Penn Institute for Economic Research
Department of Economics
University of Pennsylvania
3718 Locust Walk
Philadelphia, PA 19104-6297
pier@econ.upenn.edu
http://economics.sas.upenn.edu/pier

PIER Working Paper 16-026

Intermediation as Rent Extraction

by

Maryam Farboodi

Gregor Jarosch

Guido Menzio

http://ssrn.com/abstract=2904061

Intermediation as Rent Extraction*

MARYAM FARBOODI
Princeton University

GREGOR JAROSCH
Stanford University and NBER

GUIDO MENZIO
University of Pennsylvania and NBER

December 2016

Abstract

This paper develops a theory of asset intermediation as a pure rent extraction activity. Agents meet bilaterally in a random fashion. Agents differ with respect to their valuation of the asset's dividends and with respect to their ability to commit to take-it-or-leave-it offers. In equilibrium, agents with commitment behave as intermediaries, while agents without commitment behave as end users. Agents with commitment intermediate the asset market only because they can extract more of the gains from trade when reselling or repurchasing the asset. We study the extent of intermediation as a rent extraction activity by examining the agents' decision to invest in a technology that gives them commitment. We find that multiple equilibria may emerge, with different levels of intermediation and with lower welfare in equilibria with more intermediation. We find that a decline in trading frictions leads to more intermediation and typically lower welfare, and so does a decline in the opportunity cost of acquiring commitment. A transaction tax can restore efficiency.

JEL Codes: D11, D21, D43, E32.

Keywords: Intermediation, Rent extraction.

*We thank our audiences at Princeton, Stanford, UCLA, McGill, EIEF, Banque de France and EPFL Lausanne. We are especially grateful to Fernando Alvarez, Francesco Lippi, Paolo Martellini, Adrian Masters, Claudio Michelacci and Randy Wright for their comments. The paper was formerly circulated as "Tough Middlemen".

1 Introduction

This paper develops a theory of asset intermediation as a pure rent extraction activity, it studies the extent and determinants of this type of intermediation and its consequences for welfare. Intermediaries in asset markets trade frequently, purchase assets at relatively low prices and sell them at relatively high prices. The standard view—first formalized by Rubinstein and Wolinsky (1987)—is that intermediaries are better connected than final users and, for this reason, they trade more frequently, they can purchase assets at low prices and resell them at high prices. Sellers are willing to trade assets to intermediaries at low prices because it would take them a long time to find buyers on their own. Buyers are willing to purchase assets from intermediaries at high prices because it would take them long time to find sellers on their own. According to this view, intermediaries are faster traders and, for this reason, they can charge a bid-ask spreads. This paper takes the opposite view of intermediation: intermediaries are agents who are better at extracting gains from trade than final users and, for this reason, they can purchase assets at low prices, resell them at high prices and trade more frequently. A seller is willing to trade an asset to an intermediary because, even though it would take him the same amount of time to find a buyer on his own, he would get a worse price. A buyer is willing to purchase an asset to an intermediary because, even though it would take him the same amount of time to find a seller on his own, he would pay a higher price. According to the view advanced in this paper, intermediaries are able to charge higher bid-ask spreads and, for this reason, they trade faster. The aim of the paper is to explore the descriptive and normative implications of this view of intermediation as a pure rent extraction activity.

We develop our theory in the context of the textbook asset market model of Duffie, Gârleanu and Pedersen (2005). Specifically, we consider a market populated by heterogeneous agents who trade an asset in fixed supply. The trading process is decentralized and frictional, in the sense that agents need to search the market to find a potential trading partner. Agents are heterogeneous along two dimensions. First, some agents enjoy a high flow payoff when holding the asset, while others enjoy a low flow payoff. This dimension of heterogeneity generates a motive for trade. Second, some agents can commit to take-it-or-leave-it offers when meeting a trading partner, while others cannot commit and end up either on the receiving end of a take-it-or-leave-it offer or bargaining over the price of the asset. This dimension of heterogeneity is the main difference between our model and Duffie, Garleanu and Pedersen (2005) and the premise of our theory of intermediation as a rent extraction activity.

The first part of the paper characterizes the properties of equilibrium for given measures of agents with and without commitment. We find that the equilibrium displays a rich pattern of trade. Unsurprisingly, the equilibrium is such that low-valuation agents sell the asset to high-valuation agents irrespective of their commitment type. More surprisingly, the equilibrium is such that low-valuation agents without commitment sell to low-valuation agents with commitment and high-valuation agents with commitment sell to high-valuation agents without commitment. Intuitively, a low-valuation agent without commitment trades the asset to a low-valuation agent with commitment because the latter can sell the asset to a high-valuation trader at a higher price than the former. Similarly, a high-valuation agent with commitment trades the asset to a high-valuation agent without commitment because the former can go back to the market and purchase another unit of the asset at a lower price than the latter.

In equilibrium, agents without commitment act as final users—in the sense that they buy the asset only when they have a high valuation for it and only sell the asset when their valuation falls—and agents with commitment act as intermediaries—in the sense that they buy and sell the asset irrespective of their valuation. Also, agents with commitment trade more frequently, buy at lower prices and sell at higher prices than agents without commitment. Agents with commitment act as intermediaries not because they are better at finding trading partners (as in Rubinstein and Wolinsky 1987) but because they are better at extracting rents. In this sense, our model is a theory of intermediation as a pure rent extraction activity.

The second part of the paper studies the extent and determinants of intermediation as a rent extraction activity. To this aim, we consider the agents' decision to invest in a technology that gives them the power to commit to posted prices (e.g., a technology to delegate negotiations to salesmen without discretion over prices). We show that the return to investing in such a technology is a hump-shaped function of the fraction of agents with commitment (i.e., intermediaries) operating in the market. When the fraction of intermediaries is low, agents without commitment are protected from exploitation because they can easily meet other agents without commitment with whom they share the gains from trade. Thus, the return on investing in the commitment technology is low. When the fraction of intermediaries is high, there are few agents without commitment that can be exploited. Thus, the return on investing in the commitment technology is also low. The fact that the return on investing in the commitment technology is hump-shaped implies that there may be multiple equilibria with different levels of intermediation. Multiple equilibria can be welfare ranked, with welfare being lower the higher is the extent of intermediation, and any equilibrium in which agents spend resources to acquire the commitment technology is inefficient. These findings are easy to understand. In our model, intermediation is a pure rent extraction activity which benefits the intermediary but does not benefit society in any way. The more resources agents devote to become intermediaries, the lower is welfare. And if agents devote any resources to become intermediaries, equilibrium is inefficient.

Our most surprising findings are related to the effect of declining trading frictions on the extent of intermediation and welfare. It would seem natural to conjecture that, when trading frictions become smaller, the return from acquiring a commitment technology to extract rents would fall and so would the extent of intermediation. After all, in a Walrasian Equilibrium, being able to extract rents is worthless because perfect competition fully protects buyers and sellers from exploitation. The conjecture turns out to be wrong. We show that, when trading frictions become smaller, the return from acquiring the commitment technology rises and so does the extent of intermediation. In fact, as trading frictions get smaller, the decline in the rents that can be extracted by an agent with commitment is outweighed by the increase in the rate at which an agent with commitment encounters opportunities for rent extraction. This finding suggests that, under the view of intermediation as a rent-extraction activity, one should not expect intermediation to disappear as trading frictions become smaller and smaller because of improvements in information and communication technology. On the contrary, under the view of intermediation as a rent-seeking activity, one can explain the rise of the intermediation sector experienced (see, e.g., Philippon 2015) as the natural consequence of a decline in trading frictions. Even more surprisingly, we find that, if all agents face the same cost of acquiring the commitment technology, a decline in trading frictions lowers welfare (as long as the fraction of agents with commitment is interior). That is, the welfare gains created by a decline in trading frictions are more than eroded by the welfare costs associated with the rise of intermediation. Furthermore, we show that when the flow payoff of the asset falls agents increasingly select into rent extraction activities with adverse consequences. This formalizes a novel mechanism through which an environment of low returns affects financial activity in unintended and potentially undesirable ways. [1]

The last part of the paper studies the effect of introducing a tax on the transactions of the asset. This is a natural question since the laissez-faire equilibrium is typically inefficient. We show that the equilibrium pattern of trade depends on the size of the transaction tax. When the tax is relatively small, the pattern of trade is the same as

[1] The argument is distinct, yet similar in spirit, to the "reach for yield" mechanism first developed in Rajan (2006) where market participants move towards increasingly risky investment when facing a low risk-free rate.

in the laissez-faire equilibrium. That is, the pattern of trade includes both fundamental transactions—i.e. transactions between low and high valuation agents—and intermediation transactions—i.e. transactions between agents with the same valuation and different commitment power. When we increase the tax, the fundamental transactions still take place but the intermediation transactions break down. When we further increase the tax, all trades break down. We also show that the transaction tax lowers the return from investing in the commitment technology. For any arbitrary distribution of costs to acquire the commitment technology, the tax that maximizes welfare is such that the after-tax surplus in any fundamental trade is zero. Intuitively, this is the optimal tax level because it reproduces a key feature of Walrasian Equilibrium, namely that the surplus in any particular trade between a buyer and a seller is zero and, thus, investing in the technology to commit to prices to extract more surplus is worthless. The optimal tax does not only maximize welfare, but also implements the first-best allocation.

The paper contributes to the search-theoretic literature on intermediation started by Rubinstein and Wolinsky (1987) and Kyiotaki and Wright (1989). Rubinstein and Wolinsky (1987) study the emergence of intermediation in a product market with search frictions, in which agents might differ with respect to the rate at which they meet others. The paper argues that studying intermediation requires modelling explicitly the process and the frictions of trade and, hence, departing from the notion of Walrasian Equilibrium. The main finding is that, in equilibrium, intermediaries (agents who do not produce nor consume the good) are active only if they have a higher meeting rate than final users. Kyiotaki and Wright (1989) discover an alternative theory of intermediation while studying the emergence of commodity money. They show that agents who do not produce nor wish to consume the commodity with the lowest storage cost (the commodity that ends up being used as a medium of exchange) act as intermediaries, as they purchase the commodity from producers only to resell it to consumers. Wright and Wong (2014) highlight the many similarities between monetary and intermediation theory. Nosal, Wong and Wright (2015) generalize Rubinstein and Wolinsky (1987) by allowing consumers, producers and intermediaries to differ with respect to bargaining power, meeting rates and holding costs and find conditions under which intermediaries are active in equilibrium, as well as conditions under which intermediation is essential. Nosal, Wong and Wright (2016) push the analysis further by allowing agents to choose in real time whether to be producers or intermediaries.

A related strand of literature has focused on intermediation in asset markets using the search-theoretic framework of Duffie, Garleanu and Pedersen (2005). Farboodi, Jarosch and Shimer (2016) study the equilibrium pattern of trade in a version of Duffie, Garleanu and Pedersen (2005) where agents differ with respect to their meeting rate. They also study the agents' choice of a meeting rate and find that, in general, ex-ante identical agents make different choices. Hugonnier, Lester and Weill (2016) study the equilibrium pattern of trade in a version of Duffie, Garleanu and Pedersen (2005) where the agent's valuation for the asset is a continuous variable. They find that agents with average valuations act as intermediaries, in the sense that they purchase the asset from agents with a lower valuation and sell it to agents with higher valuation. Uslu (2016) considers in a rich unified framework heterogeneity in meeting rates and valuation.

The paper closest to ours is Masters (2008). The paper studies a version of the frictional product market of Diamond (1982) in which agents differ with respect to their cost of production and bargaining power. It shows that agents with high costs of production and high bargaining power become intermediaries, in the sense that they neither produce nor consume the final good, but rather transfer it from producers to consumers. This is the only paper we know that identifies differences in the ability to extract gains from trade as a motive for intermediation. However, this paper is very different from ours because it focuses on a product market, where the gains from trade are fundamentally static, rather than on an asset market, where the source of the gains from trade is dynamic. This is why, for instance, in Masters (2008) differences in bargaining power are not enough to create intermediation, while they are in our model.

2 Environment

We consider the market for an indivisible asset. The supply of the asset is fixed and of measure $A = 1/2$. The market for the asset is populated by a measure 1 of heterogeneous agents. An agent's type is described by a couple $\{i, j\}$, where $i = \{S, T\}$ denotes the agent's commitment power and $j = \{L, H\}$ denotes the agent's valuation of the asset. The labels S and T stand for *Soft* an *Tough*. The labels L and H stand for *Low* and *High*. The first dimension of an agent's type is permanent. The measure of agents with commitment power S is constant and equal to ϕ_S, with $\phi_S \in [0, 1]$, and the measure of agents with commitment power T is constant and equal to $\phi_T = 1 - \phi_S$. The second dimension of an agent's type is transitory. In particular, an agent's valuation switches from L to H at the Poisson rate $\sigma > 0$. Simmetrically, an agent's valuation switches from H to L at the Poisson rate σ.

An agent can either hold 0 or 1 units of the asset. An agent of type $\{i, j\}$ gets a utility of u_j per unit of time in which he holds the asset, with $u_H > u_L \geq 0$. An agent gets a utility of 0 per unit of time in which he does not hold the asset. Agents have linear utility with respect to a numeraire good, which is used as a medium of exchange in the asset market. Agents discount future utilities at the exponential rate $r > 0$.

Trade is bilateral and frictional. In particular, one agent meets another randomly selected agent at the Poisson rate λ, with $\lambda > 0$. If the meeting involves two agents without asset or two agents with the asset, there is no opportunity to trade. If an agent with the asset meets an agent without the asset, there is a trading opportunity. The terms of the trade depend on the commitment power of the two agents. In particular, if an agent of type T meets an agent of type S, the agent of type T makes a take-it-or-leave-it offer to the agent of type S. The offer consists of p units of the numeraire good to be exchanged for the ownership of the asset. If the agent of type S accepts the offer, the trade is executed. Otherwise, the trade is not completed and the agents part ways. If two agents of type T meet, one is randomly selected to make a take-it-or-leave-it offer to the other. If two agents of type S meet, they play an alternating-offer bargaining game à la Rubinstein (1982) with a risk of breakdown $\delta > 0$. We assume that the bargaining game takes place in virtual time and we consider the limit for $\delta \to 0$.

A few comments about the environment are in order. We assume that agents are heterogeneous along two dimensions. First, we assume that agents differ with respect to their valuation of the asset and that an agent's valuation changes over time. The assumption is common in the literature and is meant to capture either literally variation across agents and over time in the utility of the services of the asset or, in reduced-form, variation across agents and over time in the ability to hedge any risk associated with the payoff of the asset. The assumption is needed to generate and maintain a motive for trading the asset. Indeed, if all agents had the same valuation for the asset, then there would be no trade. If agents had different valuations but these valuations remained constant over time, the asset would eventually end up in the hands of the agents with the highest valuation and trade would stop.

Second, we assume that agents differ with respect to their ability to commit to take-it-or-leave-it offers. The assumption is the main novelty of our environment relative to the previous literature and, as we shall see, it leads to non-fundamental trades. The assumption can be interpreted as saying that some agents can commit to posted prices—because, e.g., they can delegate trade to representatives without the authority to accept/propose any price different from the one pre-specified by the agent—while some agents cannot commit to post prices and, hence, end up bargaining over the terms of trade. In section

3, we will study the equilibrium of the model taking as given the measure of agents with commitment power. In section 4, we will study the equilibrium of the model when agents can acquire the technology to commit to posted prices at some cost.

We assume that the measure of the asset is half the measure of the population and that the stochastic process for the agent's valuation guarantees that, in a stationary equilibrium, exactly half of the agents have a high valuation for the asset while the other half has a low valuation. These assumptions are made for analytical tractability, as they imply that the equilibrium will be symmetric, i.e. the measure of agents with high valuation who do not hold the asset will be equal to the measure of agents with low valuation who own the asset.

The model is deliberately simple and abstract. Its purpose is to provide a framework in which to think about the effect of heterogeneity in commitment power in a decentralized asset market. There are many examples of decentralized asset markets. One fitting example is the housing markets. In this market, trade is decentralized, agents have different and time-varying utilities from living in a particular house, and some agents—say developers and flippers—may be able to commit to take-it-or-leave-it offers, while other agents may bargain. Another example may be the fine art market. In this market, trade is typically decentralized, agents have different and time-varying valuations for the same piece of art, and some agents—say art gallerists—may be able to commit to take-it-or-leave-it offers. Finally, as pointed out by Duffie, Garleanu and Pedersen (2005), there are some financial assets (over-the-counter markets) that operate in a decentralized fashion. It is not far-fetched to think that, in these markets, some agents have more commitment power than others.

3 Equilibrium

We look for an equilibrium in which trade follows the pattern illustrated in Figure 1. That is, we look for an equilibrium in which an agent of type (S, L) never buys the asset and sells it to traders of type (S, H), (T, L) and (T, H), an agent of type (T, L) sells to traders of type (T, H) and (S, H), an agent of type (T, H) sells to traders of type (S, H), and an agent of type (S, H) never sells the asset. In words, we look for an equilibrium in which the asset flows from low to high-valuation agents, and from agents without commitment power to other agents without commitment power either directly or by way of agents who can commit. This is the pattern of trade that one would naturally expect to emerge in

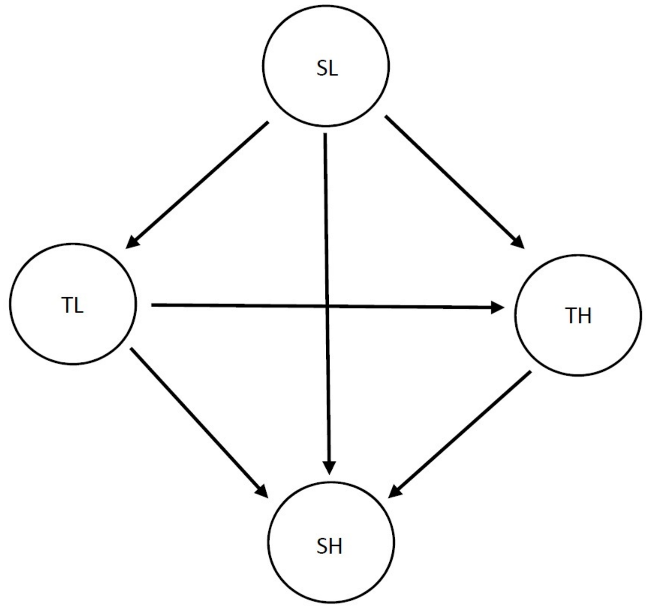


Figure 1: Pattern of Trade

equilibrium. Certainly, one would expect low-valuation agents to sell the asset to high-valuation ones, as the trade increases the utility flow provided by the asset. However, one would also expect low-valuation agents without commitment to sell the asset to low-valuation agents with commitment, as the latter are better at extracting rents than the former.

In subsection 3.1, we derive down the conditions for equilibrium. In subsection 3.2, we formalize the above intuition and prove that an equilibrium with the pattern of trade illustrated in Figure 1 always exists. In subsection 3.3, we characterize some of the key properties of equilibrium. In Appendix A, we rule out the existence of any other type of equilibrium. The main finding contained in this section is that, from the model, emerges a theory of intermediation as a pure rent extraction activity. Agents with commitment power act as intermediaries—in the sense that they purchase the asset with the intent of reselling it—while agents without commitment power act as final users—in the sense that they purchase the asset and hold it unless their valuation changes. Agents with commitment power intermediate the asset, not because they are better at finding traders who want to change their asset position, but because they are better at extracting rents from traders who want to change their position. Our theory of intermediation as pure rent extraction generates sensible implications about the trading frequency and trading prices of intermediaries compared to final users.

3.1 Equilibrium conditions

We denote as $\{V_{i,j}, U_{i,j}\}$ the equilibrium lifetime utility for agents of type (i,j). In particular, we denote as $V_{i,j}$ the lifetime utility of an agent of type (i,j) who holds the asset, as $U_{i,j}$ the lifetime utility of an agent of type (i,j) who does not hold the asset, and as $D_{i,j} = V_{i,j} - U_{i,j}$ the net value of holding the asset for an agent of type (i,j). We denote as $P_{i,j}(n,m)$ the price at which an agent of type (i,j) sells the asset to an agent of type (n,m). Finally, we denote as $\{\mu_{i,j}, \nu_{i,j}\}$ the stationary distribution of agents across types and asset holdings. In particular, $\mu_{i,j}$ denotes the measure of agents of type (i,j) who hold the asset and $\nu_{i,j}$ the measure of agents of type (i,j) without the asset.

Given the pattern of trade in Figure 1, we can show that the stationary distribution is symmetric, in the sense that the measure of agents with valuation j who hold the asset is equal to the measure of agents with valuation $\neg j$ who do not hold the asset. That is, $\mu_{i,L} = \nu_{i,H}$ and $\mu_{i,H} = \nu_{i,L}$ for $i = \{S, T\}$. We then find it useful to define λ_i as $\lambda\mu_{i,L}$ and $\hat{\lambda}_i$ as $\lambda\mu_{i,H}$. That is, λ_i is the rate at which an agent meets a trader of type i who has a low valuation but holds the asset. Since the distribution is symmetric, λ_i is also the rate at which an agent meets a trader of type i who has a high valuation but does not have the asset. Similarly, $\hat{\lambda}_i$ is the rate at which an agent meets a trader of type i who has a low valuation and does not hold the asset, as well as the rate at which he meets a trader of type i who has a high valuation and owns the asset.

3.1.1 Soft Agent

We begin by formulating the Bellman Equations that describe the lifetime utility of agents with commitment power S. First, consider an agent of type (S, L) who holds the asset. The agent's lifetime utility satisfies the Bellman Equation

$$\begin{aligned} rV_{SL} &= u_L + \sigma\left(V_{SH} - V_{SL}\right) + \lambda_S\left(P_{SL}(S,H) + U_{SL} - V_{SL}\right) \\ &\quad + \lambda_T\left(P_{SL}(T,H) + U_{SL} + V_{SL}\right) + \hat{\lambda}_T\left(P_{SL}(T,L) + U_{SL} - V_{SL}\right). \end{aligned} \tag{1}$$

The above expression is easy to understand. The agent receives a flow utility u_L. At the rate σ, the agent's valuation of the asset switches from L to H and the agent experiences a change in lifetime utility of $V_{SH} - V_{SL}$. At the rate λ_S, the agent meets a trader of type (S, H) without asset. When this happens, the agent sells the asset at the price $P_{SL}(S, H)$ and experiences a change in lifetime utility of $U_{SL} - V_{SL}$. At the rate λ_T, the agent meets a trader of type (T, H) without asset. In this case, the agent sells the asset at the price $P_{SL}(T, H)$ and his lifetime utility changes by $U_{SL} - V_{SL}$. Finally, at the rate $\hat{\lambda}_T$ the agent meets a trader of type (T, L) without asset. When this happens, the agent sells the asset at the price $P_{SL}(T, L)$ and experiences a change in lifetime utility of $U_{SL} - V_{SL}$.

When an agent of type (S, L) meets a trader of type (T, L) or (T, H), he is on the receiving end of a take-it-or-leave-it offer. The take-it-or-leave offer is such that the agent is indifferent between selling and keeping the asset. That is, $p_{SL}(T, j) + U_{SL} - V_{SL} = 0$ or, equivalently, $p_{SL}(T, j) = D_{SL}$. When the agent meets a trader of type (S, H), an alternating-offer bargaining game takes place. As it is well-known, the outcome of the alternating-offer bargaining game is a price such that the gains from trade accruing to the seller (i.e., the agent) equal the gains from trade accruing to the buyer (i.e., the trader). The gains from trade for the seller are $p_{SL}(S, H) + U_{SL} - V_{SL}$. Similarly, the gains from trade for the buyer are $-p_{SL}(S, H) + V_{SH} - U_{SH}$. Thus, the price $p_{SL}(S, H)$ is $(D_{SH} + D_{SL})/2$. Substituting the equilibrium prices in (1), we obtain

$$rV_{SL} = u_L + \sigma \left(V_{SH} - V_{SL}\right) + \lambda_S \left(D_{SH} - D_{SL}\right)/2. \tag{2}$$

Second, consider an agent of type (S, L) who does not hold the asset. The agent's lifetime utility satisfies the Bellman Equation

$$rU_{SL} = \sigma \left(U_{SH} - U_{SL}\right). \tag{3}$$

The above expression is also easy to understand. The agent receives a flow utility 0. At the rate σ, the agent's valuation of the asset switches from L to H and the agent experiences a change in lifetime utility of $U_{SH} - U_{SL}$. At the rate λ, the agent meets some trader. No matter the type of trader whom he meets, the agent does not purchase the asset and his lifetime utility does not change.

Third, consider an agent of type (S, H) who currently holds the asset. His lifetime utility satisfies the Bellman Equation

$$rV_{SH} = u_H + \sigma \left(V_{SL} - V_{SH}\right). \tag{4}$$

The above expression is analogous to (3). The agent receives a flow utility u_H. At the rate σ, the agent's valuation of the asset switches from H to L and the agent experiences a change in lifetime utility of $V_{SL} - V_{SH}$. At the rate λ, the agent meets some trader. No matter the type of trader whom he meets, the agent keeps the asset and his lifetime utility does not change.

Finally, consider an agent of type (S, H) who does not have the asset. His lifetime utility satisfies the Bellman Equation

$$\begin{aligned} rU_{SH} &= \sigma\left(U_{SL}-U_{SH}\right)+\lambda_S\left(-P_{SL}(S,H)+V_{SH}-U_{SH}\right) \\ &+\lambda_T\left(-P_{TL}(S,H)+V_{SH}-U_{SH}\right)+\hat{\lambda}_T\left(-P_{TH}(S,H)+V_{SH}-U_{SH}\right). \end{aligned} \tag{5}$$

The above expression is analogous to (1). The agent receives a flow utility 0. At the rate σ, the agent's valuation switches from H to L and the agent experiences a change in lifetime utility of $U_{SL}-U_{SH}$. At the rate λ_S, the agent buys the asset from a trader of type (S,L) at the price $-P_{SL}(S,H)$. At the rate λ_T, the agent buys the asset from a trader of type (T,L) at the price $-P_{TL}(S,H)$. At the rate $\hat{\lambda}_T$, the agent buys the asset from a trade of type (T,H) at the price $-P_{TH}(S,H)$. Whenever the agent buys the asset, he experiences a change in lifetime utility of $V_{SH}-U_{SH}$.

When an agent of type (S,H) meets a trader of type (T,L) or (T,H), he receives a take-it-or-leave-it offer. The take-it-or-leave-it offer is such that the agent is made just indifferent between accepting it and rejecting it. That is, $-P_{Tj}(S,H)+V_{SH}-U_{SH}=0$ or, equivalently, $P_{Tj}(S,H)=D_{SH}$. When the agent meets a trader of type (S,L), an alternating-offer bargaining game takes place. The outcome of the bargaining game is the price p that equates the gains from trade accruing to the two parties. That is, $P_{SL}(S,H)=(D_{SH}+D_{SL})/2$. Substituting the equilibrium prices in (5), we obtain

$$rU_{SH}=\sigma\left(U_{SL}-U_{SH}\right)+\lambda_S\left(D_{SH}-D_{SL}\right)/2. \tag{6}$$

Subtracting (3) from (2), we find that the net value D_{SL} of holding the asset for an agent of type (S,L) is given by

$$rD_{SL}=u_L+\sigma\left(D_{SH}-D_{SL}\right)+\lambda_S\left(D_{SH}-D_{SL}\right)/2. \tag{7}$$

The net value from holding the asset is given by the agent's flow utility, u_L, plus the change in net value when the agent's preferences change, $\sigma(D_{SH}-D_{SL})$, plus the value of the option to sell the asset, $\lambda_S(D_{SH}-D_{SL})/2$. Notice that the option value of selling for an agent of type (S,L) only depends on rate at which he meets traders of type (S,H) and on the associated gains from trade. Indeed, while an agent of type (S,L) also sells the asset to traders of type (T,L) and (T,H), he captures none of the gains from those trade.

Subtracting (6) from (4), we find that the net value D_{SH} from holding the asset for an agent of type (S,H) is given by

$$rD_{SH}=u_H+\sigma\left(D_{SL}-D_{SH}\right)-\lambda_S\left(D_{SH}-D_{SL}\right)/2. \tag{8}$$

The net value from holding the asset is given by the agent's flow utility, u_H, plus the change in net value when the agent's preferences change, $\sigma(D_{SL} - D_{SH})$, minus the value of the foregone option to buy the asset, $\lambda_S(D_{SH} - D_{SL})/2$. Again, the option value of buying for an agent of type (S, H) only depends on rate at which he meets traders of type (S, L) and on the associated gains from trade. Indeed, while an agent of type (S, H) also buys the asset to traders of type (T, L) and (T, H), he captures none of the gains from those trade.

3.1.2 Tough Agent

Now we formulate the Bellman Equations that describe the lifetime utility of agents with commitment power T. First, consider an agent of type (T, L) who currently holds the asset. The agent's lifetime utility satisfies the Bellman Equation

$$\begin{aligned} rV_{TL} &= u_L + \sigma\left(V_{TH} - V_{TL}\right) \\ &\quad + \lambda_S\left(P_{TL}(S,H) + U_{TL} - V_{TL}\right) + \lambda_T\left(E[P_{TL}(T,H)] + U_{TL} - V_{TL}\right). \end{aligned} \tag{9}$$

The agent receives a flow utility u_L. At the rate σ, the agent's valuation of the asset switches from L to H and the agent experiences a change in lifetime utility of $V_{TH} - V_{TL}$. At the rate λ_S, the agent meets a trader of type (S, H) without the asset. When this happens, the agent sells to the trader at the price $P_{TL}(S, H)$ and experiences a change in lifetime utility of $U_{TL} - V_{TL}$. At the rate λ_T, the agent meets a trader of type (T, H) without the asset. In this case, the agent sells the asset to the trader at the expected price $E[P_{TL}(T, H)]$ and experiences a change in lifetime utility of $U_{TL} - V_{TL}$.

Second, consider an agent of type (T, L) who does not have the asset. The agent's lifetime utility satisfies the Bellman Equation

$$rU_{TL} = \sigma\left(U_{TH} - U_{TL}\right) + \lambda_S\left(-P_{SL}(T,L) + V_{TL} - U_{TL}\right). \tag{10}$$

At the rate σ, the agent's valuation of the asset switches from L to H and the agent experiences a change in lifetime utility of $U_{TH} - U_{TL}$. At the rate λ_S, the agent meets a trader of type (S, L) with the asset. When this happens, the agent buys from the trader at the price $P_{SL}(T, L)$ and experiences a change in lifetime utility of $V_{TL} - U_{TL}$.

Third, consider an agent of type (T, H) with the asset. The agent's lifetime utility satisfies the Bellman Equation

$$rV_{TH} = u_H + \sigma\left(V_{TL} - V_{TH}\right) + \lambda_S\left(P_{TH}(S,H) + U_{TH} - V_{TH}\right). \tag{11}$$

The agent receives a flow utility u_H. At the rate σ, the agent's valuation of the asset switches from H to L and the agent experiences a change in lifetime utility of $V_{TL} - V_{TH}$. At the rate λ_S, the agent meets a trader of type (S, H) without the asset. When this happens, the agent sells to the trader at the price $P_{TH}(S, H)$ and experiences a change in lifetime utility of $U_{TH} - V_{TH}$.

Finally, the lifetime utility of an agent of type (T, H) without the asset is such that

$$\begin{aligned} rU_{TH} &= \sigma\left(U_{TL} - U_{TH}\right) \\ &+\lambda_S\left(P_{SL}(T,H) + V_{TH} - U_{TH}\right) + \lambda_T\left(E[P_{TL}(T,H)] + V_{TH} - U_{TH}\right). \end{aligned} \tag{12}$$

At the rate σ, the agent's valuation of the asset switches from H to L and the agent experiences a change in lifetime utility of $U_{TL} - U_{TH}$. At the rate λ_S, the agent meets a trader of type (S, L) with the asset. When this happens, the agent buys from the trader at the price $P_{SL}(T, H)$ and experiences a change in lifetime utility of $V_{TH} - U_{TH}$. At the rate λ_T, the agent meets a trader of type (T, L) with the asset. In this case, the agent buys the asset to the trader at the expected price $E[P_{TL}(T, H)]$ and experiences a change in lifetime utility of $V_{TH} - U_{TH}$.

When an agent of type (T, j) purchases the asset from a trader of type (S, L), he advances a take-it-or-leave-it offer that makes the agent indifferent between selling and keeping the asset. That is, $p_{S,L}(T, j) + U_{SL} - V_{SL} = 0$ or, equivalently, $p_{S,L}(T, j) = D_{SL}$. Similarly, when an agent of type (T, j) sells the asset to an agent of type (S, H), he advances a take-it-or-leave-it offer that makes the agent indifferent between buying and not buying. That is, $-p_{T,j}(S, H) + V_{SH} - U_{SH} = 0$ or, equivalently, $p_{T,j}(S, H) = D_{SH}$. When an agent of type (T, L) sells the asset to a trader of type (T, H), the price depends on who gets to make the take-it-or-leave-it offer. If the seller makes a take-it-or-leave-it offer, the price is such that $-p + V_{TH} - U_{TH} = 0$. If the buyer makes a take-it-or-leave-it offer, the price is such that $p + U_{TL} - V_{TL} = 0$. Since the seller and the buyer are equally likely to make the offer, the expected price is $E[P_{TL}(T, H)] = (D_{TH} + D_{TL})/2$.

Subtracting (10) from (9) and substituting in the equilibrium prices, we find that the net value D_{TL} of holding the asset for an agent of type (T, L) is given by

$$\begin{aligned} rD_{TL} &= u_L + \sigma\left(D_{TH} - D_{TL}\right) \\ &+\lambda_S\left(D_{SH} - D_{TL}\right) + \lambda_T\left(D_{TH} - D_{TL}\right)/2 - \lambda_S\left(D_{TL} - D_{SL}\right). \end{aligned} \tag{13}$$

The net value from holding the asset is given by the agent's flow utility, u_L, plus the change in net value when the agent's preferences change, $\sigma(D_{TH} - D_{TL})$, plus the value of the option to sell the asset, $\lambda_S(D_{SH} - D_{SL}) + \lambda_T(D_{TH} - D_{TL})/2$, net of the foregone option to buy the asset, $\lambda_S(D_{TL} - D_{SL})$.

Subtracting (12) from (11) and substituting the equilibrium prices, we find that the net value of holding the asset for an agent of type (T, H) is given by

$$\begin{aligned} rD_{TH} &= u_H + \sigma\left(D_{TL} - D_{TH}\right) \\ &+\lambda_S\left(D_{SH} - D_{TH}\right) - \lambda_T\left(D_{TH} - D_{TL}\right)/2 - \lambda_S\left(D_{TH} - D_{SL}\right). \end{aligned} \tag{14}$$

The net value from holding the asset is given by the agent's flow utility, u_H, plus the change in net value when the agent's preferences change, $\sigma\left(D_{TL} - D_{TH}\right)$, plus the value of the option to sell the asset, $\lambda_S\left(D_{SH} - D_{TH}\right)$, net of the foregone option to buy the asset, $\lambda_T\left(D_{TH} - D_{TL}\right)/2 + \lambda_S\left(D_{TH} - D_{SL}\right)$.

3.1.3 Individual Rationality

We now need to establish necessary and sufficient conditions under which the pattern of trade described in Figure 1, and assumed in the Bellman Equations for agents of type S and T, is consistent with the individual rationality of buyers and sellers.

As a preliminary step, consider a meeting between an agent of type (i, j) who holds the asset and an agent of type (m, n) who does not have the asset. Trade is individually rational if and only if the gains from trade, $D_{m,n} - D_{i,j}$, are positive. To see why this is the case, note that, if $i = S$ and $m = S$ and trade takes place, the price at which (i, j) sells the asset to (m, n) is $p = (D_{m,n} + D_{i,j})/2$. The seller finds it optimal to trade if and only if $p - D_{i,j} \geq 0$. The buyer finds it optimal to trade if and only if $-p + D_{m,n} \geq 0$. These conditions are equivalent to $D_{m,n} - D_{i,j} \geq 0$. If $i = T$ and $m = S$ and trade takes place, the price at which (i, j) sells the asset to (m, n) is $p = D_{m,n}$. The seller finds it optimal to trade if and only if $p - D_{i,j} \geq 0$ or, equivalently, $D_{m,n} - D_{i,j} \geq 0$. The buyer finds it optimal to trade because $-p + D_{m,n} = 0$. Similarly, if $i = S$ and $m = T$ and trade takes place, the price at which (i, j) sells the asset to (m, n) is $p = D_{i,j}$. Then, the seller finds it optimal to trade because $p - D_{i,j} = 0$ and the buyer finds it optimal to trade if and only if $-p + D_{m,n} \geq 0$ or, equivalently, $D_{m,n} - D_{i,j} \geq 0$. Finally, if $i = T$ and $m = T$ the price at which trade takes place depends on whether the buyer or the seller is selected to make a take-it-or-leave-it offer. In either case, both parties find it optimal to trade if and only if $D_{m,n} - D_{i,j} \geq 0$.

Since trade is individually rational for both the buyer and the seller if and only if $D_{m,n} - D_{i,j}$ are positive, the pattern described in Figure 1 is consistent with individual rationality if and only if

$$D_{SL} \leq D_{TL} \leq D_{TH} \leq D_{SH}. \tag{15}$$

To check necessity, notice that the inequalities in (15) imply that (S, L) selling the asset to (T, L), (T, H) or (S, H) is consistent with individually rationality, and so is (S, L) not buying the asset from (T, L), (T, H) or (S, H). Similarly, (T, L) selling the asset to (T, H) or (S, H) is consistent with individual rationality, and so is (T, L) not buying the asset from (T, H) or (S, H). Finally, (T, H) selling the asset to (S, H) is consistent with individual rationality and so is (T, H) not buying from (S, H). To check sufficiency, notice that if any of the inequalities in (15) is violated, some trade in Figure 1 certainly violates individual rationality.

3.1.4 Stationary Distribution

Lastly, we need to establish a set of necessary and sufficient conditions to guarantee that the distribution of agents in the market $\{\mu_{i,j}, \nu_{i,j}\}$ is stationary. The distribution is stationary if and only if the measure of agents who, during any interval of time, become asset holders of type (i, j) equals the measure of agents who, during the same interval of time, stop being asset holders of type (i, j). Similarly, the measure of agents who become asset seekers of type (i, j) must equal the measure of agents who stop being asset seekers of type (i, j). In addition, the distribution of agents in the market must be consistent with the measure of agents with commitment power S and T and with the measure of the asset circulating in the market.

First, consider the flows in and out of the group of agents of type (S, L) who do not have the asset. The flow in equals the flow out if and only if

$$\nu_{SL}\sigma = \nu_{SH}\sigma + \mu_{SL}\lambda\left(\nu_{TL} + \nu_{TH} + \nu_{SH}\right). \tag{16}$$

The left-hand side of (16) is the flow out of the group, which is given by the measure of agents of type (S, L) without the asset whose preferences switch from L to H. The right-hand side of (16) is the flow into the group, which is given by the sum of two terms. The first term is the measure of agents of type (S, H) without the asset whose preferences switch from H to L. The second term is the measure of agents of type (S, L) who own the asset and sell it.

The inflow-outflow equation for the group of agents of type (S, L) who own the asset is

$$\mu_{SL}\left[\sigma + \lambda\left(\nu_{TL} + \nu_{TH} + \nu_{SH}\right)\right] = \mu_{SH}\sigma. \tag{17}$$

The left-hand side of (17) is the flow out of the group, which is given by the measure of agents of type (S, L) who own the asset and who either sell it or who change their valuation from L to H. The right-hand side of (17) is the flow into the group, which is given by the measure of agents of type (S, H) who own the asset and whose valuation switches from H to L.

The inflow-outflow equation for the group of agents of type (T, L) who do not hold the asset is

$$\nu_{TL}\left[\sigma+\lambda\mu_{SL}\right]=\nu_{TH}\sigma+\mu_{TL}\lambda\left(\nu_{TH}+\nu_{SH}\right). \tag{18}$$

The left-hand side of (18) is the flow out of the group, which is given by the measure of agents of type (T, L) who purchase the asset or whose valuation switches from L to H. The right-hand side is the flow into the group, which is given by the sum of two terms. The first term is the measure of agents of type (T, H) without the asset whose valuation switches from H to L. The second term is the measure of agents of type (T, L) who own the asset and sell it.

The inflow-outflow equation for the group of agents of type (T, L) who own the asset is

$$\mu_{TL}\left[\sigma+\lambda\left(\nu_{TH}+\nu_{SH}\right)\right]=\mu_{TH}\sigma+\nu_{TL}\lambda\mu_{SL}. \tag{19}$$

The left-hand side of (19) is the flow out of the group, which is given by the measure of agents of type (T, L) with the asset who either sell it or whose valuation switches from L to H. The right-hand side is the flow into the group, which is given by the sum of two terms. The first term is the measure of agents of type (T, H) with the asset whose valuation switches from H to L. The second term is the measure of agents of type (T, L) without the asset who buy it.

There are four additional inflow-outflow equations: two inflow-outflow equations for the group of agents of type (S, H) who respectively own and seek the asset, and two inflow-outflow equations for the group of agents of type (T, H) who respectively have and do not have the asset. As these equations are analogous to the four inflow-outflow equations above, we will not write them down explicitly.

The stationary distribution also needs to satisfy some adding-up constraints:

$$\sum\nolimits_{j=\{L,H\}}\left(\mu_{T,j}+\nu_{T,j}\right) = \phi_T, \tag{20}$$

$$\sum\nolimits_{j=\{L,H\}}\left(\mu_{S,j}+\nu_{S,j}\right) = \phi_S, \tag{21}$$

$$\sum\nolimits_{j=\{L,H\}}\left(\mu_{S,j}+\mu_{T,j}\right) = 1/2. \tag{22}$$

The first constraint states that the stationary distribution must be such that the overall measure of agents with commitment power T must be equal to ϕ_T. The second constraint states that the stationary distribution must be such that the overall measure of agents with commitment power S must be equal to ϕ_S. The last constraint states that the stationary distribution must be such that the overall measure of agents holding the asset must equal the measure of the asset in circulation, which we assumed to be 1/2.

3.1.5 Definition of Equilibrium

We are now in the position to define a stationary equilibrium.

Definition 1 *A stationary equilibrium in which trade follows the pattern illustrated in Figure 1 is given by the agents' net values for holding the asset $\{D_{ij}\}$, the distribution of agents in the market $\{\mu_{i,j}, \nu_{i,j}\}$ such that:*

(i) Individual rationality of trade: $\{D_{ij}\}$ satisfies condition (15);

(ii) Stationarity of the distribution: $\{\mu_{i,j}, \nu_{i,j}\}$ satisfies conditions (16)-(22).

3.2 Existence and Uniqueness of Equilibrium

In order to establish the existence of an equilibrium with the pattern of trade in Figure 1, we need to check the condition for the individual rationality of the pattern of trade, to check the condition for the stationarity of the to solve for the agents' net values from holding the asset, $D_{i,j}$, and for the distribution of agents, $\{\mu_{i,j}, \nu_{i,j}\}$, and then check that the equilibrium conditions (i) and (ii) are satisfied.

The first step in establishing the existence of equilibrium is to verify the condition for the individual rationality of the pattern of trade. To this aim, consider the gains from trade $D_{SH} - D_{SL}$ between an agent of type (S, H) who seeks the asset and an agent of type (S, L) who owns the asset. From (7) and (8), it follows that the gains from trade are

$$D_{SH} - D_{SL} = \frac{u_H - u_L}{r + 2\sigma + \lambda_S} > 0. \tag{23}$$

The gains from trade $D_{SH} - D_{SL}$ are positive. They are given by the difference in the valuation of the asset between the buyer and the seller, $u_H - u_L$, capitalized by the factor $(r + 2\sigma + \lambda_S)^{-1}$, where r reflects direct discounting of future payoffs, 2σ is a discounting term due to the fact that the agents' valuation of the asset varies over time, and λ_S is a discounting term due to the fact that, by trading with each other, the two agents give up future trading opportunities. Specifically, the buyer gives up the opportunity of purchasing the asset from some other trader of type (S, L) and capturing half of the gains from that trade. The seller gives up the opportunity of selling the asset to some other trader of type (S, H) and capturing half of the gains from that trade.

Next, consider the gains from trade $D_{TH} - D_{TL}$ between an agent of type (T, H) without owns the asset and an agent of type (T, L) who owns the asset. From (13) and (14), it follows that the gains from trade are

$$D_{TH} - D_{TL} = \frac{u_H - u_L}{r + 2\sigma + 2\lambda_S + \lambda_T} > 0. \tag{24}$$

The gains from trade $D_{TH} - D_{TL}$ are positive. They are given by an expression similar to (23), except for the capitalization factor. The capitalization factor in (24) is smaller than in (23) because, by trading with each other, the two agents forego more valuable trading opportunities. The buyer gives up the opportunity of purchasing the asset from another trader of type (T, L) and capturing half of the gains from that trade. Similarly, the seller gives up the opportunity of selling the asset to another trader of type (T, H) and capturing half the gains from that trade. These foregone options explain the λ_T term in the capitalization factor. Additionally, the buyer loses and the seller acquires the option of purchasing the asset from a trader of type (S, L) and capturing all of the gains from that trade. Similarly, the seller loses and the buyer acquires the option of selling the asset to a trader of type (S, H) and capturing all of the gains from that trade. These switches of options account for the $2\lambda_S$ term in the capitalization factor.

Finally, consider the gains from trade $D_{TL} - D_{SL}$ between an agent of type (T, L) who seeks the asset and an agent of type (T, L) who holds the asset, as well as the gains from trade $D_{SH} - D_{TH}$ between an agent of type (S, H) without the asset and an agent of type (T, H) who holds the asset. Using (7)-(8) and (13)-(14), it is immediate to show that

$$\begin{aligned} D_{TL} - D_{SL} &= D_{SH} - D_{TH} \\ &= \tfrac{1}{2}\left[\frac{\lambda_T\left(D_{TH} - D_{TL}\right) + \lambda_S\left(D_{SH} - D_{SL}\right)}{r + 2\sigma + 2\lambda_S}\right] > 0. \end{aligned} \tag{25}$$

Equation (25) shows that the gains from trade $D_{TL} - D_{SL}$ are positive. This finding is perhaps surprising. Indeed, the buyer—the agent of type (T, L)—and the seller—the agent of type (S, L)—have the same valuation for the asset. However, the gains from trade are positive because (T, L) can extract more rents than (S, L) when selling the asset to somebody else. Specifically, (T, L) can capture half rather than none of the gains from trade associated with selling the asset to an agent of type (T, H). Similarly, (T, L) can capture all rather than half of the gains from trade associated with selling the asset to an agent of type (S, H). These differences explains the numerator on the right-hand side of (25). The denominator is the capitalization factor, which includes the direct discounting of future payoffs as well as the discounting due to the rate of change in the agents' preferences. Equation (25) also shows that the gains from trade $D_{SH} - D_{TH}$ are positive. Indeed, given the symmetry of the model, the gains from trade $D_{SH} - D_{TH}$ are equal to $D_{TL} - D_{SL}$. Taken together, the inequalities in (23)-(25) imply that $D_{SL} < D_{TL} < D_{TH} < D_{SH}$. Hence, condition (i) in the definition of equilibrium is satisfied and the pattern of trade is individually rational.

The second step in establishing the existence of equilibrium is to show the existence of a distribution of agents $\{\mu_{i,j}, \nu_{i,j}\}$ that satisfies the stationarity and adding-up conditions (16)-(22). To this aim, we begin by noting that the measure of agents of type (i, L) and the measure of agents of type (i, H) are each equal to half of the population of agents with commitment power i, for $i = S, T$. That is,

$$\mu_{i,L} + \nu_{i,L} = \mu_{i,H} + \nu_{i,H} = \phi_i/2, \text{ for } i = \{S, T\}. \tag{26}$$

The above result follows from the fact that the rate at which agents' preferences switch from L to H is the same as the rate at which preferences switch from H to L and the switching rate is independent of the agent's inventory of the asset and commitment power. Formally, the result can be obtained by combining the inflow-outflow equations for agents of type (i, j) with and without the asset and by using the adding up constraints (20) and (21).

Second, we notice that the measure of agents of type (i, L) with the asset is the same as the measure of agents of type (i, H) without the asset for $i = \{S, T\}$. That is,

$$\mu_{i,L} = \nu_{i,H}, \text{ for } i = \{S, T\}. \tag{27}$$

The result above states that the measure of agents who have a low valuation and are currently holding the asset is the same as the measure of agents who have a high valuation and are currently seeking the asset. Intuitively, the result follows from the symmetry of the environment and of the pattern of trade. Formally, the result is derived by combining the inflow-outflow equations for agents of type (i, L) with the asset and for agents of type (i, H) without the asset and by using the symmetry of the preference shock and the fact that there is a measure $\frac{1}{2}$ of the asset in circulation.

Third, we notice that the measure of agents of type (S, L) with the asset is given by

$$\mu_{SL} = \sqrt{\left(\frac{\sigma}{\lambda}\right)^2 + \frac{\sigma}{2\lambda} + \frac{\phi_T^2}{16}} - \left(\frac{\sigma}{\lambda} + \frac{\phi_T}{4}\right). \tag{28}$$

The above expression is obtained from (17) using $\nu_{SH} = \mu_{SL}$, $\nu_{SL} = \phi_S - \mu_{SH}$ and $\nu_{TL} + \nu_{TH} + \nu_{SH} = \phi_T/2 + \mu_{SL}$. Notice that μ_{SL} is between 0—which is the perfect assignment of the asset—and $\phi_S/4$—which is the random assignment of the asset. As one would have expected, μ_{SL} converges to zero when the ratio σ/λ of the rate of preference change to the meeting rate goes to zero. Also as expected, μ_{SL} converges to $\phi_S/4$ when σ/λ goes to infinity.

Finally, we notice that the measure of agents of type (T, L) with the asset is given by

$$\mu_{TL} = \frac{\phi_T}{4} + \sqrt{\left(\frac{\sigma}{\lambda}\right)^2 + \frac{\sigma}{2\lambda}} - \sqrt{\left(\frac{\sigma}{\lambda}\right)^2 + \frac{\sigma}{2\lambda} + \frac{\phi_T^2}{16}}. \tag{29}$$

The above expression is obtained by summing (17) and (19) and using the fact that $\nu_{SH} + \nu_{TH} = \mu_{SL} + \mu_{TL}$. Notice that$\mu_{TL}$ is between 0 and $\phi_T/4$. As one would have expected, μ_{TL} converges to zero when the ratio σ/λ goes to zero and μ_{TL} converges to $\phi_T/4$ when σ/λ goes to infinity.

Clearly, the distribution of agents $\{\mu_{i,j}, \nu_{i,j}\}$ in (26)-(29) is the only one that may satisfy the stationarity conditions and the adding-up constraints (16)-(22). Moreover, it is immediate to verify that this distribution does satisfy (16)-(22). Hence, the distribution $\{\mu_{i,j}, \nu_{i,j}\}$ in (26)-(29) satisfies condition (ii) in the definition of equilibrium. Moreover, since $\mu_{SL} = \nu_{SH}$ and $\mu_{TL} = \nu_{TH}$, we have verified the conjecture that the stationary distribution is symmetric.

We have thus established the existence and uniqueness of a stationary equilibrium in the asset market with the pattern of trade illustrated in Figure 1. In Appendix A, we rule of the existence of stationary equilibria with a different pattern of trade within the class of symmetric equilibria.

Proposition 1: Market equilibrium. *(i) There exists a unique stationary equilibrium with the pattern of trade described in Figure 1. (ii) There exists no other stationary symmetric equilibria.*

3.3 Properties of Equilibrium

The unique stationary equilibrium is such that the pattern of trade is as illustrated in Figure 1. Agents without commitment power behave as final users of the asset, in the sense that they only purchase the asset when their valuation is high, and they only sell the asset when their valuation becomes low. In contrast, agents with commitment power behave as intermediaries, in the sense that they buy the asset even when their valuation is low in order to sell it to someone with a high valuation, and they sell the asset even when their valuation is high in order to buy another unit of the asset from someone with a low valuation.

Agents with commitment power act as intermediaries because the ability to make take-it-or-leave-it offers allows them to extract a larger share of the gains from trade. An agent who has a low valuation for the asset and lacks commitment will sell the asset to an agent who also has a low valuation for the asset but who can commit to take-it-or-leave-it offers only because the agent with commitment power will be able to resell the asset at a higher price. Similarly, an agent who has a high valuation for the asset and who can commit to take-it-or-leave-it offers will sell the asset to an agent with high valuation and no ability to commit only because he is able to procure himself the asset at a lower price.

Several features of intermediation are worth pointing out. First, intermediaries trade faster than final users, even though they have the same meeting rate. To see this, we look at the average trading rate α_S for agent of type S and the average trading rate α_T for an agent of type T, where α_S and α_T are given by

$$\begin{aligned} \alpha_S &= \left[\mu_{SL}\lambda\left(\nu_{TL}+\nu_{TH}+\nu_{SL}\right)+\nu_{SH}\lambda\left(\mu_{SL}+\mu_{TL}+\mu_{TH}\right)\right]/\phi_S, \\ \alpha_T &= \left[\mu_{TL}\lambda\left(\nu_{TH}+\nu_{SH}\right)+\nu_{TL}\lambda\mu_{SL}+\mu_{TH}\lambda\nu_{SH}+\nu_{TH}\lambda\left(\mu_{SL}+\mu_{TL}\right)\right]/\phi_T. \end{aligned} \tag{30}$$

After substituting the equilibrium distribution $\{\mu_{i,j},\nu_{i,j}\}$, it is straightforward to show that α_T is greater than α_S. Intuitively, agents of type T trade faster than agents of type S because they buy and sell the asset more frequently than the rate at which their preferences change. In contrast, agents of type S buy and sell the asset at the same frequency at which they change their preferences.

Second, intermediaries buy at low and sell at high prices, while final users buy at high and sell at low prices. To see this, we solve for the equilibrium prices and find that

$$\begin{aligned} P_{SL}(S,H) &= EP_{TL}(T,H) = \frac{u_H+u_L}{2r}, \\ P_{S,L}(T,j) &= \frac{u_H+u_L}{2r} - \frac{u_H}{r+\lambda_S}, \\ P_{T,j}(S,H) &= \frac{u_H+u_L}{2r} + \frac{u_H}{r+\lambda_S}. \end{aligned} \tag{31}$$

The average market price for the asset is $(u_H+u_L)/2r$, which is the value of keeping the asset indefinitely given a valuation of $(u_H+u_L)/2$. An agent of type (S,H) purchases the asset at either the market price—if he happens to buy from a trader of type (S,L)—or above the market price—if he happens to buy from a trader of type (T,L) or (T,H). Naturally, the premium that (S,H) pays when he purchases from traders of type T is decreasing in λ_S, which is the rate at which he could have met a trader of type (S,L)

trying to sell. An agent of type (S, L) sells the asset at wither the market price—if he happens to buy from a trader of type (S, H)—or below the market price—if he happens to buy from a trader of type (T, L) or (T, H). Because of the symmetry of the model, the discount that (S, L) takes when he sells to traders of type T is the same as the premium that (S, H) pays when he buys from traders of type T. Agents of type T either trade at the asset at the market price—if they happen to trade with other agents of type T—or they buy at a discount and sell at a premium—if they happen to trade with agents of type S.

Third, intermediaries set prices, while final users either take prices as given or bargain over prices. Indeed, agents of type T make take-it-or-leave-it offers—which means that they post prices (possibly prices that depend on the identity of the counterparty). Agents of type S either are on the receiving end of take-it-or-leave-it offers—which means that they take prices as given—or they bargain over the terms of trade.

The baseline model formalizes a theory of intermediation as a pure rent extraction activity. In Rubinstein and Wolinsky (1987), Duffie, Garleanu and Pedersen (2010) and Farboodi, Jarosch and Shimer (2016), intermediaries purchase the asset from final users with low valuation because they are faster at finding final users with high valuation. In Kiyotaki and Wright (1989), intermediaries purchase the asset from final users with low valuation because they have a lower opportunity cost from keeping the asset on their books. In Hugonnier, Lester and Weill (2016), intermediaries purchase the asset from final users with low valuation and sell it to final users with high valuation because they have an intermediate valuation. In contrast to these models, in our model intermediaries buy the asset from final users with low valuation only because they can get a better price when they sell it to final users with high valuation. Our theory of intermediation produces some sensible implications on the behavior of intermediaries and final users: intermediaries trade more frequently, they trade at better prices, and they make prices rather than take prices as given.

We conclude the analysis of the baseline model with some considerations about efficiency. The equilibrium is efficient—in the sense that it maximizes the sum of the agents' lifetime utilities—if and only if, in any meeting, the asset is given to the agent who has the highest flow payoff from holding it. The equilibrium is efficient as it satisfies this criterion. However, in equilibrium, there are trades that have a strictly positive surplus and that do nothing to improve welfare. Indeed, the "intermediation trades" between agents of type T and S with the same valuation have strictly positive surplus but leave welfare unaffected.

The above observation implies that—in a richer version of the model—the equilibrium might be inefficient. In the next section, we are going to see how inefficiency emerges in a version of the model where the decision to acquire commitment power is endogenous. Here, we want to highlight possible sources of inefficiency when the fraction of agents with commitment power is taken as given. For example, imagine a version of the model in which buyer and seller have to bear a cost if they trade the asset. As long as the cost is small but positive, efficiency requires that, in any meeting between agents with different valuations, the asset is given to the agent with the highest valuation and that, in any meeting between agents with the same valuation, the asset is not traded. However, as long as the cost is small enough, the equilibrium pattern of trade is the same as in Figure 1 and hence it is inefficient. Indeed, the trades between agents of type T and S with the same valuation take place and reduce welfare.

Similarly, imagine a version of the model where agents of type T have a lower meeting rate than agents of type S. As long as the difference in meeting rates is small but positive, the equilibrium is efficient if and only if, in any meeting between agents with different valuation, the asset is given to the agent with the highest valuation and, in any meeting between agents with the same valuation but different meeting rates, the asset is given to the agent with the highest meeting rate. As long as the difference in meeting rates is small enough, the equilibrium pattern of trade is the same as in Figure 1. The trades between agents of type T and S with the same valuation have the opposite direction than the one that maximizes welfare.

4 Extent and Determinants of Intermediation

In this section, we endogenize the measure of agents who commit to posted prices—and thus act as intermediaries in the asset market—and the measure of agents who cannot commit to posted prices—and thus act as final users in the asset market. We assume that, before entering the market, agents choose whether to invest into a technology that allows them to commit to posted prices. This could be a technology that allows the agent to delegate all negotiations to a representative that is given no authority over pricing decision. Alternatively, this could be a technology that allows the agent to make his transaction history public and, in turn, allows the agent to build a reputation for commitment.

In subsection 4.1, we compute the benefit to an agent from having the commitment technology and the cost to an agent from acquiring the commitment technology. In subsection 4.2, we compute the equilibrium measure of agents who choose to acquire commitment power and, hence, to act as intermediaries in the asset market. We show that, in general, there are multiple equilibria with different degrees of intermediation, equilibria with more intermediation are associated with lower welfare, and all equilibria in which agents spend some resources to become intermediaries are inefficient. In subsection 4.3, we carry out some comparative statics exercises. First, we consider the effect of a decline in trading frictions and show that, as trading frictions fall, the equilibrium measure of intermediaries increases and that this effect might be so strong so as to lower welfare. Second, we consider the effect of a decline in the opportunity cost of acquiring commitment power. In order to sidestep issues related to transitional dynamics, we carry out the analysis in the limit for r going to zero.

4.1 Benefit and Cost of Commitment

4.1.1 Benefit of Commitment

In order to measure the benefit of commitment power, we need to solve for the lifetime utility of agents of type T and S. The annuitized lifetime utility for an agent of type (S, j) without the asset is

$$rU_{S,j} = \lambda_S \left(D_{SH} - D_{SL}\right)/4. \tag{32}$$

The above expression is obtained by solving equations (3) and (6) with respect to U_{SL} and U_{SH}. The expression is easy to understand. Conditional on his valuation being L, an agent of type S without the asset enjoys a flow utility of zero and an annuitized capital gain of zero. Conditional on his valuation being H, the agent enjoys a flow utility of zero and an annuitized capital gain of $\lambda_S(D_{SH} - D_{SL})/2$, which reflects the option value of purchasing the asset. The annuitized lifetime utility of an agent of type S is a weighted average of the flow payoff conditional on having a valuation of L and the flow payoff conditional on having a valuation of H. In the limit for $r \to 0$, the weights in the average are 1/2 and 1/2, as these are the long-run fractions of time the agent spends in the two valuation states.

The annuitized lifetime utility for an agent of type (S, j) with the asset is

$$rV_{S,j} = \left(u_H + u_L\right)/2 + \lambda_S \left(D_{SH} - D_{SL}\right)/4. \tag{33}$$

The above expression is obtained by solving equations (2) and (4) with respect to V_{SL} and V_{SH}. It is also easy to understand this expression. Conditional on his valuation being L, an agent of type S who owns the asset enjoys a flow utility of u_L and an annuitized capital gain of $\lambda_S\left(D_{SH} - D_{SL}\right)/2$, which reflects the option value of selling the asset. Conditional on his valuation being H, the agent enjoys a flow utility of u_H. The annuitized lifetime utility of the agent is the average of the two conditional payoffs.

The annuitized lifetime utility for an agent of type (T, j) without the asset is

$$\begin{aligned} rU_{Tj} &= \lambda_S \left(D_{TL} - D_{SL}\right)/2 + \lambda_S \left(D_{TH} - D_{SL}\right)/2 + \lambda_T \left(D_{TH} - D_{TL}\right)/4 \\ &= \lambda_S \left(D_{SH} - S_{SL}\right)/2 + \lambda_t \left(D_{TH} - D_{TL}\right)/4, \end{aligned} \tag{34}$$

where the first line is obtained by solving equations (10) and (12) with respect to U_{TL} and U_{TH} and the second line is obtained by noting that $D_{TH} - D_{SL} = D_{SH} - D_{TL}$. The annuitized lifetime utility for an agent of type T without the asset is the average of the agent's flow payoff conditional on having a valuation of L and of H. Conditional on a valuation of L, the agent's flow payoff is $\lambda_S \left(D_{TL} - D_{SL}\right)$. Conditional on a valuation of H, the agent's flow payoff is $\lambda_S \left(D_{TH} - D_{SL}\right) + \lambda_T \left(D_{TH} - D_{TL}\right)/2$.

Finally, the annuitized lifetime utility for an agent of type (T, j) with the asset is

$$rV_{T,j} = \left(u_L + u_H\right)/2 + \lambda_S \left(D_{SH} - D_{SL}\right)/2 + \lambda_T \left(D_{TH} - D_{TL}\right)/4. \tag{35}$$

The above expression is obtained by solving equations (9) and (11) with respect to V_{TL} and V_{TH}. The agent's flow payoff when his valuation is L is given by the sum of the flow utility u_L and the annuitized option value of selling the asset $\lambda_S(D_{SH}-D_{TL})+\lambda_T(D_{TH}-D_{TL})/2$. The agent's flow payoff conditional when his valuation is H is given by the sum of the flow utility u_H and the option value of selling the asset $\lambda_S(D_{SH} - D_{TH})$. The expression in (35) is the average of the flow payoffs conditional on valuations L and H.

From (32)-(35), it follows that the benefit B to an agent from being of type T rather than S is the same whether the agent enters the market with a valuation of L or H and whether the agent enters the market holding the asset or not. That is, $B = r(V_{T,j} - V_{S,j}) = r(U_{T,j} - U_{S,j})$ for $j = \{L, H\}$. The benefit B of commitment is given by

$$B = \left[\lambda_S \left(D_{SH} - D_{SL}\right) + \lambda_T \left(D_{TH} - D_{TL}\right)\right]/4. \tag{36}$$

The above expression is easy to understand. The first term on the right-hand side of (36) is the average of the additional rents that an agent can obtain by buying or selling the asset to a trader of type S if he has commitment power rather than not. These additional rents are equal to the meeting rate λ_S times 1/4 of the gains from trade $D_{SH} - D_{SL}$. The second term on the right-hand side of (36) is the average of the additional rents that an agent can obtain by buying of selling the asset to a trader of type T if he has commitment power rather than not. These additional rents are equal to the meeting rate λ_T times 1/4 of the gains from trade $D_{TH} - D_{TL}$.

Substituting the gains from trade $D_{SH} - D_{SL}$ with (15) and the gains from trade $D_{TH} - D_{TL}$ with (16), we can rewrite B as

$$B = \left\{ \frac{\lambda_S}{2\sigma + \lambda_S} + \frac{\lambda_T}{2\sigma + 2\lambda_S + \lambda_T} \right\} \frac{u_H - u_L}{4}, \tag{37}$$

where the meeting rates λ_S and λ_T are respectively given by

$$\lambda_S = \sqrt{\sigma^2 + \lambda\sigma/2 + (\lambda\phi_T)^2/16} - (\sigma + \lambda\phi_T/4), \tag{38}$$

$$\lambda_T = \lambda\phi_T/4 + \sqrt{\sigma^2 + \lambda\sigma/2} - \sqrt{\sigma^2 + \lambda\sigma/2 + (\lambda\phi_T)^2/16}. \tag{39}$$

The expression in (37), together with (38) and (39), gives us the benefit B of commitment power as a function on the measure ϕ_T of agents of type T. We can show that B is strictly increasing in ϕ_T over the interval $(0, \phi_T^*)$ and strictly decreasing in ϕ_T over the interval $(\phi_T^*, 1)$, where $\phi_T^* \in (0, 1)$. Moreover, we can show that B attains its maximum value $\bar{b}$ for $\phi_T = \phi_T^*$ and its minimum value $\underline{b}$ for $\phi_T = 0$ and $\phi_T = 1$. The properties of B as a function of ϕ_T are illustrated in Figure 2 below.

The finding that the benefit of commitment is hump-shaped in the fraction of traders who have commitment is somewhat surprising. In order to understand this finding, it is useful to differentiate B with respect to λ_T (which is monotonically increasing in ϕ_T) taking into account that λ_S is equal to the difference between a constant and λ_T. Formally, we have

$$\begin{aligned} \frac{dB}{d\lambda_T} = & \left\{ \left(\frac{1}{2\sigma + 2\lambda_S + \lambda_T} - \frac{1}{2\sigma + \lambda_S} \right) \right. \\ & \left. + \frac{\lambda_S}{(2\sigma + \lambda_S)^2} + \frac{\lambda_T}{(2\sigma + 2\lambda_S + \lambda_T)^2} \right\} \frac{u_H - u_L}{4} \end{aligned} \tag{40}$$

The first term on the right-hand side of (40) measures the effect of λ_T on the composition of the agent's trading opportunities. A higher λ_T increases the rate at which the agent meets traders of type T and lowers the rate at which the agent meets traders of type S. This effect is negative, as the additional rents that an agent with commitment can capture is greater when he meets traders of type S than when he meets traders of type T. The second term on the right-hand side of (40) measures the effect of λ_T on the additional rents that an agent with commitment can capture in meetings with traders of type S. This effect is positive because an increase in λ_T lowers λ_S and, consequently, it lowers the outside option of traders of type S. The last term on the right-hand side of (40) measures the effect of λ_T on the additional rents that an agent with commitment can extract in meetings with traders of type T. This effect is also positive because an increase in λ_T lowers λ_S and, consequently, the outside option of traders of type T.

When $\lambda_T = 0$, B is increasing in λ_T because the negative effect on the composition of trading opportunities facing an agent with commitment power is dominated by the positive effect on the additional rents that the agent can capture in meetings with traders of type S. When $\lambda_S = 0$, B is decreasing in λ_T lowers the negative effect on the composition of trading opportunities facing an agent with commitment power dominates the positive effect on the additional rents that the agent can capture in meetings with traders of type T. These observations imply that the benefit of commitment is first increasing and then decreasing in the fraction of agents with commitment.

4.1.2 Cost of Commitment

The cost of acquiring commitment power may vary from agent to agent, it may be the same for all agents, it may be zero for some and infinite for others. In order to accommodate all of these cases, we consider a generic distribution $F(c)$ of costs of acquiring commitment power, where $F(c)$ denotes the measure of agents who face a cost non-greater than c. Associated with the cost distribution $F(c)$, there is a correspondence $C(\phi_T) = \{c : F(c) = \phi_T\}$, where $C(\phi_T)$ denotes the ϕ_T quantile of the cost distribution. It is natural to think of $C(\phi_T)$ as the cost correspondence of acquiring commitment power.

We are sometimes going to focus on two particular specifications of the cost correspondence. The first specification is one in which the cost correspondence is perfectly inelastic. Here, we assume that every agent faces the same cost $c \geq 0$ for acquiring commitment power. Thus, the cost distribution $F(c)$ is degenerate at c, and the cost correspondence $C(\phi_T)$ is such that $C(0) = [0, c]$, $C(\phi_T) = c$ for all $\phi_T \in (0, 1)$, and $C(1) = [c, \infty]$. The specification is interesting because it captures the view that commitment power is a technology that can be acquired by any agent at the same cost and that intermediaries and final users in the asset market are, from an ex-ante perspective, identical. This specification is illustrated in Figure 2 below.

The second specification is one in which the cost correspondence is perfectly elastic. Here, we assume that a fraction $\hat{\phi}_T$ of agents can acquire commitment power at no cost and a fraction $1-\hat{\phi}_T$ of agents faces an infinite cost to acquire commitment power. That is, the cost distribution $F(c)$ is such that $F(c) = \hat{\phi}_T$ for all $c \geq 0$. Thus, the cost correspondence $C(\phi_T)$ is such that $C(\phi_T) = 0$ for all $\phi_T \in [0, \hat{\phi}_T]$ and $C(\phi_T) = \infty$ for all $\phi_T \in (\hat{\phi}_T, 1]$. The specification is interesting because it captures the view that commitment power is an innate trait rather than a technology that can be acquired at some cost. This specification is illustrated in Figure 3(a) below.

4.2 Equilibrium and Welfare

A measure ϕ_T of agents with commitment power is an equilibrium if and only if the benefit of commitment, $B(\phi_T)$, is greater than the cost of acquiring commitment for a measure ϕ_T of agents and smaller than the cost of acquiring commitment for a measure $1-\phi_T$ of agents. Formally, ϕ_T is an equilibrium if and only if $B(\phi_T) \in C(\phi_T)$. Graphically, ϕ_T is an equilibrium if and only if it is an intersection between the benefit function $B(\phi_T)$ and the cost correspondence $C(\phi_T)$.

Figure 2 illustrates the set of equilibria in the case of a perfectly inelastic cost correspondence. If the common cost of commitment c is greater than the minimum benefit of commitment, $\underline{b}$, and the maximum benefit of commitment, $\bar{b}$, there are three equilibria. In the first equilibrium (marked as E_1), the measure of agents of type T is $\phi_{T,1} = 0$. This is an equilibrium because the benefit of commitment at $\phi_{T,1} = 0$ is $\underline{b}$, which is smaller than the cost of commitment c. In this equilibrium, none of the agents acquires commitment power and, hence, the asset market is not intermediated. In the second equilibrium (marked as E_2), the measure of agents of type T is $\phi_{T,2} > \phi_{T,1}$. This is an equilibrium because the benefit of commitment at $\phi_{T,2}$ is equal to the cost c. In this equilibrium, a relatively small fraction of agents acquire commitment power and act as an intermediary in the asset market. In the third equilibrium (marked as E_3), the measure of agents of type T is $\phi_{T,3}$, with $\phi_{T,3} > \phi_{T,2}$ and $\phi_{T,3} < 1$. Again, this is an equilibrium because also at $\phi_{T,3}$ the benefit of commitment is equal to the cost c. In this equilibrium, a relatively larger fraction of agents acquire commitment power and act as intermediary. If the cost of commitment is small enough—specifically smaller than $\underline{b}$—there is a unique equilibrium in which all agents choose to acquire commitment power. Similarly, if the cost of commitment is high enough—specifically greater than $\bar{b}$—there is a unique equilibrium in which none of the agents chooses to acquire commitment power.

There are some important observations to make about the equilibrium set characterized above. First, notice that, as long as the cost of commitment is neither too low nor too high—there are multiple equilibria. The multiplicity of equilibria is due to the fact that the benefit from having commitment power in the asset market is, over some region, increasing in the measure of agents of type T. If there are no traders of type T in the market, agents of type S know that they can often trade at favorable prices. Hence, if there an no traders of type T, an individual has no incentive to acquire commitment power as this power would not allow him to force agents of type S to buy the asset at much of a premium or sell the asset at much of a discount. However, if there are some

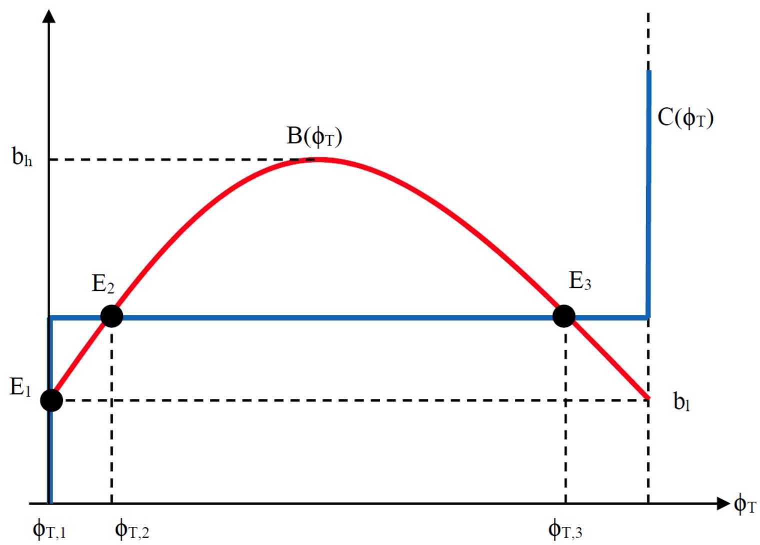
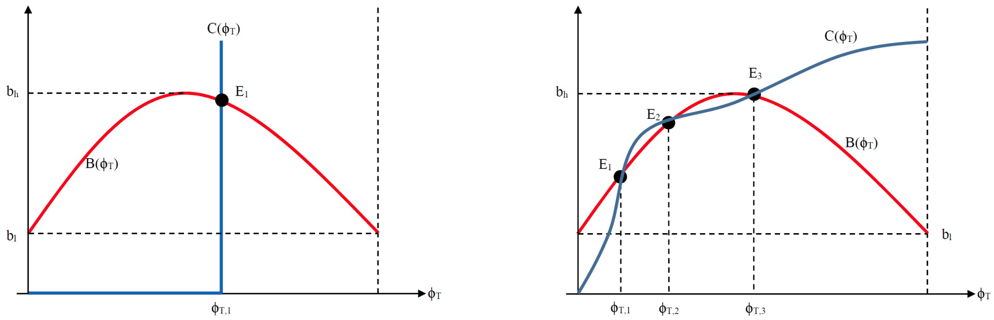


Figure 2: Equilibrium Intermediation: Inelastic cost

traders of type T in the market, agents of type S know that they will face less favorable prices. Hence, if there are some traders of type T, an individual has the incentive to acquire commitment power as this power will allow him to force agents of type S to buy the asset at a relatively high premium or to sell it at a relatively high discount.

Second notice that not all equilibria are stable. Using a standard heuristic argument, we say that an equilibrium ϕ_T^* is stable if—at ϕ_T^*—the derivative of the benefit of commitment with respect to ϕ_T is smaller than the derivative of the cost of commitment with respect to ϕ_T. Conversely, we say that an equilibrium ϕ_T^* is unstable if the derivative of the benefit of commitment is greater than the derivative of the cost of commitment. The rationale behind this notion of stability is that, if one were to exogenously throw into the market an extra ϵ of agents of type T, these agents would be worse off in a stable equilibrium, while they would be better off in an unstable one. Using this notion of stability, it is immediate to see that the odd-numbered equilibria (counting by lowest to highest ϕ_T) are all stable and that the even-numbered equilibria are all unstable. In Figure 2, for instance, equilibria E_1 and E_3 are stable, while equilibrium E_2 is unstable.

Finally, notice that, even though all agents are ex-ante identical, some stable equilibria have the feature that a fraction of agents chooses to acquire commitment power and become an intermediary and another fraction of agents chooses not to acquire commitment power and become a final user. The existence of equilibria in which ex-ante identical agents choose to specialize into either becoming intermediaries or final users is due to the fact



(a) Elastic (b) Generic

Figure 3: Equilibrium Intermediation

that the benefit from having commitment power is lowest both when there are no traders of type T in the market and when all traders in the market are of type T. Intuitively, the benefit of commitment is lowest when there are no traders of type T because, in this case, the market protects agents of type S from being exploited by agents with commitment. The benefit of commitment is lowest when there all traders are of type T because, in this case, there are no agents of type S to exploit.

Figure 3(a) illustrates the set of equilibria in the case of a cost correspondence that is perfectly elastic at some $\hat{\phi}_T$. Obviously, in this case, the unique equilibrium is such that the measure of agents of type T is $\hat{\phi}_T$. Finally, Figure 3(b) illustrates the set of equilibria in the case of a generic cost correspondence. Depending on the shape of the correspondence, there can exist a unique equilibrium in which no agent acquires commitment power, a unique equilibrium in which all agents acquire commitment power, or multiple equilibria with cardinality $2N+1$ and varying measures of agents with commitment power. Also in this case, all the odd-numbered equilibria are stable and all the even-numbered equilibria are unstable.

We summarize our findings in the following proposition.

Proposition 2. Equilibrium measure of intermediaries. *(i) Suppose that $C(\phi_T)$ is perfectly inelastic at c. Then: (a) if $c \in (0, \underline{b})$, there is a unique equilibrium with a measure $\phi_T = 1$ of agents of type T; (b) if $c \in (\underline{b}, \overline{b})$, there are three equilibria with, respectively, measures $\phi_{T,1}$, $\phi_{T,2}$ and $\phi_{T,3}$ of agents of type T, where $0 = \phi_{T,1} < \phi_{T,2} < \phi_{T,3} < 1$; (c) if $c > \overline{b}$, there is a unique equilibrium with a measure $\phi_T = 0$ of agents of type T. (ii) Suppose that $C(\phi_T)$ is perfectly elastic at $\hat{\phi}_T$. Then, there is a unique equilibrium with a measure $\phi_T = \hat{\phi}_T$ of agents of type T. (iii) For any $C(\phi_T)$, there are $2N+1$ equilibria with, respectively, measures $\phi_{T,1}$, $\phi_{T,2}$, ... $\phi_{T,2N+1}$ of agents of type T, for some $N \geq 0$.*

We now turn to examining the welfare properties of equilibrium. Consider an equilibrium with ϕ_T^* agents of type T and $1-\phi_T^*$ agents of type S. In such an equilibrium, welfare—as measured by the sum of the agents' annuitized lifetime utilities net of annuitized costs of acquiring commitment power—is given by

$$\begin{aligned} W &= \phi_T^* \left[\frac{\lambda_S}{2\sigma+\lambda_S} \frac{u_H - u_L}{2} + \frac{\lambda_T}{2\sigma + 2\lambda_S + \lambda_T} \frac{u_H - u_L}{4} \right] \\ &+ (1-\phi_T^*) \left[\frac{\lambda_S}{2\sigma+\lambda_S} \frac{u_H - u_L}{4} \right] + \frac{u_L + u_H}{4} - \int_0^{\phi_T^*} C(x)dx \end{aligned} \tag{41}$$

The first term on the right-hand side of (41) is the lifetime utility of agents of type T if none of them held the asset. The second term is the lifetime utility of agents of type S if none of them held the asset. The third term is the additional lifetime utility enjoyed by agents of type S and T who do hold the asset. And the last term is the cost borne by agents in order to acquire the commitment technology.

The key to understand the welfare properties of equilibrium is the following expression

$$\begin{aligned} W &= [\mu_{SL} + \mu_{TL}]\, u_L + [\mu_{SH} + \mu_{TH}]\, u_H - \int_0^{\phi_T^*} C(x)dx \\ &= \frac{u_H}{2} - \left[\sqrt{\left(\frac{\sigma}{\lambda}\right)^2 + \frac{\sigma}{2\lambda}} - \frac{\sigma}{\lambda} \right] (u_H - u_L) - \int_0^{\phi_T^*} C(x)dx. \end{aligned} \tag{42}$$

The first line in (42) states that welfare is equal to u_L times the measure of low-valuation agents who hold the asset plus u_H times the measure of high-valuation agents who hold the asset net of the cost borne by agents of type T to acquire the commitment technology. The second line in (42) makes use of the fact that the measure of low and high-valuation agents with the asset is independent of ϕ_T^* to show that welfare is a constant minus the cost borne by agents of type T to acquire the commitment technology. The finding is not surprising. After paying the cost to acquire commitment power, agents of type T enjoy a higher lifetime utility than agents of type S. However, agents of type T enjoy a higher lifetime utility not because they improve the allocation of the asset in any way—indeed, the fraction of agents of type L who holds the asset is independent of the measure of agents of type T—but because they extract rents from agents of type S. That is, the higher lifetime utility enjoyed by agents of type T comes entirely at the expense of the lifetime utility enjoyed by agents of type S. Therefore, after paying the cost to acquire commitment power, the sum of the lifetime utilities of agents of type T and S is a constant. The only effect of ϕ_T^* on welfare is through the cost of acquiring commitment.

Two conclusions follow directly from (42). First, any equilibrium with $\phi_T^* > 0$ is inefficient unless $\int_0^{\phi_T^*} C(x)dx = 0$. Indeed, efficiency requires that no agent pays any cost to acquire commitment power and that, in any meeting between agents with different valuations, the asset is given to the one with the highest valuation. An equilibrium always satisfies the efficiency condition on the pattern of trade (as established in Proposition 1) but it satisfies the efficiency condition on entry if and only if $\int_0^{\phi_T^*} C(x)dx = 0$. That is, intermediation is ex-post welfare neutral, but, if there are costs associated with becoming an intermediary, intermediation reduces ex-ante welfare. Second, whenever there are multiple equilibria, the equilibria can be ranked by welfare. In particular, the equilibrium with the lowest ϕ_T^* has the highest welfare, the equilibrium with the second lowest ϕ_T^* has the second highest welfare, and so on. That is, the best equilibrium is always the one with the lowest amount of intermediation.

Consider, for example, the case of an inelastic cost function that is illustrated in Figure 2. The equilibrium with a measure $\phi_{T,1}$ of intermediaries has higher welfare than the equilibrium with a measure $\phi_{T,2}$ of intermediaries, which, in turn, has higher welfare than the equilibrium with $\phi_{T,3}$ intermediaries. The equilibrium with $\phi_{T,1}$ intermediaries is efficient. The equilibria with $\phi_{T,2}$ and $\phi_{T,3}$ intermediaries are both inefficient. At the other extreme, consider the case of a perfectly elastic cost function that is illustrated in Figure 3. Here, the equilibrium is efficient because, even though, the measure of intermediaries is positive none of them has paid a cost to acquire commitment power.

We summarize our findings on welfare in the following proposition.

Proposition 3. Welfare. *Suppose there are $2N+1$ equilibria with, respectively, $\phi_{T,1}$, $\phi_{T,2}$,... $\phi_{T,2N+1}$ measures of agents of type T, where $\phi_{T,1} < \phi_{T,2} < ... < \phi_{T,2N+1}$. (i) Welfare in an equilibrium with $\phi_{T,i}$ agents of type T is strictly greater than welfare in an equilibrium with $\phi_{T,j}$ agents of type T for all $i < j$. (ii) An equilibrium with $\phi_{T,i}$ agents of type T is inefficient if and only if $\int_0^{\phi_{T,i}} C(x)dx = 0$.*

4.3 Comparative Statics

The benefit function $B(\phi_T)$ depends on the rate λ at which an agent meets a trader, on the rate σ at which an agent changes valuation, and on the difference $u_H - u_L$ between high and low valuation. However, it is easy to see that $B(\phi_T)$ does not depend on λ and σ separately, but only on their ratio $\kappa = \lambda/\sigma$, which we can interpret as an inverse measure of trading frictions. Comparative statics with respect to κ are particularly interesting from both a theoretical and an applied point of view. From the theoretical point of view, the comparative statics are interesting because one would presume that—as trading frictions vanish—the benefit from acquiring commitment power in order to extract more rents would vanish as well. Indeed, in a Walrasian equilibrium where trade is frictionless, there is no value in being able to commit to take-it-or-leave-it offers because the market pins down uniquely the price. Any bid below the market price is rejected and any offer above the market price is rejected as well. From the applied point of view, the comparative statics with respect to κ are interesting because it seems natural to think that trading frictions have gotten smaller over time and will continue to get smaller due to improvements in communication and information technology.

The cost function $C(\phi_T)$ depends on the distribution of opportunity costs for acquiring the commitment technology. These costs may be those associated with making the outcome of the agent's transactions public, so that he can sustain a reputation for being a tough negotiator. Alternatively, these costs may be those required to delegate negotiation to a salesman who has no discretion in accepting any price different than the one prespecified by the agent. Since the resources allocated to cover the cost to acquire the commitment technology, it seems natural to think that $C(\phi_T)$ depends positively on the rate of return R on alternative investments. Comparative statics with respect to R are particularly interesting because of the recent decline in real interest rates.

4.3.1 Trading Frictions

Let κ denote the ratio between the rate λ at which agents meet and the rate σ at which agents change preferences. Similarly, let κ_S denote the ratio between λ_S and σ and let κ_T denote the ratio between λ_T and σ. Using this notation, we can write the benefit of acquiring commitment power as

$$B = \left\{ \frac{\kappa_S}{2+\kappa_S} + \frac{\kappa_T}{2\sigma + 2\kappa_S + \kappa_T} \right\} \frac{u_H - u_L}{4}, \tag{43}$$

where the normalized meeting rates κ_S and κ_T are respectively given by

$$\kappa_S = \sqrt{1+\kappa/2+(\kappa\phi_T)^2/16} - (1+\kappa\phi_T/4)\,, \tag{44}$$

$$\kappa_T = \kappa\phi_T/4 + \sqrt{1+\kappa/2} - \sqrt{1+\kappa/2+(\kappa\phi_T)^2/16}. \tag{45}$$

Clearly, B, κ_S and κ_T all depend on the fraction ϕ_T of agents with commitment power. However, to keep the notation lighter we shall abstract from this dependence.

The derivative with respect to κ of the rate at which an agent meets a mismatched trader of type S is given by

$$\frac{d\kappa_S}{d\kappa} = \frac{1/2 + k\phi_T^2/8}{2\sqrt{1+\kappa/2+(\kappa\phi_T)^2/16}} - \frac{\phi_T}{4}. \tag{46}$$

The derivative above is positive. Intuitively, as the normalized meeting rate κ increases, there are two effects on κ_S. On the one hand, the meeting rate increases and this tends to raise κ_S. On the other hand, the measure μ_{SL} of mismatched traders of type S fall and this tends to lower κ_S. Since the measure of mismatched traders of type S falls less than proportionally with κ, the first effect dominates and $d\kappa_S/d\kappa$ is positive.

The derivative with respect to κ of the rate at which an agent meets a mismatched trader of type T is given by

$$\frac{d\kappa_T}{d\kappa} = \frac{\phi_T}{4} + \frac{1}{4\sqrt{1+k/2}} - \frac{1/4 + \kappa\phi_T^2/16}{\sqrt{1+\kappa/2+(\kappa\phi_T)^2/16}}. \tag{47}$$

The derivative above is also positive. There are two effects of κ on κ_T: the direct effect of κ on the rate at which agents meet and the indirect effect of κ on the measure μ_{TL} of mismatched traders of type T. The direct effect is positive. The indirect effect does maybe positive or negative, as an increase in k increases the rate at which agents of type (T,L) acquire the asset from traders of type (S,L) as well as the rate at which agents of type (T,L) sell the asset to traders of type (T,H) and (S,H). However, the direct effect always dominates and κ_T increases with κ.

The derivative of the benefit of acquiring commitment power with respect to κ is

$$\frac{dB}{d\kappa} = \left\{ \frac{1}{(2+\kappa_S)^2}\frac{d\kappa_S}{d\kappa} + \frac{1}{(2+2\kappa_S+\kappa_T)^2}\left[(1+\kappa_S)\frac{d\kappa_T}{d\kappa} - \kappa_T\frac{d\kappa_S}{d\kappa}\right]\right\}\frac{u_H-u_L}{4}. \tag{48}$$

The first term on the right-hand side of (48) is the derivative with respect to κ of the additional rents that an agent can extract from all meetings with mismatched traders of type S if he is of type T rather than S (i.e., the extra rents in each meeting with mismatched traders of type S times the frequency of meetings with mismatched traders of type S). This term is positive because the extra rents that an agent can extract from mismatched traders of type S are increasing with respect to κ_S and κ_S is increasing with respect to κ. The second term is the derivative with respect to κ of the additional rents that an agent can extract from all meetings with mismatched traders of type T if he is of type T rather than S. After substituting in $d\kappa_S/d\kappa$ and $d\kappa_T/d\kappa$, we can show that this term is also positive. Thus, the derivative of B with respect to κ is positive. In other words, the lower are search frictions in the market, the higher is the benefit of having the power to commit to take-it-or-leave-it offers.

The finding that $dB/d\kappa > 0$ runs against common intuition. Indeed, one is tempted to approximate the behavior of an economy with small frictions with the behavior of a Walrasian equilibrium. In a Walrasian equilibrium, commitment power is worthless as agents with and without commitment are fully protected from exploitation by market competition. Thus, one might conclude that—as search frictions get smaller—the benefit of commitment power should vanish too. Alas, this logic is flawed. The key mistake is that, while it is true that the rents that an agent of type T can extract from mismatched traders of type S fall as frictions become smaller, it is also true that the rate at which an agent of type T meets a mismatched trader of type S—and hence the volume of opportunities for rent extraction—also increases. As one can see immediately from the first term in (48), the positive effect of diminishing frictions on volume dominates the negative effect on margins. For the same reason, diminishing frictions have a positive effect on the second term in (48).

We now turn to examining the impact of diminishing frictions on the measure of commitment agents in the market. Figure 4 illustrates the effect of a small increase in the normalized meeting rate from κ to κ' on the measure of agents of type T when all agents face a cost of acquiring commitment power of c, with $c \in (\underline{b}, \bar{b})$. The benefit function $B(\phi_T)$ shifts up as the meeting rate increases from κ to κ'. If the increase is small enough, the number of equilibria does not change. In the first equilibrium, the measure of agents who acquire commitment power is zero for both κ and κ'. In the second equilibrium, the measure of agents who acquire commitment power falls. In the third equilibrium, the measure of agents with commitment power increases. Restricting attention to the stable equilibria (the first and the third), we can then conclude that the increase in the meeting rate unambiguously raises the equilibrium set of agents of type T. This conclusion generalizes to the case of an arbitrary cost function $C(\phi_T)$.

Proposition 4: Intermediation and trading frictions. *Let* $(\phi_{T,1}, \phi_{T,2}, ..., \phi_{T,2N+1})$ *denote the equilibrium measures of agents of type* T *given the normalized arrival rate* κ*, and let* $(\phi'_{T,1}, \phi'_{T,2}, ..., \phi'_{T,2N'+1})$ *denote the equilibrium measures of agents of type* T *given* κ'*. (i) For any* $\kappa' > \kappa$*,* $\mu'_{T,1} \geq \mu_{T,1}$ *and* $\mu'_{T,2N'+1} \geq \phi_{T,2N+1}$*. (ii) If* $N = N'$*, then* $\phi'_{T,j} \geq \phi_{T,j}$ *for all stable equilibria* $j = 1, 3, 5, ...$

Proposition 4 states that, as trading frictions become smaller and smaller, the fraction of agents who acquire commitment power increases. Since agents with commitment power find it optimal to act as intermediaries, the proposition implies that, as trading frictions become smaller and smaller, the number of intermediaries in the market grows larger and larger. This rather surprising implication follows directly from two properties of equilibrium: the benefit of rent extraction grows as frictions vanish and the agents who can extract rents are those who act as middlemen. Proposition 4 provides a novel explanation

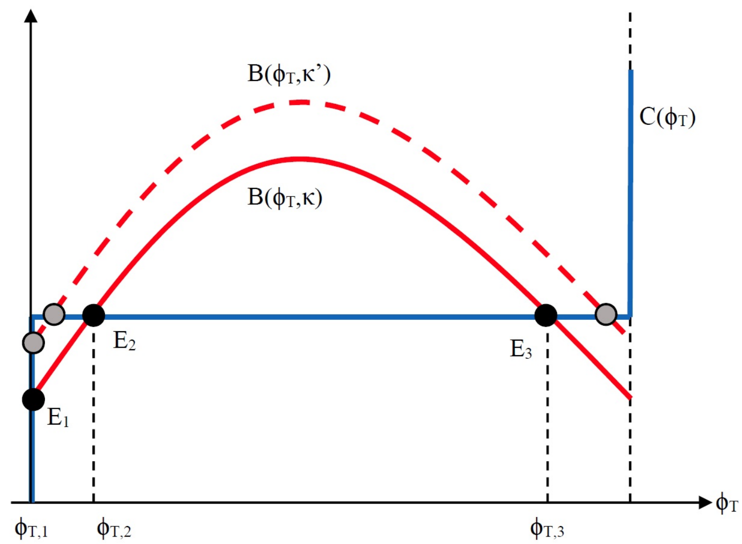


Figure 4: Decline in Trading Frictions

for the dramatic rise of the financial intermediation sector that has taken place in the US since the 1950s (see, e.g., Philippon 2015). According to our explanation, the rise in the financial intermediation has not taken place in spite of the decline in trading frictions brought about by improvements in information technology. According to our explanation, the rise in the financial sector has taken place precisely because improvements in information technology have reduced trading frictions.

It is natural to wonder about the effect of diminishing trading frictions on welfare. In general, a decline in trading frictions has two countervailing effects on welfare. On the one hand, a decline in trading frictions makes it easier for agents to adjust their asset inventories in response to preference shocks and, hence, improves the allocation of the asset. On the other hand, a decline in trading frictions induces more agents to spend resources to acquire commitment power and become intermediaries. Using the formula for W in the second line of (42), one can formally see that the derivative of welfare with respect to κ is given by

$$\frac{dW}{d\kappa} = \left\{ \left(\frac{1}{\kappa^2} + \frac{1}{2\kappa} \right)^{-1/2} \left(\frac{1}{\kappa^3} - \frac{1}{4\kappa} \right) + \frac{1}{\kappa^2} \right\} (u_H - u_L) - C(\phi_T^*) \frac{d\phi_T^*}{d\kappa}. \tag{49}$$

The first term on the right-hand side of (42) is the effect of an increase in κ on the measure of high-valuation agents holding the asset. The second term is the effect of an increase in κ on the amount of resources that are socially wasted in acquiring commitment power.

In general, the sign of (49) depends on the shape of the cost correspondence $C(\phi_T)$.

However, there are two cases in which we can make precise welfare statements. Suppose that the cost correspondence is infinitely elastic at some $\hat{\phi}_T$. In this case, the derivative of welfare with respect to the normalized meeting rate κ is given by

$$\frac{dW}{d\kappa} = \left\{ \left(\frac{1}{\kappa^2} + \frac{1}{2\kappa} \right)^{-1/2} \left(\frac{1}{\kappa^3} - \frac{1}{4\kappa} \right) + \frac{1}{\kappa^2} \right\} (u_H - u_L) \, . \tag{50}$$

The above expression is positive because, when the cost correspondence is infinitely elastic, the measure of agents with commitment is constant and the only effect of lowering k on welfare is to improve the allocation of the asset.

Now, suppose that the cost correspondence is perfectly inelastic. Further suppose that the cost c of acquiring the commitment technology is such that there are three equilibria, E_1,E_2 and E_3, as illustrated in Figure 2. Let us focus on the equilibrium E_3, which is the stable equilibrium with a positive measure of intermediaries. At the E_3 equilibrium, the measure of agents of type T is interior. Hence, the cost and the benefit of acquiring the commitment technology are equated, i.e.

$$c = \left\{ \frac{\kappa_S}{2+\kappa_S} + \frac{\kappa_T}{2+2\kappa_S+\kappa_T} \right\} \frac{u_H - u_L}{4}. \tag{51}$$

Moreover, at the E_3 equilibrium, the total benefit of commitment enjoyed by agents of type T is equal to the total cost of acquiring commitment borne by agents of type T. Hence, the expression for welfare in (41) simplifies to

$$W = \frac{\kappa_S}{2+\kappa_S} \frac{u_H - u_L}{4} + \frac{u_L + u_H}{4}. \tag{52}$$

Substituting out κ_T with (45), we can rewrite (51) as the following quadratic equation in κ_S

$$\begin{aligned} 0 = \quad & \kappa_S^2 \left[\frac{4c}{u_H - u_L} \right] + \kappa_S \left[2 + \frac{12c}{u_H - u_L} + \left(\frac{4c}{u_H - u_L} - 2 \right) \sqrt{1 + \frac{k}{2}} \right] \\ & +2 + \frac{8c}{u_H - u_L} + \left(\frac{8c}{u_H - u_L} - 2 \right) \sqrt{1 + \frac{k}{2}} \end{aligned} \tag{53}$$

The smaller of the two solution to (53) with respect to κ_S is the rate at which agents meet mismatched traders of type S in the E_3 equilibrium. When κ increases, the smaller solution to (53) with respect to κ_S falls. Hence, in the E_3 equilibrium, an increase in the meeting rate κ leads to a decline in the meeting rate κ_S of mismatched agents of type S. In turn, this implies that an increase in the meeting rate κ leads to lower welfare, since (52) is an increasing function of κ_S.

We now complete the characterization of the welfare effect of κ when the cost correspondence is perfectly inelastic. When $\kappa = 0$, the benefit function is such that $B(\phi_T) = 0$ for all ϕ_T. When $\kappa \to \infty$, the maximum of the benefit function is such that the minimum benefit is $\underline{b}^*$ and the maximum benefit is $\overline{b}^*$, with $0 < \underline{b}^* < \overline{b}^*$. Take any cost of commitment c smaller than $\underline{b}^*$. For all κ smaller than some κ^*, there is a unique equilibrium with $\phi_T = 0$. As we increase κ in the interval $(0, \kappa^*)$, the measure of agents with commitment remains constant and, as it is clear from (50), welfare strictly increases. For κ greater than κ^* and smaller than some κ^{**}, there are two stable equilibria E_1 and E_3 with, respectively, $\phi_{T,1} = 0$ and $\phi_{T,3} \in (0,1)$ measures of agents of type T. From Proposition 3, we know that welfare at E_1 is strictly greater than welfare at E_3. As we increase κ in the interval (κ^*, κ^{**}), the welfare associated with E_1 increases and welfare at E_3 falls. For κ greater than κ^{**}, there is a unique equilibrium with $\phi_T = 1$. As we increase κ in this region, welfare increases. For $c \in (\underline{b}^*, \overline{b}^*)$, the analysis is the same except that there is no cutoff κ^{**} above which the unique equilibrium is such that all agents acquire commitment power. For $c > \overline{b}^*$, the analysis is the same except that there is no cutoff κ^* above which multiple equilibria emerge.

The analysis of the welfare effects of k are summarized in the following proposition.

Proposition 5. Welfare and trading frictions. *(i) Suppose $C(\phi_T)$ is perfectly elastic. Welfare is strictly increasing in κ. (ii) Suppose $C(\phi_T)$ is perfectly inelastic at c. For all $c \in (0, \underline{b}^*)$, there exist κ^* and κ^{**} with $0 < \kappa^* < \kappa^{**}$ such that: (a) for $\kappa \in (0, \kappa^*)$, the unique equilibrium is such that $\phi_T = 0$ and welfare is strictly increasing in κ; (b) for $\kappa \in (k^*, k^{**})$, there are two stable equilibria E_1 and E_3 with measures $\phi_{T,1} = 0$ and $\phi_{T,3} \in (0,1)$ of agents of type T. Welfare is strictly greater at E_1 than at E_3 and it is strictly increasing in κ at E_1 and strictly decreasing at E_3; (c) for $\kappa > \kappa^{**}$, the unique equilibrium is such that $\phi_T = 1$ and welfare is strictly increasing in κ. For $c \in (\underline{b}^*, \overline{b}^*)$, (b) applies to all $\kappa > \kappa^*$. For $c > \overline{b}^*$, (a) applies to all $\kappa > 0$.*

Proposition 5 implies that, if agents face the same cost c of acquiring commitment power and equilibrium is such that there are both agents with and without commitment in the asset market, a decrease in trading fictions causes welfare to fall. The result may seem paradoxical, as it means that an improvement in technology leads to worse economic outcomes. However, the result is nothing but a direct implication of our simple model of intermediation as pure rent-extraction. Proposition 4 shows that a decrease in trading frictions increases the benefit of intermediating the asset to extract rents and, hence, increases the number of intermediaries in the market. Proposition 5 shows that, if agents are ex-ante homogeneous, the cost of the additional resources allocated by agents to acquire the ability to extract rents is smaller than the benefit of the improvement in

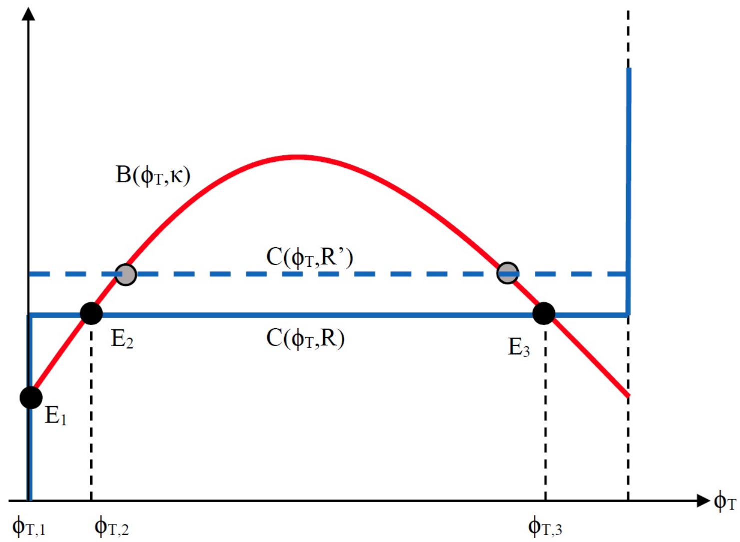


Figure 5: Decline in Rate of Return

the asset allocation afforded by the decrease in trading frictions and, hence, welfare falls.

4.3.2 Rate of Return on Alternative Investments

We now carry out comparative statics with respect to the cost correspondence $C(\phi_T)$. We assume that the correspondence depends on a parameter R, which we interpret as the rate of return on the alternative investments that agents forego when they decide to acquire the commitment technology. We assume that $C(\phi_T; R)$ is continuous, differentiable and strictly increasing with respect to R. Moreover, we assume that $\lim_{R\to 0} C(\phi_T; R) = 0$ and $\lim_{R\to\infty} C(\phi_T; R) = \infty$.

Figure 5 illustrates the effect of an increase in the rate of return on alternative investments from R to R' on the measure of agents of type T, under the assumption of that all agents face the same opportunity cost $c(R) \in (\underline{b}, \bar{b})$ for acquiring the commitment technology. If the increase in R is small enough, the number of equilibria does not change. In the first equilibrium, the measure of agents who acquire commitment power is zero for both R and R'. In the second equilibrium, the measure of agents who acquire commitment power increases. In the third equilibrium, the measure of agents with commitment power falls. Restricting attention to the stable equilibria, we conclude that the increase in the parameter R lowers the equilibrium measure of agents of type T. The conclusion generalizes to the case of an arbitrary cost function $C(\phi_T; R)$.

Proposition 6: Intermediation and return on investments. *Let* $(\phi_{T,1}, \phi_{T,2}, ..., \phi_{T,2N+1})$ *denote the equilibrium measures of agents of type* T *given the rate of return* R*, and let* $(\phi'_{T,1}, \phi'_{T,2}, ..., \phi'_{T,2N'+1})$ *denote the equilibrium measures of agents of type* T *given the rate of return* R'*. (i) For any* $R' > R$*,* $\mu'_{T,1} \geq \mu_{T,1}$ *and* $\mu'_{T,2N'+1} \geq \phi_{T,2N+1}$*. (ii) If* $N = N'$*, then* $\phi'_{T,j} \geq \phi_{T,j}$ *for all stable equilibria* $j = 1, 3, 5, ...$

We now turn to examining the effect of the rate of return on alternative investments on welfare. The rate of return R affects welfare through two channels. On the one hand, an increase in R increases the cost borne by the agents who decide to acquire the commitment technology. Through this channel, an increase in R tends to reduce welfare. On the other hand, an increase in R lowers the measure of agents who decide to acquire the commitment technology. Through this channel, an increase in R tends to increase welfare. Formally, the derivative of welfare (41) with respect to R around an equilibrium with ϕ_T agents of type T is given by

$$\frac{dW}{dR} = \int_0^{\phi_T^*} \frac{dC(x, R)}{dR} dx - C(\phi_T^*) \frac{d\phi_T^*}{dR}. \tag{54}$$

In general, the effect on welfare of an increase in the rate of return R depends on the particular shape of the cost correspondence $C(\phi_T, R)$. To say something more precise, we consider the case of a perfectly inelastic cost correspondence. Further, let us consider the case in which the cost $c(R)$ of acquiring the commitment technology is such that there are three equilibria, E_1, E_2 and E_3, as illustrated in Figure 2. Let us first focus on the equilibrium E_3, which is the stable equilibrium with a positive measure of intermediaries. At the E_3 equilibrium, the rate κ_S at which agents meet mismatched traders of type S is given by the smallest solution to the quadratic equation

$$\begin{aligned} 0 = \quad & \kappa_S^2 \left[\frac{4c(R)}{u_H - u_L} \right] + \kappa_S \left[2 + \frac{12c(R)}{u_H - u_L} + \left(\frac{4c(R)}{u_H - u_L} - 2 \right) \sqrt{1 + \frac{k}{2}} \right] \\ & + 2 + \frac{8c(R)}{u_H - u_L} + \left(\frac{8c(R)}{u_H - u_L} - 2 \right) \sqrt{1 + \frac{k}{2}} \end{aligned} \tag{55}$$

When R increases, the smaller solution to (55) with respect to κ_S goes up. Hence, in the E_3 equilibrium, an increase in the rate of return on alternative investments leads to an increase in the rate at which agents meet mismatched traders of type S. In turn, this implies that an increase in R leads to higher welfare, since (52) shows that W is an increasing function of κ_S.

We now complete the characterization of the welfare effect of R when the cost correspondence is perfectly inelastic. Let R^* be such that $c(R^*) = \underline{b}$ and let R^{**} be such that

$c(R^{**}) = \bar{b}$. For any $R \in (0, R^*)$, there is a unique equilibrium with $\phi_T = 1$. As we increase R in this region, welfare strictly declines because the measure of agents of type T does not change while the resources they spend on acquiring commitment power increase. For any $R \in (R^*, R^{**})$, there are two stable equilibria E_1 and E_3 with, respectively, $\phi_{T,1} = 0$ and $\phi_{T,3} \in (0, 1)$ measures of agents of type T. Welfare at E_1 is strictly greater than welfare at E_3. As we increase R in the interval (R^*, R^{**}), the welfare at E_1 remains constant as no resources are spent on acquiring commitment power. The welfare associated at E_3 strictly increases. For any R greater than R^{**}, there is a unique equilibrium with $\phi_T = 0$. As we increase R in this region, welfare remains constant.

The analysis of the welfare effects of R is summarized in the following proposition.

Proposition 7. Welfare and return on investments. *Assume $C(\phi_T; R)$ is perfectly inelastic at $c(R)$. There exist R^* and R^{**} with $0 < R^* < R^{**}$ such that: (a) for $R \in (0, R^*)$, the unique equilibrium is such that $\phi_T = 1$ and welfare is strictly decreasing in R; (b) for $R \in (R^*, R^{**})$, there are two stable equilibria E_1 and E_3 with measures $\phi_{T,1} = 0$ and $\phi_{T,3} \in (0, 1)$ of agents of type T. Welfare is strictly greater at E_1 than at E_3 and it is independent of R at E_1 and strictly increasing in R at E_3, (c) for $R > R^{**}$, the unique equilibrium is $\phi_T = 0$ and welfare is independent of R.*

Proposition 6 states that a decrease in R induces more agents to acquire commitment power and become intermediaries. The result suggests that a decline in the fundamental payoff of the asset leads to an increase in the fraction of agents who choose to acquire commitment power and become intermediaries in the asset market. A reduction in R could for instance reflect an environment with a declining return on productive investments; alternatively, it may capture a regime of low real interest rates. Our results capture, in a stylized fashion, the notion that in such circumstances agents will increasingly engage in non-productive activities such as financial intermediation for the purpose of rent extraction. That is, with declining real payoffs, rent seeking becomes a relatively more attractive activity. Proposition 7 then states that, if all agents have equal access to the rent-extraction technology, the shift towards rent extraction associated with a decline in R might lower welfare.

The results is related to the influential notion of "reach for yield" developed in Rajan (2006) which argues that with low risk-free returns investors tend to rebalance their portfolios towards more risky assets.[2] We add to this a connected, yet distinct and novel mechanism through which low rates may affect financial activity in an unintended way,

[2]See Cociuba, Shukayev and Ueberfeldt (2016) for an overview. Jiménez, Ongena, Peydró and Saurina (2014) provide empirical evidence.

namely that financial market participants move towards rent extraction when facing a period of low-returns. The zero-sum character of such activities makes them socially undesirable if there is a (fixed) cost associated with acquiring the necessary skills or technology as is summarized in proposition 7. This observation naturally leads us to study policy tools that could potentially alleviate the adverse effects of rent extraction in the next section.

5 Transaction Tax

In this section, we consider the effect of introducing a transaction tax in our asset market model. Specifically, we consider a fixed tax τ that buyer and seller need to pay to the government whenever they trade the asset. The government rebates the revenues of the transaction tax to the market participants in a lump-sum fashion. In subsections 5.1 through 5.3, we study the effect of the transaction tax on the equilibrium pattern of trade taking as given the measures of agents of type S and T. We find that, depending on the size of the tax, the equilibrium pattern of trade may be the same as in the laissez-faire equilibrium, it may involve only a subset of the trades that emerge in the laissez-faire equilibrium, or it may lead to a complete shut-down of all trade. In subsection 5.4, we study the effect of the transaction tax on the equilibrium measure of agents of type S and T and on welfare. We then characterize the transaction tax that maximizes welfare. We show that the optimal transaction tax reproduces a key feature of Walrasian Equilibrium. Namely, the optimal transaction tax is such that, just as in a Walrasian Equilibrium, the surplus in any particular transaction between a buyer and a seller is zero. We also show that the optimal transaction makes the equilibrium efficient. Since the optimal transaction tax implements the efficient allocation, we are justified in restricting attention to this particular policy instrument rather than to formulate and solve a general mechanism design problem.

5.1 Intermediation Equilibrium

We first look for conditions on the transaction tax under which there exists an equilibrium in which both fundamental trades—i.e. the trades between a seller with low valuation and a buyer with high valuation—and intermediation trades—i.e. the trades between buyers and sellers with the same valuation and different commitment power—take place. This is an equilibrium in which, notwithstanding the presence of a transaction tax, the pattern of trade remains is the same as in the laissez-faire equilibrium described in Section 3. We shall refer to this as the intermediation equilibrium.

In an intermediation equilibrium, the gains from trade $D_{i,H} - D_{i,L} - \tau$ between an agent of type (i, L) with the asset and an agent of type (i, H) without the asset are given by

$$
\begin{aligned}
D_{SH} - D_{SL} - \tau &= \frac{u_H - u_L - (r + 2\sigma)\tau}{r + 2\sigma + \lambda_S}, &\quad (56)\\
D_{TH} - D_{TL} - \tau &= \frac{u_H - u_L - (r + 2\sigma + 2\lambda_S)\tau}{r + 2\sigma + 2\lambda_S + \lambda_T}. &\quad (57)
\end{aligned}
$$

Clearly, when the transaction tax τ is set to zero, the expressions in (56) and (57) are the same as in the laissez-faire equilibrium. When τ is positive, the expressions in (56) and (57) are smaller than in the laissez-faire equilibrium.

The gains from trade $D_{TL} - D_{SL} - \tau$ between an agent of type (S, L) with the asset and an agent of type (T, L) without the asset are equal to $D_{SH} - D_{TH} - \tau$—which are the gains from trade between an agent of type (T, H) with the asset and an agent of type (S, H) without the asset. The gains associated to these trades are

$$
\begin{aligned}
&D_{TL} - D_{SL} - \tau = D_{SH} - D_{TH} - \tau\\
&\quad = \frac{\lambda_S \left(D_{SH} - D_{SL} - \tau\right) + \lambda_T \left(D_{TH} - D_{TL} - \tau\right) - 2(r + 2\sigma + \lambda_S)\tau}{2\left(r + 2\sigma + 2\lambda_S\right)} \qquad (58)
\end{aligned}
$$

Again, when the transaction tax τ is zero, the expression in (58) is the same as in the laissez-faire equilibrium. When τ is positive, the expression in (58) is smaller than it is in the laissez-faire equilibrium. Note that the transaction tax τ affects (58) not only indirectly through the negative effect of the tax on the extra rents that (T, L) can capture relative to (S, L) when he sells the asset to (T, H) and (S, H). The transaction tax τ also affects (58) directly because, when (S, L) passes the asset to (T, L), the two parties have to pay the tax.

Finally, the gains from trade $D_{TH} - D_{SL} - \tau$ between an agent of type (S, L) with the asset and an agent of type (T, H) without the asset are equal to $D_{SH} - D_{TL} - \tau$—which are the gains from trade between an agent of type (S, H) without the asset and an agent of type (T, L) with the asset. The gains from these trades are given by

$$
\begin{aligned}
D_{TH} - D_{SL} - \tau &= D_{SH} - D_{TL} - \tau\\
&= D_{TH} - D_{TL} - \tau + D_{TL} - D_{SL} \qquad (59)
\end{aligned}
$$

When τ is zero, the gains from trade between (S, L) and (T, H) are equal to the sum of the gains from trade between (S, L) and (T, L) and the gains from trade between (T, L) and (T, H). However, when τ is positive, the gains from trade between (S, L) and (T, H) are greater than this sum, as the trade between (S, L) and (T, H) is hit by the transaction tax once rather than twice.

The stationary distribution $\{\mu_{i,j}, \nu_{i,j}\}$ in an intermediation equilibrium is the same as in the laissez-faire equilibrium as the pattern of trade is the same. Therefore, we have

$$\mu_{i,j} + \nu_{i,j} = \phi_i/2, \text{ for } j = \{L, H\} \text{ and } i = \{S, T\}, \tag{60}$$

$$\mu_{i,L} = \nu_{i,H} \text{ for } i = \{S, T\}, \tag{61}$$

$$\mu_{SL} = \sqrt{\left(\frac{\sigma}{\lambda}\right)^2 + \frac{\sigma}{2\lambda} + \frac{\phi_T^2}{16}} - \left(\frac{\sigma}{\lambda} + \frac{\phi_T}{4}\right), \tag{62}$$

$$\mu_{TL} = \frac{\phi_T}{4} + \sqrt{\left(\frac{\sigma}{\lambda}\right)^2 + \frac{\sigma}{2\lambda}} - \sqrt{\left(\frac{\sigma}{\lambda}\right)^2 + \frac{\sigma}{2\lambda} + \frac{\phi_T^2}{16}}. \tag{63}$$

Having characterized the gains from trade and the stationary distribution, we can now derive the conditions on the transaction tax under which an intermediation equilibrium exists. First, note that (S, L) sells the asset to (S, H) and (T, L) sells the asset to (T, H) if and only if (56) and (57) are positive, which is the case if and only if

$$\tau \leq \frac{u_H - u_L}{r + 2\sigma + 2\lambda_S}. \tag{64}$$

Second, note that (S, L) sells the asset to (T, L) and (T, H) sells the asset to (S, H) if and only if (58) is positive, which is the case if and only if

$$\tau \leq \frac{(\lambda_S + \lambda_T)(u_H - u_L)}{(\lambda_S + \lambda_T)(r + 2\sigma) + 2(r + 2\sigma + \lambda_S)(r + 2\sigma + \lambda_S + \lambda_T)}. \tag{65}$$

Third, note that if (64) and (65) are positive, then (S, L) sells the asset to (T, H) and (T, L) sells the asset to (S, H).

Since the right-hand side of (65) is smaller than the right-hand side of (64), we can conclude that an intermediation equilibrium exists if and only if the transaction tax τ satisfies condition (65). That is, an intermediation equilibrium exists if and only if the transaction tax is sufficiently small to make sure that the intermediation trades—i.e. the trades between (S, L) and (T, L) and between (T, H) and (S, H)—are profitable. The finding is consistent with our observations on the effect of τ on the gains associated with the different trades, which made clear that a transaction tax would impact the intermediation trades more than the fundamental trades.

5.2 Fundamental Equilibrium

We now look for conditions on the transaction tax under which there exists an equilibrium in which the fundamental trades—i.e. the trades between a seller with low valuation and a buyer with high valuation—take place, while the intermediation trades—i.e. the trades between buyers and sellers with the same valuation and different commitment power—do not. We shall refer to this as a fundamental equilibrium.

We start the analysis of a fundamental equilibrium by characterizing the gains associated with the different trading opportunities that arise in the model economy. It is easy to show that the gains from trade $D_{i,H} - D_{i,L} - \tau$ between an agent of type (i, L) with the asset and an agent of type (i, H) without the asset are given by

$$D_{SH} - D_{SL} - \tau = \frac{u_H - u_L - (r + 2\sigma)\tau}{r + 2\sigma + \lambda_S}, \tag{66}$$

$$D_{TH} - D_{TL} - \tau = \frac{u_H - u_L - (r + 2\sigma)\tau}{(r + 2\sigma + \lambda_S)(r + 2\sigma + \lambda_S + \lambda_T)}. \tag{67}$$

The expression in (66) is the same expression as in an intermediation equilibrium. This is due to the fact that intermediation trades do not affect the lifetime utility of agents of type S. However, the expression (67) is different than in an intermediation equilibrium. In fact, in a fundamental equilibrium, the trade between (T, L) and (T, H) does not lead (T, L) to enjoy a capital gain from acquiring the option of buying the asset from (S, L) and it does not lead (T, H) to enjoy a capital gain from acquiring the option of selling the asset to (S, H).

The gains from trade $D_{TH} - D_{SL} - \tau$ between an agent of type (S, L) with the asset and an agent of type (T, H) without the asset are equal to $D_{SH} - D_{TL} - \tau$—which are the gains from trade between an agent of type (S, H) without the asset and an agent of type (T, L) with the asset. The gains from these trades are given by

$$\begin{aligned} D_{TH} - D_{SL} - \tau &= D_{SH} - D_{TL} - \tau \\ &= \left[1 - \frac{1}{2}\frac{\lambda_S + \lambda_T}{r + 2\sigma + \lambda_S + \lambda_T}\right]\frac{u_H - u_L - (r + 2\sigma)\tau}{r + 2\sigma + \lambda_S}. \end{aligned} \tag{68}$$

The expression in (68) is different than in an intermediation equilibrium. Again, this is because, in a fundamental equilibrium, (T, H) does not enjoy a capital gain associated with the option of reselling the asset to (S, H). Similarly, (T, L) does not enjoy a capital gain associated with the option of repurchasing the asset from (S, L).

Finally, the gains from trade $D_{TL} - D_{SL} - \tau$ between a potential buyer of type (T, L) and a potential seller of type (S, L) are equal to $D_{SH} - D_{TH} - \tau$—which are the gains from trade between a potential buyer of type (S, H) and a potential seller of type (T, H). The gains from these trades are given by

$$\begin{aligned} D_{TL} - D_{SL} - \tau &= D_{SH} - D_{TH} - \tau \\ &= \frac{1}{2} \frac{(\lambda_S + \lambda_T)\left[u_H - u_L - (r + 2\sigma)\tau\right]}{(r + 2\sigma + \lambda_S + \lambda_T)(r + 2\sigma + \lambda_S)} - \tau. \end{aligned} \tag{69}$$

Also the above expression is different than in an intermediation equilibrium. The difference here is due to the fact that, when (T, L) entertains the decision of purchasing from (S, L), he understands that he does not lose anything by giving up the option of purchasing the asset from somebody else. Similarly, when (T, H) entertains the decision of selling to (S, H), he understands that he does not lose anything by giving up the option of selling the asset to somebody else.

Next, we characterize the stationary distribution $\{\mu_{i,j}, \nu_{i,j}\}$ in a fundamental equilibrium. Using the fact that the agent's valuation follows a symmetric switching process that is independent from the agent's commitment power, we can show that the measure of agents of type i with valuation L and the measure of agents of type i with valuation H are each equal to one half of the overall measure of agents of type i. Using the fact that the measure of the asset is half of the measure of the population and the pattern of trade is symmetric, we can show that the measure of agents of type (i, L) with the asset is equal to the measure of agents of type (i, H) without the asset. Formally, we have

$$\mu_{i,j} + \nu_{i,j} = \phi_i/2, \text{ for } j = \{L, H\} \text{ and } i = \{S, T\}, \tag{70}$$

$$\mu_{i,L} = \nu_{i,H} \text{ for } i = \{S, T\}. \tag{71}$$

We can then solve the inflow-outflow equation for the measure of agents of type (S, L) and (T, L) who own the asset and find

$$\mu_{iL} = \phi_i \left[\sqrt{\left(\frac{\sigma}{\lambda}\right)^2 + \frac{\sigma}{2\lambda}} - \frac{\sigma}{\lambda} \right], \text{ for } i = \{S, T\}. \tag{72}$$

It is useful to compare the stationary distribution in a fundamental equilibrium and in an intermediation equilibrium. The sum of μ_{SL} and μ_{TL} is the same in a fundamental as in an intermediation equilibrium, because—when taken as a unit—agents of type (S, L) and (T, L) acquire the asset and sell the asset at the same rate in the two types of equilibria. However, μ_{SL} and μ_{TL} are different. In a fundamental equilibrium, the pattern of trade of agents of type (S, L) is the same as the pattern of trade of agents of type (T, L) and, hence, $\mu_{SL}/\phi_S = \mu_{TL}/\phi_T$. In an intermediation equilibrium, agents of type (S, L) sell more often the asset than agents of type (T, L) and, hence, $\mu_{SL}/\phi_S < \mu_{TL}/\phi_T$. Therefore, while

$\lambda_S + \lambda_T$ is the same in a fundamental equilibrium as in an intermediation equilibrium, λ_S is greater (and λ_T smaller) in a fundamental equilibrium.

Having characterized the gains from trade and the stationary distribution in a fundamental equilibrium, we can now derive the conditions on the transaction tax such that such an equilibrium exists. To this aim, note that the fundamental trades take place if and only if (66)-(68) are positive, which is the case if and only if τ is such that

$$\tau \leq \frac{u_H - u_L}{r + 2\sigma}. \tag{73}$$

The intermediation trades do not take place if and only if (69) is negative, which is the case if and only if τ is such that

$$\tau \geq \frac{(\lambda_S + \lambda_T)(u_H - u_L)}{(\lambda_S + \lambda_T)(r + 2\sigma) + 2(r + 2\sigma + \lambda_S)(r + 2\sigma + \lambda_S + \lambda_T)}. \tag{74}$$

Therefore, for any value of the transaction tax τ greater than (74) and smaller than (73), there exists a fundamental equilibrium.

5.3 No Trade Equilibrium

We now consider an equilibrium in which the asset is never traded. In a no-trade equilibrium, the gains from trade in all fundamental transactions—i.e. transactions between an agent of type (i, L) with the asset and an agent of type (m, H) without the asset—are identical and given by

$$D_{m,H} - D_{i,L} - \tau = \frac{u_H - u_L - (r + 2\sigma)\tau}{r + 2\sigma}. \tag{75}$$

The expression in (75) is intuitive. In a no-trade equilibrium, the agents expect to never trade the asset again. Hence, in any fundamental transaction, the gains from trade are simply equal to the difference between the lifetime utility from holding the asset forever for a high-valuation agent and for a low-valuation agent net of the tax T.

The gains from trade in all intermediation transactions—i.e. transactions between an agent of type (i, j) with the asset and an agent of type (i, n) without the asset—are all identical and given by

$$D_{i,n} - D_{i,j} - \tau = -\tau. \tag{76}$$

The expression in (76) is also intuitive. In a no-trade equilibrium, the agents expect to never trade the asset again. Hence, in any intermediation transaction, the transaction does not generate any improvement in the lifetime utility provided by the holder of the asset. However, the transaction generates a cost associated with the tax τ.

In a no-trade equilibrium, the stationary distribution $\{\mu_{i,j}, \nu_{i,j}\}$ is such that

$$\mu_{S,j} + \mu_{T,j} = 1/4, \text{ for } j = \{L, H\}. \tag{77}$$

Since no trade takes place in equilibrium, each agent who comes into the market with the asset holds it forever. Since each agent spends half of his life in state L and the other half in state H, it follows that the measure of high-valuation agents with the asset is equal to half of the measure A of the asset. Similarly, the measure of low-valuation agents with the asset is equal to half of A. Notice that, in a no-trade equilibrium, the stationary measure of agents of type τ with the asset—as well as the stationary measure of agents of type S with the asset—is the same as in the initial distribution.

Having characterized the gains from trade and the stationary distribution, we can now find conditions on the transaction tax τ under which a no-trade equilibrium exists. To this aim, notice that a no-trade equilibrium exists if and only if the gains from trade (75) from any fundamental transaction and the gains from trade (76) from any intermediation transaction are negative. In turn, this is the case if and only if the transaction tax τ is such that

$$\tau \geq \frac{u_H - u_L}{r + 2\sigma}. \tag{78}$$

5.4 Optimal Transaction Tax

In this section, we study the effect of the transaction tax on welfare. First, we consider the case in which the government sets a relatively small transaction tax. Specifically, we consider the case in which τ is such that

$$0 < \tau < \frac{\hat{\lambda}}{2\sigma\hat{\lambda} + 2\left(2\sigma + \hat{\lambda}\right)^2}, \tag{79}$$

where $\hat{\lambda}$ is defined as

$$\hat{\lambda} \equiv \sqrt{\sigma^2 + \sigma\lambda/2} - \sigma.$$

It is easy to verify that, when the transaction tax τ satisfies (79), the unique equilibrium—for any measure ϕ_T of agents with commitment power—is the intermediation equilibrium characterized in subsection 5.1.

In order to determine the equilibrium measure of agents with commitment power, we need to compute the benefit and cost of commitment. In an intermediation equilibrium, the benefit of commitment is given by

$$\begin{aligned} B = \quad & \left[\frac{\lambda_S}{2\sigma + \lambda_S} + \frac{\lambda_T}{2\sigma + 2\lambda_S + \lambda_T}\right] \frac{u_H - u_L}{4} \\ & - \left[\frac{\lambda_S \sigma}{2(2\sigma + \lambda_S)} + \frac{\lambda_T(\sigma + \lambda_S)}{2\sigma + 2\lambda_S + \lambda_T} + \lambda_S\right] \frac{\tau}{2}, \end{aligned} \tag{80}$$

where $\lambda_S = \lambda\mu_{SL}$, $\lambda_T = \lambda\mu_{TL}$ and μ_{SL} and μ_{TL} are the stationary measures of mismatched agents in an intermediation equilibrium. The first line on the right-hand side of (xx) is the benefit of commitment in the laissez faire economy. The second line is a negative term that is linearly decreasing with respect to the transaction tax τ. The cost of acquiring commitment power is given by the cost correspondence $C(\phi_T)$. The equilibrium measure of agents of type T is a ϕ_T^* such that the benefit of commitment for the marginal agent of type T, $B(\phi_T^*)$, is equal to the cost of commitment for the marginal agent of type T, $C(\phi_T^*)$. Note that, since the benefit of commitment is lower than in the laissez-faire equilibrium, the equilibrium measure of agents of type T is lower (in all stable equilibria) with a transaction tax than in the laissez-faire equilibrium. Since the benefit of commitment falls with τ, the equilibrium measure of agents of type T is decreasing (in all stable equilibria) with respect to the size of the tax.

Given an equilibrium measure ϕ_T^* of agents of type T, welfare is given by

$$W = \frac{u_H}{2} - \left[\sqrt{\left(\frac{\sigma}{\lambda}\right)^2 + \frac{\sigma}{2\lambda}} - \frac{\sigma}{\lambda}\right](u_H - u_L) - \int_0^{\phi_T^*} C(x)dx, \tag{81}$$

where the above expression makes use of the characterization of the stationary distribution in an intermediation equilibrium. For a given ϕ_T^*, the expression above is the same as in the laissez-faire equilibrium. Hence, the welfare gain generated by the transaction tax comes not from an improvement in the allocation of the asset, but from the reduction in the resources that agents spend to acquire the commitment technology.

Next, we consider the case in which the government sets a relatively high value for the transaction tax. Specifically, we consider the case in which τ is such that[3]

$$\frac{\hat{\lambda}}{2\sigma\hat{\lambda}+4\sigma\left(2\sigma+\hat{\lambda}\right)}<\tau<\frac{1}{2\sigma}. \tag{82}$$

When the transaction tax τ satisfies (82), the unique equilibrium—for any measure ϕ_T of agents with commitment power—is the fundamental equilibrium characterized in subsection 5.2.

In a fundamental equilibrium, the benefit of commitment is given by

$$B=\left[\lambda_S+\lambda_T\frac{2\sigma}{2\sigma+\hat{\lambda}}\right]\frac{\sigma\left(u_H-u_L-2\sigma\tau\right)}{2(2\sigma+\lambda_S)^2}, \tag{83}$$

where $\lambda_S=\lambda\mu_{SL}$, $\lambda_T=\lambda\mu_{TL}$ and μ_{SL} and μ_{TL} are the measures of mismatched agents in a fundamental equilibrium. The equilibrium measure of agents of type T is a ϕ_T^* such that the benefit of commitment for the marginal agent of type T, $B(\phi_T^*)$, is equal to the cost of commitment for the marginal agent of type T, $C(\phi_T^*)$. Note that the benefit of commitment in a fundamental equilibrium is smaller than in the laissez-faire equilibrium. Hence, the equilibrium measure of agents of type T is lower (in all stable equilibria) with than without a transaction tax. Moreover, note that the benefit of commitment in a fundamental equilibrium is proportional to $u_H-u_L-2\sigma\tau$. Hence, the equilibrium measure of agents of type T is lower (in all stable equilibria) the higher is the transaction tax.

Given an equilibrium measure ϕ_T^* of agents of type T, welfare is given by

$$W=\frac{u_H}{2}-\left[\sqrt{\left(\frac{\sigma}{\lambda}\right)^2+\frac{\sigma}{2\lambda}}-\frac{\sigma}{\lambda}\right](u_H-u_L)-\int_0^{\phi_T^*}C(x)dx. \tag{84}$$

Notice that, for a given ϕ_T^*, welfare is the same in a fundamental equilibrium as in an intermediation equilibrium and as in the laissez-faire equilibrium. This is because, in all three types of equilibria, the sum of the measure of low-valuation agents who hold the asset is exactly the same. This observation implies that, also in the case of a fundamental equilibrium, the welfare gain generated by the transaction tax comes only from its effect on the measure of agents who decide to acquire the commitment technology and become intermediaries.

[3] The attentive reader may have noticed that there is a gap between the upper bound of (79) and the lower bound of (82). When the transaction tax τ falls in this gap, there might be an intermediation equilibrium, a fundamental equilibrium or coexistence between an intermediation and a fundamental equilibrium depending on ϕ_T. We do not present the analysis of this case, as it is not necessary for our results on the optimal transaction tax.

Finally, we consider the case of a transaction tax τ such that

$$\tau \geq \frac{u_H - u_L}{2\sigma}. \tag{85}$$

When τ satisfies (85), the unique equilibrium is—for any given measure ϕ_T of agents with commitment power—the no-trade equilibrium characterized in subsection 5.3. In a no-trade equilibrium, calculating welfare is very simple. In fact, in a no-trade equilibrium, the benefit of acquiring commitment power is zero since being able to commit to take-it-or-leave-it offers is worthless when the asset does not circulate in the market. Hence, whatever the equilibrium measure of agents of type T might be, the resources that these agents spend on acquiring commitment power must be zero. Moreover, in a no-trade equilibrium, the allocation of the asset among low-valuation and high-valuation agents is uniform, as agents do not trade the asset when their valuation changes. These observations immediately imply that welfare is given by $W = (u_L + u_H)/4$.

We are now in a position to characterize the transaction tax that maximizes welfare. To this aim, notice that the allocation of the asset is efficient—in the sense that the asset is always passed from low-valuation to high-valuation agents—in an intermediation equilibrium and in a fundamental equilibrium, while it is inefficient in a no-trade equilibrium. Also, notice that the amount of resources used by agents to acquire the commitment technology is efficient—in the sense that it is equal to zero—in the no-trade equilibrium, while it is generally inefficient in an intermediation and in a fundamental equilibrium. This suggests the existence of a trade-off between transaction taxes supporting different types of equilibria. However, when the transaction tax τ converges from below to $(u_H - u_L)/2\sigma$, the fundamental equilibrium exists (uniquely) and is such that the benefit from acquiring commitment power is zero. Hence, the amount of resources wasted on acquiring commitment power goes to zero and both the allocation of the asset and the amount of resources allocated to acquire commitment power are efficient. Since a transaction tax $\tau \to (u_H - u_L)/2\sigma$ attains efficiency, it is also welfare maximizing.

We have thus established the following result.

Proposition 8: Optimal transaction tax. *(i) For any cost correspondence $C(\phi_T)$, welfare is maximized by setting the transaction tax τ* to *$(u_H - u_L)/2\sigma - \epsilon$ for $\epsilon > 0$ and arbitrarily small. (ii) For any cost correspondence $C(\phi_T)$, the equilibrium is efficient when the transaction tax τ is set to $(u_H - u_L)/2\sigma - \epsilon$ for $\epsilon > 0$ and arbitrarily small.*

Some comments about Proposition 8 are in order. First, note that the optimal tax τ is such that the after-tax gains from trade are equal to zero in any fundamental transactions—i.e. in any transaction between a low-valuation seller and a high-valuation buyer. This property of the optimal tax is intuitive. In a Walrasian Equilibrium, the ability to extract more of the surplus from a trade is completely worthless because, in any trade between a particular buyer and a particular seller, the surplus is zero. Instead of trading with a particular seller, the buyer can always go to the centralized marketplace and purchase the asset at the market price. Similarly, instead of trading with a particular buyer, the seller can always go to the centralized marketplace and sell the asset at the same market price. Hence, in a Walrasian Equilibrium, the surplus in any trade between a particular buyer and a particular seller is zero. The optimal tax reproduces this feature of the Walrasian Equilibrium by making the after-tax gains from trade equal to zero. Like a centralized marketplace, the optimal transaction tax protects agents without commitment power from exploitation at the hands of agents with the ability to make take-it-or-leave-it offers. However, while a centralized marketplace protects agents without commitment by making their outside option better, the tax protects agents without commitment by worsening their inside option (i.e., the net value of trade).

Second, note that the optimal transaction tax is robust to the distribution of costs to acquire commitment power facing different market participants. Indeed, no matter what the cost correspondence $C(\phi_T)$ might be, the optimal tax is $\tau = (u_H - u_L)/2\sigma$. This property means that the optimal tax can be implemented even when the government does not know the commitment cost correspondence and even if the government does not know whether commitment power is an innate individual trait or an acquired skill. This property also means that the government need not change the transaction tax when the commitment cost correspondence changes because of, say, changes in the return on alternative investment opportunities. However, the formula for the optimal transaction tax does depend on other details of the market, such as the difference in valuation between different agents and the frequency at which these valuations change. Moreover, if there are more than two levels of valuation for the asset, implementing the efficient allocation would require a more sophisticated policy than a single transaction tax.

Third, it is useful to interpret Proposition 8 through the lens of the mechanism design approach to optimal taxation. The optimal transaction tax implements the unconstrained efficient allocation. Moreover, the optimal transaction tax is a very simple instrument, in the sense that it does not require the government to observe the price at which different transactions are executed, the commitment type of the history of trade of different market participants. The transaction tax only requires the government to observe when the asset is traded. From these observations, it follows that the optimal transaction tax implements the solution to an optimal mechanism problem, as long as the mechanism knows when the asset is traded. In particular, the mechanism does not need to observe prices, commitment types, or trading histories. One would have thought that the optimal mechanism might involve a transfer to the mechanism that depends on the history of trade of the buyer and the seller. Instead, Proposition 8 shows that, even though formulating the mechanism design problem might be quite complicated, the solution is very simple, at least in our simple environment.

Clearly these findings relate closely to a large body of existing work on financial transaction taxes following Tobin (1978). However, the theoretical foundations have largely been centered around two themes: first, going back to Keynes (1936), excessive price volatility in financial markets (see, for instance, Summers and Summers (1989)); second, and more closely related, around efforts to gain informational advantages that lead to a rat race for information with no aggregate benefits (see Stiglitz (1989)). We share with the latter theme the notion that costly efforts to generate private returns can be wasteful if the private returns exceed the social ones.[4] The mechanism modeled here, however, differs in at least two dimensions. First, it directly formalizes intermediation in a frictional environment as a rent extraction activity. Second, it models rent-extraction in its purest form, namely as a (costly) effort that directly aims at extracting surplus from other market participants. We thus view our results as a cleanly formalized and novel case for potential benefits of a financial transaction tax.

6 Conclusions

We developed a theory of intermediation in asset markets as a pure rent-extraction activity. We considered a frictional market populated by agents who are heterogeneous with respect to their valuation of the asset's dividend and with respect to their ability to commit to posted prices. We showed that the equilibrium pattern of trade is such that agents with commitment act as intermediaries—in the sense that they buy and sell the asset irrespective of their valuation—while agents without commitment act as end users—in the sense that they buy the asset only when their valuation is high and keep it until their valuation turns low. As typical intermediaries in the real world, agents with commitment trade more frequently than end users (although they have the same meeting rate), purchase the asset at lower prices and sell the asset at higher prices than end users. Agents with commitment intermediate the asset market only because they can

[4]Burman et al. (2015) point to the claim that a financial transaction tax "would reduce the diversion of valuable human capital into pure rent-seeking activities of little or no social value".

extract more of the gains from trade than agents without commitment. For instance, a low-valuation agent without commitment trades the asset to a low-valuation agent with commitment because the latter can re-sell the asset at a higher price.

We then endogenized the measures of agents with and without commitment by studying the agent's decision to invest in a commitment technology. We showed that the benefit of investing in the commitment technology is hump-shaped in the measure of agents with commitment in the market, leading to the possibility of multiple equilibria with different levels of intermediation. We showed that equilibria with more intermediation have lower welfare and that any equilibrium in which resources are devoted to the commitment technology is inefficient. We showed that a decline in trading frictions leads to an increase in the return from investing in the commitment technology and, hence, to an increase in the extent of intermediation. We also showed that, in some natural cases, a decline in trading frictions leads to lower welfare, as the increase in the costs associated with acquiring the commitment technology outweighs the benefits associated with faster trade. We also showed that a decline in the rate of return on investments alternative to the commitment technology leads to an increase in intermediation and, in some natural cases, leads to a decline in welfare. These comparative statics invite two observations. First, as progress in information and communication technology keeps lowering trading frictions, we should expect the intermediation sector to keep growing. Second, in times of low rates of return on investment, we should expect the intermediation sector to expand. Both phenomena might cause lower welfare.

Finally, we studied the effect of introducing a transaction tax. We showed that the transaction tax reduces the incentives to acquire the commitment technology and, hence, the extent of intermediation. We showed that, depending on the size of the tax, the equilibrium pattern of trade might be the same as in the laissez-faire equilibrium, it might involve only fundamental trades, or it might involve no trade at all. We found that the tax that maximizes welfare is such that the after-tax gains from trade in fundamental transactions are zero. That is, the tax that maximizes welfare reproduces artificially a key property of Warasian Equilibrium: in any meeting between a buyer and a seller, the surplus is zero. The welfare-maximizing tax also implements the first-best allocation and it does so without requiring any information on the history of trade of individual agents, on the price at which individual transactions take place, or on the cost facing individual agents to acquire the commitment technology. Therefore, even though formulating a mechanism design problem for the structure of the asset market might be very complicated, the solution of such problem is not.

References

[1] Burman, L., W. Gale, S. Gault, B. Kim, J. Nunns and S. Rosenthal. 2015. "Financial Transaction Taxes in Theory and Practice," Tax Policy Center.

[2] Cociuba, S., M. Shukayev and A. Ueberfeldt. 2016. "Collateralized Borrowing and Risk Taking at Low Interest Rates," *European Economic Review*, 85: 62-83.

[3] Duffie, D., N. Gârleanu, and L. Pedersen. 2005. "Over-the-Counter Markets," *Econometrica*, 73: 1815-1847.

[4] Farboodi, M., G. Jarosch and R. Shimer. 2016. "Meeting Technologies in Asset Markets." Mimeo, University of Chicago.

[5] Hugonnier, J., B. Lester and P. Weill. 2016. "Heterogeneity in Decentralized Asset Markets." Mimeo, University of California Los Angeles.

[6] Jiménez, G., S. Ongena, J. Peydró and J. Saurina. 2014. "Hazardous Times for Monetary Policy: What Do Twenty-Three Million Bank Loans Say About the Effects of Monetary Policy on Credit Risk-Taking? " *Econometrica*, 82: 463-505.

[7] Keynes, J. 1936. "The General Theory of Employment, Interest, and Money."

[8] Kyiotaki, N., and R. Wright. 1989. "On Money as a Medium of Exchange." *Journal of Political Economy*, 97: 927-954.

[9] Masters, A. 2008. "Unpleasant Middlemen," *Journal of Economic Behavior and Organization*, 68: 581-594.

[10] Nosal, E., R. Wright and Y. Wong. 2015. "More on Middlemen: Equilibrium Entry and Efficiency in Intermediated Markets." *Journal of Money, Credit and Banking.*

[11] Nosal, E., R. Wright and Y. Wong. 2016. "Who Wants to be a Middleman?" Mimeo, University of Wisconsin Madison.

[12] Philippon, T. 2015. "Has the US Financial Industry Become Less Efficient?" *American Economic Review.*

[13] Rajan, R. 2006. "Has Finance Made the World Riskier?" *European Financial Management.*

[14] Rubinstein, A. 1982. "Perfect Equilibrium in a Bargaining Model." *Econometrica.*

[15] Rubinstein, A., and A. Wolinsky. 1987. "Middlemen," *Quarterly Journal of Economics.*

[16] Stiglitz, J. 1989. "Using Tax Policy To Curb Speculative Short-Term Trading." *Journal of Financial Services Research*, 3: 101-115.

[17] Summers, L. and V. Summers. 1989. "When financial markets work too well: A cautious case for a securities transactions tax." *Journal of Financial Services Research*, 3: 261-286.

[18] Tobin, J. 1978. "A Proposal for Monetary Reform." *Eastern Economics Journal*, 4: 153-159.

[19] Uslu, S. 2016. "Pricing and Liquidity in Decentralized Asset Markets." Mimeo, University of California Los Angeles.

[20] Wright, R., and Y. Wong. 2014. "Buyers, Sellers, and Middlemen: Variations on Search-Theoretic Themes." *International Economic Review*, 55: 375-397.