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"The Impact of Capital-Based Regulation on Bank Risk-Taking: A Dynamic Model"

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The Impact of Capital-Based Regulation on Bank Risk-Taking: A Dynamic Model*

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Abstract

In this paper, we model the dynamic portfolio choice problem facing banks, calibrate the model using empirical data from the banking industry for 1984-1993, and assess quantitatively the impact of recent regulatory developments related to bank capital. The model suggests that the new regulatory environment may have the unintended consequence of inducing banks, especially undercapitalized ones, to invest in riskier assets. This holds both under higher capital requirements (even if risk-based) and under capital-based deposit-insurance premia.

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1. Introduction

In this study, we consider the impact of recent developments in bank capital regulations on the risk-taking behavior of banks.

In recent years, in the wake of the savings and loan crisis in the U.S., more stringent and complex capital regulation has been brought to bear upon federally-insured depository institutions. In 1988 the federal regulatory agencies adopted new capital rules. As described in Avery and Berger 1991, a major effect of these rules was to raise the ratio of capital to total assets and to risk-weighted assets.\(^1\)

Another important initiative, the Federal Deposit Insurance Corporation Improvement Act (FDICIA), was legislated in 1991. One important aspect of the new regulation was its requirement that the FDIC implement “risk-related” pricing of deposit insurance. Thus, in 1993 fixed insurance-premia were replaced by premia tied to capital ratios and supervisory risk-ratings. Thereby, banks with lower capital ratios were made to pay higher premia.

These regulatory reforms were aimed at discouraging bank risk-taking, preventing bank failures, and ensuring continued solvency of the deposit insurance fund. Excessive risk-taking (also called the “moral hazard” problem) arises because the government deposit-guarantee allows banks to make riskier loans without having to pay higher interest rates on deposits. As a result, banks may be prone to take on excessive risk.\(^2\) Indeed, there is mounting evidence that excessive risk taking has been a problem under the deposit insurance contract; see Berlin, Saunders, and Udell (1991) and references therein.

The goal of this paper is to construct an analytical framework capturing the risk-taking behavior of banks and the effect that the new regulations have on it. Our framework focuses on two issues. First, it recognizes the fact that different banks may respond differently to the new regulations. Well-capitalized banks may reduce risk-taking in an effort to avoid the insurance-premium surcharge they would pay should their capital fall below the regulatory requirement. This

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\(^1\)Bank capital shields the deposit-insurance fund from liability by absorbing bank losses and preventing bank insolvency. For this reason, capital regulation has long been a mainstay of banking supervision. Formal capital guidelines, including minimum required ratios of capital to total assets, were instituted by the federal regulatory agencies in 1981. Prior to that, federal regulators had supervised bank capital on a case-by-case basis. See Wall (1989) for further discussion.

\(^2\)To policy makers and regulators, a potential social cost of bank risk-taking is the possibility that a major bank failure or a series of failures could impose external costs on financial markets. See Bhattacharya and Thakor (1993) and Berger et. al (1994) for further discussion.
would be in line with the regulatory intent. Undercapitalized banks, on the other hand, may increase their risk-taking. They may do so because the surcharge reduces their earnings and, therefore, makes them more willing to risk bankruptcy. Furthermore, given that they are close to bankruptcy, risk-taking might be the best recourse to increase their capital. Thus, the efficacy of the new regulations may very well depend on how well-capitalized a bank is.

The second issue is to quantify the impact of the new regulations. How much difference they make depends on various market parameters, for instance, the deposit-loan rate spread and the actual insurance premia that banks pay; the higher is the spread, the more profitable is the bank, and the less likely it is to jeopardize its profits by taking risk. Accordingly, after we construct the analytical framework, we calibrate it with parameter values which come from the banking industry, and assess the efficacy of the regulations under these values.

To this end, we first model the dynamic portfolio-choice problem facing banks. The model considers banks which operate in a multiperiod setting with the objective of maximizing the discounted value of their profits. In each period, and based on its capital position, a bank makes a portfolio choice; i.e., it decides how to allocate its assets between risky and safe investments. Then—as a result of the bank’s portfolio choice, its pre-existing capital position, and the realization of returns on its loans (which is a random variable)—the bank’s capital position for the next period is determined, and the bank faces the same problem again. Thus, the model provides a link between a bank’s capital position and its risk-taking behavior.\(^3\)

To explore the incentive effects of the FDIC’s deposit insurance pricing scheme, we incorporate capital-based premia into our model. Thus, we posit that banks are assessed higher premia if their capital-to-asset ratio falls below the regulatory requirement. The model also provides a framework for analyzing the impact of risk-based capital regulation, whereby a bank’s capital requirement depends on its portfolio choice. These features are easily embedded into the model, since the opportunities a bank faces are allowed to depend on its capital position and on the portfolio choices it makes.

After constructing a theoretical model embodying these features, we calibrate

\(^3\)If the bank becomes insolvent, it ceases to operate. Thus, one aspect of this model is that the bank will want to remain solvent and generate future profits, and this will partially offset the moral-hazard problem that the bank is subject to (wanting to exploit a deposit insurance subsidy.) Keeley (1990) presents empirical findings consistent with the view that the incentive to protect future profits is a moderating influence on bank risk-taking.
it using a set of parameter values which come from empirical data on the banking sector during the period 1984-1993. We then solve the model numerically and apply it to analyze the impact on bank risk-taking of increased capital requirements, capital-based premia differentials, and risk-based capital requirements. The model yields a variety of interesting implications in regard to the choice of bank portfolios and the efficacy of capital-based regulation.

The first finding is that the relationship between capital and risk-taking is U-shaped. Severely under-capitalized banks take maximum risk. Then, as a bank's capital rises, it takes less risk. Then, as capital continues to rise, it will take more risk again. Thus, risk-taking first decreases, then increases. This finding is robust to various values of market and regulatory parameters.

Second, we find that a premium surcharge imposed on undercapitalized banks worsens the moral-hazard problem, boosting their incentive to take on risk. Surprisingly, however, we find that a premium surcharge has no appreciable impact on the behavior of a well-capitalized bank. Therefore, contrary to the intent of regulators, premium surcharges increase bank risk-taking.

Third, we find that an increased capital requirement generally leads to increased risk-taking. In the case of a minimum standard for the ratio of capital to total assets (henceforth referred to as a flat standard), a bank that was well-capitalized before the regulatory change will take on additional portfolio risk. A higher capital requirement may also induce more risk-taking among banks that were under-capitalized. Therefore, while an increased capital requirement has the potential of reducing insolvencies (were banks to maintain their portfolio choices), banks offset this effect by taking on more risk. The net result is that the probability of insolvency remains intact.

Fourth, the model suggests that a risk-based capital rule functions much like a flat rule, in that a bank generally does not choose to reduce risk in order to achieve compliance. When a flat standard is replaced by an effectively more stringent risk-based standard, an ex-ante well-capitalized bank tends to move in the direction of increasing both capital and risk in order to comply with the new rule. This implication of the model is consistent with empirical studies that have examined

Of course, the added capital will mitigate the impact of the increased portfolio risk. Nevertheless, the effect of regulatory capital requirements on bank portfolio risk is an important question, because, as emphasized by Berger et al. (1995), "binding regulatory capital requirements...involve a long-run social tradeoff between the benefits of reducing risk of negative externalities from bank failures and the costs of reducing bank intermediation." To the extent that capital requirements provide incentives for increased risk-taking, the tradeoff becomes more severe.
banks' response to the introduction of risk-based capital standards. These studies generally have not found evidence of a shift toward reduced risk-taking (Berger and Udell 1994; Hancock and Wilcox 1994).

Our study differs from previous studies of the moral hazard problem and capital regulation along the following dimensions. Most previous studies formalize the moral hazard problem within a static framework where bank capital is set at the level required by regulation (Kahane 1977; Kareken and Wallace 1978; Koehn and Santomero 1980; Kim and Santomero 1988; Furlong and Keeley 1989, 1990; Crouhy and Galai 1991; Gennotte and Pyle 1991). Several of these studies examine the effect of a higher (flat) capital standard and find that banks respond by choosing higher-risk portfolios. The static framework, however, precludes consideration of intertemporal consequences of risk-taking, precludes cross-sectional predictions regarding the behavior of both well-capitalized and undercapitalized banks, and cannot be applied to analyze the impact of risk-based capital regulation. Ritchken, Thomson, DeGennaro, and Li (1993) introduce a dynamic model of the moral hazard problem that bears some similarity to ours. In their model, a bank revises its portfolio choices dynamically in response to changes in its capital position, and this dynamic flexibility allows the bank to more fully exploit the deposit insurer. Issues pertaining to capital regulation are not addressed in that study, however.

We proceed as follows. The basic model is developed in section 2. Section 3 deals with calibration of the model. In section 4, we numerically solve the model under the assumption of no premium surcharge. In section 5, we analyze the impact of a higher capital requirement and the effects of capital-based premia. In section 6, we consider two extensions of the model and provide suggestions for future research. In the first extension, the bank chooses its desired capital target for the next period, which may be greater than or equal to the regulator's requirement, which may be risk-based. With this version of the model we address several issues, including the impact of a risk-based capital standard. In the second extension, we allow the bank to receive capital infusions, relaxing an earlier constraint on external financing. Section 7 concludes.

2. The Basic Model

We consider the dynamics of bank portfolio choices in an infinite-horizon model. To simplify the model, and commensurate with the data we have, we allow banks to choose their portfolio composition only (as opposed to choosing their portfolio...
size5). Accordingly, bank size is fixed and normalized at 1.

Banks are subject to a flat (not risk-based) minimum capital requirement—also called a “regulatory standard”—$C^*$. Assets are funded by capital $C$ and deposits $D$; $C + D = 1$. At the beginning of each period, the bank chooses a portfolio composition consisting of $R$ units of the risky asset and $S$ units of the safe asset; $R + S = 1$.

The cost of deposits is given by the function $r(C)$, where

\[ r(C) = r_0 \text{ if } C \geq C^* \text{ and } r(C) = r_0 + \pi \text{ if } C < C^*; \pi \geq 0. \quad (2.1) \]

When bank capital satisfies the regulatory standard $C^*$, the cost of deposits is $r_0$, which incorporates both interest paid to depositors and the base deposit insurance-premium. If the regulatory standard is not met, $C < C^*$, a premium surcharge $\pi \geq 0$ is assessed.

The safe asset earns a certain, end-of-period, return $x > 1$ per unit of investment, while the risky asset earns a random return $y$. Ex-ante, the risky asset promises the return $y_0 > x$ per unit invested, but ex-post, a fraction $u$ of the investment in the risky asset yields a return of 0; i.e., this fraction is lost. The remaining fraction, $1 - u$, yields the promised return $y_0$.6 Thus, the realized return on the risky asset is $y(u) \equiv y_0(1 - u)$. The fractional loss $u$ is a random variable taking values between 0 and 1, drawn from a distribution with density function $g(u)$ and cumulative distribution function $G(u)$.

In any period, the bank’s owners (stockholders) earn the residual return on the bank’s investments after the bank has paid its depositors and its deposit insurance premium and met the minimum capital requirement. Formally, let $z(C, R, u)$ denote the return net of payments to depositors that is implied by a beginning of period capital level $C$, a portfolio choice $R$, and a loss realization $u$:

\[ z(C, R, u) \equiv x(1 - R) + y(u)R - r(C)(1 - C). \quad (2.2) \]

5 A more general model might include the choice of portfolio composition and size. In that instance a bank may respond to unfavorable loan payoffs by “downsizing”, i.e., reducing its investments and keeping its capital within the regulatory requirement. Either way, there is a cost associated with unfavorable loan payoffs. In one instance, it is the increased insurance premium; in the other it is the adjustment to a suboptimal size. We choose to focus on the first cost because data on premia surcharges are readily available, while data on the relationship between size and profitability is harder to come by.

6 The returns $x$ and $y_0$ are net of loan-production costs or other non-interest expenses associated with financial intermediation.
If \( z(C, R, u) \geq C^* \), stockholders earn \( z(C, R, u) - C^* \). If \( 0 < z(C, R, u) < C^* \), stockholders earn zero, and the entire net return that period goes toward next period’s capital.\(^7\) If \( z(C, R, u) \leq 0 \), the bank ceases to exist and the FDIC pays off depositors after claiming the return on the bank’s asset portfolio.

The set of fractional losses consistent with continued bank solvency is bounded above by \( u_A \), where \( u_A \) satisfies \( z(C, R, u) = 0 \). Similarly, the set of fractional losses consistent with positive stockholder earnings is bounded above by \( u_B \), where \( u_B \) satisfies \( z(C, R, u) = C^* \). Thus, using 2.2:

\[
    u_A = \frac{\left[ x(1 - R) + y_0 R - r(C)(1 - C) \right]}{y_0 R}, \tag{2.3}
\]

\[
    u_B = u_A - \left( \frac{C^*}{y_0 R} \right). \tag{2.4}
\]

Note that \( u_A \) and \( u_B \) are functions of \( C \) and \( R \).

The bank’s optimal investment in the risky asset will depend only on \( C \), the state variable. The optimal investment function, denoted \( R(C) \), is determined along with the value function \( V(C) \) as the solution to the dynamic programming problem:

\[
    V(C) = \max_R \left\{ \int_0^{u_B} [z(C, R, u) - C^*]g(u)du + \delta V(C^*) \int_0^{u_B} g(u)du + \delta \int_{u_A}^{u_B} V(z(C, R, u))g(u)du \right\}, \tag{2.5}
\]

where \( \delta \) denotes the rate at which stockholders discount future earnings. The maximand in 2.5 can be understood as follows. The first term represents expected current-period earnings, since stockholders earn \( z(C, R, u) - C^* \) in the event of a favorable realization, \( u \leq u_B \), and earn zero otherwise. The second term represents the continuation value when the bank meets the capital requirement at the end of the current period, weighted by the probability that this will be the case. The third term is the expected continuation value when the bank cannot meet the capital requirement (but is still solvent).

\(^7\)This assumption is consistent with regulatory requirements mandated by FDICIA, whereby undercapitalized banks are prohibited from paying dividends or paying management fees to a parent holding company.
Let \( E[u \mid u \leq u_B] \equiv \int_0^u g(u) du \). Since \( z(C, R, u) - C^* = y_0 R(u_B - u) \), we can rewrite 2.5 in the more amenable form:

\[
V(C) = \max_R \{ u_B y_0 R G(u_B) - y_0 R E[u \mid u \leq u_B] + \delta V(C^*) G(u_B) \}
\]

\[
+ \delta \int_{u_B}^{u_A} V(z(C, R, u)) g(u) du \}.
\]

Since 2.6 is not analytically solvable, we generate a numerical solution. Towards that, we discretize the problem as follows. Define \( n = C^*/0.002 \) points along the range \([0, C^*]\):

\[
C_1 = 0.002; C_{i+1} = C_i + 0.002, \ i = 1, \ldots, n.
\]  

(2.7)  
(The number \( n \) will vary, depending on the regulatory requirement, \( C^* \).)  
We also define 20 points \( R_j \) in \([0, 1]\):

\[
R_1 = 0.05; R_{j+1} = R_j + 0.05, \ j = 1, \ldots, 20.
\]  

(2.8)  
To each \( C_i, R_j \), we attribute \( n \) points \( u_k \) in \([u_B, u_A]\):

\[
u_k = u_A - (0.002) k / (1 + y_0) R_j, \ k = 1, \ldots, n.
\]  

(2.9)  
u_k represents the fractional loss that would leave the bank with \((0.002)k\) units of capital at the end of the period, given that the bank began the period with \( C_i \) units of capital and a portfolio choice \( R_j \). Note that \( C_n = C^* \), \( u_0 = u_A \), and \( u_n = u_B \).

The numerical solution to 2.6 will then be a set of portfolio positions \( R_i^* = R^*(C_i) \) and a set of discounted present values \( V_i^* \equiv V^*(C_i) \), \( i = 1, \ldots, n \), such that \( R_i^* \) solves:

\[
\max_{R_j} \{ u_B y_0 R_j G(u_B) - y_0 R_j E[u \mid u \leq u_B] + \delta V_n^* G(u_B) \}
\]

\[
+ \sum_{k=1}^{n-1} \delta [(V_k^* + V_{k+1}^*) / 2] G(u_{k+1}) - G(u_k)] \},
\]

and such that \( V_i^* \) satisfies (to close approximation):

\[
V_i^* = u_B y_0 R_i^* G(u_B) - y_0 R_i^* E[u \mid u \leq u_B] + \delta V_n^* G(u_B)
\]

\[
+ \sum_{k=1}^{n-1} \delta [(V_k^* + V_{k+1}^*) / 2] G(u_{k+1}) - G(u_k)] \}
\]

(2.10)  
8
for each \( i \). Note that \( u_B, u_k, \) and \( u_{k+1} \) in 2.10 are evaluated at \( C_i \) and \( R_j \), while \( u_B, u_k, \) and \( u_{k+1} \) in 2.11 are evaluated at \( C_i \) and \( R^*_i \). Condition 2.10 states that for each \( i \), \( R^*_i \) is the portfolio allocation that maximizes \( V(C_i) \), the expected value of current and discounted future earnings. The corresponding maximum value of \( V(C_i) \) is \( V_i^* \) as defined by 2.11.

### 3. Calibration of the Model

Computation of a numerical solution to 2.10 and 2.11 is straightforward once a probability distribution \( G(u) \) is specified and parameter values are assigned.\(^8\) Parameters values to be assigned include: The minimum capital requirement \( C^* \), the deposit interest rate \( r_0 \), the return \( x \) on the safe asset, the ex ante promised return on the risky asset \( y_0 \), the discount factor \( \delta \), and the parameters of a specified probability distribution, \( G(u) \). In this section, we assign values (or ranges of values) consistent with observed data from the banking industry. Later in the paper, we indicate the effects of varying these values.

#### 3.1. Parameter values other than \( G \)

We initially set \( C^* = 0.06 \), which was the regulatory standard prior to the reform. We calibrate \( r_0 \), the interest payment plus base insurance-premium per dollar of deposits, as follows. We draw on a panel data-set consisting of end-of-year, Call Report data from the years 1984 through 1993. Every U.S. commercial bank having at least $300 million in assets and at least a 6 percent ratio of equity capital to assets as of year-end 1984 is included in the panel.\(^9\) First, we compute the sample means, by year, of interest expense per dollar of deposits. \(^{10}\) To these, we add the mean effective insurance-premium payment (per dollar of deposits by year.) Then, we compute the mean of these means across years, to obtain the empirical estimate \( r_0 = 1.050 \). (We employ this two-step procedure in order to

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\(^8\) The FORTRAN programs used to compute the solutions discussed below are obtainable from the authors upon request.

\(^9\) Attention is confined to this period because modifications to the Call Reports that were instituted in 1985 introduced certain inconsistencies with earlier years' data. Extreme values were deleted from this panel data-set.

\(^{10}\) Interest expense per dollar of deposits for year \( t \) is total deposit-interest-expenses incurred during \( [t - l, t) \), i.e., during the year prior to date \( t \), divided by the average total deposits for the reporting dates \( t - 1 \) and \( t \).
avoid placing disproportionate weight on earlier years, since the number of banks in the panel was declining over time.

The safe asset is represented by 6-month treasury bills. We set the safe asset’s return $x$ equal to 1.060, the average interest rate on 6-month treasury bills over the period 1985-1993.

Direct measurement of the return to banks’ risky investments, $y_0$, is impractical. Hence, we selected values for $y_0$ that are consistent with the returns on publicly-issued debt, rated B or lower. The typical spreads between the return to these debts and 6-months treasury bills have ranged from 5% to 6%; see, for example, Carey and Luckner (1994). Accordingly, calibrations considered below include $y_0 = 1.113$, $y_0 = 1.1175$ and $y_0 = 1.122$.

Our calibration of the discount factor is based on historical stock-market returns. During 1971-1990, the annual excess stock-market return over 6-month treasury bills averaged 0.037 (Campbell and Hamao 1992). We add this to the average 6-month t-bill rate during 1984-1993, 1.060, and then take the reciprocal, to obtain $\delta = 0.91$.

3.2. Simulation procedure for estimating the loss distribution, $G(u)$

Following McAllister and Mingo (1996) and Jones (1995), a Monte Carlo implementation of a multi-factor model is used to estimate the probability distribution of losses on the risky asset. We do this following a two-step procedure. First we determine the probability of default on individual projects. Then, conditional on default, we estimate the amount lost. Combining these two steps gives us the probability distribution, $G(u)$.

In more detail, we equate the risky asset with a portfolio of 100 loans, each of which is invested in a project that yields a random return. The return on the project associated with loan $i$, denoted $r_i$, is assumed to depend linearly on an economic, “market risk,” factor $w$ and an idiosyncratic risk factor $\varepsilon_i$:

$$r_i = bw + \varepsilon_i.$$  \hfill (3.1)

The economic factor $w$ is assumed to be a normal random variable with mean 0 and variance 1. The idiosyncratic factors $\varepsilon_i$ are assumed to be normal random variables that are independent of each other and of $w$, having mean 0 and variance $s^2$. The correlation between individual project returns in the portfolio is $b^2$.

If the realized return on a project is below the cutoff point $y_0$, the loan goes into default. Therefore, the probability of default is $d = Pr(r_i < y_0 \mid b^2, s^2)$. Note
that once $b^2$ and $s^2$ are specified, the model implies a probability of default $d$ on an individual loan. Alternatively, if $d$ and $b^2$ are specified, then $s^2$ is determined by the model.

The fractional loss on a loan, conditional on default, is denoted $L$ and is allowed to depend on general economic conditions. Specifically, we posit a piecewise linear relationship between $L$ and $w$:

$$L = \begin{cases} 
L_0 \text{ if } w \geq 0, \\
L = L_0 + L_1(w - w_1) \text{ if } w_1 < w < 0, \\
L = L_0 + L_1 + L_2(w - w_2) \text{ if } w_2 < w < w_1 \text{ and } L = L_0 + L_1 + L_2 \text{ if } w < w_2.
\end{cases}$$

(3.2)

Given this specification, we numerically calculate the (unconditional) probability distribution of total losses on a hypothetical loan portfolio, after assigning reasonable values to the parameters $b^2$ and $d$ in 3.1 and $L_0$, $L_1$, $L_2$, $w_1$, and $w_2$ in 3.2. The calculations involve, first, randomly drawing realizations for the market and idiosyncratic risk factors to determine which loans default. The default rate is then multiplied by the rate of loss in the event of default to yield a loss rate on the portfolio. To calibrate the loss probability distribution in this way, 10,000 simulated portfolio loss rates are constructed.\(^{11}\)

3.3. Assignment of numerical values to parameters underlying $G$

We mostly hold the correlation parameter constant at $b^2 = 0.25$, a value suggested as reasonable by Jones (1995) and McAllister and Mingo (1996) (as reported below we have also experimented with other values, without appreciable effect on the results). With respect to the parameter $d$, the individual default probability for the loans comprising our model's risky asset, we assume that this is at least as great as the probability of default associated with B-rated bonds. According to Moody's 1994, default rates within one year of issue on B-rated bonds historically have averaged around 8 percent. We experimented thus with $d$'s ranging from $d = 0.08$ to $d = 0.093$.

We set $L_0 = 0.30$, $L_1 = 0.40$, $L_2 = 0.10$, $w_1 = -1.50$, and $w_2 = -2.00$: i.e., the loss given default is 30 percent when the economy is strong, it increases up to a maximum of 80 percent when the economy weakens, and it has an expected value of 41 percent.\(^{12}\)

\(^{11}\)We adopted the particular approach employed by Jones (1995). We thank David Jones for supplying us with a copy of his simulation program.

\(^{12}\)The dependence of loss-given-default on $w$ has a modest effect on the shape of the loss distribution.
This expected loss-given-default is corroborated by other, independent, data. For instance, a report by the Society of Actuaries (1993) finds an average of 44 percent loss severity (loss per unit of credit exposure) on defaulted private placement bonds for issuers rated BB or lower. Furthermore, drawing on the panel data-set introduced previously, we find that the dollar amount of commercial loans, past due 90 days or more or non-accruing, as a proportion of total dollars loaned, averaged 0.009, while net losses per dollar loaned averaged 0.023. The latter number divided by the former, which may be viewed as indicative of the typical loss per dollar of defaulted loans, is 0.38.

3.4. Piecewise linear approximation to $G(u)$

Loss distributions generated using the simulation procedure just described exhibit a characteristic shape. They exhibit a mass point at zero, corresponding to well-performing loans. Then the distributions exhibit a moderate rise through the median and a very sharp rise through the upper percentiles; losses exceeding 20 percent are confined to the extreme upper tail of the distribution.

Given this, we approximated $G$ by means of a piecewise linear distribution. In other words, we utilized distributions of the form:

$$
G(u) = \begin{cases} 
[u/u^i]G(u^i) & \text{if } 0 \leq u \leq u^i, \\
G(u^i) + [(u - u^i)/(u^{i+1} - u^i)][G(u^{i+1}) - G(u^i)] & \text{if } u^i \leq u \leq u^{i+1}, \ i = 1, \ldots, 7,
\end{cases} $$

where $u^1, \ldots, u^8$ denote the 1st, 5th, 25th, 50th, 75th, 95th, 99th and 100th percentiles, respectively. We chose these percentiles in accordance with the shape of the distributions as described above. We assigned values to $u^6$ and $u^7$ that were somewhat larger than the corresponding percentiles of the simulated distributions. The result are robust to alternative specifications of loss-given-default.

\footnote{The proportion of loans in default is calculated annually as the end-of-year ratio of loans 90 days or more past due or non-accruing to total loans. A bank's net loss rate is calculated annually as the ratio of net charge-offs (charge-offs minus recoveries) during the year divided by the average of beginning and end-of-year total loans. To obtain the average proportion of loans in default and the average net loss rate for the panel, we compute the mean across banks for each year, and then compute the mean of these means across years.}

distribution, similar to the effect of a small increase in the correlation parameter $b^2$. The results are robust to alternative specifications of loss-given-default.
to adjust for the strong concavity of the tails of the simulated loss distributions. We let $u^8$ be determined by the condition $G'(u^8) = 1$.\footnote{For instance, we set $u^8$ equal to the mean value of $u$ between the 90th and 99th percentiles of the simulated distribution.}

Values of $u^1$ through $u^8$ for four distributions utilized below are shown in table 1, along with the mean values $E[u]$. Loss distribution (i) in table 1 is the piecewise linear approximation to the simulated distribution based on an 8 percent individual default probability ($d = .080$) and a 25 percent correlation between loan rates of return ($b^2 = .25$). Distributions (ii) and (iii) are based on $d = .0875$ and $d = .093$, respectively, with $b^2 = .25$, while (iv) assumes $d = .093$ and $b^2 = .20$.

To check for consistency, we computed the expected return on the risky asset for the various values of $y_0$ specified above and the simulated $G$ from table 1. When $y_0 = 1.113$ and with the loss distribution (i) of table 1, we get $E[y(u)] = 1.062$. With $y_0 = 1.1175$ and loss distribution (ii), we get $E[y(u)] = 1.061$. With $y_0 = 1.122$ and loss distributions (iii) and (iv), respectively, we get $E[y(u)] = 1.061$ and $E[y(u)] = 1.063$. As one would expect, the implied expected returns exceed the risk-free return (0.06) by a small amount (10 to 30 basis points), reflecting specialized knowledge of banking firms.

4. Solutions to the Model Assuming a Fixed Insurance Premium

4.1. U-shape of the solution

In this section, we focus on solutions obtained with $\pi = 0$, i.e., where no premium surcharge is assessed when bank capital falls below the regulatory standard $C^\ast$. We also hold $C^\ast$ constant at 0.06, which was the regulatory standard prior to the introduction of risk-based standards.

In figure 1a, we depict the solution obtained when the risky asset is calibrated with $y_0 = 1.113$ and with loan-loss distribution (i) of table 1. This solution exhibits a characteristic, U-shaped relationship between the amount of risk a bank takes and the bank's current capital position. A severely undercapitalized bank takes maximal risk. Then, as capital rises beyond a certain level, the solution jumps to a much lower level of risk-taking. Subsequently, as capital rises the bank takes on more risk. Therefore, risk-taking first decreases and then increases.

The U-shaped pattern can be given an intuitive interpretation as follows. To begin with, the cost of investing in the risky asset is the loss of future profits in
the event of insolvency. This is counter-balanced by the benefit of shifting the cost of insolvency (paying depositors and the insurance premium) to the FDIC, and by the more attractive return on the risky asset ($E[y(u)] > x$). At low capital levels, this trade-off yields a corner solution (maximal risk-taking), since undercapitalized banks are the most likely to benefit from "cost-shifting," and since risk-taking may be their best way to recapitalize. It is worth noting that this result provides a formal rationale for the prompt corrective action provisions of the FDICIA, which require progressively more strict regulatory intervention as a bank's capital declines.\footnote{The prompt corrective action regulations define five capital zones. Banks in capital zone 1 (well capitalized) face no mandatory restrictions on activities. Those in zone 2 (adequately capitalized) are subject to increased regulatory scrutiny, including more frequent supervisory exams and prior FDIC approval to accept brokered deposits. Banks in capital zone 3 (undercapitalized) face several mandatory restrictions; for instance, these banks are prohibited from accepting brokered deposits and from paying dividends or management fees, and are subject to restrictions on asset growth. Those in zone 4 (significantly undercapitalized) are subject to the same restrictions as those in zone 3 plus several additional ones. Banks in capital zone 5 (critically undercapitalized) generally must be placed in receivership or conservatorship within 90 days after being assigned to this category.}

At higher capital levels, an additional factor affecting the marginal return to risk-taking comes into play—the possibility that the bank will experience a loss of capital without becoming insolvent. In this event, the bank does not get the cost-shifting benefit (at least not in the present period). This outcome reduces the bank's incentive to invest in the risky asset. The bank then switches from maximal risk-taking to much more limited risk-taking.\footnote{Insolvency risk is the only aspect of risk that is relevant at lower capital levels because the risky asset is characterized by a loss distribution that is highly skewed and has a long tail (it is leptokurtic). Thus, losses only rarely occur but tend to be large when they do occur.} This yields an interior solution (intermediate level of risk-taking), as in the model of DeGennaro et. al (1995).

Then, at still higher capital levels, incremental investment in the risky asset is associated with smaller incremental risk of becoming insolvent (the bank has a bigger "cushion"). In addition, the expected return to the risky asset is higher than the return to the safe asset. Therefore, the incentive to invest in the risky asset rises again. This generates a positive relationship between capital and risk at high levels of capital.
4.2. Results under alternative calibrations

To verify the robustness of this finding, we have experimented with various other parametrizations, generating the following results.

(a) Increasing the degree of risk in the calibration of the risky asset yields the solutions depicted in figures 2a and 3a. For the solutions depicted in figure 2a, we calibrated the risky asset with \( y_0 = 1.1175 \) and loan-loss distribution (ii) of table 1; figure 3a corresponds to \( y_0 = 1.122 \) and loan-loss distribution (iii) of table 1. The characteristic U-shaped pattern is still seen, although the range of maximal risk-taking among undercapitalized banks is wider.

(b) An increase in the promised return \( y_0 \) on the risky asset entails an expansion of the range of maximal risk-taking. This is due, naturally, to the greater attractiveness of the risky asset. For example, in the case of the solution depicted in figure 2a, when we reduce \( y_0 \) by 10 basis points to \( y_0 = 1.1165 \), the range of maximal risk-taking contracts from .038 to .034 at the upper-end.\(^{17}\)

(c) The range of maximal risk-taking tends to be quite responsive to \( r_0 \), or equivalently, to the spread between the deposit interest rate \( r_0 \) and the return \( x \) on the safe asset: A larger rate spread implies a smaller range of maximal risk-taking. For example, the solution depicted in figure 2a is replaced by that depicted in figure 4 when we reduce \( r_0 \) by 5 basis points, to 1.0495.\(^{18}\) When \( r_0 \) is further reduced to 1.0490, the range of maximal risk-taking disappears entirely (and is replaced by minimal risk-taking). The logic behind this is that as the spread between \( r_0 \) and \( x \) increases, banks become more profitable and, accordingly, will try to remain in business by taking less risk.

(d) The loan-loss distributions considered thus far are based on a market risk (correlation) parameter \( b^2 = 0.25 \). We experimented with loan-loss distributions based on alternative specifications of the correlation parameter such as that shown

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\(^{17}\)In the absence of a premium surcharge on undercapitalized banks, the solution collapses to minimal risk-taking at all capital levels for \( y_0 \leq 1.1145 \) (\( E[y(u)] \leq 1.058 \)). The solution converges to maximal risk-taking at all capital levels for \( y_0 \geq 1.1225 \) (\( E[y(u)] \geq 1.065 \)). Calibrations of \( y_0 \) that yield minimal (or maximal) risk-taking at all capital levels would seem inconsistent with an equilibrium in the asset market. For instance, with all banks investing solely in the safe asset, one would expect the contractual interest rate on the risky asset, \( y_0 \), to rise.

\(^{18}\)Likewise, when we increase \( r_0 \) by 5 basis points, the range of maximal risk-taking expands up to capital level .048.
in column (iv) of table 1. We found that a decrease in $b^2$ yields a contraction of the range of maximal risk-taking among undercapitalized banks; compare figure 3a to figure 5. Moreover, a decrease in $b^2$ is associated with increased risk-taking by well-capitalized banks: investment in the risky asset increases from 0.35 to 0.40.\footnote{In some cases, in the absence of a premium surcharge, the range of maximal risk-taking disappears entirely (and instead, a range of minimal risk-taking occurs) when $b^2$ is reduced.}

These effects of changing the correlation parameter can be explained as follows. With a decrease in $b^2$, the largest potential losses occur with reduced probability. This tends to encourage risk-taking by well-capitalized banks. Concomitantly, the mean loss declines and the median loss increases. This reduces the cost-shifting subsidy and tends to deter risk-taking by undercapitalized banks.

5. Solutions to the Model Under Varying Regulatory Parameters

We now present solutions for alternative values of the regulatory parameters, reflecting the effects of regulatory reforms.

5.1. Varying the insurance premium

We begin the analysis with the impact of capital-based deposit insurance premia. As noted, in 1993 the FDIC implemented "risk-related" pricing of deposit insurance, whereby banks with lower capital ratios and those assessed by examiners to be more risky pay higher premia.\footnote{The FDIC bases insurance premia on bank capital-ratios and on supervisory risk-ratings that reflect examiner evaluations of bank earnings, asset quality, liquidity, and management. Essentially, our analysis assumes that premium assessments are based primarily on \textit{ex-post} indicators, whereby banks undertaking increased risk are assessed higher premia only in the event that their risk-taking results in losses.} When first introduced, the premium differential between the lowest and highest risk-categories was 8 basis points, with an average differential of about 5 basis points.\footnote{There is now a wider differential as a result of reductions in insurance premia for institutions in the lowest risk-category.}

The solution depicted in figure 1b assumes a premium differential of 5 basis points ($\pi = 0.0005$), where the calibration is otherwise identical to that in figure 1a. A comparison of figures 1a and 1b indicates that the primary effect of an insurance premium differential is to \textit{aggravate} the moral hazard problem...
among severely undercapitalized banks: Banks at capital levels 0.010 through 0.026 switch from very limited risk-taking (\(R^* \leq 0.15\)) to holding only risky assets (\(R^* = 1.0\)). An increase in the premium differential \(\pi\) has no impact, however, on the behavior a well-capitalized bank (capital level 0.06). In other words, contrary to what one might expect, an insurance premium differential exhibits no deterrent effect on risk-taking among well-capitalized banks.

Moreover, an increase in the premium differential has a substantial impact on failure probabilities among under-capitalized banks, which are computed from 2.3 given the solution to the model. For instance, in the case depicted in figures 1a and 1b, when banks at capital level 0.010 switch to maximal risk-taking as a result of the premium surcharge, their probability of failing after one period increases from 0.0067 to 0.2291. For banks at capital level 0.026, the probability of failing after one period increases from 0.0090 to 0.1969.

Increasing the degree of risk in the calibration of the risky asset does not qualitatively affect these results. The solution depicted in figures 2b and 3b assume a premium differential of 5 basis points (\(\pi = 0.0005\)), where the calibrations are otherwise identical to those in figures 2a and 3a, respectively. We again find that the effect of an insurance premium-differential is to widen the range of maximal risk-taking, although this effect is now somewhat less dramatic. Again, the insurance premium-differential has no deterrent effect on risk-taking among well-capitalized banks.

An insurance premium-surcharge on undercapitalized banks is associated with an expanded range of maximal risk-taking because it boosts their risk-taking incentive: it increases the bank’s payment due per dollar of deposits (and, therefore, increases the cost-shifting “subsidy”). Further, the premium surcharge reduces the expected present value of future profits, thus lowering the opportunity cost of risk-taking.

Although capital-based premia were intended to be a deterrent to risk-taking, in our model they fail to do so for well-capitalized banks. This result can be understood as follows. The insurance-premium surcharge reduces the net present value of an undercapitalized bank relative to that of a well-capitalized bank and this, in principle, should deter risk-taking. However, given the values of the parameters in our data set (e.g., \(x, G, \rho_0\)), this effect turns out to be weak. Consequently, the quantitative effect is negligible, which implies no change in the behavior of well-capitalized banks.
5.2. The Impact of a higher capital requirement

Now consider the effect of increases in the regulatory standard on a bank's risk-taking and its probability of failing after one period. First, consider the effects on well-capitalized banks (banks that maintain compliance with the capital standard as it changes.) Table 2 depicts the results obtained when parameters other than the capital standard are calibrated as in figure 2a. A small increase in the capital standard above 0.06 does not lead to increased risk-taking among well-capitalized banks, and thus their probability of failure is reduced. Further increases in the capital requirement, however, lead to increased risk-taking, which offsets the favorable effect of higher capital on the probability of failure. For instance, a bank complying with an increase in the regulatory capital requirement from $C^* = 0.06$ to $C^* = 0.08$ increases its investment in the risky asset from 0.35 to 0.45, which leaves its probability of failure unchanged at 0.098. Therefore, more-than-marginal changes in the capital requirement affect the behavior of well-capitalized banks (in the direction of more risk-taking), but have little effect on the probability of failing after one period.

The effect of an increase in the capital standard on the entire solution are shown in figure 6a. This solution replaces that depicted in figure 2a when the capital standard is raised to $C^* = 0.08$ (with other parameters held constant.) Note that the increase in risk-taking that occurs when a bank moves from the ex-ante well-capitalized level ($C^* = 0.06$ in figure 2a) to the ex-post well-capitalized level ($C^* = 0.08$ in figure 6a) is consistent with the overall U-shape pattern of the solutions, whereby risk-taking increases with capitalization. We also observe that the range of maximal risk-taking among undercapitalized banks tends to expand as the capital standard is raised. Further, introduction of a premium surcharge has the same effect under an 8-percent standard as under a 6-percent standard—it expands the range of maximal risk-taking, as shown in figure 6b.

5.3. Results under alternative calibrations

Experimentation with alternative calibrations indicates that the qualitative effects of a premium differential or a higher capital standard are robust. Changing the calibration primarily affects the size of the range of maximal risk-taking among undercapitalized banks.

An increase in the premium surcharge levied on undercapitalized banks generally results in an expansion of the range of maximal risk-taking. This generally is the case unless this range already encompasses all capital levels below the reg-
ulatory standard.

6. Extensions of the Basic Model

6.1. Endogenizing the bank’s capital target

Thus far, we have assumed that the bank will hold no more capital than is required by regulation. We now relax this assumption. Suppose that each period, the bank chooses a target level of capital $K$ for the subsequent period, subject to the regulatory standard $C^*$. If the bank desires to begin the next period with a capital “cushion” above the regulatory standard, it will choose $K > C^*$. In that case, $K$ serves as a self-imposed capital requirement, replacing $C^*$. Thus, when $z(C, R, u) \geq \max[K, C^*]$, stockholders earn $z(C, R, u) - \max[K, C^*]$. When $0 < z(C, R, u) < \max[K, C^*]$, stockholders earn zero, and the entire net return that period goes toward next period’s capital.

To represent this scenario, we replace $C^*$ with $K$ in 2.4 and on the right hand side of 2.5. The bank makes now two choices: optimal investment in the risky asset, $R^*(C)$, and optimal capital target, $K^*(C)$, both of which depend on the state variable $C$. They are determined along with the value function $V(C)$ as the solution to the dynamic programming problem:

$$V(C) = \max_{R,K} \left\{ \int_0^{u_B} [z(C, R, u) - K] g(u) du + \delta V(K) \int_0^{u_B} g(u) du \right\}$$

subject to the constraint:

$$K \geq C^*.$$ 

where $u_B$ is now defined by:

$$u_B = u_A - K/y_0 R.$$ 

A bank’s optimal capital level in the absence of regulation may be obtained by solving the above model with $C^* = 0$. For each of the calibrations used earlier, we find that the bank would choose to hold very little capital in the absence of a regulatory requirement. For example, with $C^* = 0$ and otherwise
using the calibration from figure 2a, we find that the bank will maintain a capital
target \( K^*(C) = 0.004 \) for any \( C \). This confirms our implicit assumption that the
regulatory requirement is binding.

Although a bank would hold very little capital in the absence of regulation,
when subject to a regulatory standard the bank may desire to maintain more
than the required amount of capital. This follows from the fact that the bank’s
objective function is non-concave since that the marginal cost of holding capital
above the required level may decline as the capital requirement is raised. To
evaluate banks’ incentive to hold more capital, and whether our previous results
depend on the presumed absence of such an incentive, we (re-) solved the above
model for each of the calibrations used earlier. Essentially the same results are
obtained. We find that the optimal capital target \( K^*(C) \) generally coincides with
\( C^* \) and when it does not, it only exceeds \( C^* \) by a small amount. For example,
solving the model with the calibration from figure 1a shows that the bank will
maintain a capital target \( K^*(C) = 0.062 \), implying a small cushion over the capital
standard \( C^* = 0.060 \). At capital level 0.062, the bank continues to invest 0.40 in
the risky asset.

6.2. Risk-based capital requirements

In 1988, federal regulatory agencies adopted “risk-based” capital standards that,
for many banks, were more stringent than the prior standards (Avery and Berger
1991). Under the prior standards, primary capital had to be at least 6 percent
of total balance-sheet assets or the bank would face supervisory action. Under
the risk-based standards, differing weights are assigned to various categories of
bank assets (e.g. home mortgage loans, treasury bills, commercial loans) prior to
summing the assets, to reflect differences in credit risk. The regulations adopted in
1988 required that total capital be at least 8 percent of risk-weighted assets (where
loan-loss reserves were no longer to be fully included as a component of measured
capital). In addition, these regulations set standards for tier-one capital (a more
restrictive definition of capital) in relation to risk-weighted assets and for tier-one

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22 Primary capital was defined to include equity, loan loss reserves, preferred stock, and various kinds of debentures; see Wall (1989) for details.

23 The 8 percent risk-based standard adopted in 1988 was a minimum capital standard for banks. The regulations provided and continue to provide substantial leeway for regulators to set more stringent standards for all but the strongest institutions (Peek and Rosengren 1995a,b).
capital in relation to total assets.\textsuperscript{24,25}

The above extension of the basic model provides a ready framework for examining the effects of a risk-based capital standard: a risk-based standard may be represented by a constraint on permissible combinations of $K$ and $R$ in place of 6.2. We shall assume that this constraint takes the form:

$$K \geq C^* + c(R - R_0)(0.002/0.05), \quad (6.4)$$

where $C^*$, $R_0$ and $c$ are parameters determining the stringency of the requirement. (We multiply by $0.002/0.05$ because 0.002 is one unit of capital and 0.05 is one unit of the risky asset in the discretized version of the model.)

$R_0$ is the threshold level of risk-taking beyond which required capital increases with the proportion invested in the risky asset, and $c$ is the rate at which required capital increases with the proportion invested in the risky asset once this threshold is exceeded. At levels of risk-taking at or below $R_0$, the capital requirement is fixed at $C^*$. The capital requirement becomes more stringent with an increase in $C^*$ or $c$, or with a smaller $R_0$.

We examine the effect of a risk-based standard by solving the dynamic programming problem 6.1 subject to the constraint 6.4. The solution yields a unique capital level $C^0$ such that $K^*(C^0) = C^0$; i.e., at $C^0$, the bank converges to its capital target (and, a fortiori, meets the regulatory standard.) The bank seeks to maintain this level of capitalization over the long run—it is analogous to the well-capitalized level in the original model. We shall refer to $C^0$ as the bank’s “long-run capital target.”

The solutions for various calibrations of the model suggest that increasing the stringency of a risk-based standard has an analogous effect to imposing a higher flat standard (raising $C^*$ with $c = 0$). Indeed, if a flat standard ($c = 0$) is replaced with an effectively more stringent, risk-based standard, an ex-ante well-capitalized level in relation to risk-weighted assets and tier-one capital in relation to total assets, respectively.

\textsuperscript{24}In addition, the 1988 regulation required banks to hold some capital against off-balance sheet activities. Banks were directed to comply with the new standards by 1992. See Wall (1989) for details.

\textsuperscript{25}As noted previously (see footnote 22), FDICIA introduced a distinction between well capitalized and adequately capitalized (along with three distinct categories of undercapitalized), whereby the latter are subject to closer regulatory scrutiny. Thus, for instance, under current regulations, total capital has to be at least 8 percent of risk-weighted assets for a bank to be considered adequately capitalized, and at least 10 percent of risk-weighted assets for a bank to be considered well capitalized. In addition, there are tests for well capitalized vs. adequately capitalized that are based on tier-one capital in relation to risk-weighted assets and tier-one capital in relation to total assets, respectively.
capitalized bank moves in the direction of increasing both its capital and its risk as it moves toward compliance (toward its long-run capital target.) Banks do not choose to achieve compliance by taking on less portfolio risk than they otherwise would prefer at their long-run target capital level. This implication of the model is consistent with empirical findings, as noted earlier.

Moreover, the solutions given a risk-based standard exhibit the same overall shape as those depicted earlier. Also, under a risk-based standard as under a flat standard, the primary effect of an insurance-premium surcharge on undercapitalized banks is a widening of the range of maximal risk-taking.²⁶

For example, consider the impact of alternative capital standards using the calibration from figure 1a. Table 3 shows, for various flat and risk-based standards, the bank's long-run capital target as implied by the solution to the model. The corresponding portfolio choice also is indicated. The first row of the table shows that, given a flat capital requirement, $C^* = 0.06$, the bank's long-run capital target is $C = 0.062$ (it will hold a "capital cushion", as noted previously), at which point it will invest 0.40 in the risky asset. Each risk-based standard entails the same or higher capital and equal or greater risk compared to the flat standard $C^* = 0.06$, with just one exception. The exceptional case ($c = 4$ and $R_0 = 7$) is where the standard dictates an extremely steep trade-off between capital and risk. Moreover, in most cases, there is an equivalent flat standard for each risk-based standard.

In sum, this analysis suggests that the impact of a risk-based capital requirement on bank risk-taking is not much different from that of a flat capital requirement. Of course, the analysis should be viewed as preliminary, because risk-based capital rules—in reality—are more complex than the model represents. In particular, as mentioned above, several broad risk-categories of assets are defined for

²⁶The solution under a risk-based standard does not necessarily coincide with the solution under a comparable flat standard, but differences are minor. (By "comparable", we mean a flat standard that coincides with the long-run target capital level under the risk-based standard.) In some cases, to maintain compliance with a risk-based standard and avoid paying a premium surcharge when its capital has fallen to just below the long-run target level, a bank will take on less risk than it would when subject to a comparable flat standard.

Further, in some cases, the range of maximal risk-taking among undercapitalized banks is slightly smaller under the risk-based standard. Such cases arise, however, only if the risk-based standard is assumed to apply at lower capital levels. It may be more realistic to assume that banks that are significantly undercapitalized become subject to a flat standard. Evidence suggests that in the case of significantly undercapitalized banks, examiners have tended to focus on capital in relation to total assets (the leverage ratio) rather than risk-weighted assets (Prek and Rosengren 1995b,c.)
purposes of computing regulatory capital ratios. Thus, for fuller consideration
of the impact of risk-based capital requirements, a model with more than two
risk-categories of assets would be appropriate.

6.3. Access to External Capital

Next, we generalize the basic model by granting the bank access to external capital. Specifically, we assume that, in any period, the bank chooses a capital-infusion policy $I$ as well as a portfolio allocation $R$. The capital-infusion policy determines the source of funds for next period's capital. If $I = 1$, the bank's owners provide capital "infusion" (e.g., the owners proceed to liquidate some of their outside assets). If $I = 0$, the bank uses internal financing as in the basic model. Any capital infusion, however, is conditional on the bank remaining solvent during the period.

For simplicity, we assume that the amount of any infusion brings the bank's capital back to the minimum capital requirement, $C^*$. A capital-infusion is treated as a negative dividend; that is, it is subtracted from current period earnings. Thus, in the case $I = 1$, the current period dividend equals $z(C, R, u) - C^*$ and the bank begins the next period with capital $C^*$, provided the bank remains solvent. Hence, we replace $C^*$ with $(1 - I)C^*$ in 2.4. The bank's optimal investment in the risky asset and the optimal capital-policy both depend on the state variable $C$ and are determined along with the value function $V(C)$ as the solution to the dynamic programming problem 2.6, where $u_B$ is now defined by:

$$\text{UB} = U_0 - (1 - I)C^*/y_0 R. \quad (6.5)$$

Solving this model for each of the calibrations used earlier, we find that the portfolio risk-choices $R^*$ implied by the solutions are identical or nearly identical to those of the original model. Thus, the ability to raise capital externally has little impact on decisions pertaining to portfolio risk.

On the other hand, this model yields new implications regarding the impact of the regulatory parameters on a bank's capital-policy. First, a premium surcharge on undercapitalized banks provides an added incentive for those banks to raise capital externally. This is not altogether surprising, since the premium surcharge

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27 There is no loss of generality here. When this assumption is relaxed to allow smaller capital infusions, we find that optimal capital policies involve either an infusion of the entire amount $C^*$ or no capital infusion.

28 Note that we can no longer set $z(C, R, u) - C^* = y_0 R(u_B - u)$ in 2.5 and obtain 2.6. Instead, we set $z(C, R, u) - C^* = y_0 R(u_B - u) - IC^*$. 
represents a cost of remaining undercapitalized. This effect is illustrated in figure 7a and 7b, which depict the solutions obtained using the calibrations from figures 2a and 2b. The presence of an asterisk indicates that the optimal capital-policy is $I(C) = 1$, a capital infusion. The range of capital levels for which capital infusions are optimal is substantially larger in figure 7b, reflecting the impact of the premium surcharge. One may also note that the solutions with respect to portfolio risk are the same as in figures 2a and 2b.

Thus, one effect of a premium differential is a wider range of maximal risk-taking, and another is that undercapitalized banks are apt to seek infusions of capital from outside sources. Provided the bank does not become insolvent before this additional capital is forthcoming, the infusion of capital will have a risk-mitigating effect. Due to the capital infusion, the bank returns to the well-capitalized level; hence, the probability of failure decreases. Therefore, if banks that have become undercapitalized have ready access to external sources of capital (and that's a big "if"), the effect of a premium differential on banks' capital policies has the potential to mitigate the increase in risk due to their portfolio responses.

The effect of a higher capital requirement on banks' capital-policies is less sanguine. We find that, in general, undercapitalized banks become less inclined to seek infusions of capital from external sources when the capital requirement is raised.

6.4. Suggestions for future research

Several additional issues may be of interest for future research, where the analysis would require further development of the basic model. Two such issues have already been noted. First, would a risk-based capital requirement have different implications in a model with multiple risk-categories of assets? In particular, would banks be more inclined to achieve compliance by adjusting their risk-taking rather than their capital? Second, the model could be extended by allowing banks to adjust their asset size, and this could be applied to questions pertaining to capital-induced "credit crunches".

Another possible topic for future research is the impact of an increased capital-requirement in an industry-equilibrium context. The above analysis indicates that higher capital-requirements are accompanied by increased risk-taking. This suggests that the favorable effect on bank safety and soundness of increased capital are offset by banks' portfolio-adjustments. However, the net result has not been
quantified. One possible approach would be to posit an entry process (such as an assumption that one bank enters for each that fails), and then to solve for the steady-state distribution of bank capital levels and the steady-state failure rates under alternative regulatory assumptions. This remains a topic for future research.

7. Concluding Remarks

This paper sets up a model of the banking firm, calibrates it using realistic parameter values, and applies it to analyze the impacts on bank risk-taking of increased capital standards, capital-based premia differentials, and risk-based capital requirements. A bank is assumed to operate in a multi-period setting; the bank's capital and its portfolio choices may fluctuate over time depending on the realized returns on loans. Thus, we consider the dynamics of bank portfolio choices and the behavior of well-capitalized as well as undercapitalized banks.

A general implication of the model is that the amount of risk a bank undertakes depends on the bank's current capital position, where the relationship is roughly U-shaped. In particular, a severely under-capitalized bank tends to take on maximal risk. This result suggests that moral hazard is a serious problem among banks near to insolvency; thus, it provides a formal rationale for the prompt corrective action provisions of FDICIA. As capital rises to a more modestly undercapitalized level, maximal risk-taking typically is replaced by much more limited risk-taking. Then, as capital rises to the regulatory standard, a bank tends to take on more risk.

In the case of a flat capital requirement, if the capital requirement is increased, then an ex-ante well capitalized bank will take on additional portfolio risk as it adds capital to comply with the new standard. This is consistent with the overall U-shape of the solution, whereby beyond the lowest capital levels, risk-taking tends to increase with capitalization.

The model has striking implications with respect to the impact of capital-based deposit insurance premia. A primary intent of the Congress in mandating risk-related pricing of deposit insurance was to create a disincentive against banks engaging in risky activities. In our model, however, the premium surcharge has no appreciable impact on the behavior of a well-capitalized bank. Moreover, a
premium surcharge aggravates the moral hazard problem among undercapitalized banks, as reflected in a substantial widening of the capital range over which maximal risk-taking occurs.

On the other hand, a premium surcharge may encourage undercapitalized banks to seek infusions of capital from outside sources. This effect of a premium differential on banks' capital-policies has the potential to mitigate the increase in risk due to their portfolio responses.

The model suggests that an increased risk-based capital standard is analogous to a higher flat standard. That is, an ex-ante well-capitalized bank generally will respond to the increased standard by raising additional capital and taking on more portfolio risk.

Although significantly undercapitalized banks in our model respond to capital-based insurance premia by increasing the riskiness of their portfolios, it should be noted that the prompt corrective action provisions of FDICIA are intended to promote effective regulatory responses to such behavior. Nevertheless, our model suggests that some of the recent regulatory initiatives could have some unintended consequences.
Table 1  
Calibrations of the Loan Distribution

<table>
<thead>
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<th>Percentile</th>
<th>G(u)</th>
<th>(i) $d=0.080$ $b^2=0.25$</th>
<th>(ii) $d=0.875$ $b^2=0.25$</th>
<th>(iii) $d=0.093$ $b^2=0.25$</th>
<th>(iv) $d=0.093$ $b^2=0.29$</th>
</tr>
</thead>
<tbody>
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<td>$u_1$</td>
<td>0.01</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
</tr>
<tr>
<td>$u_2$</td>
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<td>0.0002</td>
<td>0.0002</td>
<td>0.0002</td>
<td>0.0002</td>
</tr>
<tr>
<td>$u_3$</td>
<td>0.25</td>
<td>0.007</td>
<td>0.007</td>
<td>0.008</td>
<td>0.011</td>
</tr>
<tr>
<td>$u_4$</td>
<td>0.50</td>
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<td>0.022</td>
<td>0.025</td>
<td>0.027</td>
</tr>
<tr>
<td>$u_5$</td>
<td>0.75</td>
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<td>0.066</td>
<td>0.066</td>
</tr>
<tr>
<td>$u_6$</td>
<td>0.95</td>
<td>0.15</td>
<td>0.17</td>
<td>0.18</td>
<td>0.16</td>
</tr>
<tr>
<td>$u_7$</td>
<td>0.99</td>
<td>0.21</td>
<td>0.23</td>
<td>0.24</td>
<td>0.22</td>
</tr>
<tr>
<td>$u_8$</td>
<td>1.00</td>
<td>0.83</td>
<td>0.70</td>
<td>0.65</td>
<td>0.82</td>
</tr>
<tr>
<td>$\text{E}(u)$</td>
<td></td>
<td>0.047</td>
<td>0.051</td>
<td>0.054</td>
<td>0.053</td>
</tr>
</tbody>
</table>
Table 2
Risk-Taking and Probability of Failure for Well-Capitalized Banks

<table>
<thead>
<tr>
<th>Required Capital Level C*</th>
<th>Portfolio R*(C*)</th>
<th>Risk</th>
<th>Probability of End-of-Period Failure u_A</th>
</tr>
</thead>
<tbody>
<tr>
<td>.060</td>
<td>0.35</td>
<td>0</td>
<td>0.0098</td>
</tr>
<tr>
<td>.064</td>
<td>0.35</td>
<td>0</td>
<td>0.0096</td>
</tr>
<tr>
<td>.068</td>
<td>0.40</td>
<td>0</td>
<td>0.0099</td>
</tr>
<tr>
<td>.072</td>
<td>0.40</td>
<td>0</td>
<td>0.0097</td>
</tr>
<tr>
<td>.076</td>
<td>0.45</td>
<td>0</td>
<td>0.0100</td>
</tr>
<tr>
<td>.080</td>
<td>0.45</td>
<td>0</td>
<td>0.0098</td>
</tr>
</tbody>
</table>

Calibration: $r_0 = 1.050$, $x = 1.1175$, $d = 0.0875$, $b^2 = 0.25$, $E[u] = 0.051$, $E[y(u)] = 1.061$. 
### Table 3

**The Impact of a Risk-Based Capital Requirement**

<table>
<thead>
<tr>
<th>Capital Standard</th>
<th>Long-Run Capital Target C₀</th>
<th>Portfolio R*(C₀)</th>
<th>Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>flat: C*=0.060</td>
<td>0.062</td>
<td>0.40</td>
<td></td>
</tr>
<tr>
<td>flat: C*=0.062</td>
<td>0.062</td>
<td>0.40</td>
<td></td>
</tr>
<tr>
<td>flat: C*=0.064</td>
<td>0.064</td>
<td>0.40</td>
<td></td>
</tr>
<tr>
<td>flat: C*=0.066</td>
<td>0.070</td>
<td>0.40</td>
<td></td>
</tr>
<tr>
<td>flat: C*=0.070</td>
<td>0.070</td>
<td>0.45</td>
<td></td>
</tr>
<tr>
<td>flat: C*=0.072</td>
<td>0.072</td>
<td>0.45</td>
<td></td>
</tr>
<tr>
<td>flat: C*=0.080</td>
<td>0.080</td>
<td>0.50</td>
<td></td>
</tr>
<tr>
<td>risk-based: C*-0.06; c=1; R₀=0.020</td>
<td>0.070</td>
<td>0.45</td>
<td></td>
</tr>
<tr>
<td>risk-based: C*-0.06; c=1; R₀=0.030</td>
<td>0.064</td>
<td>0.40</td>
<td></td>
</tr>
<tr>
<td>risk-based: C*-0.06; c=1; R₀=0.035</td>
<td>0.062</td>
<td>0.40</td>
<td></td>
</tr>
<tr>
<td>risk-based: C*-0.06; c=2; R₀=0.025</td>
<td>0.080</td>
<td>0.50</td>
<td></td>
</tr>
<tr>
<td>risk-based: C*-0.06; c=2; R₀=0.030</td>
<td>0.072</td>
<td>0.45</td>
<td></td>
</tr>
<tr>
<td>risk-based: C*-0.06; c=2; R₀=0.035</td>
<td>0.064</td>
<td>0.40</td>
<td></td>
</tr>
<tr>
<td>risk-based: C*-0.06; c=3; R₀=0.035</td>
<td>0.066</td>
<td>0.40</td>
<td></td>
</tr>
<tr>
<td>risk-based: C*-0.06; c=4; R₀=0.035</td>
<td>0.060</td>
<td>0.35</td>
<td></td>
</tr>
</tbody>
</table>

Calibration: \( r_0 = 1.050, x = 1060, y_0 = 1.113, d = 0.080, \delta^2 = 0.25, E[u] = 0.047, E[y(u)] = 1.062. \)
References


Calibration: $C^* = 0.06; g_0 = 1.050; r = 1.06; \gamma_g = 1.113; d = 0.08; b^2 = 0.25; E[u] = 0.047; E[y(u)] = 1.062$
Figure 2
Impact of a Premium Differential

Calibration: $C^* = 0.06; \rho = 1.050; r = 1.06; \gamma_0 = 1.1175; d = 0.0875; b^2 = 0.25; E[u] = 0.051; E[y(u)] = 1.061
Figure 3
Impact of a Premium Differential

Calibration: $C^* = 0.06; \rho_0 = 1.050; r = 1.06; y_0 = 1.122; d = 0.093; b^2 = 0.25; E[u] = 0.054; E[y(u)] = 1.061$
Figure 4

The Solution Under Alternative Calibrations

Calibration: $C^* = 0.06; \rho_0 = 1.0495; r = 1.06; y_0 = 1.1175; d = 0.0875; b^2 = 0.25; E[u] = 0.051; E[y(u)] = 1.061$
Figure 5
The Solution Under Alternative Calibrations

Calibration: $C^*=0.06; \rho_p = 1.050; r=1.06; y_0 = 1.122; d=0.093; b^2 = 0.20; E[u]=0.053; E[y(u)]=1.063$
Figure 6
Impact of an Increased Capital Standard

Calibration: $C^* = 0.08; \rho_0 = 1.050; r = 1.06; y_0 = 1.1175; d = 0.0875; b^2 = 0.25; E[u] = 0.051; E[y(u)] = 1.061$
Figure 7
Impact of a Premium Differential

Calibration: $C^* = 0.06; \rho = 1.050; r = 1.06; \gamma = 1.1175; d = 0.0875; b^2 = 0.25; E[u] = 0.054; E[y(u)] = 1.061$