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Reputation and competition*

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Abstract

I consider repeated games with both moral hazard and adverse selection where a continuum of agents compete. It is shown that equilibria with reputation -where high effort is always exerted- may be sustained under imperfect information; the existence of such equilibria contradicts the standard results without competition. An explicit characterization of these equilibria is provided, as a discussion of the role of the environment.

1 Introduction

We are what we repeatedly do. Excellence, then, is not an art, but a habit.

Aristotle, Nicomachean Ethics

Reputation is usually de ned in game theory as the perception others have of the players characteristics (utility function or pro t function) which

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determines its choice of strategy. Hence, reputation phenomena can be studied by introducing two distinct types of agents. Thus, suppose that the agents facing each other in a repeated game are firms and consumers, and that firms might be either incurably "lazy" or willing, if it pays to do so, to make costly efforts to increase the utility its consumers will experience. For instance, a higher effort might increase the quality of a good or service for the consumer. Suppose further that such a higher quality is unobservable before purchase and only noisily reveals the effort choice. Is there any way for a firm to find it optimal to exert costly efforts? And is there any way in which this firm could find it optimal to keep doing so over time? These questions, central to the literature on reputation, have been tackled by Fudenberg and Levine [6] and Mailath and Samuelson [12]. The standard problem created by imperfect information, first posed by Holmström [7] in a labor context, is the following: once a consumer is convinced that she is facing a firm making repeatedly high effort, the incentives of this firm to really do so are decreasing. If the quality experienced by the consumer is low in some period, this consumer will attribute this to bad luck, since she is pretty sure that the firm made a high effort. So why shouldn't the firm just rest on its laurels for a few periods and "enjoy" its reputation? Of course, this argument unravels any equilibrium where high effort is always sustained. To explain how such behavior could indeed be part of an equilibrium, most of the authors have used tricks to bound beliefs away from one. For instance, one can assume that in every period there is a fixed, exogenous probability that the type of the firm might change (see, however, Mailath and Samuelson [13]).

This paper shows that competition among firms can alternatively explain such equilibria, without introducing bounds on beliefs. Indeed, casual empiricism suggests that such equilibria exist in competitive settings. The canonical example for such a situation is the repeated interaction between consumers and restaurants. Some cooks are intrinsically bad, some are better. But the better cooks can also produce bad meals and producing better meals entails a cost. Moreover, even a better cook can be unlucky for various reasons (like the unpredictable quality of the ingredients). Nevertheless, the best restaurants are obviously repeatedly exerting efforts, although there is no doubt that the experience of a bad meal might not induce a change in their status.

This paper demonstrates how the dynamics of competition endogenously generate the necessary constraints that force firms to perpetually exert high effort. Consumers not only choose according to the beliefs about the firm with which they trade, but also according to the beliefs that they entertain about other rms. It does not matter how good a rm is thought to be, but only whether it is thought to be at least as good as its rivals. Suppose that, as soon as a consumer does not think so about her rm any more, she leaves it for any rm that managed to keep its clients. Then only those rms that have the best records keep clients and operate. Thus, the behavior of other consumers yields sufficient information to update consumers beliefs, even though consumers only observe the outcomes of their current rm. This behavior also maintains the homogeneity of the operating rms. This forces rms to exert high effort, to keep up with the standards of excellence of the operating rivals.

There is however a price to pay: no matter how good a rm is thought to be, it may be compelled to exit. In fact, a vanishing fraction of bad rms forces a constant, positive fraction of good rms to exit in every period. But this threat to a rm also provides incentives to its rivals, eager to attract additional consumers. Prices rise over time. They initially are very low, below cost, and gradually rise to an asymptotic level that exceeds the cost of high effort by a premium.

Besides studying the existence of equilibria with reputation-building , this paper also sheds light on how prices, size and age convey information about rm s reputation. The effort level might be interpreted as an unobserved and only imperfectly revealed quality level. This paper can then be regarded as an extension of C. Shapiro s classical paper [15] on premiums for high quality products. I can then interpret the results as a formalization of the following insight of T. Scitovszky:

> *The economic theory of consumers choice is based on the assumption that the consumer knows what he buys. He is presumed to be an expert buyer who can appraise the quality of the various goods, offered for sale and chooses between them by contrasting, one against the other, the price and quality of each good. This assumption was probably a reasonable one in the early days of industrial capitalism when modern economic theory began[..]. The size of a rm, its age, even its nancial success are often regarded as indices of the quality of its produce. Hence the importance producers attach to goodwill and trade marks, hence the much advertised claims of some of them to being the biggest or oldest rm in their trade. T. Scitovszky [14].*

One related paper is the one by Tadelis [16] (see also Kreps [10]). His framework is close to the present one, although he doesn t consider that some agents might have different choices (that is, in his model, a good cook is constrained to always make high effort). More importantly, his motivation is to explain how trademarks and goodwills can acquire value. The related literature in industrial organization is very large, and the interested reader is referred to the papers of Allen [1], Rogerson [11], C. Shapiro [14] and Klein and Le- er [8].

The next section introduces the model. Section 3 presents the results. The last section considers some extensions and discusses the conclusions.

2 The model

I consider two types of rms, dubbed good and bad . Good rms can exert either high effort, at a per consumer cost of $0 < c < 1$, or low effort at zero cost. Bad rms can only exert low effort (also at zero cost). A rm s type is private information. High effort leads to a probability α of a good outcome (and to a probability $1 - \alpha$ of a bad outcome), while low effort leads to a probability β of a good outcome (respectively $1 - \beta$). Assume that $1 > \alpha > \beta > 0$. The common discount factor is denoted $\delta \in (0, 1)$. Both types of rms maximize the discounted, expected stream of pro ts. I assume however that a good rm, if indifferent between both effort levels, chooses high effort. Moreover, if any rm is indifferent between operating or exiting (which yields zero pro ts), then it decides to exit (except in the initial period). At the beginning of any period, a rm either announces a price or exits (but it cannot exit for the forthcoming period after setting a price). Once a rm has exited, it is assumed that it cannot reenter. The total, initial mass of rms is 1, and the proportion of good rms is $\phi_0 \in (0, 1)$.

Consumers are identical, and their total mass is one. Each rm can serve a continuum of consumers. Time is discrete, indexed by $t = 0, 1, 2, ..$ and the horizon is in nite. Consumers are Bayesian rational (they have beliefs over the rms type and use all the available information to update their beliefs according to Bayes rule), know ϕ_0, but don t know the types of the rms. In the rst period, consumers observe prices posted by the rms, and are randomly matched within the set of rms exhibiting prices they prefer. The consumers are expected utility maximizers and derive a higher utility from enjoying a good outcome than a bad outcome. Without loss of generality, normalize the utility of consuming a good outcome to one and the utility of a bad outcome to zero. Every consumer has the possibility to engage in one trade per period (which involves a payment p from the consumer to the rm in exchange for the outcome of the effort of the rm). The utility derived by a consumer of not engaging in any trade (or reservation utility, or outside option) is $1 > \gamma \geqq \beta$. The outside option can be thought as the value to the consumer of her next best alternative. For instance, $\gamma = \beta$ is the outside option if the alternative consists of a separate competitive sector composed only of bad rms. This ensures that low effort does not yield a higher utility to the consumer than her outside option. I assume also that no contractual arrangement is possible, that is, payment cannot be made contingent on the outcome or on any other resolution of uncertainty.

Consumers can switch from a rm to any other rm at the end of any period. There is no cost for a consumer to switch rms. In case of indifference between switching or staying, I suppose that the consumer stays with the same rm.

A consumer who has decided to switch rms observes the prices set by all rms when deciding with which rm to trade.[1] A consumer who decides to stay with some rm j knows the sequence of prices and outcomes of this rm since she began trading with it. However, she does not observe prices or outcomes of any other rms during her relationship with rm j. Moreover, if the consumer decides to leave rm j, she will henceforth be unable to distinguish rm j from other rms in the market. That is, while she knows which outcomes she has enjoyed (and at which prices) since the initial period ($t = 0$), she can only distinguish two kinds of rms before deciding whether to switch rms: the one with which she just traded (and for which she knows the prices and outcomes since she joined it), and all the others, for which, in case she decides to switch, she only knows the price for the forthcoming period. An important implication is that even if she knows the price distribution in every period (from switching in every period), she cannot identify the price path of any rm.

A consumer who has decided to stay with the same rm can decide, upon observing the price posted by the rm, whether to trade with the rm. However, if she chooses not to trade, she cannot trade with any other rm

[1] Note that the consumer only observes these prices after having decided to leave; that is, a consumer who decides to stay with her current rm does not observe the price distribution.

in that period; accordingly, her utility for that period is β if she decides not to trade. Of course, a consumer who switched rms can also decide not to trade for the following period. A consumer who does not trade in period t with her rm does not observe the outcome of the rm s effort choice, and decides to stay or switch at the end of the period. Although this is irrelevant to what follows, we also assume that consumers do not observe the decisions of other consumers.

At the beginning of any period, upon observing how many consumers decided to stay, each rm sets a price (possibly negative) or chooses to exit. Consumers then decide whether to trade. Each rm then exerts some effort after observing the number of consumers who have decided to trade with it. All the consumers of a given rm receive the same outcome. The rm also observes this outcome (but not the outcomes experienced by the other rms). The prices of the rms are set simultaneously, after which the rms can observe the prices set by other rms (and hence, of course, how many decided to operate). Firms do not price discriminate and, indeed, they cannot distinguish between consumers who stayed and consumers who switched and chose them. Firms with no consumers at the beginning of a period are assumed to exit.[2]

The timing is summarized below.

−−−−**5′**−−−−−**1**−−−−−**2**−−−−−**3**−−−−−**4**−−−−−**5**−− >

−−*t-1*−}{−−−−−−−−−−−−− *period* *t*−−−−−−−−−−−−}{−*t+1*

5′ - Consumers decide to stay or switch.

1 - Firms set price, after observing the number of consumers who stay

2 - Consumers who switched choose rm,

and consumers who stayed decide whether to trade.

3 - Firms choose effort level,

after observing the number of consumers who trade.

4 - Firms and consumers observe outcome.

5 - Consumers decide to stay or switch.

[2] Although this assumption seems to be crucial, it can be dispensed with. For instance, it is unnecessary whenever switching consumers can observe whether rms have any consumers that stay.

I focus on equilibria where the decisions of consumers depend only on the pricing of firms, their beliefs over the type of their current firm and their beliefs over the distribution of types in the market. The pricing decisions of the firms and their effort choices depend on consumers' decision to stay and trade, which in turn depend on the previous outcome and on the previous price. These strategies are Markovian, since past outcomes and past prices can be summarized by consumers beliefs. Notice that the strategies under study do not even depend on the previous effort level. Indeed, that effort level does not affect the firm's objective function, and effort remains private information, so that this restriction can be interpreted as requiring firms to only condition their strategies to strategic variables, that is variables that do affect their future payoff, directly or through consumers beliefs, when the variable is observed. Consumers can have different expectations according to the price they face. A strategy for a consumer in every period specifies two decisions. First, the consumer has to decide whether to trade: if she stayed with the same firm at the end of the previous period, she decides if she prefers to trade or not. If she decided to switch firms at the end of the previous period, she has to decide whether to trade, and if so, with which firm. Second, if she traded in that period, she has to decide at the end of the period whether to stay with the firm for the forthcoming period, or to switch.

A strategy for a firm in every period similarly specifies two decisions. First, the firm has to decide whether to operate, and if so, which price to post. Second, a good firm decides whether to exert high effort. I am interested in establishing conditions under which equilibria exist where good firms always exert high effort.

Given our interest in these equilibria, I need only consider histories of the following form: t-histories are represented by a t-vector of outcomes enjoyed by consumer i or produced by firm j, and prices paid or posted: $h_t^k \in H_t^k = \left\{X_\nu^k, p_\nu^k\right\}_{\nu=1,..,t}, k = i,\ j,\ X_\nu^k \in \{G, B, \emptyset\},\ p_\nu^k \in \mathbb{R} \cup \{\emptyset\}$, where p_ν^k is the price paid in period ν and X_ν^k is the quality experienced in period ν: either good (G) or bad (B); when the consumer refused any price or/and the firm exits (recall that the consumer decides whether to stay prior to the decision of the firm to operate in the following period), I denote the corresponding value taken by these variables by $\emptyset$. Moreover, I denote by $E_\nu^j = H, L$ the effort level exerted by firm j in period ν, where H is short for high and L for low. The restriction of the t-histories to the outcomes is referred as to the <u>t-history of outcomes</u>.

I study perfect Bayesian equilibrium; consumers maximize their utility given their beliefs, which are correct in equilibrium and rms maximize pro ts (given other rms and consumers behavior). I focus on symmetric equilibria, where every rm of a given type choose the same strategy, and similarly for the consumers.

I denote by ϕ_t^i the belief of, i.e. the probability assigned by consumer i in period t that her current rm is good. Note that this posterior probability includes the exit behavior of the rms. The belief of consumer i in period t over the effort level of her current rm j is thus $\phi_t^i \tau_t^j$, given that consumer i believes rm j chooses effort levels according to strategy τ (τ_t^j thus denotes the probability with which rm j exerts high effort in period t). Note that if the consumer believes good rms always choose high effort then the two beliefs are identical. Finally, I write $\phi\left(\phi_t^i \mid X\right)$ to refer to the (Bayesian) updating rule applied to belief ϕ_t^i after experiencing $X \in \{B, G\}$, *assuming that* good rms always exert high effort, and I write $\phi^{(k)}\left(\phi_t^i \mid X\right)$ to refer to the Bayesian update applied to belief ϕ_t^i after experiencing a string of k realizations equal to $X \in \{B, G\}$. For instance, $\phi^{(t)}(\phi_0 \mid G)$ is the belief at the beginning of period t of a consumer who only experienced good outcomes.[3]

The <u>value</u> V_t of a rm in period t is the maximal expected discounted stream of pro ts it can achieve from period t on.

In this model, competition generates incentives for the rms to sustain high effort. The reward, or carrot of sustaining effort lies in the opportunity of attracting the consumers of the competitors. The stick , of course, is the threat of losing consumers, were the rm to disappoint them. I am interested in determining under which conditions these incentives might be sufficient to re-establish high effort as part of an equilibrium behavior, despite the imperfect information available to the consumers. Further, I show that the stick can be sometimes sufficient to obtain such a result. The following section then investigates the converse question: can the carrots of price increases or growth be sufficient to induce high effort? Finally, I introduce the possibility of name trading, and discuss how this affects the different results.

[3] De ne $\phi^{(0)}(\phi_0 \mid X) = \phi_0$ for any $\phi_0 \in (0,1)$ and $X \in \{B, G\}$.

3 Two central results: from rm competition to rm selection

In this subsection, I investigate the conditions under which consumers are loyal. Moreover, consumer behavior is shown to have far-reaching consequences for rm performance.

Lemma 1 *: Suppose that consumer i believes that, in any period, all rms charge the same price. Consumer i leaves her current rm j as soon as her posterior belief $\phi^i \tau^j$ is smaller than her belief over the other rms.*

Since the price charged by the other rms is the same, there is no cost of switching rms, and consumers are expected utility maximizers, this is obvious. (Note that, since consumers don t communicate, the belief consumer i has over the rms she is not matched with, is the same across rms).

As is usual in models of this type, there are equilibria where low effort is always exerted by all the rms after period t and all consumers believe that no effort will be exerted after period t. Instead, I am interested in those equilibria where high effort is always sustained, which I call <u>high effort equilibria</u>.

I say that an equilibrium is <u>nonrevealing</u> if in any period all those rms which decide to operate charge the same price. I also say that a rm j is <u>surviving in period t</u> if it has not decided to exit in any period up to t.

Proposition 1 *: In any nonrevealing high effort perfect Bayesian equilibrium, the only surviving rms in period t have histories of outcomes $h_t = (G, G, G, .., G)$. That is, they experienced a good outcome in every period.*

Proof. In what follows we nd convenient to write ϕ_t for $\phi^{(t)}(\phi_0|G)$. At the end of period 0, consumers having had a bad outcome can only gain by switching to another rm, since the probability of facing a high effort level from, say, rm j in period 1 is higher conditional upon observing a good outcome of rm j in period 0 than upon observing a bad outcome. Hence, consumers can only gain by leaving a rm with which they had a bad outcome, and, since good rms make high effort, consumers have strictly positive expected gains of doing so. Since every consumer is behaving in this way, all consumers trade in $t = 1$ with rms having had a good outcome in the initial period, all the rms having had bad outcomes have no consumer at the beginning of period 1, and thus exit, and hence $\phi_1^i = \phi_1$.

Suppose that this is true in period t. In particular, $\phi_t^i = \phi_t$. Now, suppose consumer i experiences a bad outcome in period $t+1$. Since $\phi(\phi_t \mid B) \leq \phi(\phi_t \mid G)$, I have:

$$\begin{aligned}\phi(\phi_t \mid B) &< E\left[\phi_{t+1} \mid \phi_t\right] = (\alpha\phi_t + \beta(1-\phi_t))\,\phi(\phi_t \mid G) \\ &\quad + ((1-\alpha)\,\phi_t + (1-\beta)(1-\phi_t))\,\phi(\phi_t \mid B)\end{aligned}$$

This inequality implies that the posterior belief of agent i over the effort level of her current firm is lower than what she can expect by switching firm. By the previous lemma, consumer i will leave her current firm. Since every consumer behaves in this way, consumer i knows that any firm which still operates must have also experienced a good outcome in period $t+1$. Hence the $t+1$-histories of outcomes are $h_{t+1} = (G, G, G, ..)$ and $\phi_{t+1}^i = \phi_{t+1}$, which concludes the argument. ■

This proposition means that consumers are optimally behaving myopically, even though in principle there is value to staying with this firm, and amazingly information that may be valuable is thrown away. It is the driving force of the model. The world it describes is without "mercy", any failure leads to bankruptcy. The result derives from focusing on nonrevealing equilibria, more precisely, on the assumption that consumers believe that the prices of the firms do not reflect their previous outcomes. Indeed, depending on the application one has in mind, it might seem more plausible that unsuccessful firm might bargain with its consumers, offering them a lower price than would a successful firm to compensate for the lower belief over the effort level they have. Such a procedure will be briefly examined in the last part of the paper.

The strength of this argument is illustrated by considering the case of more than two, but still finitely many, outcomes. Then only those firms experiencing the best outcome would survive. This breaks down if there are switching costs or if the average belief over the effort level of the remaining firms is noisy. If there are switching costs and more than two outcomes, firms which experience a lower outcome might not be forced to exit. Indeed, eventually, even firms which experience the worst possible outcome won't exit. This is because, for any given cost, there is a time where beliefs of consumers are so close to one that the second best outcome need not induce the consumer to leave, because posterior does not change much in response to different outcomes. But this in turn induces heterogeneity among histories of operating firms, drives down the average belief of consumers over firms, so that a rm with a particularly successful history will then be able to experience lower outcomes without exiting.

Notice that, in contrast to standard models in repeated games, there are no equilibria characterized by some phase of punishment; for instance, there is no equilibrium where consumers would leave their rm after two bad outcomes; this should seemingly reduce the possibility of sustaining high-effort, by reducing the choice of strategies that could be optimal for the consumers. Without competition, one can construct equilibria where consumers use some statistics to decide whether to stay or leave, and where rms sustain high-effort depending on these statistics. But using such strategies imply some cost to the consumer during the punishment phases, that no consumer wants to bear, and need not bear, under competition. Thus, the game with competition reduces to an ultimatum game, which paradoxically enables high-effort to be sustained, as is shown below. High effort is always sustained, and the strategies of both consumers and rms are very simple.

Notice also that the argument just made relies on the fact that the probability distribution over outcomes is atomic. If the outcome space is in nite (and absolutely continuous densities over it induced by the effort level, such that the density of high effort rst order stochastically dominates the density of low effort) then there is no equilibrium where consumers behave symmetrically, unless no rm makes high effort. To see this, notice that, in the absence of switching costs, a consumer only stays with rm j if the belief over effort she has about rm j is at least as large as the (average) belief she has over the effort level of the remaining rms. But the threshold level of the belief over effort at which a consumer should be indifferent between staying and switching would always be lower than the average belief over the effort level of the rms which will survive (precisely because the belief over the effort level of those rms must be at least above this threshold level), a contradiction.

4 The general model

Given the last claim that we have seen, time is a sufficient statistic (since all the rms surviving in period t have the same t-history). Hence $V(\phi_t) = V_t$. Suppose that, if all the prices charged by the surviving rms is the same, the additional consumers coming to them will be equally shared. De ne n_t to be the number of consumers per (surviving) rms in period t. Obviously, the perspective of having more consumers in the future provides an additional incentive to exert high effort. Moreover, recall that the outcome of each firm in every period is perfectly correlated across consumers of this particular firm.

Since from period t to the following, only a fraction $\alpha\phi_t + \beta(1-\phi_t)$ survive, for any time $t \geq 0$,

$$\frac{n_t}{n_{t+1}} = \alpha\phi_t + \beta(1-\phi_t).$$

Let V_t^H (respectively V_t^L) be the value of a firm *per consumer* in period t of exerting high (respectively low) effort in all the subsequent periods; define V_t to be the maximal value per consumer in period t over the set of possible strategies for a good firm from period t on. Consider the strategy of exerting high effort in every period. The per consumer gain of a one-shot deviation to low effort in period t is:

$$p_t + \beta\delta\frac{n_{t+1}}{n_t}V_{t+1}^H,$$

whereas the per consumer gain of exerting high effort in period t is:

$$p_t - c + \alpha\delta\frac{n_{t+1}}{n_t}V_{t+1}^H.$$

Hence, high-effort can be sustained in equilibrium only if, for any $t \geq 0$,

$$\frac{n_{t+1}}{n_t}V_{t+1}^H \geq \frac{c}{(\alpha-\beta)\delta}.$$

A competitive equilibrium is:

Definition 1 A *(symmetric) competitive equilibrium with reputation* is a sequence $\{p_t, \psi_t\}_{t=0}^{\infty}$ of prices and beliefs such that prices are profit maximizing for the firms given the beliefs, consumers choose firms to maximize their expected utility, beliefs are correct,

$$V_0^H = 0,\ V_0^L = 0,$$

and, for any $t \geq 0$,

$$\frac{n_{t+1}}{n_t}V_{t+1}^H = \frac{c}{(\alpha-\beta)\delta}.$$

The last part of the de nition says that the incentive compatibility constraint is binding in every period, that is, the prices charged are the lowest possible given the incentive constraints. The motivation for this restriction is the following: rms are competing through prices in every period to attract the consumers who left the rm with which they were trading in the previous period. Since in every period, there is a positive mass of such consumers, while the mass of consumers staying with a particular rm is negligible, any higher price would not be compatible with equilibrium, *provided* that the beliefs of the consumers do not assign a higher probability of the rm to be a good rm for higher prices. But both types of rms would bene t from a price increase, so that such a belief speci cation would not be particularly attractive. Observe also that there is one more equation than unknowns so that it is not clear *a priori* whether such equilibria exist. In fact, it is easy to see that, provided that good rms exert high effort, one equation is redundant; for instance, assuming that bad rms earn overall zero pro ts, and assuming that incentive compatibility binds in every period imply that good rms are indifferent between high effort and low effort, so that exerting high effort also yields overall zero pro ts. The zero pro t condition in period 0 for bad rms and for good rms always exerting high effort seems natural to require in analogy to the traditional theory of perfect competition. For instance, introducing a very small, but positive measure of consumers in period 0 that only care about the price, but not the quality, would imply that overall pro ts of the both types of rms must be zero in equilibrium. It is by no means necessary to require the incentive compatibility to hold with equality, but it helps pinning down the prices; the qualitative features of the equilibrium can be obtained without equality.

These conditions can be rewritten as :

$$\sum_{t=0}^{\infty} (\beta\delta)^t (n_t p_t) = 0, \tag{1}$$

$$\sum_{t=0}^{\infty} (\alpha\delta)^t n_t (p_t - c) = 0, \tag{2}$$

and for every $t \geq 0$,

$$n_{t+1} V_{t+1}^H = \frac{n_t c}{(\alpha - \beta)\delta}. \tag{3}$$

I show in the appendix that this system can be uniquely solved. Recall that $\phi_t \equiv \phi^{(t)}(\phi_0 \mid G)$.

Lemma 2 *Equations (1), (2) and (3) have a unique solution given by* $\{p^*\}_{t=0}^{\infty}$*, where* $p_0^* = -\frac{\beta}{\alpha-\beta}c$*, and,* $\forall t \in \mathbb{N}$*,*

$$p_t^* = \frac{c}{(\alpha-\beta)\,\delta}\left(\alpha\frac{\phi_{t-1}}{\phi_t} - \beta\delta\right).$$

We define now belief functions for consumer i to be a collection of functions mapping prices into probabilities. That is, for any $t \geq 0$, $\psi_t^i : \mathbb{R}\cup\{\emptyset\} \to [0,1]$. $\psi_t^i(p)$ (or ψ_t^i) is the belief of consumer i over the effort level of her current firm, say j, upon observing price p, that is, $\psi_t^i = \tau_t^j \phi_t^i$.

Define the beliefs $\{\psi_t\}_{t=0}^{\infty}$ to be the collection of belief functions:

-if consumer i switched firms at the end of period $t-1$, for any firm and associated price p_t she observes, let $\psi_t^i(p_t) = \phi_t$ if $p_t = p_t^*$. Otherwise, let $\psi_t^i(p_t) = 0$.[4]

-if the consumer stayed with her firm of the last period, let $\psi_t^i(p_t) = \phi\left(\psi_{t-1} \mid G\right)$ if $p_t = p_t^*$ (where $\psi_0 = \phi_0$). Otherwise, let $\psi_t^i(p_t) = 0$.

The equilibrium strategies are as follows:

A consumer who decided to stay at the end of period $t-1$ accepts to trade if and only if the price posted is either negative (in which case expected utility is larger than her outside option for any beliefs) or equal to p_t^*. A consumer who decided to switch at the end of period $t-1$ chooses to trade if there is some firm posting such a price, and if so, among those, she chooses a firm maximizing her utility given her beliefs.[5] At the end of period t, a consumer stays with a firm if and only if the price she paid was p_t^* and she experienced a good outcome.

A firm sets a price equal to p_t^* if there is some consumer who decided to stay at the end of period $t-1$ (recall that a firm exits otherwise). Indeed, the only prices at which consumers accept to trade are negative prices or equilibrium prices. A good firm exerts then high effort if and only if it has posted p_t^* and at least some consumer accepted to trade with it.

The motivation of these beliefs and strategies is as follows: firms are expected to follow the equilibrium pricing; if the price posted is lower, the firm violates the incentive constraint, so that it makes sense to assume that

[4] Of course, $\psi_t^i(\emptyset) = 0$.

[5] That is, let $\underline{p_t}$ be the infimum of the prices posted. If some firm posts p_t^* and $\underline{p_t} \neq p_t^*$, the consumer chooses p_t^* if $\underline{p_t} + (\alpha-\beta)\,\phi_0 > p_t^*$. If there is no firm charging p_t^*, the consumer chooses $\underline{p_t}$ if $\underline{p_t} \leq 0$. Otherwise, she does not trade.

its consumers believe that the rm is going to exert low effort; if the price posted is higher, then the consumer pays more than it would pay elsewhere, so she had better leave that rm; note that a higher price is not viewed as a signal that the rm is more likely to exert high effort (than a rm following equilibrium pricing), since both types of rms bene t from higher prices. Nor does a negative price signal future negative prices, which seems intuitive since rms have to break even. In either case, it seems reasonable for the consumer to want to leave the rm as soon as possible, which in turn leads the rm to exert low effort; consumers accept to trade with rms applying the equilibrium pricing or posting a negative price (since then, even if the consumer knows that low effort will be exerted, her expected utility from trading is larger than her reservation utility). In view of the last lemma, consumers should leave rms which followed the equilibrium pricing but with which they experienced a bad outcome, and stay if they had a good outcome, in which case their beliefs are simply the Bayesian update, given a good outcome, of the belief they had in the previous period.

The following assumption on the parameters ensures existence of equilibria with reputation.

Assumption A1:

$$c \leq \frac{(\alpha-\beta)\,\delta\,((\alpha-\beta)\,\phi_1+\beta-\gamma)}{(\alpha-\beta)\,\phi_1+\beta\,(1-\delta)}$$

Lemma 3 *: Suppose A1 holds. The sequence $\{p_t^*, \psi_t\}_{t=0}^{\infty}$ is a competitive equilibrium of the complete model. Given these prices, all the good rms produce high effort in every period where they are called upon to play. In equilibrium, consumers leave their rm at the end of a period if and only if they experienced a bad outcome previously. Firms exit as soon as they have no consumer in some period.*

Lemma 4 *: prices $\{p^*\}_{t=0}^{\infty}$ are strictly increasing over time and they converge to:*

$$\bar{p} = \frac{(\alpha-\beta\delta)}{(\alpha-\beta)\,\delta}c > c.$$

Under A1, $\bar{p} \leqq 1-\gamma$.

Proofs are in appendix. The assumption A1 ensures that consumers prefer to trade than to use their outside option. In fact, A1 is both necessary and sufficient for the existence of high effort equilibria (given that price pro le). Notice that the right hand side of the bound on c is increasing in δ, and bounded above by $\alpha - \beta + \frac{\beta - \gamma}{\phi_1}$.[6] This means that high effort equilibria are more likely to emerge when high effort induces a signi cantly higher probability of survival than low effort does, and when trade occurs frequently. Also, provided that γ is close to β, assumption A1 is always satis ed when β is low enough, that is, when low effort is very likely to lead to exit. Notice nally that a high effort equilibrium cannot arise if ϕ_0 is too low, that is, when the probability that a rm is bad is too high to allow prices to be high enough to convey incentives.

The negative price in period 0 is the cheapest way to prevent low effort night- iers , because the low success probability of a rm producing low-effort can be reinterpreted as a higher discount rate (since low probability of a good outcome, given the structure of the model, is equivalent to a high hazard rate). Prices gradually rise, rewarding the surviving rms. This is due to the fact that the growth of the rms is larger in the early periods (low-effort rms which have a high probability to have a bad outcome gradually disappear), so that the incentives provided by the perspectives of growth allow prices to be driven down (from their asymptotic level) by competition without violating the incentive constraints. However, over time, growth decreases towards its asymptotic level (α^{-1}), so that the incentive compatibility requires higher returns to high effort, that is, the prices increase towards their asymptotic level. Paradoxically, although the fraction of bad rms converges to zero, it still forces more than a fraction $1-\alpha$ of rms to abandon the market in every period. Good rms might have to wait until they record positive revenue per period, and this waiting time, not surprisingly, increases as the initial share of Bad rms in the population increases. One would have expected this waiting time to decrease with $\alpha - \beta$, which captures how quickly the probability of a surviving rm to be of the good type increases over time, but the model doesn t yield such a result.

Notice that the value of the rm is given by:

$$n_t V_t = \frac{n_{t-1} c}{(\alpha - \beta)\,\delta}.$$

Accordingly, it is growing over time without bound. This is driven by the increase in consumers, and might explain why the value of goodwill and

[6] Given that the outside utility is γ, it was already obvious that c should not exceed $1-\gamma$.

trademarks, while related to the price, need not be constrained by it. Notice also that the equilibrium price is decreasing in δ. That is, the premium which is required for a reputation to be sustained decreases with the discount rate: the shorter the time periods, the quicker a firm might recover the costs of an investment, the more likely the investment will be made. The role of α and β is straightforward.

Obviously, the particular price path chosen depends on the notion of refinement that we adopted, namely, on the focus on equilibria where the incentive compatibility is tight in every period. Without this refinement, prices need not be monotonic. However, there would be an upward trend in these prices, due to the two effects at work here. First, incentives require that, from any t on, the discounted level of future prices exceed cost.[7] In fact, the asymptotic price $\bar{p}$ we found is precisely the minimal level of prices satisfying incentive compatibility, if prices were constant over time. Second, since competition drives overall profits to zero, prices cannot be consistently larger than cost. Given that profits are discounted, these lower prices are posted in the early periods, giving rise to the increasing profile. The gradual character of this increase is due to the monotonically decreasing growth rate.

4.1 The partial model

Since consumers are leaving the bad or unlucky firms, the surviving firms are experiencing growth of their consumer base. However, in order to disentangle the effects of the different incentives on the nature of the equilibrium, I examine now briefly what happens if unsuccessful firms are disappearing, but the associated consumers don't go to any other firm.[8]

As in the general model, denote V_t^H and V_t^L the value for a firm of always exerting respectively high effort and low effort in any period, starting at t.

[7] The "real" discount factor to be applied should indeed be $(\alpha - \beta)\,\delta$.

[8] Alternatively, we can assume that there is a death rate among consumers which exactly offsets the growth of consumers of the general model. The (time-dependent) death rate θ_t which makes things add up is given by:

$$\theta_t = 1 - \frac{1}{\beta} \frac{1 + (\alpha/\beta)^{t+1} \frac{\phi_0}{1-\phi_0}}{1 + (\alpha/\beta)^{t} \frac{\phi_0}{1-\phi_0}}$$

Note however that this is a very different formulation of the problem, although the results are the same. In particular, the number of consumers staying with any surviving firm is decreasing.

In order for V_t^H to equal V_t, it must be that no one shot-deviation from high effort to low effort is ever pro table. The value of a one-shot deviation in period t is:

$$p_t + \delta\beta V_{t+1}^H, \tag{4}$$

whereas the value of not deviating is

$$p_t - c + \delta\alpha V_{t+1}^H, \tag{5}$$

Hence, for high effort to be sustained in every period, it must that for any $t \geq 0$,

$$V_{t+1}^H \geq \frac{c}{\delta\left(\alpha - \beta\right)}. \tag{6}$$

That is, since $V_{t+1}^H = \sum_{i=0}^{\infty} \left(\alpha\delta\right)^i \left(p_{t+i+1} - c\right)$, only if, for any $t \geq 0$,

$$\sum_{i=0}^{\infty} \left(\alpha\delta\right)^i p_{t+i+1} - \sum_{i=0}^{\infty} \left(\alpha\delta\right)^i p_i \geq 0.$$

As Fudenberg and Kreps [4] pointed out, reputation is just a cost-bene t analysis. Indeed, this condition illustrates the economic nature of reputation: it is an investment which will only be undertaken if the expected return exceeds the expected cost of it. Although there is now no compelling reason to focus attention on equilibria where the incentive constraints are binding for any period (unless one considers the alternative perspective that this model is just a version of the general model where there is a death rate of consumers which offsets the growth effect due to competition), I will do so to compare the results to those of the general model.

De nition 2 :A *(symmetric) competitive equilibrium with reputation* is a sequence $\{p_t, \psi_t\}_{t=0}^{\infty}$ of prices and beliefs such that prices are pro t maximizing for the rms given the beliefs, consumers choose rms to maximize their expected utility, beliefs are rational, $V_0^H = 0$, $V_0^L = 0$ and, for any $t \geq 0$,

$$\sum_{i=0}^{\infty} \left(\alpha\delta\right)^i p_{t+i+1} - \sum_{i=0}^{\infty} \left(\alpha\delta\right)^i p_i = \frac{c}{\delta\left(\alpha - \beta\right)},$$

Notice that:

$$V_0^L = 0 \Leftrightarrow \sum_{i=0}^{\infty} (\beta\delta)^i p_i = 0, \tag{7}$$

and

$$V_0^H = 0 \Leftrightarrow \sum_{i=0}^{\infty} (\alpha\delta)^i (p_i - c) = 0. \tag{8}$$

Given the latter constraint, the incentive constraint can be further simpli ed to, for any time t:

$$\sum_{i=0}^{\infty} (\alpha\delta)^i p_{t+i+1} = \frac{(1-\beta\delta)\, c}{\delta(\alpha-\beta)(1-\alpha\delta)}. \tag{9}$$

It is not hard (and can be found in appendix) to solve recursively this system of equations which uniquely determines the path of prices. Indeed, de ne $\{p_t^*\}_{t=0}^{\infty}$ to be the prices $p_0^* = -\frac{\beta}{\alpha-\beta}c$ and for any time $t \geq 1$, $p_t^* = p^* = \frac{(1-\beta\delta)}{(\alpha-\beta)\delta}c$. Beliefs $\{\psi_t\}_{t=0}^{\infty}$ are de ned exactly as in the general model.

The following assumption is the analog of A1. It is a necessary and sufficient condition for a high effort equilibrium to exist:

Assumption A2:

$$c \leq \frac{\delta(\alpha-\beta)((\alpha-\beta)\phi_1 + \beta - \gamma)}{1-\beta\delta}.$$

Lemma 5 *: Suppose that A2 is satis ed. Then the sequence $\{p_t^*, \psi_t\}_{t=0}^{\infty}$ is a competitive equilibrium with reputation of the partial model. Given these prices, all the good rms produce high effort in every period where they are called upon to play.*

This shows that sticks can be sufficient incentives for the good rms to exert high effort. Unsurprisingly, the constraints on the parameter for such an equilibrium to exist can be shown to be stronger than the one in the general model. To see this, notice that the prices of the partial model are always at least as large as the prices of the general model. Thus, the consumer is more likely to prefer trading (to his outside option) in the general model than in the partial one.

5 Discussion

5.1 A world with mercy?

In this subsection I consider variations where unsuccessful rms need not disappear, since it is often not realistic to assume that unsuccessful rms are forced into bankruptcy as soon as they experience bad outcome. In particular, such a feature is not accurate in the case of our benchmark example, the restaurant market. Take for instance an award-winning place which fails to renew its award. This restaurant will likely suffer both a loss in its consumer base, and the inability to raise its prices; however, the restaurant will surely not exit. It seems interesting to explore whether the previous results are robust to such extensions.

One possibility is to get one step further than before, and construct equilibria where rms exit after two bad outcomes. It is not difficult to construct such an equilibrium, and examine under which conditions high effort can be sustained. Instead of solving only for the equilibrium path of prices posted by rms which never had a bad outcome, we also need to determine the price path posted by a rm after its rst bad outcome, set in such a way that consumers remain indifferent between staying with that rm, despite the lower posterior, and switching to a rm without any bad outcomes. Incentive compatibility is assumed to be holding with equality for rms which never had any bad outcomes.[9] For simplicity, we assume that the consumer growth, which depends on the pattern of exits, is uniform across rms, that is, a consumer who switched is equally likely to join any surviving rm following the equilibrium strategy (irrespective of it having none or one bad outcome in its history, as can be inferred from the equilibrium prices).

Suppose in what follows that good rms exert high effort. Since outcomes are independent, the posterior attached in period t to a rm having one bad outcome does not depend on when that bad outcome occurred. This implies that $p_t^*(t') = p_t^*(t'') \equiv q_t^*$ for any t', $t'' < t$, where $p_{t_1}^*(t_2)$, $t_1 < t_2$, is the equilibrium price charged in period t_2 by a rm having had one bad outcome in period t_1 (and good outcomes in all the other periods). We also let p_t^* be the equilibrium price charged in period t by a rm which never had any bad outcome. Let $\phi_t^0 = \phi^{(t)}(\phi_0|G)$ and $\phi_t^1 = \phi^{(t-1)}(\phi(\phi_0|B)|G)$ be the

[9] For rms having had a bad outcome, since prices are pinned down by the arbitrage between the two types of rms, we do not, of course, expect incentive compatibility to hold with equality.

beliefs over the effort level in period t of a firm having had respectively no bad outcome and one bad outcome up to period t, where as before ϕ_0 is the initial proportion of good firms. Since consumers are indifferent between staying and switching, we have, for any $t > 0$:

$$\phi_t^0 - \phi_t^1 = p_t^* - q_t^*$$

One can then solve for the equilibrium prices, as is done in appendix. The description of the equilibrium strategies is very similar to the one of the general model, with consumers leaving after two bad outcomes, and firms correspondingly exiting after such histories, and hence I omit it.

A first difference relates to the growth of the consumer size. While asymptotically, the number of exits per period is increasing, as it must be in early periods when most of the firms have had no bad outcome, it need not be so in between, since bad firms, more likely to exit, rapidly disappear. The complexity of those dynamics tends to obscure the results, but the main features are similar to the general model: the initial price is set such that profits are zero, and hence is much lower than the later prices, which have to yield a sufficient premium for high effort to be sustained. While price premia need not be increasing over the whole domain, they ultimately do converge to

$$\bar{p} - c = \left(\frac{\alpha\,(1-\delta)}{\delta}\right)^2 \frac{c}{(1-\alpha)\,(\alpha-\beta)},$$

which is arbitrarily small whenever the discount factor is close enough to one, and increases without bound when α tends to one, or β tends to α : that is, high effort equilibria can be sustained when firms are very patient, and cannot be sustained when high effort is riskless or statistically indistinguishable from low effort.

Instead of going one step beyond, and allowing firms to survive despite a bad outcome, one could study what happens when two bad outcomes are allowed, or three, and so forth. In the limit, one can wonder whether high effort can be supported as an equilibrium when firms never exits. A complete analysis of such a situation is beyond the scope of this paper, but the following thought experiment suggests that even in that case, high effort equilibria can arise in situations similar to the one previously described (δ close to 1, $\alpha - \beta$ large enough, α small enough). For the sake of comparison, let us assume that the growth path of the consumer size is identical to the one of the general model for a firm which is always successful; suppose that whenever a rm has a bad outcome, it takes one step back with respect to the price path it would follow if it were always successful, $\{p_t\}_{t=0}^{\infty}$. That is, the rm re-applies the period t price in period $t+1$. In this case, assume also that the number of consumers remains constant from period t to period $t+1$. Let us abstract from the issue of whether such a behavior is utility maximizing for the consumers. Notice that this is the mildest conceivable punishment from the point of view of the rm; indeed, after a failure of rm j in period t, the belief over effort of its consumers is smaller than the equilibrium belief that they had about their rm at the beginning of period t. Put differently, these prices are strictly higher than what a rm should expect to get after a bad outcome if they were derived from correct beliefs updated according to Bayes rule. Correspondingly, what we derive are conditions on the incentives of the rms that are stronger than they would be in any equilibrium where consumers behavior is taken into account. As before, we assume that the incentive compatibility constraints hold with equality in every period; it is then shown in appendix that the prices $\{p^*\}_{t=0}^{\infty}$are $p_0^* = -\frac{\beta}{\alpha-\beta}c$ and

$$p_t^* = \frac{c}{(\alpha-\beta)\,\delta}\left((1-\alpha\delta)\,\frac{\sum_{i=0}^{t-1} n^i}{n^t} - \beta\delta\right).$$

These prices are strictly increasing, and they converge to:

$$\bar{p} = \left(1 + \frac{\alpha\,(1-\delta)}{(\alpha-\beta)\,(1-\alpha)\,\delta}\right) c > c.$$

Hence, they do not exceed the maximal willingness to pay of the consumers whenever

$$\left(1 + \frac{\alpha\,(1-\delta)}{(\alpha-\beta)\,(1-\alpha)\,\delta}\right) c \leq 1-\gamma.$$

This thought experiment emphasizes the importance of the growth of the consumer base. Indeed, suppose, as in the partial model, that consumer size remains constant in this derivation. Since the price has to be bounded above and there is no other source of revenue growth, there will be almost no loss of revenue in being shifted one period back when the price is close enough to its asymptotic level. The gain per consumer of exerting low effort, however, is still c. Ultimately therefore, the incentives for high effort cannot be satis ed.

If the two asymptotic price level of respectively the general model and the thought experiment are compared, notice that the former can be written as:

$$\bar{p} = \frac{(\alpha - \beta\delta)}{(\alpha - \beta)\,\delta} c = \left(1 + \frac{\alpha\,(1 - \delta)}{(\alpha - \beta)\,\delta}\right) c,$$

while the latter has been seen to be:

$$\bar{p} = \left(1 + \frac{\alpha\,(1 - \delta)}{(\alpha - \beta)\,(1 - \alpha)\,\delta}\right) c.$$

Observe that when α is close to zero, these asymptotic prices are very close.[10] One interpretation is that, when α is very low, the cost of shifting back one period is high, because a rm must be very lucky to return to its initial position. To put it another way, luck becomes very valuable, and jeopardizing ones position becomes increasingly costly.

Interestingly also, the higher the discount rate δ, the more likely an equilibrium with reputation exists. This is a result reminiscent of Shapiro [15]; when periods of time are shorter, rms reap more rapidly the returns on their investments in reputation. Thus reputation is likely to emerge as an equilibrium phenomenon.[11]

5.2 Swapping Reputations

The reader might note that the possibility of trading names of rms has not been explored here. If names can be traded, their value to the seller (if the seller extracts all the surplus) is $n_t V_t$ in the general model, which is strictly increasing in time. Since the good type of rm is indifferent at

[10] The prices derived in the experiment (and their asymptotic level) are however larger than the equilibrium prices of the general model (and their asymptotic level); since by ruling out exits, we reduce the incentives to exert high effort, future prices have to increase faster than in the general model to provide alternative incentives.

[11] Of course in applications, one might infer α from δ (or the contrary), since the probability of a good outcome should be measured per unit of time. Moreover, such an inference can be done on the basis of the average existence length T of a rm in the market under study, since

$$ET = \frac{1}{(1 - \alpha)^2}$$

for a rm doing always high effort (Replace α with β for a rm doing low effort). Moreover, this restricts the interpretation of comparative statics, since both α and δ are, for all practical purposes, functions of time.

any date t between either high or low effort, it must be that both good and bad types of rms are equally willing to buy this rm. However, this is only a partial analysis of the problem. Clearly, such ownership changes would affect consumers beliefs. If the trade of names is unobservable (to the consumer), and trade can occur with positive probability in any period, consumers beliefs are bounded away from one (see [12] for a discussion of such a phenomenon in this kind of setting). If the trade is observable, then this would be equivalent to the general model with a new rm starting and an older rm disappearing. In the former case, that of unobservable trade, the price pro le would be affected while it is unchanged in the latter.

For concreteness, consider the general model, and suppose that in any period, a rm has a probability $\lambda \in (0,1)$ of being sold to an unknown buyer of the good type with (exogenous) probability $\theta \in (0,1)$ (and of bad type with probability $1-\theta$). Suppose that the transaction is not observable, that is the consumer cannot determine if a rm has changed ownership. The updating rule for the beliefs would accordingly change from ϕ to ψ, where:

$$\psi(x) = (1-\lambda)\,\phi(x) + \lambda\theta, \forall x \in (0,1).$$

which implies that beliefs are always bounded away from one. Let $\left\{\phi_i^T\right\}_{i=0}^{\infty}$ (where T denotes Trade) be the pro le of beliefs upon always observing good outcomes. Note that the preliminary proposition regarding rm selection still holds, i.e. any surviving rm experienced good outcomes in every period. One can thus derive a price path $\left\{p_i^T\right\}_{i=0}^{\infty}$ corresponding to this economy with reputation trade. In this case, except possibly for a nite number of initial periods, ϕ_i^T is lower than $\phi_i^* \; \forall i$ (where $\{\phi_i^*\}_{i=0}^{\infty}$ denotes the belief path of the general model). Now,

$$p_t^T = \frac{c}{(\alpha-\beta)\,\delta}\left(\alpha\frac{\phi_{t-1}^T}{\phi_t^T} - \beta\delta\right) = \frac{c}{(\alpha-\beta)\,\delta}\left((\alpha-\beta)\,\phi_{t-1}^T + \beta - \beta\delta\right) \leq p_t^*.$$

for all t, except possibly for a nite number of initial periods. That is, the equilibrium price of the model with reputation swapping is lower than in the general model. Similarly, it is easy to verify that the value per consumer of the rm is lower given the possibility of name trade and that the total value of the rm is larger (except possibly again for a nite number of initial periods).[12]

[12] This latter conclusion can also be drawn in the framework of the model with renegotiation.

Thus reputation trading increases the value of those rms surviving long enough. One intuition follows: since rms have a positive probability, in any period, of being sold into the management of a bad type, the aggregate mass of rms running bankruptcy is larger than in the general model. Hence, the number of consumers joining any surviving rm is larger, so that the (total) value to the rm of surviving an additional period is larger. Price is thereby driven down by competition, since more incentives for high effort can be conveyed by future opportunities.[13]

6 Conclusion

This model provides a framework in which competition generates a more realistic result than that rst obtained by Holmström; contrary to existing literature this paper shows that simple Markov equilibria with high effort can be sustained under imperfect information.[14] The intuition is the following: consumers might become progressively convinced that the rm they are facing is a good one; however, if this is the case, they will believe that the surviving competitors of that rm are equally good. The outside option endogenously generated by competition prevents the rm from abusing the trust of its consumers. The message is simple; in a competitive environment, one should never rest on his laurels. Both the fear of losing clients and the hope of attracting new ones are strong incentives for a rm to exert effort, which may be interpreted as high quality production, technological upgrading, or simple hard work.

Factors which reduce these incentives or the intensity of competition also reduce the set of parameters for which such equilibria exist. In particular, this is the case if bankruptcy, or exit, is less threatening; but even when a rm is never threatened to exit, that is, when the consequence of failure results only in smaller prices, high effort can be consistently exerted in equilibrium, *provided* that growth perspectives are strong enough: the opportunity of attracting new consumers can provide alternative incentives to price increases, or the threat of bankruptcy.

Ultimately, there is a high price to pay to ensure the existence of equilibria

[13] One objection is that that any rm might also be led to sell with probability λ; but since it is assumed that the seller gets the whole surplus of the trade, that is, the value of the rm, this does not affect the incentives to exert effort.

[14] See however Mailath and Samuelson [12].

where high effort is always sustained; namely, an ever shrinking proportion of bad rms forces a constant fraction of better but unlucky rms to exit. As Andrew Carnegie puts it, while the law [of competition] may be sometimes hard for the individual, it is best for the race, because it ensures the survival of the ttest in every department.

One could nally interpret the model in the following way: types and effort could be considered to represent long-term commitment and spot commitment respectively (or irreversible and reversible investment).[15] For instance, suppose that the type of the rm is understood to be the educational achievements of an agent; one might then wonder what pattern of educational investment could endogenously emerge in our model. The answer is disturbing: since the system of equations characterizing an equilibrium is overdetermined, the overall expected pro t to a high type rm is zero, whether this condition follows from the de nition of the equilibrium or not. As a consequence, due to competition, no costly long-term investment would be freely undertaken. In other words, every agent would choose the minimal education level. This result is an interesting puzzle which is left to further research.

[15] I am grateful to Andy Postlewaite for this suggestion.

Appendix

- Lemma 5:

Lemma 5 is immediate: Indeed, from

$$\sum_{i=0}^{\infty} (\alpha\delta)^i p_{t+i+1} = \frac{(1-\beta\delta)}{\delta(\alpha-\beta)(1-\alpha\delta)}c, \tag{10}$$

for any $t \geq 1$, observe that p_t can be taken to be constant for $t \geq 1$, i.e.

$$p = \frac{(1-\beta\delta)}{\delta(\alpha-\beta)}c.$$

Moreover, from (9), one gets that

$$p_0 = -\frac{\beta}{\alpha-\beta}c.$$

It is readily veri ed that these values satisfy (7).

Since (11) and (9) yield that, $\forall t \geq 1$,

$$\sum_{i=0}^{t} (\beta\delta)^i p_i + (\beta\delta)^{t+1} \frac{(1-\beta\delta)}{\delta(\alpha-\beta)(1-\alpha\delta)}c = 0.$$

Uniqueness follows. To check that these speci cations constitute an equilibrium, see the end of the next lemma. The proof is almost verbatim the same.

- Lemma 3:

Lemma 3 is hardly more involved. I show that $p_0 = -\frac{\beta}{\alpha-\beta}c$ and, $\forall t \geq 1$,

$$p_t = \frac{c}{(\alpha-\beta)\delta}\left(\alpha\frac{\phi_{t-1}}{\phi_t} - \beta\delta\right)$$

solve (1), (2), (3).

Notice that

$$p_t = \frac{c}{(\alpha-\beta)\delta}\left(\alpha\frac{\phi_{t-1}}{\phi_t} - \beta\delta\right) \Leftrightarrow n_t p_t = \frac{c}{(\alpha-\beta)\delta}(n_{t-1} - \beta\delta n_t),$$

so that

$$\begin{aligned}\sum_{i=0}^{\infty}(\alpha\delta)^i n_i p_i &= -\frac{\beta}{\alpha-\beta}c+\frac{\alpha c}{(\alpha-\beta)}\left(\sum_{i=0}^{\infty}(\alpha\delta)^i(n_i-\beta\delta n_{i+1})\right)\\ &= -\frac{\beta}{\alpha-\beta}c+\frac{\alpha c}{(\alpha-\beta)}\left(n_0+\frac{(\alpha-\beta)}{\alpha}\sum_{i=0}^{\infty}(\alpha\delta)^{i+1}n_{i+1}\right)\\ &= \sum_{i=0}^{\infty}(\alpha\delta)^i n_i c.\end{aligned}$$

Moreover, $\forall t \geq 1$,

$$\begin{aligned}&\sum_{i=0}^{\infty}(\alpha\delta)^{i+1}n_{t+i+1}(p_{t+i+1}-c)\\ =\ &\frac{c}{\delta(\alpha-\beta)}\left(\sum_{i=0}^{\infty}(\alpha\delta)^{i+1}\left(n_{t+i}-\beta\delta n_{t+i+1}\right)\right)-\sum_{i=0}^{\infty}(\alpha\delta)^{i+1}n_{t+i+1}c\\ =\ &\frac{c}{\delta(\alpha-\beta)}\left(\alpha\delta n_t+\delta(\alpha-\beta)\sum_{i=0}^{\infty}(\alpha\delta)^{i+1}n_{t+i+1}\right)-\sum_{i=0}^{\infty}(\alpha\delta)^{i+1}n_{t+i+1}c\\ =\ &\frac{\alpha n_t}{\alpha-\beta}c.\end{aligned}$$

Checking the third constraint is super uous by construction. Moreover, uniqueness is derived as before.

To check that this is an equilibrium:

First, it is necessary to check that consumers accept to trade at these prices. That is, it is necessary that, $\forall t$:

$$p_t^* \leq \alpha\phi_t+\beta(1-\phi_t)-\gamma=(\alpha-\beta)\phi_t+\beta-\gamma.$$

Tedious algebra shows that this is equivalent to:

$$c \leq \frac{(\alpha-\beta)\delta((\alpha-\beta)\phi_1+\beta-\gamma)}{(\alpha-\beta)\phi_1+\beta(1-\delta)}.$$

which is satis ed by A1.

For the rm: suppose that rm j in period t sets a price higher than p_t^*. Given the consumers belief, the clientèle of j is better off leaving it (and going to rms charging p_t^*

and having some consumers), so that rm j will become indistinguishable from a starting rm and have zero value from next period on. Moreover, the only possible prices at which consumers accept to trade in the current period are negative prices (which might be higher than p_t^*), so that a deviation is not pro table. Setting a lower price yields lower revenue since the rm does no attract any additional consumer, unless the price is negative, at which the rm anyway runs losses. This concludes the argument. Suppose that a rm has no clientèle any more. In the general model, the rm is then supposed to exit.

For the consumer: given that good rms are making always high effort with probability one, the belief speci cation indeed satis es rationality and Bayes rule. Leaving rms which experienced a bad outcome is a straightforward consequence of maximizing expected utility and of their belief that the rm will exit in the next period. The acceptance rule and the choice rule are also obviously optimal given the beliefs of the consumers.

For the partial model, the assumption on the parameters A2 which is both necessary and sufficient for the existence of an equilibrium with reputation is also a consequence of $p_t^* \leq (\alpha - \beta)\phi_t + \beta - \gamma$. Straightforward algebra shows that this is satis ed for any t if and only if:

$$c \leq \frac{\delta(\alpha-\beta)((\alpha-\beta)\phi_1+\beta-\gamma)}{1-\beta\delta}.$$

- the model where one bad outcome is allowed:

Recall that p_t^0 is the (equilibrium) price charged by a rm which never had a bad outcome in period t. Similarly, we de ne $p_t^1 \equiv q_t^*$ is the price charged by a rm which had exactly one bad outcome up to period t, in period t. Let $\{n_t\}_{t=0}^{\infty}$, $n_0 = 1$, be the size of the consumer base (per rm) in period t.[16] We de ne V_t^1 (respectively V_t^0) to be the (optimal) per consumer value of a rm which had exactly one (resp. no) bad outcome up to period t, in period t. We have the following relationships holding:

$$n_t V_t^1 = n_t\left(p_t^1 - c\right) + \alpha\delta n_{t+1}V_{t+1}^1, \tag{11}$$

$$n_t V_t^0 = n_t\left(p_t^0 - c\right) + \alpha\delta n_{t+1}V_{t+1}^0 + (1-\alpha)\delta n_{t+1}V_{t+1}^1, \tag{12}$$

$$n_0 V_0^0 = 0, \tag{13}$$

$$n_t c = n_{t+1}(\alpha-\beta)\delta\left(V_{t+1}^0 - V_{t+1}^1\right), \tag{14}$$

$$\Delta\phi_t \equiv \phi_t^0 - \phi_t^1 = p_t^0 - p_t^1. \tag{15}$$

Substracting (11) from (12) we get:

$$n_t\left(V_t^0 - V_t^1\right) = n_t\left(p_t^0 - p_t^1\right) + \alpha\delta\left(V_{t+1}^0 - V_{t+1}^1\right) + (1-\alpha)\delta n_{t+1}V_{t+1}^1$$

[16] Recall that we assume that this does not depend on the particular history of a rm.

Substituting for $V_t^0 - V_t^1$, $V_{t+1}^0 - V_{t+1}^1$ and for $p_t^0 - p_t^1$ using equations (14) and (15), writing down the resulting equations for t and $t+1$, substituting $n_{t+1}V_{t+1}^1$in the former using equation (11), and substracting from it the latter multiplied by $\alpha\delta$, rearranging, one gets:

$$\begin{aligned} & (1-\alpha)\,\delta n_{t+1}\left(p_{t+1}^1 - c\right) \\ = \; & \left((n_{t-1} - \alpha\delta n_t) - \alpha\delta\left(n_t - \alpha\delta n_{t+1}\right)\right)\frac{c}{(\alpha-\beta)\,\delta} - \left(n_t\Delta\phi_t - \alpha\delta n_{t+1}\Delta\phi_{t+1}\right) \end{aligned} \tag{16}$$

from which we obtain p_t^1, and using (15), p_t^0 for any $t > 1$. Using (13), one then obtains p_0^0. For completeness, it is easy to check that:

$$n_t = \frac{1}{\phi_0\left(1+(1-\alpha)\,t\right)\alpha^t + (1-\phi_0)\left(1+(1-\beta)\,t\right)\beta^t},$$

as well as

$$\Delta\phi_t = \phi_0\alpha^t\left(\frac{1}{\phi_0\alpha^t + (1-\phi_0)\,\beta^t} - \frac{(1-\alpha)\,\alpha\beta}{\phi_0\,(1-\alpha)\,\beta\alpha^t + (1-\phi_0)\,(1-\beta)\,\alpha\beta^t}\right).$$

One can then easily verify that

$$\bar{p} - c = \left(\frac{\alpha\,(1-\delta)}{\delta}\right)^2\frac{c}{(1-\alpha)\,(\alpha-\beta)},$$

study the properties from the price pro le and derive the appropriate restrictions on the parameters.

- The thought experiment: rms never exit in equilibrium; the following necessary conditions for an equilibrium with reputation are the natural counterparts of those of the preceding section.

$$\sum_{t=0}^{\infty}(\beta\delta)^t\left(n_t p_t\right) = 0. \tag{17}$$

$$\sum_{t=0}^{\infty}(\alpha\delta)^t\left(n_t p_t - c\right) = 0. \tag{18}$$

$$n_{t+1}V_{t+1} - n_t V_t = \frac{n_t c}{(\alpha-\beta)\,\delta}. \tag{19}$$

In what follows, I will usually write p_t for p_t^*.

I proceed as before, checking that

$$p_0 = -\frac{\beta}{\alpha-\beta}c$$

and

$$p_t = \frac{c}{\delta\left(\alpha-\beta\right)}\left(\left(1-\alpha\delta\right)\frac{\sum_{i=0}^{t-1} n_i}{n_t} - \beta\delta\right)$$

solve (7), (8), (9).

Notice that:

$$\begin{aligned}
& \sum_{i=0}^{\infty}\left(\alpha\delta\right)^i n_i p_i \\
= \quad & -\frac{\beta}{\alpha-\beta}c + \frac{c}{\left(\alpha-\beta\right)\delta}\left(\sum_{i=0}^{\infty}\left(\alpha\delta\right)^{i+1}\left(\left(1-\alpha\delta\right)\sum_{j=0}^{i} n_j - \beta\delta n_{i+1}\right)\right) \\
= \quad & -\frac{\beta}{\alpha-\beta}c + \frac{c}{\left(\alpha-\beta\right)\delta}\left(\alpha\delta n_0 + \sum_{i=1}^{\infty}\left(\alpha\delta\right)^i\left(\left(1-\alpha\delta\right)\sum_{j=0}^{i}\left(\alpha\delta\right)^{j+1} - \beta\delta\right) n_i\right) \\
= \quad & \left(1+\sum_{i=1}^{\infty}\left(\alpha\delta\right)^i n_i\right)c = \sum_{i=0}^{\infty}\left(\alpha\delta\right)^i n_i c,
\end{aligned}$$

and that, $\forall t \geq 1$:

$$\begin{aligned}
& n_{t+1}V_{t+1} \qquad (20) \\
= \quad & \sum_{j=0}^{\infty}\left(\alpha\delta\right)^j n_{t+j+1}\left(p_{t+j+1} - c\right) \\
= \quad & \sum_{j=0}^{\infty}\left(\alpha\delta\right)^j \left(\left(1-\alpha\delta\right)\sum_{i=0}^{t+j} n_i - \alpha\delta n_{t+j+1}\right) \\
= \quad & \frac{c}{\left(\alpha-\beta\right)\delta}\left(\left(1-\alpha\delta\right)\left(\sum_{k=0}^{\infty}\left(\alpha\delta\right)^k\right)\sum_{i=0}^{t} n_i\right) \\
& + \frac{c}{\left(\alpha-\beta\right)\delta}\sum_{i=t+1}^{\infty}\left(\alpha\delta\left(\sum_{k=0}^{\infty}\left(\alpha\delta\right)^k\right)\left(1-\alpha\delta\right)n_i - \alpha\delta n_i\right) \\
= \quad & \frac{c}{\left(\alpha-\beta\right)\delta}\sum_{i=0}^{t} n_i,
\end{aligned}$$

where the fact is used that

$$n_t\left(p_t - c\right) = \frac{c}{(\alpha - \beta)}\left(\left(1-\alpha\delta\right)\left(\sum_{i=0}^{t-1} n_i\right) - \alpha\delta n_t\right).$$

On the other hand, one has that:

$$\begin{aligned} \frac{n_t}{(\alpha-\beta)\,\delta}c + n_t\left(p_t - c\right) &= \frac{(1-\alpha\delta)\,c}{(\alpha-\beta)\,\delta}\sum_{i=0}^{t-1} n_i + \frac{(1-\alpha\delta)\,c}{(\alpha-\beta)\,\delta}n_t \qquad (21) \\ &= \frac{(1-\alpha\delta)\,c}{(\alpha-\beta)\,\delta}\sum_{i=0}^{t} n_i \end{aligned}$$

From (1a) and (2a), one immediately gets:

$$(1-\alpha\delta)\sum_{j=0}^{\infty}(\alpha\delta)^j\, n_{t+j+1}\left(p_{t+j+1} - c\right) = \frac{n_t}{(\alpha-\beta)\,\delta}c + n_t\left(p_t - c\right)$$

which is the incentive constraint.

- Properties of $\{p_t^*\}_{i=0}^{\infty}$:

I establish that the price pro le of the thought experiment is increasing. Indeed, de ne $\forall t \geq 1$:

$$S_t = \frac{\sum_{i=0}^{t-1} n_i}{n_t}.$$

Now,

$$p_t = \frac{c}{(\alpha-\beta)\,\delta}\left(\left(1-\alpha\delta\right)S_t - \beta\delta\right),$$

so that showing that the prices are increasing is equivalent to showing that S_t is increasing.

Rewrite

$$S_t = \frac{\sum_{i=0}^{t-1} n_i}{n_t} = \sum_{i=0}^{t-1}\prod_{j=i}^{t-1}\left(\left(\alpha-\beta\right)\phi_j - \beta\right),$$

so that S_t is seen to satisfy the recurrence relation:

$$S_{t+1} = \left(\left(\alpha-\beta\right)\phi_t - \beta\right)\left(S_t + 1\right).$$

Hence S_t is increasing if and only if $\forall t \geq 1$,

$$S_t \leq \frac{(\alpha-\beta)\,\phi_t - \beta}{1 - \left(\left(\alpha-\beta\right)\phi_t - \beta\right)}.$$

$(\phi_0 \in (0,1) \Rightarrow \forall t \geq 1, \phi_t \in (0,1))$

I prove the inequality by induction; I prove that $S_1 \leq S_2$, which implies that $S_1 \leq \frac{(\alpha-\beta)\phi_1-\beta}{1-((\alpha-\beta)\phi_1-\beta)}$.

I wish to show that

$$\frac{n_0}{n_1} \leq \frac{n_0+n_1}{n_2},$$

which is equivalent to showing that:

$$\alpha\phi_0 + \beta(1-\phi_0) \leq (\alpha\phi_0 + \beta(1-\phi_0))(\alpha\phi_1 + \beta(1-\phi_1)) + \alpha\phi_0 + \beta(1-\phi_0).$$

Rearranging and factorizing yields the equivalent (dropping the zero subscript):

$$g(\phi) \equiv (\alpha-\beta)^2(1-\alpha-\beta)\phi^2 - (\alpha-\beta)\left(\alpha-\beta+\alpha\beta+2\beta^2\right)\phi - \beta^3 \leq 0.$$

$g(0) = -\beta^3 \leq 0$, $g(1) = -\alpha^3$, so that if $\alpha+\beta \leq 1$, the condition is true $\forall\phi \in (0,1)$. Notice moreover that, in the case where $\alpha+\beta \geq 1$, the sum of the roots is negative ($= \frac{\left(\alpha-\beta+\alpha\beta+2\beta^2\right)}{(\alpha-\beta)(1-\alpha-\beta)}$), so that one root at least is negative (in fact both since the products of the roots is positive), and hence the condition is also true $\forall\phi \in (0,1)$.

Suppose that, for $t \in \mathbb{N}$,

$$S_t \leq \frac{(\alpha-\beta)\phi_t - \beta}{1-((\alpha-\beta)\phi_t-\beta)}.$$

Then

$$\begin{aligned} S_{t+1} &= ((\alpha-\beta)\phi_t-\beta)(S_t+1) \\ &\leq \frac{(\alpha-\beta)\phi_t-\beta}{1-((\alpha-\beta)\phi_t-\beta)} \\ &\leq \frac{(\alpha-\beta)\phi_{t+1}-\beta}{1-\left((\alpha-\beta)\phi_{t+1}-\beta\right)}, \end{aligned}$$

because $f : x \longrightarrow \frac{x}{1-x}$ is increasing and ϕ_t is increasing in t. I can thus conclude that prices are increasing. In order to show that the prices converge over time, it is enough to show that S_t is converging. But this is immediate, since

$$S_{t+1} = ((\alpha-\beta)\phi_t-\beta)(S_t+1) \leq \alpha(S_t+1) \leq \alpha^t S_1 + \frac{1-\alpha^{t+1}}{1-\alpha} \leq \frac{1-\alpha^{t+2}}{1-\alpha},$$

so that $\{S_t\}$ is increasing and bounded.

From the incentive compatibility constraint, I also get:

$$n_t p_t = n_{t-1}p_{t-1} + \frac{c}{(\alpha-\beta)\delta}\left((1-\delta(\alpha-\beta))n_{t-1} - \beta\delta n_t\right).$$

Since p_t converges and $\lim_{t\to\infty}\frac{n_{t-1}}{n_t} = \alpha$, I get $\bar{p}$.

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