \tag{27}$$

So the initial supposition is false, proving (a).

Implication (b) is proved similarly. The initial supposition is now a weak inequality, causing (23) to become a weak inequality. Equality (24) still holds. Because $A_1 > \hat{\lambda}$ and $\rho > 0$, $U^1(\cdot, s, y)$ is more risk averse than $\hat{U}(\cdot, s, y)$ and $U_3^1, \hat{U}_3 > 0$. Thus, (24) implies that (26) is now a strict inequality. That, together with (3) and (23), again imply the impossibility (27), proving (b). ■

Proof of Lemma 1. The bid function is defined by (3): $U(-b(x), x, x) = 0$. The reserve function is defined by (2), or rather,

$$\int_0^s U(-r(s), s, y)dG(y) = 0. \tag{28}$$

Differentiating (28) with respect to s yields

$$r'(s) = \frac{U(-r(s), s, s)g(s) + \int_0^s U_2(-r(s), s, y)dG(y)}{\int_0^s U_1(-r(s), s, y)dG(y)}. \tag{29}$$

Since $U_1 > 0$ and $U_2 > 0$, and the first term of the numerator is nonnegative by (28) and $U_3 \geq 0$, we have $r'(s) > 0$ for all $s \in (0, 1]$.

We first show that under A0-A2,

$$r'(s) \leq \frac{1 - e^{-\lambda\gamma(s)}}{\lambda} \frac{g(s)}{G(s)} + \frac{1}{G(s)} \int_0^s v_s(s, y)dG(y). \tag{30}$$

The first step in deriving this inequality is to note that, by A1, Pratt (1964, equation (21)), and the fact that u is weakly more risk averse than the CARA utility function $\hat{u}(x) = (1 - e^{-\lambda x})/\lambda$, we have

$$\frac{u(v-r) - u(v-b)}{u'(v-b)} \leq \frac{\hat{u}(v-r) - \hat{u}(v-b)}{\hat{u}'(v-b)} = \frac{1 - e^{-\lambda(b-r)}}{\lambda}.$$

This implies that for any v, r, and $b \geq r$,

$$u(v-r) - u(v-b) \leq \frac{1 - e^{-\lambda(b-r)}}{\lambda} u'(v-b).$$

Substituting $\hat{v}(s, Y)$ for v and taking expectations conditional on $Y_1 = s$ yields

$$\frac{U(-r,s,s) - U(-b,s,s)}{U_1(-b,s,s)} \leq \frac{1 - e^{-\lambda(b-r)}}{\lambda}. \tag{31}$$

Jewitt (1987) shows that A1 implies $-U_1$ is weakly more risk averse than $-U$. Thus, since (3) and (28) imply

$$U(-b(s),s,s) = \frac{1}{G(s)} \int_0^s U(-r(s),s,y)dG(y),$$

we have

$$U_1(-b(s),s,s) \leq \frac{1}{G(s)} \int_0^s U_1(-r(s),s,y)dG(y). \tag{32}$$

Now, by (3) we see that the first term on the right side of (29) can be written as

$$\frac{U(-r(s),s,s)g(s)}{\int_0^s U_1(-r(s),s,y)dG(y)} = \frac{[U(-r(s),s,s) - U(-b(s),s,s)]\, g(s)}{\int_0^s U_1(-r(s),s,y)dG(y)}.$$

From this and (31)-(32) we obtain

$$\frac{U(-r(s),s,s)g(s)}{\int_0^s U_1(-r(s),s,y)dG(y)} \leq \frac{1 - e^{-\lambda(b(s)-r(s))}}{\lambda} \frac{g(s)}{G(s)}. \tag{33}$$

Furthermore, notice that

$$\begin{aligned}
& \frac{1}{G(s)} \int_0^s U_2(-r(s),s,y)dG(y) \\
= & \frac{1}{G(s)} \int_0^s \mathbb{E}\left[u'(\hat{v}(s,Y) - r(s))\hat{v}_s(s,Y) \; |Y_1 = y\right] dG(y) \\
\leq & \frac{1}{G(s)} \int_0^s \mathbb{E}\left[u'(\hat{v}(s,Y) - r(s)) \; |Y_1 = y\right] dG(y) \times \frac{1}{G(s)} \int_0^s \mathbb{E}\left[\hat{v}_s(s,Y) \; |Y_1 = y\right] dG(y) \\
= & \frac{1}{G(s)} \int_0^s U_1(-r(s),s,y)dG(y) \times \frac{1}{G(s)} \int_0^s v_s(s,y)dG(y),
\end{aligned} \tag{34}$$

where the inequality follows from Chebyshev's sum inequality, since $(\lambda, m) \gg 0$ implies U_1 is nonincreasing in y and A2 implies v_x is nondecreasing in y. Substituting (33) and (34) into (29) we obtain (30).

We now find sufficient conditions for $\pi_s < 0$, or equivalently, $-\pi_s > 0$. By (6) and (30), we have

$$\begin{aligned}
-\pi_s(s) &= g(s)\gamma(s) - r'(s)G(s) \\
&\geq g(s)\left(\gamma(s) + \frac{e^{-\lambda\gamma(s)} - 1}{\lambda}\right) - \int_0^s v_s(s,y)dG(y) \\
&= H\left(\lambda\gamma(s)\right) g(s)\gamma(s) - \int_0^s v_s(s,y)dG(y),
\end{aligned} \tag{35}$$

where H is defined in (18) and satisfies $H'(x) = (e^x - x - 1)e^{-x}/x^2 > 0$, $H(0) = 0$, and $H(\infty) = 1$.

Now, since $\lambda > 0$, Proposition 1 (iii) and (12) imply that

$$\gamma(s) > \gamma_0(s) = \int_0^s \frac{G(y)}{G(s)} \frac{v_y(s,y)}{v_s(s,y)} v_s(s,y) dy$$

for all $s > 0$. By the definition of m in (20), we have

$$\frac{v_y(s,y)}{v_s(s,y)} \geq m,$$

and hence

$$\gamma(s) > m \int_0^s \frac{G(y)}{G(s)} v_s(s,y) dy = m \frac{1}{G(s)} \int_0^s \frac{G(y)}{g(y)} v_s(s,y) dG(y). \tag{36}$$

Since A2 implies that $v_s(s,y)$ is nondecreasing in y, and A0 implies $G(y)/g(y) = y/(n-1)$ is increasing in y, applying Chebyshev's sum inequality to (36) yields

$$\begin{aligned}
\gamma(s) &> m \frac{1}{G(s)} \int_0^s \frac{G(y)}{g(y)} dG(y) \times \frac{1}{G(s)} \int_0^s v_s(s,y) dG(y) \\
&= \frac{ms}{n} \times \frac{1}{G(s)} \int_0^s v_s(s,y) dG(y) \qquad (37) \\
&\geq \frac{ms}{n}, \qquad (38)
\end{aligned}$$

where the second inequality holds because (9) implies $v_s \geq 1$. Replacing the first $\gamma(s)$ in (35) with the term in (38) and the second $\gamma(s)$ in (35) with the term in (37), we obtain for $s, \lambda, m > 0$ that

$$\begin{aligned}
-\pi_s(s) &> \left(H\left(\frac{\lambda m s}{n}\right) \frac{(n-1)m}{n} - 1 \right) G(s) \times \frac{1}{G(s)} \int_0^s v_s(s,y) dG(y) \\
&= M(s,\lambda,m) \int_0^s v_s(s,y) dG(y),
\end{aligned}$$

using (17). Thus, if $M(s,\lambda,m) \geq 0$, we obtain the desired inequality,

$$-\pi_s(s) > M(s,\lambda,m) \int_0^s dG(y) = M(s,\lambda,m) G(s),$$

since $v_s(s,y) \geq 1$. ■

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