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An Equilibrium Model of Firm Growth and Industry Dynamics

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Abstract

We develop a model of firm size in which firms are unable to access as many consumers as they want. Newly arrived consumers match randomly with firms. Subsequently, consumers must pay search costs to be able to switch firms. This cost promotes an inertial tendency for consumers to remain with the firm they currently buy from. Consequently, established firms enjoy a proprietary relationship with respect to their old customers while new entrants only have access to first time consumers, who are as yet unattached to any firm. As time goes on, firms acquire successive generations of new consumers, and their stock of loyal customers grows gradually. Thus, more senior firms command higher market shares.

We construct an industry model based on this hypothesis and show that the aggregate implications of the model are consistent with empirical facts about industry dynamics (e.g., Dunne, Roberts and Samuelson, (1988, 1989) or Davis and Haltiwanger (1992)): Larger and older firms are less likely to exit than younger and smaller firms. In our model this results from the fact that the option value of remaining operative in the aftermath of a high cost-shock is greater for an older firm than for a younger firm because the value of a cost turnaround is greater for the former (which has already accumulated a large customer-base) than for the latter (which has yet to do so). This enables older (and larger) firms to survive adverse cost-shocks

which force the exit of younger (and smaller) firms. For similar reasons, R&D expenditures are larger for larger firms, as per the empirical findings surveyed by Cohen and Lewis (1989).

1. Introduction

Firms, like people, are born small. Even a McDonalds or a Microsoft are very small at their inception and require the passage of time to achieve their spectacular size. This paper develops an equilibrium model of firm growth, explores its aggregate implications and applies it to important empirical and theoretical issues in industrial organization.

We consider markets in which it is costly for consumers who have previously purchased from one firm to switch to a competing firm, even when the two firms' products are functionally identical (Klemperer, 1987). The "brand loyalty" that results from such switching costs gives firms market power over their repeat purchasers and implies that a firm's current market share determines future profit. Obvious reasons for switching costs include the transaction cost of changing a supplier (as in the case of closing an account with one bank and opening a new account with a competitor), the cost of learning to use a new brand (as in the case of competing computer operating systems or software), uncertainty about the quality of untested brands, psychological costs of breaking a habit and information costs of learning the prices or existence of competing brands (see Klemperer (1995) for additional reasons and examples).

Any of these examples of switching costs would drive the type of growth dynamics our model describes. For concreteness, however, we focus on the latter, assuming that consumers must incur search costs to learn about the prices of "new sellers" with which they have not previously transacted. New consumers and firms continuously enter the market. A newly arrived consumer is randomly matched with a firm. Subsequently, the cost of searching for a new firm and the prices that firms charge (in equilibrium) lock the consumer in with her original firm. Thus established firms enjoy a proprietary relationship with repeat purchasers and compete with new entrants for first time consumers, who are as yet unattached to any firm. As it acquires successive generations of new consumers, a firm's stock of repeat purchasers grows. Thus, the longer its tenure in the market, the greater is the firm's market share.

We explore the aggregate implications of this model and, especially, its ability to account for stylized facts about industry dynamics. An extensive and impressive empirical literature (e.g., Dunne, Roberts and Samuelson, (1988, 1989)) finds the extent of firm turnover to be quite striking even in mature and narrowly-defined industries. Firm size and age have been identified among the determinants most strongly associated with this turnover: larger and older firms are less likely to exit than younger and smaller firms. Our model of firm growth accounts for these stylized facts in a very natural way. In our setting, the effect of an idiosyncratic productivity shock on a firm's exit decision depends negatively on its age. The reason for this is that the value of remaining operative in the wake of an adverse cost-shock is determined by the *option value* of a cost turnaround, which is higher for older firms (which have already accumulated a large customer base) than for younger firms (which have yet to do so). Therefore, older (and larger) firms can survive adverse cost-shocks which force the exit of younger (and smaller) firms.

Our model can also account for the existence and properties of price dispersion. Roberts and Supina (1996) have recently documented the following facts: (i) The same product is sold at different prices by different firms. (ii) This price dispersion is persistent over time; low-priced firms tend to remain low-priced and vice versa.[1] And, (iii) prices are contemporaneously correlated with marginal costs, firm sizes and profitability; low-priced firms tend to have lower marginal costs, a larger volume of sales and higher profits. All these are features of the equilibrium we construct. Finally, the model can shed light on the positive correlation between size and R&D expenditures, as per the findings of Cohen and Lewis (1989).

We proceed as follows. To simplify exposition and assimilation, we first provide a simplified version of the model in section 2. In this version, all firms, irrespective of production cost or market share, charge identical prices, and consumers do not actively search in equilibrium. This paves the way for, and is followed by, the more comprehensive model of section 4 in which equilibrium prices

[1]This feature should be distinguished from the type of price dispersion documented by Villas-Boas (1995) for the case of retail stores. In his study, the rank-order of firms in the price dispersion are highly variable and are based on the fact that retail stores run sales and the identity of the lowest-priced seller (as well as the value of the lowest price) varies a great deal from period to period. The theoretical model which Villas-Boas findings confirm is Varian (1980).

and firms' size-distribution are derived simultaneously.

2. A Model With Unit Demand

Time is discrete. There is a continuum of firms producing an identical product. At each period, b identical new consumers—or "newborns"—enter the market. At each period of her life, a consumer demands one indivisible unit of the product if its price is less than or equal to $\overline{p}$, and zero otherwise[2]. A consumer is subject to a constant death probability of d at each period.

In the first period of her life, a consumer is costlessly and randomly matched with a firm. Subsequently, she can costlessly return to the firm from which she bought in the previous period. Switching to a new firm, however, is costly. It is assumed that consumers know only the distribution of prices in the market, but not individual prices. To learn the price of, and buy from, a new firm costs $\sigma > 0$. We call σ the "search cost". We assume σ is the same for all consumers. Within any period, a consumer may sequentially sample the prices of an unlimited number of new firms at the constant cost of σ per firm.

A firm bears three types of costs. First, to enter the market, the firm must pay K, which is sunk subsequent to entry. Second, a fixed cost of F must be borne to be operative at any period. This cost can be saved by exiting the market at the beginning of the period; re-entry, however, requires paying K once more. Third, to produce, the firm must pay a constant per-unit cost, c. c is determined stochastically, and can assume one of three values: c_L, c_M and c_H, $c_H > c_M > c_L$, where $c_H > \overline{p} \geq c_M$.

We assume that c varies from one period to the next in a Markovian fashion, and let γ_{ij} be the probability that a firm whose current cost is c_i turns c_j at the following period, i, $j = L$, M, H. We assume that $0 < \gamma_{Lj} < 1$, $0 < \gamma_{Mj} < 1$, $j = L$, M, H, and $\gamma_{HH} = 1$. That is, a c_L-firm may turn c_M or c_H and, similarly, a c_M-firm may turn c_L or c_H. But c_H is an absorbing state; once the cost escalates to c_H it never goes down again.

The assumption that $\gamma_{HH} = 1$ and that $c_H > \overline{p}$ ensures—in the simplest possible way—that continual entry and exit persist in the steady state, the empirically relevant case. It is further

[2]In section 4 we develop a model with a strictly decreasing demand funtion, and consumers who buy variable quantities, depending on the price.

assumed that the distribution $\gamma_{M\bullet}$ stochastically dominates $\gamma_{L\bullet}$, i.e., that $\gamma_{LL} > \gamma_{ML}$ and $\gamma_{LL} + \gamma_{LM} > \gamma_{ML} + \gamma_{MM}$. This ensures that a c_L-firm is more likely to have a lower cost in the next period than a c_M-firm. Therefore, a c_L-firm has a greater discounted value than a c_M-firm of the same size.

There is an infinite pool of potential entrants and free entry. We assume that potential entrants know their first-period marginal cost before entering (before paying K). Of course, future costs (beyond the first period), being random, are unpredictable. Free entry and an infinite pool of potential entrants assures that (i) an actual entrant's expected profit is zero, and (ii) all actual entrants are c_L-firms.

Firms are distinguished by current marginal costs and age—the time elapsed since entry. We refer to such pair as a **firm type**. Firm age is perfectly correlated with firm size (how many customers the firm has), as will become evident below. Therefore, firm type is equivalently described by cost and size.

We seek prices and flows of entry and exit which give rise to a **free-entry, steady-state equilibrium**. In such an equilibrium, there is a constant number of firms of each type, and a price associated with each type of firm, such that firms' individually optimal pricing and entry/exit decisions reproduce the steady-state distribution upon which they are based.

3. Analysis

3.1. Preliminaries

As is well known, the assumption that all consumers have unit demands and positive search costs ensures that the unique equilibrium price of each firm is the monopoly price, $\overline{p}$ (Diamond, 1971). This facilitates the analysis in two significant ways. First, it fixes the equilibrium price independently of the distribution of firm-types. Second, since switching is costly and $\overline{p}$ extracts all the surplus, a customer stays with its original firm (the firm with which it was first matched) as long as its price does not exceed $\overline{p}$. Once the original firm suffers a c_H cost-shock, it must exit the market (by the assumption that c_H is absorbing and that it exceeds $\overline{p}$), at which point all its customers must also exit the market. Thus consumers are not actively searching in equilibrium[3].

Since customers are "loyal" to their first firm, new entrants can sell only to newborns, who distribute randomly between all firms (regardless of age). Let x be the number of new customers that each firm acquires in each period, and let n be the equilibrium number of firms. Then $x = b/n$.

Let us fix an x (which we will eventually endogenize), and analyze the optimal entry/exit behavior of firms given this x. In each period a firm acquires x new customers and loses a fraction d of its old customers. Thus a firm of age t has a total of

$$z = x + x(1-d) + ... + x(1-d)^{t-1} = x\frac{1-(1-d)^t}{d} \tag{3.1}$$

customers. Equation (3.1) reflects the idea that a firm accumulates customers *gradually*: The older the firm is the more customers it has. We call z the "customer stock", the "customer base" or the "market share" of a firm. Clearly, customer stock and firm age are monotonically related. Hence, firm is equivalently specified by its current cost and customer stock (rather than its age.)

The fact that a firm accumulates customers only gradually implies that a firm's value—its discounted future profit—depends not only on its current cost but also its customer stock, z. Let $R_L(z)$ ($R_M(z)$) be the discounted value of a low- (medium-) cost firm with z customers. By exiting, the firm loses all its accumulated customers but saves the fixed cost, F, attaining a profit of zero[4]. If it does not exit, the firm pays F and sells z units in the current period. In addition, it retains $(1-d)z$ customers to the next period (those who don't die), and acquires its share of newborns, x. Therefore, its next-period value is $R_L(x + (1-d)z), R_M(x + (1-d)z)$, or zero, depending on whether its next-period cost is c_L, c_M, or c_H. Consequently, the value of a firm obeys the following recursive relationships:

$$R_L(z) = Max\left\{0, -F + z(\overline{p} - c_L) + \delta\left[\gamma_{LL}R_L(x + z(1-d)) + \gamma_{LM}R_M(x + z(1-d))\right]\right\}, \tag{3.2}$$

$$R_M(z) = Max\left\{0, -F + z(\overline{p} - c_M) + \delta\left[\gamma_{ML}R_L(x + z(1-d)) + \gamma_{MM}R_M(x + z(1-d))\right]\right\}. \tag{3.3}$$

By dynamic programming techniques (Stokey, Lucas and Prescott (1993), ch. 4):

[3] In section 4 we extend the model, allowing for active consumer search in equilibrium.

[4] The firm returns to the pool of potential entrants, whose value—in equilibrium—is zero.

(i) There exists a unique solution to equations (3.2) and (3.3).

(ii) The solution is continuous and increasing in z (the state variable), x and $\overline{p}$ (the parameters).

(iii) $R_L(z) > R_M(z)$ (this follows from stochastic dominance of $\gamma_{M\bullet}$ over $\gamma_{L\bullet}$).

Given property (ii) there exists a unique $x^* > 0$ so that

$$R_L(x^*) = K. \tag{3.4}$$

x^* is the number of customers that each firm must receive per period so that low-cost entrants break even on their initial investment, K.

3.2. Construction of a steady-state equilibrium

A steady-state equilibrium is constructed as follows. By free-entry all new entrants must earn zero expected profit. By equation (3.4), this is guaranteed as soon as the period flow of consumers that each firm receives is x^*. We now show that there exists a y^*, so that when y^* new firms enter each period, and when firms exit according to the optimal exit-rule, a steady-state is induced at which each firm acquires exactly x^* new consumers. Given this the market is in equilibrium: (i) Each firm is maximizing with respect to pricing and entry/exit decisions given the behavior of consumers and other firms. And, (ii) each consumer is minimizing his expenditures given the price distribution in the market.

Let us determine first the optimal exit rule: Given x^* there exists a $z^*(z^* > x^*)$ so that

$$R_M(z^*) = 0. \tag{3.5}$$

z^* has the property that a c_M-firm optimally exits if it has less than z^* customers and remains operative if it has more than z^* customers.

Given z^* there exists a critical age, call it t^*, so that only firms of age t^* or greater will have accumulated at least z^* customers. t^* is the minimum t with the property that

$$x^* \frac{1-(1-d)^{t+1}}{d} \geq z^*. \tag{3.6}$$

Thus a firm exits if and only if it becomes c_M less than t^* periods after entering the industry, or if it becomes c_H (at any date). (3.5) and (3.6) are alternative characterizations of optimal exit behavior. The next proposition shows how y^* is chosen so that, at the steady-state it induces, each active firm gets a flow of x^* new customers.

Proposition 3.1. *There exists an entry rate y^* so that, at the steady-state it induces, all new entrants earn zero profits, and all incumbent firms exit optimally.*

Proof. *Consider a constant flow of entry, y. If y new firms enter each period, then the number of c_L-firms of age less than t^*, is:*

$$n_L^y = y + y\gamma_{LL} + y\gamma_{LL}^2 + ... + y\gamma_{LL}^{t^*-1} = \frac{1-\gamma_{LL}^{t^*}}{1-\gamma_{LL}} y \equiv \beta y. \tag{3.7}$$

Let t be some date and let n_i be the number of firms with cost c_i, $i = L,\ M$, at this date. Then, the evolution of n_L and n_M between t and $t+1$ are determined as follows:

$$\begin{aligned} n_L^{'} &= \gamma_{LL} n_L + \gamma_{ML} n_M + y, \\ n_M^{'} &= \gamma_{LM}(n_L - n_L^y) + \gamma_{MM} n_M, \end{aligned} \tag{3.8}$$

where $n_i^{'}$, for $i = L,\ M$, is period-$t+1$ number of c_i-firms. A steady-state is defined by $n_i^{'} = n_i$, i.e., the number of firms of each type remains constant. If we substitute this into the LHS of (3.8), substitute from (3.7) for n_L^y and rearrange we obtain:

$$\begin{aligned} (1-\gamma_{LL})n_L - \gamma_{ML} n_M &= y, \\ -\gamma_{LM} n_L + (1-\gamma_{MM}) n_M &= -\beta\gamma_{LM} y. \end{aligned} \tag{3.9}$$

The unique solution of these linear equations is:

$$\begin{aligned} n_L &= \frac{[1-\gamma_{MM} - \beta\gamma_{LM}\gamma_{ML}]y}{(1-\gamma_{LL})(1-\gamma_{MM}) - \gamma_{LM}\gamma_{ML}}, \\ n_M &= \frac{[\gamma_{LM} - \beta\gamma_{LM}(1-\gamma_{LL})]y}{(1-\gamma_{LL})(1-\gamma_{MM}) - \gamma_{LM}\gamma_{ML}}, \\ n &\equiv n_L + n_M = \frac{[1-\gamma_{MM} + \gamma_{LM} - \beta\gamma_{LM}\gamma_{ML} - \beta\gamma_{LM}(1-\gamma_{LL})]y}{(1-\gamma_{LL})(1-\gamma_{MM}) - \gamma_{LM}\gamma_{ML}} \equiv Ay. \end{aligned} \tag{3.10}$$

This solution is well defined since $1-\gamma_{ii} > \gamma_{ij}$ for $i \neq j$, which implies the denominator is positive.

Therefore, we have shown that for any y there is a unique steady-state number of firms, n, and that n is proportional to y: $n = Ay$, where A depends on x^* only (A depends on β (from 3.10),

β depends on t^* (from 3.7), t^* depends on z^* (from 3.6), and z^* depends on x^* (from 3.5).) Since newborns are uniformly divided among firms we have $x = b/n = b/A(x^*)y$. Free-entry requires that $x = x^*$. Thus if we set $y^* = b/A(x^*)x^*$ and if y^* firms enter each period, the steady state is such that the ex-ante profit of entrants is zero. ■

3.3. Firm Size and the Probability of Exit

Empirical studies of industry dynamics find that exit probabilities are decreasing in both size and age (see Dunne, Roberts and Samuelson, 1988, 1989 or Davis and Haltiwanger 1992). Our model is consistent with this finding. On the basis of the preceding analysis, we may distinguish three possible cases. At one extreme, suppose $z^* \leq (2-d)x^*$. Then, since a firm can only become c_M after its first period, c_M-firms never exit. At the other extreme, suppose that $z^* > x^*/d$, which is the limit size of an infinitely-old firm. Then all c_M-firms exit, irrespective of age. In both these cases, the exit probability is independent of size (equivalently age). When $z^* \leq (2-d)x^*$, only c_H-firms exit. In that case, the exit probability of a c_L-firm (c_M-firm) is γ_{LH} (γ_{MH}), i.e., is independent of its size. When $z^* > x^*/d$, *both* c_M- and c_H-firms exit. Then, a c_L-firm's exit probability, $\gamma_{LM} + \gamma_{LH}$, is again independent of size.

The third, and most interesting, possibility is that $x^*/d > z^* > (2-d)x^*$. In this case, a c_L-firm's exit probability depends on its size. The probability that a c_L-firm of size less than z^* exits at the following period is $\gamma_{LM} + \gamma_{LH}$, while the corresponding probability for a c_L-firm of size greater than or equal to z^* is only γ_{LH}. On the other hand, the exit probability of c_M-firms, all of which are of size greater than z^*, is γ_{MH} and is thus independent of size. On average, considering both c_L- and c_M-firms, the exit probability is decreasing in size. Equivalently, the hazard rate—the probability of exit conditional on age—is decreasing, which is in accordance with the empirical literature cited above.

This property results from the fact that in our model, a firm's value increases with age. There are two reasons for this. First, age increases current sales (by increasing the firm's customer stock). A more subtle reason concerns a c_M-firm's option value derived from its prospect of turning c_L in the future. This value increases with age, because a cost turnaround will increase unit profit on a larger market share, the older is the firm. This is seen most clearly when $\overline{p} = c_M$. Then the cost of staying in the market one more period is F for all c_M-firms—regardless of size. However, the benefit is increasing in size, because a larger firm materializes a bigger profit upon a cost turnaround. Therefore, c_M-firms of sufficiently large size stay in the market, while their smaller counterparts do not.

It is instructive to compare this reasoning with the one in Jovanovic (1982) and Hopenhyn (1992). In those models, the hazard rate is decreasing because large firms have lower marginal-cost than small firms and, hence, are less likely to exit. Here, large firms do not have lower marginal cost than small ones (in fact—as we point out below—they have *higher* marginal cost.) However, their competitive advantage is in having accumulated a large customer stock, which is a time-consuming process, and which they are reluctant to "give up" by exiting the market. Put differently, in Jovanovic (1982) and Hopenhyn (1992) firm size is not an innate characteristic of a firm, merely a reflection of its cost, whereas here firms are distinguished by cost *and* size.

This accords well with the oft-mentioned notion that clientele is an asset of the firm or, equivalently, that the value of the firm reflects not just its capital assets or its technological know-how, but also the number of clients it managed to lock in. Below we further explore the implications of having size as a distinct firm characteristic, showing that large firms have a stronger incentive to invest in R&D.

3.4. The Rate of Growth and Gibrat's Law

Our assumption that firms grow by a fixed number of consumers each period implies that the growth rate is inversely related to size. This is at variance with Gibrat's law, according to which the growth rate is independent of size[5]. Our model could equally—and perhaps even more plausibly—be reformulated to accommodate Gibrat's law by assuming that the number of new consumers a firm attracts is proportional to its size, say because a first time buyer is more likely to hear of a large firm than a small one. For example, if newborns locate a firm by asking around, then large firms

[5] It is worth stressing, however, that Gibrat's "Law" is a conveniet assumption, and not an empirical law. Several studies have tried to verify the empirical validity of this law, resulting in mixed results; see Ijiri and Simon (1977). One study which accords with the assumption of this paper, that the rate of growth is negtively related to size, is Vining (1976).

capture more newborns, due to the fact that large firms are being "advertised' by more people. Such a formulation would only *reinforce* the advantage that large firms have over small ones and hence that they are less likely to exit upon an adverse cost-shock.

3.5. Are small firms leaner and meaner?

A popular conception is that small firms are more cost efficient than large ones. For example, recently there had been numerous articles in the popular press (as well as policy proposals), suggesting subsidies to small firms as means of generating new jobs. One rationale that these articles suggest is that small firms are more "nimble" and "dynamic", and that they are bound to become the large corporation of tomorrow—if they are given a chance to overcome "initial hurdles". The preceding discussion implies one sense in which our model is consistent with such a view. In equilibrium, all firms of size less than z^* are c_L (since c_M-firms of this size exit), but firms of size z^* or larger are both c_L and c_M. Thus smaller firms have lower marginal costs on average. This cannot happen in models where firms can "access" as many consumers as they wish. For instance, in the competitive models of Jovanovic (1982) or Hopenhyn (1992) large firms always have lower marginal costs than small ones; in fact, the marginal cost in those models is perfectly (and negatively) correlated with size.

However, two caveats are in order. First, the *average* cost of small firms in our model may be larger—due to the fixed cost, F, which must be spread over a smaller customer base.

Second, we have assumed that all firms are equally vulnerable to adverse cost-shocks, regardless of size (in other words, γ_{ij}'s are *exogenously* fixed in our model, independent of size). A large empirical literature, surveyed by Cohen and Lewis (1989), suggests that absolute R&D expenditures are positively correlated with size (though the case for a correlation between size and *relative* expenditures is controversial). If so, large firms might well be better insulated against adverse cost-shocks as a result of greater investments in cost-reducing technologies. If this effect is accounted for, the relationship between size and marginal cost becomes less clear cut. On the one hand, all firms of size z^* or less are low cost. On the other hand, in the class of firms of size greater than z^*, marginal costs are more likely to decrease with size because larger firms invest more in cost reduction measures. So the net result is indeterminate.

Following up on this logic, the next subsection constructs a simple example to show that our model provides theoretical support for the existence of a relationship between a firm's size and its investment in cost-reduction measures.

3.6. Investing in Cost Reduction

If in our model, firms can invest in cost-reducing innovations, expenditures on such investment will increase with firm size. This is for the same reason that the hazard rate is monotonic. The larger the current market share, the greater the future market share to which the cost saving is expected to apply and hence the higher the return on its investment.

We illustrate this with a simple example. Suppose that at the end of each period a firm invests w, $w \geq 0$, which determines the probability of becoming—or remaining—a c_L-firm in the next period. Let $f(w)$ denote this probability. We assume that $f(\bullet)$ is concave, increasing, differentiable and takes values in [0,1]. If the firm is not successful in this endeavor it becomes c_M or c_H with probabilities γ_M and γ_H, where $0 < \gamma_M,\ \gamma_H < 1$ and $\gamma_M + \gamma_H = 1$. To simplify notation and calculations, assume that $f(w)$, γ_M and γ_H are the same for low- and medium-cost firms[6].

Let $\Pr(c_i \mid c_j, w)$ be the probability that a firm's next-period cost is c_i, given an investment of w and a cost of c_j in the present period. In analogy to the basic model, assume $\Pr(c_H \mid c_H, w) = 1$; once a firm becomes c_H, no reversal is possible, regardless of how much is invested in R&D. Then, we have:

$\Pr(c_L \mid c_L, w) = \Pr(c_L \mid c_M, w) = f(w)$,
$\Pr(c_M \mid c_L, w) = \Pr(c_M \mid c_M, w) = \gamma_M(1 - f(w))$ and
$\Pr(c_H \mid c_L, w) = \Pr(c_H \mid c_M, w) = \gamma_H(1 - f(w))$.

So for low- and medium-cost firms, the expected future cost is lower the more it invests. All other assumptions are as before. Then the value of low- and medium-cost firms are given by:

$$R_L(z) \;=\; Max\{0, -F + z(\overline{p} - c_L) \tag{3.11}$$

[6] This assumption focuses attention the the "pure" size effect. Another plausible story is that c_L-firms have a technological edge over c_M-firms because they only have to *maintain* their cost at its current level rather than lower it. This story is captured in our formulation by letting f depend on the current cost as well as the R&D expenditures.

$$+ \underset{w}{Max} \{-w + \delta [f(w)R_L(x + z(1-d)) + \gamma_M(1 - f(w))R_M(x + z(1-d))]\}\},$$

$$R_M(z) = Max\{0, -F + z(\overline{p} - c_M) + \underset{w}{Max} (-w + \delta [f(w)R_L(x + z(1-d)) + \gamma_M(1 - f(w))R_M(x + z(1-d))]\}\}. \quad (3.12)$$

Differentiating inside the braces, the first-order conditions for w are given by:

$$-1 + \delta[f'(w)R_L(x + z(1-d)) - \gamma_M f'(w)R_M(x + z(1-d))] = 0. \quad (3.13)$$

Low- and medium-cost firms of the same size invest identically[7].

Proposition 3.2. *$dw/dz > 0$.*

Proof. *By (3.13), the optimal w is determined by*

$$f'(w) = 1/\delta[R_L(x + z(1-d)) - \gamma_M R_M(x + z(1-d))].$$

Thus the optimal w is increasing in z if $R_L(z) - \gamma_M R_M(z)$ is increasing in z. Since low- and medium-cost firms of the same size choose identical w and since R_i's are increasing in z, we have from (3.11) and (3.12) that $R_L(z) - \gamma_M R_M(z) = Max\{R_L(z), z[(1-\gamma_M)\overline{p} - c_L + \gamma_M c_M)]$ which is increasing in z. ■

In our setting, a larger firm invests more because it has more to gain from achieving (or maintaining) a low cost at the next period. The cogency of this reasoning depends on the distinction between costs and market share in our model. In a perfectly competitive market (e.g., Jovanovic (1982) and Hopenhyn (1992)), or even in an oligopoly market (e.g., Pakes (1996)), in which costs and market shares are perfectly correlated, there is no natural relationship between size and the incentive to invest in cost reduction. Since a currently small, high-cost firm can achieve as large a market share by becoming low cost as that of a currently large, low-cost firm, it should have no less of an incentive to invest in lowering costs.

[7] This is due to the simplifying assumption that future costs depend only on R&D investment. More generally, the optimal investment will depend on both the current cost and the amount invested.

4. More general model

A problem with the model of section 2 is that, since firms extract all of the consumers' surplus, there also exists an equilibrium in which there is no trade (consumers have no incentive to participate in the market). Even more seriously, if consumers incur even a small cost of matching with the first firm, only the no-trade equilibrium would survive.

To overcome these problems, this section presents a more general model in which consumers gain positive surplus from trade. While preserving the main features of the earlier model, it generates a richer and more realistic set of predictions. In particular, under the more general model, market equilibrium is characterized by price dispersion, with prices of medium-cost firms exceeding those of low-cost firms by a markup which depends on the model's parameters. Correspondingly, low-cost firms produce greater output than medium-cost firms of the same age (in the previous model output depended on age only and not on cost). These predictions accord well with some recent evidence on the correlations between prices, firm sizes and marginal costs in homogenous-product industries; see Roberts and Supina (1996).

In the more general model, consumers buy variable quantities. Specifically, all consumers have an identical downward sloping demand function, $D(p)$. $D(\bullet)$ has a finite intercept, $\overline{p}$, on the price axis, $\overline{p} \equiv Inf\,\{p \mid D(p) = 0\} < \infty$, and is strictly decreasing for $p < \overline{p}$. Let $S(p)$ be a consumer's surplus under $D(p)$, $S(p) \equiv \int_{p}^{\overline{p}} D(p)dp$. Let $\pi_i(p) \equiv (p - c_i)D(p)$, $i = c_L,\ c_M$, be the profit per customer of a low- and medium-cost firm, respectively. $\pi_i(p)$ are assumed to be concave. Let p_L^m and p_M^m be the monopoly prices of a c_L- and a c_M-firm, respectively. By the concavity of π_i, $p_M^m > p_L^m$.

Since consumers realize a surplus over and above the price they pay, consumers whose firm exits may search for a new firm. To ensure that this occurs in equilibrium we assume $\sigma < S(p_M^m)$. That is, we assume the search cost is less than the one-period surplus a consumer gets when all medium-cost firms charge the monopoly price, p_M^m. It will be shown below that a medium-cost firm never charges more than p_M^m and a low-cost firm charges less than that; therefore, the minimum payoff to search is $S(p_M^m)$ which more than justifies incurring the one-time search cost σ.[8] We

[8] If σ is large consumers may not search and we're back to the previous (and simpler) formulation. In general, one

continue to assume (as we did in the previous section) that $c_H > \overline{p}$ and that $Pr(c_H|c_H) = 1$. These assumptions ensure that c_H-firms exit at once. It is convenient to assume that $p_L^m \geq c_M$. That is, a low-cost firm's monopoly price covers a medium-cost firm's marginal cost. This assumption focuses attention on the (more interesting) case in which some c_M-firms survive. Otherwise, the only steady-state might be one where only c_L-firms are active in equilibrium.

The model is reformulated in continuous time[9]. There is a flow (per-unit time) of new consumers, b, and an exponential death rate, d, $0 < b,\ d < \infty$. Technological shocks occur at exponentially-distributed times, the rate of arrival being 1. Hence, shocks occur at separated points in time but they can take place at any date. When a shock occurs, the cost of production changes from c_i to c_j with probability γ_{ij}.

Since firms with different costs have different monopoly prices, the low-cost firms charge a lower price than the medium-cost firms, and consumers search optimally given this price distribution, as in Reinganum (1979). Since the search cost is positive for all consumers, Diamond's (1971) result applies to the low-cost firms which charge p_L^m (the monopoly price is independent of the number of customers since marginal cost is independent of quantity produced). The price of the medium-cost firms is denoted by p and is determined as part of the steady-state equilibrium.

We proceed as follows. First, we take p as given and characterize the steady-state distribution of firm types consistent with p. Then we characterize the properties which p must satisfy to be an equilibrium price, given that distribution. Finally we use a fixed-point argument to demonstrate the simultaneous existence of a steady-state distribution of firm-types and p, such that p is individually optimal, given the steady-state distribution and such that the choice of p by medium-cost firms reproduces the steady-state distribution.

Consider a firm that starts with z_0 customers, gets a flow of x new customers and losses a flow dz of its customer stock, z. Then its customer stock evolves according to the differential equation, $z'(t) = x - dz(t), z(0) = z_0$. The solution to this equation is $z(t) = \frac{x}{d} - \left(\frac{x}{d} - z_0\right) e^{-dt}$. Therefore,

has to determine endogenously whether consumers whose firm exits decide to search for a new firm or not.

[9]This is necessary since to establish the existence of an equilibrium we need to invoke a fixed-point argument which relies on continuity, and continuity is not guaranteed unless the critical age of exit (t^* in the previous section) is allowed to vary continuously.

taking p as given, the value of the firm satisfies the following functional equations:

$$R_L(z) = Max\{0, \int_0^\infty e^{-t}[(\pi_L^m \frac{x}{d} - F)\frac{1-e^{-rt}}{r} - \pi_L^m(\frac{x}{d} - z)\frac{1-e^{-(r+d)t}}{r+d}$$
$$+e^{-rt}\left(\gamma_{LL}R_L(z(t)) + \gamma_{LM}R_M(z(t))\right)]dt\},$$

$$R_M(z) = Max\{0, \int_0^\infty e^{-t}[\pi(p)(\frac{x}{d} - F)\frac{1-e^{-rt}}{r} - \pi(p)(\frac{x}{d} - z)\frac{1-e^{-(r+d)t}}{r+d}$$
$$+e^{-rt}\left(\gamma_{ML}R_L(z(t)) + \gamma_{MM}R_M(z(t))\right)]dt\}.$$

In these equations, e^{-t} is the density of a technological shock at t, $\pi_L^m \equiv \pi_L(p_L^m)$ is a low-cost firm's monopoly profit and $\pi(p) = \pi_M(p)$ is a medium-cost firm's profit when it charges p.

Again, dynamic programming arguments imply the existence of a unique, monotonic and continuous solution to these equations, which implies the existence of a unique x^* so that:

$$R_L(0; x^*, p) = K. \tag{4.1}$$

x^* is the flow of entry so that new entrants face zero expected profit.
Given this x^* there exists a unique z^* so that

$$R_M(z^*; x^*, p) = 0. \tag{4.2}$$

z^* is the customer stock so that firms which have accumulated z^* are indifferent between exiting and staying.
Given z^* there exists a unique t^* so that a firm will have accumulated z^* customers by the time it reaches age t^*. t^* is the solution to the equation:

$$\frac{x^*}{d}(1 - e^{-dt}) = z^* \,(\text{If } z^* > x^*/d, \text{ set } t^* = \infty)\,. \tag{4.3}$$

As in the previous section, the optimal exit rule is that firms of age $t < t^*$ exit as soon as they become c_M or c_H, while firms of age $t > t^*$ exit only if they become c_H. For future reference we record the following.

Lemma 4.1. *t^* is continuous in p.*

Proof. *This follows from the continuity of the functions in (4.2) and (4.3).* ■.

Call firms with cost c_L and age $\tau < t^*$—c_L^τ -types. Firms with cost c_L and above age t^* are called c_L^∞-types and firms with cost c_M are called c_M-types. The exit probability of firms in the last two categories is independent of age, hence there is no need to keep track of their ages.

The fraction of new entrants that reach age t before becoming c_M or c_H is $e^{-(1-\gamma_{LL})t} = \sum_{i=0}^{\infty} \gamma_{LL}^i e^{-t} \frac{t^i}{i!}$ (the index i measures how many shocks a firm experiences in the interval $[0, t]$ and all these shocks are required to take the cost from c_L to c_L, which occurs with probability γ_{LL}^i). Therefore, denoting the flow of new entrants by y, the measure of firms in the age-group $[0, t^*]$ (the group of firms that exit upon receiving a c_M-shock) is

$$n_L^y = y \int_0^{t^*} e^{-(1-\gamma_{LL})t} dt = y \frac{1 - e^{-(1-\gamma_{LL})t^*}}{1 - \gamma_{LL}} \equiv \beta y. \tag{4.4}$$

The frequency $a(\tau)$ of c_L^τ-types relative to the measure of firms in the age group $[0, t^*]$ is

$$a(\tau) = \frac{(1 - \gamma_{LL}) e^{-(1-\gamma_{LL})\tau}}{1 - e^{-(1-\gamma_{LL})t^*}} = \frac{e^{-(1-\gamma_{LL})\tau}}{\beta}. \tag{4.5}$$

Proposition 4.2. *Fix a p and assume all c_M-firms charge p. Assume also that all customers of c_M-firms accept p without search. (i) Then there exists an entry rate y^* so that, at the steady-state it induces, all new entrants earn zero profits, and all incumbent firms exit optimally. (ii) y^* and the steady-state associated with it are continuous in p.*

Proof. *Let y denote a constant flow of new entry. We show how y can be adjusted so that each firm gets a flow x^* of new customers in the steady-state. This is analogous to the determination of y in the previous section, except that the number of searching customers is endogenous (instead of being equal to the number of newborns, b) and has to be determined along with the number of firms in the steady-state, n_L and n_M.*

Consider the evolution of the number of firms, n_L and n_M. Let Δ be a small time interval. Then, given the assumption of exponential arrival, within this time interval a firm experiences 1 shock with probability Δ, 0 shocks with probability $1 - \Delta$, and more than 1 shock with probability that is an order of magnitude less than Δ (i.e., $\lim_{\Delta \to 0}$[Pr(more than one shock within Δ)/Δ]= 0). Therefore, starting with n_L and n_M firms, the number of firms after Δ units of time is:

$$n_L' = [1 - \Delta(1 - \gamma_{LL})] n_L + \Delta \gamma_{ML} n_M + \Delta y \tag{4.6}$$
$$n_M' = \Delta \gamma_{LM} (n_L - n_L^y) + [1 - \Delta(1 - \gamma_{MM})] n_M.$$

A steady-state is defined by $n_L^{'} = n_L$ and $n_L' = n_M$. Substituting this and (4.4) into (4.6) and rearranging, we get

$$\Delta(1-\gamma_{LL})n_L - \Delta\gamma_{ML}n_M = \Delta y, \tag{4.7}$$
$$-\Delta\gamma_{LM}n_L + \Delta(1-\gamma_{MM})n_M = -\Delta\beta\gamma_{LM}y.$$

Cancelling Δ gives the system of equations (3.9). Therefore, the number of firms in the steady-state is the same as in the discrete case:

$$n_L = \frac{[1-\gamma_{MM}-\beta\gamma_{LM}\gamma_{ML}]y}{(1-\gamma_{LL})(1-\gamma_{MM})-\gamma_{LM}\gamma_{ML}}, \tag{4.8}$$
$$n_M = \frac{[\gamma_{LM}-\beta\gamma_{LM}(1-\gamma_{LL})]y}{(1-\gamma_{LL})(1-\gamma_{MM})-\gamma_{LM}\gamma_{ML}},$$
$$n \equiv n_L + n_M = \frac{[1-\gamma_{MM}+\gamma_{LM}-\beta\gamma_{LM}\gamma_{ML}-\beta\gamma_{LM}(1-\gamma_{LL})]y}{(1-\gamma_{LL})(1-\gamma_{MM})-\gamma_{LM}\gamma_{ML}} \equiv Ay.$$

Define

$$\lambda = \frac{n_L}{n_L+n_M} = \frac{1-\gamma_{MM}-\beta\gamma_{LM}\gamma_{ML}}{1-\gamma_{MM}+\gamma_{LM}-\beta\gamma_{LM}\gamma_{ML}-\beta\gamma_{LM}(1-\gamma_{LL})}, \tag{4.9}$$
$$\alpha = \frac{n_L - n_L^y}{n_L} = \frac{1-\beta-\gamma_{MM}-\beta\gamma_{LM}\gamma_{ML}}{1-\gamma_{MM}-\beta\gamma_{LM}\gamma_{ML}}.$$

λ represents the fraction of low-cost firms among all firms; α represents the fraction of large low-cost firms among all low-cost firms.

Let D_L^y be the number of customers attached to firms in the age group $[0, t^*]$. Then,

$$D_L^y = \int_0^{t^*} \frac{x}{d}(1-e^{-dt})ye^{-(1-\gamma_{LL})t}dt = \frac{xy}{d}\int_0^{t^*}[e^{-(1-\gamma_{LL})t} - e^{-(d+1-\gamma_{LL})t}]dt \tag{4.10}$$
$$= \frac{xy}{d}[\frac{1-e^{-(1-\gamma_{LL})t^*}}{1-\gamma_{LL}} - \frac{1-e^{-(d+1-\gamma_{LL})t^*}}{d+1-\gamma_{LL}}]$$
$$= \frac{xy}{d}\frac{d-(d+1-\gamma_{LL})e^{-(1-\gamma_{LL})t^*}+(1-\gamma_{LL})e^{-(d+1-\gamma_{LL})t^*}}{(1-\gamma_{LL})(d+1-\gamma_{LL})} \equiv \frac{xy}{d}B.$$

Let s be the flow of searchers. Then, since searchers are divided uniformly across firms, $x = s/n = s/Ay$, where A is defined in (4.8). Therefore we can write (4.10) as

$$D_L^y = \frac{sB}{dA}. \tag{4.11}$$

Let D_L and D_M be the number of customers attached to c_L- and c_M-firms. Consider a small time interval, Δ. Then, after Δ units of time, we have:

$$D_L^{'} = D_L(1 - d\Delta)[1 - \Delta(1 - \gamma_{LL})] + D_M(1 - d\Delta)\Delta\gamma_{ML} + \Delta\lambda s,$$
$$D_M' = D_M(1 - d\Delta)[1 - \Delta(1 - \gamma_{MM})] + (D_L - D_L^y)(1 - d\Delta)\Delta\gamma_{LM} + \Delta(1 - \lambda)s.$$

In the steady state $D_L' = D_L$ and $D_M' = D_M$. Substituting this condition, canceling Δ and solving for D_L and D_M, we have:

$$D_L = \frac{\lambda s(d + 1 - \gamma_{MM}) + [(1 - \lambda)s - \frac{sB}{dA}\gamma_{LM}]\gamma_{ML}}{(d + 1 - \gamma_{LL})(d + 1 - \gamma_{MM}) - \gamma_{LM}\gamma_{ML}}, \quad (4.12)$$
$$D_M = \frac{(d + 1 - \gamma_{LL})[(1 - \lambda)s - \frac{sB}{dA}\gamma_{LM}] + \gamma_{LM}\lambda s}{(d + 1 - \gamma_{LL})(d + 1 - \gamma_{MM}) - \gamma_{LM}\gamma_{ML}},$$

where B is defined in (4.10).

Since customers of c_M-firms do not search, the only searchers are newborns, customers of c_L- and c_M-firms whose cost escalated to c_H and customers of c_L^τ-firms whose cost escalated to c_M. accordingly, the flow of searchers with Δ equals:

$$s\Delta = b\Delta + D_L\gamma_{LH}\Delta + D_M\gamma_{MH}\Delta + D_L^y\gamma_{LM}\Delta.$$

Canceling Δ, substituting for D_L and D_M from (4.12) and for D_L^y from (4.11), we obtain:

$$\begin{aligned} s &= b + \frac{\lambda s(d + 1 - \gamma_{MM}) + [(1 - \lambda)s - \frac{sB}{dA}\gamma_{LM}]\gamma_{ML}}{(d + 1 - \gamma_{LL})(d + 1 - \gamma_{MM}) - \gamma_{LM}\gamma_{ML}}\gamma_{LH} \\ &\quad + \frac{(d + 1 - \gamma_{LL})[(1 - \lambda)s - \frac{sB}{dA}\gamma_{LM}] + \gamma_{LM}\lambda s}{(d + 1 - \gamma_{LL})(d + 1 - \gamma_{MM}) - \gamma_{LM}\gamma_{ML}}\gamma_{MH} + \frac{sB}{dA}\gamma_{LM}. \end{aligned}$$

This implies:

$$\begin{aligned} b &= s\{1 - \frac{\lambda(d + 1 - \gamma_{MM}) + [1 - \lambda - \frac{B}{dA}\gamma_{LM}]\gamma_{ML}}{(d + 1 - \gamma_{LL})(d + 1 - \gamma_{MM}) - \gamma_{LM}\gamma_{ML}}\gamma_{LH} \\ &\quad + \frac{(d + 1 - \gamma_{LL})[1 - \lambda - \frac{B}{dA}\gamma_{LM}] + \gamma_{LM}\lambda}{(d + 1 - \gamma_{LL})(d + 1 - \gamma_{MM}) - \gamma_{LM}\gamma_{ML}}\gamma_{MH} + \frac{B}{dA}\gamma_{LM}\}. \end{aligned}$$

And thus

$$x = \frac{s}{n} = \frac{b}{[1 - \frac{\lambda(d+1-\gamma_{MM})+[1-\lambda-\frac{B}{dA}\gamma_{LM}]\gamma_{ML}}{(d+1-\gamma_{LL})(d+1-\gamma_{MM})-\gamma_{LM}\gamma_{ML}}\gamma_{LH} + \frac{(d+1-\gamma_{LL})[1-\lambda-\frac{B}{dA}\gamma_{LM}]+\gamma_{LM}\lambda}{(d+1-\gamma_{LL})(d+1-\gamma_{MM})-\gamma_{LM}\gamma_{ML}}\gamma_{MH} + \frac{B}{dA}\gamma_{LM}]Ay}.$$

So given x^*, as defined by the zero profit condition (4.1), we can choose:

$$y^* = \frac{b}{[1 - \frac{\lambda(d+1-\gamma_{MM})+[1-\lambda-\frac{B}{dA}\gamma_{LM}]\gamma_{ML}}{(d+1-\gamma_{LL})(d+1-\gamma_{MM})-\gamma_{LM}\gamma_{ML}}\gamma_{LH} + \frac{(d+1-\gamma_{LL})[1-\lambda-\frac{B}{dA}\gamma_{LM}]+\gamma_{LM}\lambda}{(d+1-\gamma_{LL})(d+1-\gamma_{MM})-\gamma_{LM}\gamma_{ML}}\gamma_{MH} + \frac{B}{dA}\gamma_{LM}]Ax^*},$$

ensuring that each firm gets a flow x^* of new customers.

(ii) Since all variables on the RHS of the last expression are continuous in p (see also Lemma 4.1), y^* is also continuous in p. For the same reason, α and λ, as defined in (4.9), are continuous in p. ■

Thus, corresponding to an arbitrary p, there exists a zero-profit steady-state distribution over firm-types in which the fraction of c_L-types is λ, the fraction of c_L-types above age t^* is α (see (4.9)), and the age distribution among the c_L^τ-types is $a(\tau)$ (see (4.5)).

Let us now fix a firm-type distribution (λ, α, t^*, $a(\tau)$) and construct a p which is consistent with the maximization of firms' and consumers' problems. Consider a consumer's search problem. When a consumer decides to stop searching and attach himself to a firm, his future utility will depend on changes in its price and the possibility that it will exit at some future date. Therefore, the data relevant to the consumer's decision is the price distribution across firms and the probabilities over future prices and exit. More specifically, the relevant data includes:

(i) The prices charged by firms: p_L^m for low-cost firms and (the yet to be determined) p for medium-cost firms. These prices determine the period surplus to the consumer until his firm experiences the next cost shock. They also indicate the current cost of his firm and hence the evolution of costs (and hence prices) in the future (given that costs are following Markovian transitions).

(ii) For low cost firms, the age τ. This data is relevant since young (below t^*) low-cost firms exit when they become c_M, necessitating costly future search.

Therefore, for the purpose of consumers' search decision, a firm's type is appropriately defined as its current cost and, among c_L-firms, its age τ.

We assume that consumers are able to observe the size of firms they visit[10]. This enables them

[10]If consumers are unable to observe the size of the firm from which a quote is obtained, they will take an expected value over the equilibrium size distribution of firms. This would result in more complex expressions, but without changing the results.

to determine the firm's type and, thereby, the value of buying from it.[11]

Let $V_s(p)$ be the value of searching once and then proceeding optimally when medium-cost firms' price is p. Then:

$$V_s(p) = -\sigma + \lambda\alpha V_L^{\infty}(p) + \lambda(1-\alpha)\int_0^{\infty} V_L^{\tau}(p)a(\tau)d\tau + (1-\lambda)V_M(p), \tag{4.13}$$

where $V_L^{\infty}(p)$ ($V_L^{\tau}(p)$) is the value of being attached to a low-cost firm of age$>t^*$ (of age $\tau < t^*$), and $V_M(p)$ is the value of being attached to a medium-cost firm. V_i's satisfy the following functional equations:

$$V_L^{\infty}(p) = \int_0^{\infty} e^{-t}\left\{S(p_L^m)\frac{1-e^{-rt}}{r} + e^{-rt}\left[\gamma_{LL}V_L^{\infty}(p) + \gamma_{LM}V_M(p) + \gamma_{LH}V_s(p)\right]\right\}dt, \tag{4.14}$$

$$\begin{aligned} V_L^{\tau}(p) \;=\; & \int_0^{t^*-\tau} e^{-t}\left\{S(p_L^m)\frac{1-e^{-rt}}{r} + e^{-rt}\left[\gamma_{LL}V_L^{t+\tau}(p) + (\gamma_{LM}+\gamma_{LH})V_s(p)\right]\right\}dt \\ & + \int_{t^*-\tau}^{\infty} e^{-t}\left\{S(p_L^m)\frac{1-e^{-rt}}{r} + e^{-rt}\left[\gamma_{LL}V_L^{\infty}(p) + \gamma_{LM}V_M(p) + \gamma_{LH}V_s(p)\right]\right\}dt, \end{aligned} \tag{4.15}$$

and

$$V_M(p) = Max\{V_s(p), \int_0^{\infty} e^{-t}\left\{S(p)\frac{1-e^{-rt}}{r} + e^{-rt}[\gamma_{ML}V_L^{\infty}(p) + \gamma_{MM}V_M(p) + \gamma_{MH}V_s(p)]\right\}dt\}. \tag{4.16}$$

Since no firm charges less than p_L^m, it is not optimal for a consumer attached to a low-cost firm (whose price is p_L^m), whatever its size, to search. A consumer attached to a medium-cost firm optimally searches if and only if $V_M(p) < V_s(p)$. In what follows, the argument p is suppressed from the $V_i(\bullet)$ functions whenever this leads to no ambiguity.

[11] When consumers observe the price of a firm they are able to infer its cost since cost and price are 1-1 related in equilibrium. Likewise, size and age are 1-1 related (see equation (4.3)). Therefore, by observing the price and the size of a firm, the consumer is able to infer the firm's type. Furthermore, the consumer is assumed to know the transition probabilities over costs. Therefore, once a consumer has observed the price and size of a firm, he is able to form a rational forecast over its future prices and the probabilities that it will exit at various future dates. This enables him to determine the value of buying from this firm.

Proposition 4.3. *Fix a firm-type distribution,* $(\lambda, \alpha, t^*, a(\tau))$*. (i) Then for any* $p \in [p_L^m, p_M^m]$ *there exists a unique solution,* (V_L, V_M, V_s)*, to equations (4.13)-(4.16). (ii) The solution is continuous in* p*,* $V_s - V_M$ *is increasing in* p *and* $V_s(p_L^m) - V_M(p_L^m) < 0$*. (iii) If* $V_s(p_M^m) - V_M(p_M^m) \geq 0$*, there is a* p' *so that* $V_s(p') - V_M(p') = 0$*; otherwise,* $V_s(p) - V_M(p) < 0$ *for all* $p \in [p_L^m, p_M^m]$*. In the first instance, customers of* c_M*-firms are indifferent between buying at* p' *and searching. In the second instance, they prefer buying at* p_M^m*.*

Proof. *By dynamic programming, for a given* p *and* $(\lambda, \alpha, t^*, a(\tau))$*, the system of equations (4.13) - (4.16) has a unique solution, which is continuous in* p*.* $V_s - V_M$ *is increasing in* p *because the higher is the price at* c_M*-firms, the more attractive is the search option. If* $p = p_L^m$*, all firms charge the same price, there is no incentive to search and* $V_s(p_L^m) - V_M(p_L^m) < 0$*. If* $V_s(p_M^m) - V_M(p_M^m) < 0$*, then* $V_s - V_M$ *is negative throughout* $[p_L^m, p_M^m]$ *(by the monotonicity of* $V_s - V_M$*). Otherwise, by continuity, there must be a* p' *so that* $V_s(p') - V_M(p') = 0$*.* ■

Proposition 4.2 established that, corresponding to an arbitrary p, there exists a steady-state firm-type distribution $(\lambda, \alpha, t^*, a(t))$. Proposition 4.3 established that corresponding to an arbitrary distribution, there exists a p with the property that p is accepted by consumers without search and is profit-maximizing for c_M-firms. A **steady-state equilibrium** is a tuple $(p^*, (\lambda, \alpha, t^*, a(\tau)))$ which is consistent with both conditions: p^* is individually optimal given $(\lambda, \alpha, t^*, a(\tau))$, and the choice of p^* by all medium cost firms reproduces $(\lambda, \alpha, t^*, a(\tau))$. The next proposition establishes the existence of such an equilibrium.

Theorem 4.4. *There exists a steady-state equilibrium. In this equilibrium, the price of low-cost firms is* p_L^m *and that of medium-cost firms is* $p^*, p^* > p_L^m$*, which consumers accept without search.*

Proof. *Fix a* $p \in [p_L^m, p_M^m]$*. Assume all* c_M*-firms charge* p *and that their customers accept* p *without search (this will be shown to hold for the equilibrium* p*). Then, by proposition 4.2, there exists a flow of entry,* y^**, and a distribution over firm types,* $(\lambda, \alpha, t^*, a(\tau))$*, so that each firm gets a constant flow of customers,* x^**, for which the entry/exit conditions, (4.1) and (4.2), are satisfied.*

Consider now consumers' search problem given p *and* $(\lambda, \alpha, t^*, a(\tau))$*. If* $V_M(p) < V_s(p)$*, then, by proposition 4.3, there exists a price,* $p'' \in (p_L^m, p)$*, so that under* p'' *and* $(\lambda, \alpha, t^*, a(\tau))$*, we have*

$V_M(p'') = V_s(p'')$. Let:

$$p' = \begin{cases} min\{p'', p_M^m\} & \text{if } V_M(p) < V_s(p) \\ p & \text{otherwise} \end{cases}.$$

Then, p' is such that $V_M(p') \geq V_s(p')$, i.e., consumers accept p' without search.

This defines a mapping, call it $\Psi(p)$, so that when consumers face $p' = \Psi(p)$ and $(\lambda, \alpha, t^*, a(\tau))$, we have $V_M(p') \geq V_s(p')$. Since the mapping from p to $(\lambda, \alpha, t^*, a(\tau))$ is continuous (see proposition 4.2) and since consumers' and firms' objectives are continuous, Ψ is continuous on the compact interval $[p_L^m, p_M^m]$. Therefore, it has a fixed point, p^*.

Assume now that all c_M-firms charge p^*. If $p^* < p_M^m$ then, by construction, $V_M(p^*) = V_s(p^*)$. Assume some firm deviates, raising its price above p^*. Then, given the monotonicity of $V_s - V_M$ in p, this will render $V_M < V_s$, consumers will search and the deviating firm will end up with $z = 0$. Since R_M is increasing in z, such deviation cannot be profitable. Consider a price decrease. Then the deviating firm does not get any more customers than it did before the price decrease (firms are nonatomic), and by concavity of the period payoff-function, $\pi(p)$, it losses profits on existing customers. Therefore, a price decrease cannot be profitable, either. If $p^* = p_M^m$, c_M-firms are already charging the monopoly price so certainly there is no incentive to deviate. So we have shown that p^* is a maximizing price for all c_M-firms. $p^* > p_L^m$ because, by proposition 4.3 (ii), $V_s(p_L^m) - V_M(p_L^m) < 0$, which contradicts the definition of p^*.

Under p^*, consumers' search problem is such that $V_M(p^*) \geq V_s(p^*)$ (by construction). Thus, customers of medium-cost firms are maximizing by choosing not to search. This confirms our initial assumption, so each firm gets indeed the same flow of new customers which, according to proposition 4.2, sustains the steady-state, $(\lambda, \alpha, t^*, a(\tau))$. Furthermore, conditions (4.1) and (4.2) are satisfied (again by construction), so firms are indeed maximizing with respect to price and entry/exit decisions. ■

The main difference between this version and the simpler model of section 2 is that there is more than one price which clears the market, i.e., prices are dispersed in equilibrium. In particular, the prices of medium-cost firms exceed those of low-cost firms by a markup which depends the model's parameters. By the same token, the output of medium-cost firms is less than that of low-cost firms of the same age (because each consumer demands less at the higher price). That is, low- and medium-cost firms of the same age have identical numbers of loyal customers but the low-cost firms have a larger share of output. Thus, in the richer version of the model, prices are correlated with firm sizes and with marginal costs. This type of correlations match those found in Roberts and Supina (1996).[12]

5. Conclusion

Since Gibrat, industrial economists have been aware of the need to explain size differences between firms. We have presented a simple model in which firms with identical characteristics nevertheless achieve different market shares, as a function of their tenure in the market. The main ingredient of the model is the linkage between current and future market share deriving from consumer switching costs. Thus in our formulation, a firm's market size is something quite distinct from its technological know-how. By contrast, in existing models of industry dynamics (Jovanovic (1982), Hopenhayn (1992) or Lambson (1992)), which are set in perfectly competitive environments, market shares depend only current production cost, so that cost and size are perfectly correlated variables.

[12]This cannot happen in competitive models, e.g., Jovanovic (1982) or Hopenhyn (1992). In those models a unique price clers the market, so there is no scope for correlation between prices and firm sizes or marginal costs.

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