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“How Central Banks End Crises”

by

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How Central Banks End Crises

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Abstract

To end a financial crisis, the central bank is to lend freely, against good collateral, at a high rate, according to Bagehot’s Rule. We argue that in theory and in practice there is a missing ingredient to Bagehot’s Rule: secrecy. Re-creating confidence requires that the central bank lend in secret, hiding the identities of the borrowers, to prevent information about individual collateral from being produced and to create an information externality by raising the perceived value of average collateral. Ironically, the participation of “bad” borrowers, with low quality collateral, in the central bank’s lending program is a desirable part of re-creating confidence because it creates stigma. Stigma is critical to sustain secrecy because no borrower wants to reveal his participation in the lending program, and it is limited by the central bank charging a high rate for its loans.

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1 Introduction

How do financial crises end? How is confidence restored? The classic answer to these questions was provided by Walter Bagehot (1873): The central bank should lend freely, at a high rate, and on good collateral. In the recent financial crisis, Ben Bernanke, Mervyn King and Mario Draghi, the respective heads of the Federal Reserve System, the Bank of England, and the European Central Bank, reported that they followed this advice; see Bernanke (2014a and 2014b), King (2010) and Draghi (2013). But, there was more to it than that. All three institutions also engaged in anonymous or secret lending to banks. In this paper we argue that there is a missing ingredient in Bagehot’s rule: secrecy, which produces an information externality that recreates “confidence.”

It is not obvious why Bagehot’s advice would work to restore confidence, or would be expected to work. Intuitively, it seems that the idea is that if a bank can borrow cash from the central bank by posting collateral, it can then repay depositors or lenders who want their cash back during a bank run. The idea seems to be that if enough cash is handed out, depositors become convinced that the cash is there and there is no reason to withdraw their cash. The run ends. But, the details of this are murky, and it does not seem to be the whole story. In determining why Bagehot’s advice can restore confidence, and how it can restore it at the lowest possible cost in terms of public funds, there is another part to the rule which we observe in practice, secrecy.

During the financial crisis of 2007-2008, the Federal Reserve introduced a number of new lending programs: the Term Auction Facility, the Term Securities Lending Facility, and the Primary Dealers Credit Facility. These facilities were designed to hide the identity of the borrowers by using auctions. Secrecy was also integral to the special crisis lending programs of the Bank of England and the European Central Bank. Plenderleith (2012), asked by the Bank of England to review their Emergency

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1It is widely agreed that Thornton (1802) articulated the role of the lender-of-last-resort first. There is a very large literature on Bagehot/Thornton and the lender-of-last-resort. In the main text we only summarize a very small part of this literature. For overviews see Bordo (1990), Goodhart (1988 and 1995), Freixas et al. (1999), Capie and Wood (2007) and Bignon, Flandreau, and Ugolini (2009).

2Bernanke (2010): “. . . [because of] the competitive format of the auctions, the TAF [Term Auction Facility] has not suffered the stigma of the conventional discount window” (p.2). Also, see Armantier et al. (2011). Also, the Troubled Asset Relief Program (TARP) of the U.S. Treasury involved secret lending.

Lending Facilities (ELA) during the financial crisis, wrote: “Was secrecy appropriate in 2008? In light of the fragility of the markets at the time . . . it was right to endeavor to keep ELA [Emergency Liquidity Assistance] operations in 2008 covert. None of those interviewed for this Review suggested otherwise . . . in conditions of more severe systemic disturbance, as in 2008, ELA is likely to be more effective if provided covertly” (p. 70).

Secret lending is the basis for the discount window, a facility used by many central banks around the world. Even before the Federal Reserve came into existence, the private bank clearinghouse lending during banking panics in the U.S. was done in secret, and individual bank-specific information was cut off by the clearinghouse. Further, the assets of member banks were essentially pooled by issuing a new claim, the clearing house loan certificate, which was a joint claim, further hiding the identities of borrowing members. (See Gorton and Tallman (2014)). Secrecy is pervasive in central banking lending programs and seems to be an implicit part of Bagehot’s rule.

Bagehot did not mention secrecy because “. . . a key feature of the British system, its in-built protective device for anonymity was overlooked [by Bagehot]” (Capie (2007), p. 313). Capie explains that in England geographically between the country banks and the Bank of England was a ring of discount houses. Also, see Capie (2002). If a country bank needed money, it could borrow from its discount house, which in turn might borrow from the Bank of England. In this way, it was not known where the money from the Bank of England was going.\footnote{King (1936) provides more discussion on the industrial organization of British banking in the 19th century. Also see Pressnell (1956). The Bank of England did not always get along with the discount houses, and there is a complicated history to their interaction. See, e.g., Flandreau and Ugolini (2011).}

The ability of a central bank to “restore confidence” can only be discussed in the context of a concept of a “crisis” which explains what it means to “lose confidence” in the first place. In this paper, we follow Dang, Gorton, and Holmström (2013) and Gorton and Ordonez (2014) who view a crisis as an event in which information-insensitive bank debt becomes information-sensitive when there is a bad public signal. “Information-insensitive” means that no agent has an incentive to expend resources to produce private information about the collateral backing the debt (the bank loan portfolio backing demand deposits or a bond used as collateral in a sale and repurchase agreement (repo)). “Confidence” means information-insensitive. The arrival of public bad news can cause the production of such private information to be-
come profitable, causing the bank debt to no longer be useful as money due to fears of adverse selection.

In this paper we argue that the central bank must lend in secret, hiding the identities of the borrowers in a financial crisis. If this can be accomplished, then lenders only know the average quality of bank assets in the economy, leading to lending which would not otherwise occur. But how can the central bank maintain secrets? Central banks should offer to lend to induce “bad” borrowers to take advantage of the discount window, inducing “stigma” in the market such that borrowers do not have incentives to reveal their identities.\(^5\) Stigma refers to the costs to a bank of being viewed as weak, resulting in higher borrowing costs and the possibility of facing a run.\(^6\) For example, the UK parliament attributed the run on Northern Rock to a leak by BBC that the bank had asked for and received emergency loans from the Bank of England.\(^7\) Central banks should also use haircuts on borrowers’ collateral, not to protect itself against losses, but to regulate the amount of “bad” borrowers participating.

As in Gorton and Ordonez (2014), there are no explicit financial intermediaries in the model. The roles of “banks” and “money” are implicitly retained in a model of households making short-term, collateralized loans directly to firms. Information-insensitivity of the debt is the crucial issue. Firms can borrow using secured debt or unsecured debt. Secured debt (repo) backs the loan with a specific bond. During a crisis, if this bond is a Treasury, it can reveal that the borrower went to the discount window. Unsecured borrowing refers to a loan backed by the entire portfolio of the borrower. Loans backed by a portfolio of assets are “banks.” Such a portfolio is opaque, and for this reason banks are regulated (see Dang et al. (2014)).

In the recent financial crisis borrowers switched from secured to unsecured borrowing. The asset-backed securities (ABS) used as collateral for repo migrated from broker-dealer banks and insurance companies to commercial banks and the central bank. Repo financing shrank by $1.5 trillion. (See He, Zhang, and Krishnamurthy (2010).) On September 21, 2008 it was announced that the investment banks would become bank holding companies, being subjected to stricter regulatory oversight and

\(^5\)Bernanke (2009a): “In August 2007,. . . banks were reluctant to rely on discount window credit to address their funding needs. The banks’ concern was that their recourse to the discount window, if it became known, might lead market participants to infer weaknesses/the so-called stigma problem.”

\(^6\)If the central bank lends at a “penalty rate,” then a borrowing bank sends a negative signal about its self-perceived credit worthiness were this to be revealed. See Furfine (2003).

\(^7\)See http://www.publications.parliament.uk/pa/cm200708/cmselect/cmtreasy/56/5602.htm.

We start the analysis by examining secured borrowing, which can be interpreted as the role of repo in the recent financial crisis. The equilibrium can (efficiently) be one in which no information is produced about the collateral backing the loans, which is either “good” or “bad.” Although some collateral is “bad” it can still be used to obtain loans because the loans are information-insensitive. This is so even though it is common knowledge that firms with “bad” collateral are receiving loans. This is efficient because the firms with bad collateral also receive loans and produce, increasing consumption. The underlying problem in the economy is a scarcity of good collateral (“safe debt” to back repo, for example). When good collateral is scarce, an efficient substitute is to avoid learning which collateral is good and which is bad. Good and bad collateral are pooled, which can result in a high enough perceived value of average collateral so that all firms can obtain loans.

The arrival of bad news, however, can cause households to want to produce information about the collateral. This is the crisis. Without central bank intervention, producing information about the collateral will result in a collapse of production and consumption as firms with bad collateral will not get loans (as in Gorton and Ordonez (2014)). The role of the central bank’s lending policy is to prevent information from being produced and, in this way, prevent the collapse of production and consumption. The central bank does not want the amount of collateral to suddenly shrink. How can the central bank do this? We show that confidence can be restored at a lower cost, not because of the specific loans to specific borrowers, but because the central bank’s lending creates an information externality.

The externality is created as follows. First, the lending is secret so that it is not known which firms borrowed from the central bank. Second, attracting the participation of unproductive borrowers with low quality collateral (moral hazard) induces borrowing firms to not want to reveal their identities, showing that their collateral is a government bond, due to the presence of stigma. Third, since secured funding with a government bond incurs the stigma cost, borrowers no longer use secured funding (repo) because it reveals that they borrowed from the central bank (when they offer a government bond as collateral). Finally, the benefits of producing information decreases: lenders producing information at a cost may waste their resources, finding
that the borrower went to the discount window. Together this raises the perceived average quality of collateral in the economy, so that households lend without producing information and there is no collapse of production and consumption.


The paper proceeds as follows. In Section 2 we specify the model, including the choice of information-insensitive on information-sensitive debt, and the role of a central bank. Section 3 concerns the equilibrium when the economy is in a crisis and the central bank discount window opens. First, we determine the equilibrium for a fixed collateral haircut, and second, the central bank maximizes welfare by choosing the haircut. Section 4 concludes.

2 Model

We study a two-period setting. The economy is composed of a government (central bank), a mass 1 of risk-neutral households with endowment $K$ of a numeraire good in the first period, a mass 1 of risk-neutral firms with managerial skills $E^*$ and a unit of land each, also in the first period. The numeraire cannot be stored.

A fraction $f$ of firms are entrepreneurs with a production function that transforms numeraire and managerial skills into more numeraire, stochastically, according to the following production function,

$$K' = \begin{cases} \min\{K, E^*\} & \text{with prob. } q \\ 0 & \text{with prob. } (1-q) \end{cases}$$
We assume \( qA > 1 \), so it is ex-ante optimal to finance the project up to an optimal scale \( K^* = E^* \). Since entrepreneurs have the investment opportunity but no numeraire to produce, they need to borrow numeraire from households. Even though we assume firms borrow directly from households to finance a productive investment, we can also think of the firm as a bank that borrows from households to channel funds to productive investments.

We assume that the realization of the project is not verifiable by private agents. Borrowers may use secured or unsecured loans. Secured loans are backed by a specific piece of collateral, the land. Unsecured loans are backed with an opaque portfolio of assets; here again just land. Later, when discussing crises, we will add government bonds as another asset that can be used as collateral. Unsecured loans are feasible if the borrower is a regulated bank, which is then examined and monitored, but at a cost. We assume that initially borrowers are not “banks” and borrow just using secured loans. It will become clear later, when we add government bonds, that secured borrowing is indeed the optimal choice during “normal” times.

The land that entrepreneurs hold can be used as collateral. A fraction \( \tilde{p} \) of entrepreneurs hold land that delivers numeraire \( C \) (good land) at the end of the period, while a fraction \( (1 - \tilde{p}) \) hold land that does not deliver any numeraire at the end of the period (bad land). We assume no agent knows the type of each unit of land, but households can learn about it at a cost \( \gamma \) in terms of numeraire.

The remaining fraction \( 1 - f \) of firms are non-productive. Even though they have managerial skills they do not have any productive investment opportunity available to use those skills. Even though these firms do not know the type of their land, they do know their land is good with probability \( p \), which is observable for each firm and drawn from a uniform distribution with support \([0, \tilde{p}]\). This distribution has an upper bound \( \tilde{p} \) and it is uniform for analytical simplicity, but neither of these two assumptions is critical for the results, just for the exposition.

In the second period households have \( Z \) units of labor supply, with linear disutility of labor. This labor can be used to produce \( Y = ZL^\alpha \). This implies that optimally \( L^* = (Z\alpha)^{\frac{1}{1-\alpha}} \) and \( Y^* = Z(Z\alpha)^{\frac{\alpha}{1-\alpha}} \).

\(^9\)The productivity \( Z \) is just a scalar that will determine the cost of distortions from interventions.
2.1 Optimal loan for a single entrepreneur

Consider an entrepreneur with land that is good with probability $p$. Loans that trigger information production about the land unit, called "information-sensitive" debt, are costly – since borrowers have to compensate lenders for their information cost $\gamma$. However, borrowers may still prefer to take loans that trigger information production because preventing information production by reducing the size of the loan may be too costly.

2.1.1 Information-Sensitive Debt

Lenders can learn the true value of the borrower’s land by using $\gamma$ of numeraire.\footnote{Assuming that borrowers can also produce information about the quality of land does not modify the main insights. See Gorton and Ordonez (2013) for such extension.} Assuming lenders are risk neutral and competitive, then:\footnote{Risk neutrality is without loss of generality since we will show that the loan is risk-free.}

$$p(qR_{IS} + (1-q)x_{IS}C - K) = \gamma,$$

where $K$ is the size of the loan, $R_{IS}$ is the face value of the debt and $x_{IS}$ is the fraction of land posted by the firm as collateral. The subscript $IS$ denotes an "information-sensitive" loan. In this setting debt is risk-free, that is firms will pay the same in the case of success or failure, this is $R_{IS} > x_{IS}C$. Otherwise, if $R_{IS} > x_{IS}C$, firms always default, handing over the collateral rather than repaying the debt. On the other hand, if $R_{IS} < x_{IS}C$ firms always obtain $C$ directly from holding the collateral and repay lenders $R_{IS}$. This condition pins down the fraction of collateral posted by a firm, as a function of $p$ and independent of $q$:

$$R_{IS} = x_{IS}C \Rightarrow x_{IS} = \frac{pK + \gamma}{pC} \leq 1.$$

Note that, since the interest rates and the fraction of collateral that has to be posted do not depend on $q$ because debt is risk-free, firms cannot signal their $q$ by offering to pay different interest rates. Intuitively, since collateral prevents default completely, loan terms cannot be used to signal the probability of default.

Expected total profits are then $p(qAK - x_{IS}C) + pC$. Then, substituting $x_{IS}$ in equilibrium, expected net profits (net of the land value $pC$) from information-sensitive debt
when lenders acquire information are:

\[ E(\pi|p, q, IS) = \max\{pK^*(qA - 1) - \gamma, 0\}. \]  

(1)

### 2.1.2 Information-Insensitive Debt

Another possibility is for entrepreneurs to borrow without triggering information acquisition. We assume information is private immediately after being obtained and becomes public at the end of the period. Still, the agent can credibly disclose his private information immediately if it is beneficial to do so. This introduces incentives for lenders to obtain information before the loan is negotiated and to take advantage of such private information before it becomes common knowledge.

Still it should be the case that lenders break even in equilibrium

\[ qR_{II} + (1 - q)px_{II}C = K, \]

subject to debt being risk-free, \( R_{II} = x_{II}pC. \) Then

\[ x_{II} = \frac{K}{pC} \leq 1. \]

For this contract to be information-insensitive, we have to guarantee that lenders do not have incentives to deviate and check the value of collateral privately. Lenders want to deviate because they can lend with beneficial contract provisions if the collateral is good, and not lend at all if the collateral is bad. Then, lenders want to deviate if the expected gains from acquiring information, evaluated at \( x_{II} \) and \( R_{II} \), are greater than the losses \( \gamma \) from acquiring information,

\[ p(qR_{II} + (1 - q)x_{II}C - K) > \gamma \quad \Rightarrow \quad (1 - p)(1 - q)K > \gamma. \]

More specifically, by acquiring information the lender only lends if the collateral is good, which happens with probability \( p \). If there is default, which occurs with probability \( (1 - q) \), the lender gets \( x_{II}C \) for collateral that was obtained at \( px_{II}C = K \), making a net gain of \( (1 - p)x_{II}C = (1 - p)\frac{K}{p} \). The condition that guarantees that lenders do not want to produce information when facing information-insensitive debt can then
be expressed in terms of the loan size,

\[ K < \frac{\gamma}{(1 - p)(1 - q)}. \]  

(2)

Hence, the loan size from information-insensitive debt is

\[ K(p|q, II) = \min\left\{ K^*, \frac{\gamma}{(1 - p)(1 - q)}, pC \right\} \]  

(3)

and, if feasible, expected profits, net of the land value \( pC \) are

\[ E(\pi|p, q, II) = K(p|q, II)(qA - 1). \]  

(4)

2.1.3 Optimal Financing and Information

Figure 1, which is the same as in Gorton and Ordonez (2014), shows the ex-ante expected profits in both regimes (information-sensitive and insensitive), net of the expected value of land, for each possible \( p \). The dotted blue line shows the net expected profits in the information-sensitive regime (equation 1), while the solid black function shows the net expected profits in the information-insensitive regime (equation 4). Entrepreneurs choose to raise funds forcing information acquisition about collateral in the information-sensitive \( IS \) range of beliefs \( p \) and avoiding information acquisition about the collateral in the information-insensitive \( II \) range of beliefs \( p \).

The cutoffs highlighted in Figure 1 are the same as in Gorton and Ordonez (2014) and are determined in the following way:

1. The cutoff \( p^H \) is the point below which firms have to reduce borrowing below the optimal scale \( K^* \) to prevent information acquisition:

\[ p^H = 1 - \frac{\gamma}{K^*(1 - q)}. \]  

(5)

2. The cutoff \( p^L_{II} \) comes from the point below which beliefs are so low that borrowing \( pC \) does not induce information acquisition.

\[ p^L_{II} = \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{\gamma}{C(1 - q)}}. \]  

(6)
3. The cutoff \( p_{IS}^L \) comes from the point below which borrowing that induces information acquisition does not compensate the cost of producing information:

\[
p_{IS}^L = \frac{\gamma}{K^*(qA - 1)}. \tag{7}
\]

4. Cutoffs \( p_{Ch}^C \) and \( p_{Cl}^C \) are obtained from equalizing the profit functions under information-sensitive and insensitive debt, and solving the quadratic equation:

\[
\gamma = \left[ pK^* - \frac{\gamma}{(1 - p)(1 - q)} \right] (qA - 1). \tag{8}
\]

We can summarize the expected loan sizes for different beliefs \( p \) that maximize ex-
pected profits (the upper envelope of the functions in Figure 1), by

\[
K(p) = \begin{cases} 
  K^* & \text{if } p^H < p \\
  \gamma p K^*/(1-p)(1-q) & \text{if } p^H < p < p^C_h \\
  pK^* - \gamma p^L_i/(qA-1) & \text{if } p^C_l < p < p^C_h \\
  pC & \text{if } p < p^L_{II}.
\end{cases}
\]

(9)

2.2 Crises and Interventions

For simplicity in the analysis we assume \( \hat{p} \) can only take one of two values. During normal times, \( \hat{p} = p^H \) such that \( p^H > p^H \) (first region above) and all entrepreneurs can obtain the optimal loan \( K^* \) without triggering information. During crises, \( \hat{p} = p^L \) such that \( p^C_l < p^L < p^C_h \) (third region above) and entrepreneurs prefer to borrow inducing information production, and then only a fraction \( p^L \) of entrepreneurs obtain the optimal loan \( K^* \) while the rest cannot produce.

We model crises as a shock that reduces the expected value of collateral and is also characterized by “chaos”. This chaos is captured by an idiosyncratic probability \( \varepsilon \), only observable to each borrower, that the collateral type will be revealed before the loan takes place. This probability follows a distribution \( \varepsilon \sim U[0, 1] \). The assumption of a uniform distribution is not relevant for the results, but useful for the exposition. An example of the chaos we have in mind is the sudden revelations of losses on bank portfolios during the financial crisis of 2007-2008. In September 2008, for instance, Morgan Stanley was considering a merger with Wachovia, viewed as a strong partner, because of growing doubts about Morgan Stanley’s future. See White and Sorkin (2008). Then Wachovia’s losses were revealed and within a month Wachovia Corp. was acquired by Wells Fargo (see Horwitz (2009)).

As a benchmark, in normal times the economy achieves the maximum potential consumption: agents consume \( \overline{K} \) and all entrepreneurs (a fraction \( f \) of the population) borrow on the optimal scale and produce an additional \( K^*(qA - 1) \) numeraire in the first period. They supply labor optimally to produce \( Y^* \) in the second period. In total,

\[
W_N = \overline{K} + f K^*(qA - 1) + Y^*.
\]
During crises, absent government intervention, the economy consumes

\[ W_C = \bar{K} + \bar{p}_L K^*(qA - 1) - \gamma + Y^*. \]

This is clearly smaller than consumption in normal times since \( K(p) \) is smaller than \( K^* \); not only because just a fraction \( p_L \) of entrepreneurs obtain a loan of size \( K^* \), but also because resources \( \gamma \) are spent on information production.

2.3 The Central Bank

We assume a Central Bank can intervene during crises with the following timing:\(^{12}\)

1. The Central Bank intervenes by opening a *discount window*. It exchanges government bonds for land, specifically \( B \) bonds per unit of land. In case a borrower goes to the discount window, the government takes possession of the land and there is no probability the land type is revealed to the market (i.e., \( \varepsilon \) becomes 0).

2. Borrowers choose to become regulated banks or not and then approach lenders. Either the loans will be specifically collateralized or loans will be backed by a portfolio (of land and government bonds) the composition of which is not observable, but is known to be overseen by government regulators.\(^ {13}\)

3. The government does not reveal who went to the discount window. Each borrower knows his own asset composition while lenders only infer the fraction \( y \) of entrepreneurs and the fraction \( y' \) of non-productive agents who went to the discount window in equilibrium. So they know the aggregate volume of government bonds in the hands of borrowers.

4. At the end of the first period successful borrowers who went to the discount window repay their loans using the proceeds from production and retain their bonds to redeem at the end of the second period. If those borrowers do not repay their loans, lenders take possession of the bonds and redeem them at the end of the second period. Successful borrowers who did not go to the discount

\(^{12}\) We use the terms “government” and “central bank” interchangeably.

\(^{13}\) This setting also captures the participation of investors and depositors in financial institutions depending on their beliefs about the portfolio composition of those financial interventions.
window repay lenders and retain the land and consume its output. If they defaulted on their loans, they hand the land over to the lenders who consume its output at the end of the first period.

5. The government uses the numeraire generated by the land in their possession at the end of the first period plus taxes collected in the second period to make the bond repayments.

Step 2 is the critical step. In equilibrium, borrowers will not seek to borrow in the market directly with collateral (via repo) because when they offer a government bond as collateral, they make themselves vulnerable to stigma. Therefore, borrowers choose to become regulated entities (as, for example, Goldman Sachs and Morgan Stanley did during the crisis). The ability to make the portfolio backing the loan go from one bond to a portfolio is the critical point. Again, this is what happened with private bank clearing houses prior to the Fed, when they pooled all their assets.

From this point of view, “becoming regulated” in our setting concretely means that lenders cannot distinguish “land” from government bonds in the bank’s portfolio, at a cost of government monitoring.

### 3 Recreating Confidence with Secrecy

We solve the Central Bank problem in two steps. First, we compute the equilibrium and total output in the economy as a function of the total volume of bonds $B$ introduced by the Central Bank through the discount windows. Then we allow the government to choose the optimal $B^*$ that maximizes welfare in equilibrium. In what follows we define relevant elements and describe important properties that are useful when solving for the equilibrium.

#### 3.1 Preliminaries

We assume the Central Bank cannot differentiate between productive and non-productive firms. As will be seen, the Central Bank has no incentive to produce information about land type. When the Central Bank offers a bond $B$ per unit land,
all non-productive agents with land value $pC < B$ will borrow from the discount window. The Central Bank exchanges bonds for land conditional on the firms actually borrowing, so the non-productive agents will borrow in the market. However, the non-productive agents just store the numeraire to repay the loan later and then strategically default on their loan from the Central Bank. They keep the government bonds and redeem them later for a profit.

In any equilibrium in which the Central Bank is successful in secretly maintaining the identities of which firms participated at the discount window and which did not, a loan obtained by an individual who went to the discount window is identical to a loan obtained by an individual who did not go to the discount window. Still these two strategies differ in terms of payoffs. The cost for an entrepreneur of not going to the discount window is the probability that the land type gets exogenously revealed before the loan is negotiated, in which case he loses the chance to obtain a loan if the land is bad $(\varepsilon(1 - p_L)K^*(qA - 1))$. The cost for an entrepreneur going to the discount window is the potential premium the government can charge for the bonds (the difference $p_LC - B$), or the “haircut” for the bonds defined by $1 - \frac{B}{p_LC}$. We use the terms “premium” and “haircut” interchangeably.

When a lender meets an entrepreneur, he expects the entrepreneur to have assets with an expected value that depends on the fraction $y$ of entrepreneurs going to the discount window and obtaining $B$ bonds,

$$yB + (1 - y)p_L C.$$

Note that entrepreneurs would have the same expected value of assets if instead all entrepreneurs participated at the discount window but they only trade a fraction $y$ of land in exchange for bonds. This would be consistent with an equilibrium where all entrepreneurs are homogeneous. However, in our case entrepreneurs are heterogeneous in $\varepsilon$, which implies that an entrepreneur would like to exchange all the land, or nothing, because of the possibility that information about land would be revealed.

Given this expectation of a given entrepreneur’s value of assets, lenders break even even when giving a loan $K$ if

$$\frac{f}{f + (1 - f)y'}[qR + (1 - q)x[yB + (1 - y)p_L C] + \frac{(1 - f)y'}{f + (1 - f)y'}R = K$$
where $R$ is the repayment required for a loan of $K$ and $x$ is the fraction of total assets pledged as collateral, where the composition of assets is non-observable to lenders.

In the presence of a discount window, there are more loans granted, both to entrepreneurs (a mass $f$) and to non-productive agents that go to the discount window and then borrow so as not to reveal themselves as nonproductive (a mass $(1 - f)y'$). The break-even condition above shows that the fraction of borrowers that is entrepreneurs $\left(\frac{f}{f + (1 - f)y'}\right)$ may default by the randomness of their production functions, while the rest of the borrowers, who are non-productive, always repay since they only borrow to pool with the entrepreneurs.

In equilibrium debt will be risk free. Otherwise, if the repayment is higher than the expected value of the assets, borrowers will always default and if the repayment is lower than the expected value of the assets, borrowers will always keep the assets and repay with their proceedings. This immediately implies that $R = x[yB + (1 - y)pLC]$. From the break-even condition we can then obtain the fraction of assets that are pledged in equilibrium,

$$x = \frac{K}{yB + (1 - y)pLC}$$

which is just the loan over the expected asset value, so $R = K$.

Now we can compute the incentives of lenders to privately acquire information about the land type. Recall that the cost of producing such private information is $\gamma$. The benefits of acquiring information are given by the following: With probability $\frac{f}{f + (1 - f)y'}$ the borrower is an entrepreneur. In this case with probability $p_L$ land is good and the lender still obtains $R$ with probability $q$, but with probability $1 - q$ the lender obtains a bond if the entrepreneur participated in the discount window or good land if the entrepreneur did not participate at the discount window. With probability $1 - p_L$ the lender finds out the land is bad and prefers not to lend since the expected value of the asset in case the firm defaults is lower than the loan. Finally, with probability $\frac{(1 - f)y'}{f + (1 - f)y'}$ the lender faces a non-productive firm that always repays the loan.

In this setting we assume that lenders can privately acquire information about the portfolio of the borrower (whether the borrower has bonds or not, and in case the borrower does not have bonds, whether the land is good quality or not) also at a cost $\gamma$. The expected payoff for lenders not acquiring information is just 0 by the break-even conditions in a contract that assumes no information acquisition.
In contrast, when lenders find out the asset type, with probability \( fy + (1 - f)y' \) the borrower has bonds so, lenders prefer to lend as if they did not find out, getting a payoff of:

\[
\frac{fy}{fy + (1 - f)y'} (qR + (1 - q)xB - K) + \frac{(1 - f)y'}{fy + (1 - f)y'} (R - K) - \gamma.
\]

When lenders find out, with probability \( \frac{f(1-y)}{f + (1-f)y'} (1 - p_L) \), that the borrower has bad land, lenders prefer not to lend, getting a payoff \(-\gamma\). Finally, when lenders find out, with probability \( \frac{f(1-y)}{f + (1-f)y'} p_L \), that the borrower has good land, lenders prefer to lend as if they did not know, getting a payoff of:

\[
qR + (1 - q)xC - K - \gamma.
\]

Considering that \( R = K \), this implies that there are no incentives to acquire information as long as:

\[
\left[ \frac{fy}{f + (1 - f)y'} \right] (qR + (1 - q)xB - K) + \left[ \frac{f(1-y)}{f + (1-f)y'} p_L \right] (qR + (1 - q)xC - K) - \gamma \leq 0.
\]

Rearranging

\[
[fy + f(1-y)p_L] (qK - K) + f(1-q)x(yB + (1-y)p_MC) \leq \gamma (f + (1-f)y').
\]

Since \( x(yB + (1-y)p_MC) = K \), there is no information acquisition as long as

\[
K \leq \frac{\gamma}{(1-q)(1-p_L)} \left[ \frac{f + (1-f)y'}{f(1-y')} \right].
\]  (10)

**Proposition 1** Information acquisition is less likely with Central Bank intervention when there are many non-productive agents (i.e., low \( f \)) and when there are many entrepreneurs and non-productive agents participating in the discount window (i.e., high \( y \) and \( y' \) respectively).

This Proposition is straightforward from comparing the condition for no information acquisition in the absence of intervention (equation 2) and in the presence of intervention (equation 10). It is also straightforward to check that condition (10) is relaxed with lower \( f \) and higher \( y \) and \( y' \).

It is useful to rewrite equation (10) in terms of the haircut. Participation at the discount window will depend on the bonds the central bank offers per unit of land.
Define

\[ B = \tilde{p}C, \]

such that a government choosing the discount \( \tilde{p} \) implicitly chooses how many bonds \( B \) to offer per unit of land. The haircut is \( 1 - \frac{B}{pL} \), as mentioned above. Here the central bank will choose \( \tilde{p} \), which is isomorphic to the haircut, so we will speak of choosing \( \tilde{p} \) as choosing the haircut. Since non-productive firms going to the discount window are those with \( B = \tilde{p}C > pC \), from the uniform distribution assumption,

\[ y' = \frac{\tilde{p}}{pL}. \]

Then, there is no information acquisition when

\[ K \leq \frac{\gamma}{(1 - q)(1 - pL)} \left[ 1 + \frac{(1 - f) \tilde{p}}{f \frac{pL}{G_1(\tilde{p})}} \right] \left[ 1 - \frac{1}{(1 - y(\tilde{p}))} \right]. \tag{11} \]

Note that without intervention (that is when \( \tilde{p} = 0 \), and then \( B = 0 \) and \( y = 0 \)), \( G_1 = G_2 = 1 \) and this is the same condition for no information acquisition as the condition derived in equation (2).

In contrast, with intervention (i.e., when \( \tilde{p} > 0 \)) there are fewer incentives to acquire information. On the one hand, \( G_1 \) captures the increase in expected costs of producing information about collateral because of the participation of non-productive agents in borrowing. Since the pool of borrowers has some non-productive agents with bonds, lenders may waste resources finding out about their assets because only if they participated in the discount window will they be willing to borrow. On the other hand, \( G_2 \) captures the increase in expected costs of producing information about collateral because of the introduction of government bonds among entrepreneurs. Since some entrepreneurs have government bonds instead of land, lenders may waste resources finding out their assets. This result confirms Proposition 1.

Finally, we define stigma. Since lenders know that a fraction \( \frac{(1-f)y'}{(1-f)y'+fy} \) of borrowers going to the discount window are non-productive, if lenders knew that a borrower had participated in the discount window, then they assign a probability \( \frac{(1-f)y'}{(1-f)y'+fy} \) that the firm is non-productive in the future. Since we are not constructing a fully dynamic setting, we do not model how this perception about the future gets translated into
lower expected payoffs. Instead, we will just assume that there is a stigma cost from revealing participation at the discount window and that such a cost is linear in the probability of being non-productive.

\[ \Xi(\tilde{p}) = \chi \frac{(1-f)y'(\tilde{p})}{(1-f)y'(\tilde{p}) + fy(\tilde{p})} \]

where \( \chi \) captures the strength of stigma in terms of how the perception of being non-productive translates into lower expected payoffs. In the subsequent analysis what matters is the size of the stigma cost, not its functional form. The stigma cost could as well be a constant but its linearity is convenient for the derivations.\(^{14}\)

3.2 Equilibrium

Now we solve for the equilibrium strategies of lenders (in terms of acquiring information) and of borrowers (in terms of participating at the discount window) as a function of the haircut \( B = \tilde{p}C \). Then we characterize welfare for each \( \tilde{p} \in [0,p_L] \) such that the government optimally chooses a discount (or haircut) \( \tilde{p}^* \) that maximizes welfare.

Define \( \sigma(\tilde{p}) \) to be the probability that lenders privately acquire information about the quality of land that belongs to a particular borrower, such that

\[ \sigma(\tilde{p}) = \begin{cases} 0 & \text{if } K < \frac{\gamma}{(1-q)(1-p_L)}G_1(\tilde{p})G_2(\tilde{p}) \\ [0,1] & \text{if } K = \frac{\gamma}{(1-q)(1-p_L)}G_1(\tilde{p})G_2(\tilde{p}) \\ 1 & \text{if } K > \frac{\gamma}{(1-q)(1-p_L)}G_1(\tilde{p})G_2(\tilde{p}) \end{cases} \]  

(12)

Define \( y(\tilde{p}) \) to be the probability that entrepreneurs go to the discount window, such that

\[ y(\tilde{p}) = \begin{cases} 0 & \text{if } E(\pi|nw) > E(\pi|w) \\ [0,1] & \text{if } E(\pi|nw) = E(\pi|w) \\ 1 & \text{if } E(\pi|nw) < E(\pi|w) \end{cases} \]  

(13)

where, we have defined \( L \equiv p_LK^*(qA-1) - \gamma \) to be the expected gains from borrowing

\(^{14}\)See Armantier et al. (2011), Ennis and Weinberg (2010) and Furfine (2003) for a discussion about the modeling of stigma costs.
when information about land is produced or revealed, and $H(K) \equiv K(qA - 1)$ to be the expected gains from borrowing $K$ without information acquisition about the land; and define $D(\bar{p}) \equiv (p_L - \bar{p})C$ to be the discount for entrepreneurs from participating at the discount window. Then:

$$E(\pi|nw) = \sigma(\bar{p})L + (1 - \sigma(\bar{p}))(1 - \varepsilon)H(K) + \varepsilon L$$

and

$$E(\pi|w) = \sigma(\bar{p})[H(K) - D(\bar{p}) - \Xi(\bar{p})] + (1 - \sigma(\bar{p}))[H(K) - D(\bar{p})].$$

The next three lemmas characterize the optimal strategies as a function of $\bar{p}$.

**Lemma 1** There exists a cutoff $\bar{p}_h < 1$ such that, for all $\bar{p} \in [\bar{p}_h, 1]$, lenders do not acquire information (that is, $\sigma(\bar{p}) = 0$) and more borrowers go to the discount windows when the discount is lower (that is, $y(\bar{p})$ increases with $\bar{p}$).

**Proof** We characterize the region of $\bar{p}$ for which it is an equilibrium that lenders do not acquire information about the borrower’s portfolio when the loan is $K$, i.e., $\sigma^*(\bar{p}) = 0$. As discussed, the condition for this to be an equilibrium is

$$K \leq \frac{\gamma}{(1 - q)(1 - p_L)} \left[ 1 + \frac{(1 - f) \bar{p}}{f} \right] \frac{1}{(1 - y(\bar{p}))}.$$ 

Evaluating $E(\pi|nw)$ and $E(\pi|w)$ at $\sigma(\bar{p}) = 0$, there is a marginal $\varepsilon^*(\bar{p})$ such that entrepreneurs $\varepsilon > \varepsilon^*(\bar{p})$ strictly prefer to go to the discount window instead of facing the probability of having information revealed about their land, where

$$\varepsilon^*(\bar{p}) = \frac{D(\bar{p})}{H(K) - L}.$$ 

Given our assumption of a uniform distribution of $\varepsilon$, this determines the fraction of entrepreneurs that go to the discount window

$$1 - y^*(\bar{p}) = \varepsilon^*(\bar{p}).$$

Since $K$ determines $1 - y^*(\bar{p})$, it enters both sides of the condition for information acquisition. A lower $K$ relaxes the constraint and reduces the incentives to acquire information, but at the same time reduces the number of entrepreneurs borrowing at
the discount window (increases $1 - y^*(\tilde{p})$ for a given $\tilde{p}$), increasing the incentives to acquire information. Replacing $y(\tilde{p})$ in the condition for no information acquisition to isolate $K$, the condition becomes

$$K \geq \frac{\Gamma(\tilde{p})L}{\Gamma(\tilde{p})(qA - 1) - 1}$$

where

$$\Gamma(\tilde{p}) \equiv \frac{\gamma}{(1 - q)(1 - p_L)} \frac{G_1(\tilde{p})}{D(\tilde{p})}.$$  

This condition is depicted in Figure 2, where the shaded region shows the feasible borrowing of $K \in [0, K^*]$ that does not induce information acquisition.\(^{15}\)

Figure 2: $K$ under which there is no information acquisition

As can be seen, the optimal loan $K^*$ can only be sustained in equilibrium when $\tilde{p} > \tilde{p}_h$, or when the discount $D(\tilde{p})$ is relatively low. At the extreme, when $\tilde{p} = p_L$ (and there is no discount), $\Gamma(p_L) = \infty$ and the condition for no information acquisition is $K \geq \frac{L}{qA - 1}$, which is clearly satisfied for $K^*$.

As $\tilde{p}$ declines (the discount increases), the level of $K$ that avoids information acquisi-

\(^{15}\)The figure assumes $\Gamma(\tilde{p} = 0)(qA - 1) > 1$, which implies that the asymptote of the function is defined at a negative $\tilde{p}$. Assuming otherwise just introduces an additional irrelevant region where $K < 0$ is needed to avoid information.
tion also increases. The reason is that $G_1(\tilde{p})$ declines (there are fewer non-productive firms borrowing at the discount window) and $D(\tilde{p})$ increases (fewer entrepreneurs borrowing at the discount window), reducing $\Gamma(\tilde{p})$, and making the condition to avoid information more stringent. If the condition is binding, an increase in $K$ is needed that increases $H(K)$, inducing a relatively large fraction of entrepreneurs to borrow at the discount window to discourage information acquisition.

Hence, it is feasible for firms to borrow $K^*$ without inducing information acquisition only for relatively high levels of $\tilde{p}$. In particular, borrowing $K^*$ does not induce information acquisition as long as $\tilde{p} \geq \tilde{p}_h$, where $\tilde{p}_h$ determines the discount that makes the condition for information acquisition hold with equality when $K = K^*$. More explicitly

$$\tilde{p}_h \equiv \frac{p_L - \gamma \frac{qA-1}{C}}{1 + \frac{1-f}{(1-q)(1-p_L)} \frac{qA-1}{C}} < p_L.$$  

Intuitively, when the discount is low ($\tilde{p}$ is large), many entrepreneurs choose to borrow at the discount window because the cost in terms of exchanging land for bonds at a low haircut more than compensates for the risk of information about the land being revealed. Given this, lenders do not have incentives to acquire information about the borrower’s asset portfolio.

**Lemma 2** There exists a cutoff $\tilde{p}_l < \tilde{p}_h$ such that, for all $\tilde{p} \in (\tilde{p}_l, \tilde{p}_h)$, lenders are more likely to acquire information and borrowers to go to the discount window when the discount increases (that is, both $\sigma(\tilde{p})$ and $y(\tilde{p})$ decrease with $\tilde{p}$).

**Proof**

What happens for $\tilde{p} < \tilde{p}_h$? As discussed, reducing $K$ to discourage information acquisition does not work, as it does in Gorton and Ordonez (2014). The reason for this counterintuitive result comes from the endogenous participation of firms at the discount window. By reducing $K$, the effect of reducing $y$ and $y'$ is stronger than the effect of reducing the loan size for information acquisition, thus increasing the incentives to acquire information. This implies that it is optimal to maintain the optimal loan $K^*$ and increase $y$ by allowing some information acquisition in equilibrium.
Then $\sigma(\tilde{p}) = 0$ is not an equilibrium when $\tilde{p} < \tilde{p}_h$ and $K^*$. However, $\sigma(\tilde{p}) = 1$ is not an equilibrium either when the stigma cost is not large. When $\sigma(\tilde{p}_h) = 0$, at $\tilde{p}_h$,

$$\varepsilon^*(\tilde{p}_h)[H(K^*) - L] = D(\tilde{p}_h).$$

When $\sigma(\tilde{p}_h) = 1$, $E(\pi|nw) = L$ and $E(\pi|w) = H(K^*) - D(\tilde{p}_h) - \Xi(\tilde{p}_h)$. If stigma is not large $E(\pi|w) > E(\pi|nw)$ when evaluated at $\tilde{p}_h$, which would imply $y(\tilde{p})|\sigma(\tilde{p}) = 1) = 1$ and there would be no incentives to acquire information.

Intuitively, at $\tilde{p}$ slightly lower than $\tilde{p}_h$, the discount is large enough such that, if lenders do not acquire information, then entrepreneurs’ participation is lower at the discount window, but then this induces lenders to acquire information. In contrast, if lenders do acquire information, then entrepreneurs prefer to participate at the discount window, but then this induces lenders not to acquire information. Hence, when the stigma cost is not large, there is a range of $\tilde{p} < \tilde{p}_h$ where there is no equilibrium in pure strategies for lenders.

What level of $y(\tilde{p})$ makes lenders indifferent between generating information or not when the loan is $K^*$?

$$K^* = \frac{\gamma}{(1-q)(1-p_L)}G_1(\tilde{p}) \left[ \frac{1}{(1-y(\tilde{p}))} \right].$$

From this equation is clear that there is a function $y^* = g(\tilde{p})$ that shows the $y$ level that makes this equation hold with equality for each level of $\tilde{p}$. Using the implicit function theorem,

$$\frac{d(1-y)}{d\tilde{p}} = -\frac{\partial F(\tilde{p})}{\partial \sigma} = -\frac{(1-f)(1-q)(1-p_L)}{fp_L\gamma G_1(\tilde{p})} < 0.$$ 

This result immediately implies that $y(\tilde{p})$ decreases with $\tilde{p}$ in the range of $\tilde{p}$ that is sustained by a mixed strategy equilibrium for lenders.

What is the $\sigma^*(\tilde{p})$ that sustains $y^* = g(\tilde{p})$ for each $\tilde{p}$ in such a range? Since $y^* = g(\tilde{p}) = 1 - \varepsilon^*$, then $\sigma^*$ should be determined by the randomization of information acquisition by lenders that effectively induces a fraction $y^*$ of entrepreneurs to participate at the discount window. Since $\varepsilon^*$ is the $\varepsilon$ at which entrepreneurs are indifferent between participating of the discount window or not in the equilibrium that induces $y^*$, then
\(\sigma^*\) is pinned down by
\[
\sigma^* L + (1 - \sigma^*)[(1 - \varepsilon^*)H(K^*) + \varepsilon^* L] = \sigma^* [H(K^*) - D - \Xi] + (1 - \sigma^*)[H(K^*) - D].
\]

Then
\[
\sigma^*(\tilde{p}) = \frac{D(\tilde{p}) - (1 - y^*(\tilde{p}))(H(K^*) - L)}{y^*(\tilde{p})(H(K^*) - L) - \Xi(\tilde{p})}.
\]

Taking the derivative of \(\sigma^*(\tilde{p})\) with respect to \(\tilde{p}\),
\[
\frac{\partial \sigma^*(\tilde{p})}{\partial \tilde{p}} = - \left[ \frac{C}{y^*(H(K^*) - L)} + \frac{\partial(1 - y^*)}{\partial \tilde{p}} \right] < 0.
\]

Finally, we define \(\tilde{p}_l\) such that lenders always prefer to acquire information about the borrower’s portfolio, \(\sigma(\tilde{p}) = 1\). From the definition above, this happens when
\[
D(\tilde{p}) - (1 - y^*(\tilde{p}))(H(K^*) - L) \geq y^*(\tilde{p})(H(K^*) - L) - \Xi(\tilde{p})
\]

or
\[
D(\tilde{p}) \geq H(K^*) - L - \Xi(\tilde{p}).
\]

When \(\chi\) is small enough, then it is clear that, for all \(\tilde{p} < \tilde{p}_l\) it is the case that \(\sigma^*(\tilde{p}) = 1\).

By construction this implies that \(E(\pi|nw) > E(\pi|w)\) and \(y^* = 0\). Since lenders are indifferent between producing information or not at a positive \(y^*\), they strictly prefer to produce information when \(y^* = 0\), then \(\sigma^* = 1\) is an equilibrium.

Is \(\tilde{p}_l > 0\)? Not necessarily. When \(\tilde{p} = 0\), \(\Xi(\tilde{p} = 0) = 0\). Then, if
\[
p_L C < H(K^*) - L,
\]
the equilibrium probability of information acquisition by lenders is \(\sigma^* < 1\).

Q.E.D.

When the discount is in the intermediate range, the equilibrium cannot involve pure strategies by lenders. Since participation at the discount window when lenders do not acquire information is low, lenders have incentives to acquire information. In contrast, if lenders do acquire information, borrowers have more incentives to borrow at the discount window, which discourages information acquisition. Hence, lenders have to be indifferent between producing information or not. As the discount in-
creases in this range, borrowers incentives to participate in the discount window have
to be compensated for an increase in the probability lenders acquire information.
Finally, the next Lemma just complements the previous two, conditional on $\tilde{p}_l > 0$.

**Lemma 3** If $\tilde{p}_l > 0$, for all $\tilde{p} \in [0, \tilde{p}_l]$, lenders always acquire information and borrowers never go to the discount windows (this is, $\sigma(\tilde{p}) = 1$ and $y(\tilde{p}) = 0$).

The proof comes from the construction of $\tilde{p}_l > 0$ in the proof of Lemma 2. Intuitively, when the discount is large enough, when lenders acquire information for sure borrowers prefer not to borrow at the discount window. This reinforces the incentives for lenders to acquire information.

Finally, it is straightforward to check that no agent would like to deviate from the opaque policy of the government in terms of disclosing its participation, or lack thereof, at the discount window. Entrepreneurs participating at the discount window do not want to reveal their participation, otherwise they have to pay the stigma cost without getting any benefit (in case the lender does not acquire information they receive $K^*$ and in case the lender acquires information they also receive $K^*$). Similarly entrepreneurs not participating at the discount window do not want to reveal their lack of participation, otherwise they have a higher chance that their land is monitored because lenders will always try to get information about the quality of their land once they know they hold land as collateral. Finally, non-productive firms going to the discount window do not have incentives to reveal their participation since otherwise they do not get the benefits of acquiring bonds at a profit.

The equilibrium strategies derived in Lemmas 1-3 are illustrated in Figure 3. On the horizontal axis we show the discount $D(\tilde{p})$, the red solid function shows the equilibrium probability that lenders acquire information, $\sigma(\tilde{p})$, and the black dashed function shows the equilibrium probability that borrowers participate in the discount window, $y(\tilde{p})$. The region $[0, D(\tilde{p}_h)]$ shows the equilibrium strategies in Lemma 1, the region $[D(\tilde{p}_h), D(\tilde{p}_l)]$ the equilibrium strategies in Lemma 2 and the region $[D(\tilde{p}_l), p_L C]$ the equilibrium strategies in Lemma 3.

### 3.3 Welfare

Given the equilibrium strategies for each $\tilde{p}$, we can compute the total production (or welfare in our setting) for each $\tilde{p}$. First, we need to compute the distortions in terms
of taxation that are involved for each $\tilde{p}$. We have been examining the case where a firm goes to the discount window posts its land as collateral and obtains a Treasury bond.$^{16}$ Treasury bonds must be paid off at maturity. In this subsection we first treat the central bank and the government (fiscal authority) as a single consolidated agent (though this may be implicit). In this case, the choice of haircut by the central bank can have implications for fiscal policy.

Let $T(\tilde{p})$ be the total promised bond repayments by the government minus the expected value of collateral obtained by the government via the discount window:

$$T(\tilde{p}) = [(fy + (1 - f)y')\tilde{p}C] - \left[ fyPLC + (1 - f)y'\frac{\tilde{p}C}{2} \right]$$

or,

$$T(\tilde{p}) = \left[ (1 - f)\frac{\tilde{p}}{PL} - fy(\tilde{p})(PL - \tilde{p}) \right] C.$$

If $T(\tilde{p}) > 0$, the government needs to raise resources by taxing production in the final period, $\tau Y = T$. However, this is distortionary because labor supply conditional on a tax rate $\tau$ is $L(\tau) = (Z\alpha(1 - \tau))^{\frac{1}{1-\alpha}}$. Then, the tax rate needed to raise $T$ is the one

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$^{16}$Borrowing a Treasury bond, posting land as collateral, corresponds to the Feds Term Securities Lending Facility; see Hrung and Seligman (2011). The central bank could also lend money, using land as collateral. These are observationally equivalent under our assumption that the firm/banks are now regulated.
that solves
\[ T(\tilde{p}) = \tau^*(\tilde{p})Y(\tau^*(\tilde{p})) = \tau^*(\tilde{p})(Z\alpha(1 - \tau^*(\tilde{p})))^{\frac{\alpha}{1-\alpha}}, \]
and then
\[ Y(\tilde{p}) = Z(Z\alpha(1 - \tau^*(\tilde{p})))^{\frac{\alpha}{1-\alpha}}. \]
This policy is only feasible when there is enough production at the end of the period to pay for these taxes. In other words, the discount window promises are only feasible when the output under the tax rate that maximizes resources (τ) are such that \( Y(\tau) = \alpha^{\frac{\alpha}{1-\alpha}} > T(\tilde{p}) \).

How does taxation depend on the discount \( \tilde{p} \)? Taking the derivative of total required taxation with respect to \( \tilde{p} \)
\[ \frac{\partial T(\tilde{p})}{\partial \tilde{p}} = \left[ fy + (1 - f)\frac{\tilde{p}}{p_L} - f(p_L - \tilde{p})\frac{\partial y(\tilde{p})}{\partial \tilde{p}} \right]. \]
Given Proposition 1, this derivative is guaranteed to be positive only for \( \tilde{p} < \tilde{p}_h \), since in that range \( \frac{\partial y(\tilde{p})}{\partial \tilde{p}} \leq 0 \).

Total output for each \( \tilde{p} \) is then
\[ W(\tilde{p}) = f[(1 - \sigma^*(\tilde{p})(1 - y(\tilde{p})))]H(K^*) + \sigma^*(\tilde{p})(1 - y(\tilde{p}))L] + Z(Z\alpha(1 - \tau^*(\tilde{p})))^{\frac{\alpha}{1-\alpha}}. \]

Total production in the economy is purely a function of the discount that the government introduces for its bonds, \( \tilde{p} \). This discount affects both the fraction of individuals participating at the discount window, \( y^*(\tilde{p}) \), the information production in the economy, \( \sigma^*(\tilde{p}) \) and the implied distortionary taxation \( \tau^*(\tilde{p}) \).

Recall that the first best output (the one in “normal” times) is
\[ W^{fb} = fH(K^*) + Z(Z\alpha)^{\frac{\alpha}{1-\alpha}}. \]
In “crisis” times, when there is no intervention (\( ni \)), total output is
\[ W^{ni} = fL + Z(Z\alpha)^{\frac{\alpha}{1-\alpha}}. \]
Assuming \( \tilde{p}_l > 0 \), then when \( \tilde{p} = 0, y = 0, \sigma = 1 \) and \( \tau = 0 \), this implements the \( W^{ni} \) allocation. At the other extreme, when \( \tilde{p} = p_L, y = 1, y' = 1 \), and \( \sigma = 0 \), it is possible
to implement the optimal allocation from entrepreneurs but at the cost of attracting non-productive agents and having to introduce distortionary taxation.

These equations completely characterize the equilibrium. There are certain properties that are summarized in the following proposition.

**Lemma 4** The optimal discount is always positive, that is \( \tilde{p}^* < \tilde{p}_l \leq p_L \).

**Proof** Take the derivative of \( W \) with respect to \( \tilde{p} \) evaluated at \( \tilde{p} = p_L \), where we know \( \sigma^*(\tilde{p}) \) and \( \frac{\partial \sigma^*(\tilde{p})}{\partial \tilde{p}} = 0 \).

\[
\frac{\partial W}{\partial \tilde{p}} \bigg|_{\tilde{p}=p_L} = -\frac{\alpha}{1-\alpha} Y(\tau^*(p_L)) \left( Z \alpha \frac{\partial \tau^*(\tilde{p})}{\partial \tilde{p}} \right) < 0
\]

since the derivative of \( T(\tilde{p}) \) with respect to \( \tilde{p} \) when evaluated at \( \tilde{p} = p_L \) is

\[
\frac{\partial T(\tilde{p})}{\partial \tilde{p}} \bigg|_{\tilde{p}=p_L} = 1
\]

Since welfare evaluated at \( W(\tilde{p}_l) > W(p_L) \), then \( \tilde{p}^* < \tilde{p}_l \leq p_L \).

The previous Lemma characterizes the optimal level of the discount. Even though this optimal level is positive and restores confidence, in the sense of avoiding information acquisition about all collateral, it is not clear whether the optimal discount restores confidence completely, in the sense of avoiding information acquisition completely. The next Proposition shows the condition under which the optimal intervention restores confidence completely.

**Proposition 2** Defining confidence as the probability that lenders do not acquire information, \((1 - \sigma(\tilde{p}))\), it is optimal to recreate confidence completely (that is, \( \sigma(\tilde{p}^*) = 0 \)) if \( \tilde{p}^* > \tilde{p}_h \) and it is optimal to recreate confidence partially (that is, \( \sigma(\tilde{p}^*) > 0 \)) if \( \tilde{p}_l > \tilde{p}^* < \tilde{p}_h \).

The next figure shows an example of the shape of total production. It is not guaranteed that this function is single peaked. However, as Lemma 4 shows, the function is increasing when evaluated at \( \tilde{p} = p_L \) (or \( D = 0 \)), so it has a discontinuous decline at \( \tilde{p}_l \) and increases with \( D \) in the range characterized by \([0, \tilde{p}_l]\).

If the central bank is independent of the fiscal authority, then the analysis is the same as above if the fiscal authority is willing to support the central bank. If not, the central bank can still choose the minimum discount that creates the desired perceived
average collateral quality, but it may have to absorb losses. A central bank can have negative equity. Alternatively, if the central bank wants to avoid expected losses, perhaps for political reasons, then its choice of discount is bounded and it may not be able to lend a sufficient amount to prevent the crisis.

4 Conclusions

The central bank aims to re-create confidence during a crisis, so that output does not fall. A financial crisis is an event in which information-insensitive collateral is on the verge of becoming information-sensitive, an event in which agents question the value of collateral (asset-backed securities in the recent financial crisis; loan portfolios in the pre-Fed bank runs) and want their cash back. Since good collateral is pooled with bad collateral, if information about collateral quality is produced it will result in a decline in production and consumption because firms with bad collateral will not be able to borrow. The central bank wants to prevent this from happening. It wants to prevent information from being produced.

How do financial crises end? In this paper we amend Bagehot’s rule to include secrecy, three kinds of secrecy in particular: (1) the central bank must lend in secret,
hiding the identities of the borrowers; (2) the borrowers must not reveal their identities; and (3) borrowers must have a way to hide the central bank borrowing, e.g., in a portfolio. This secrecy produces an information externality. Lenders only know the average quality of bank assets in the economy, leading to lending to firms which would not otherwise occur. This secrecy corresponds to what we have observed in crises.

Anonymous and secret central bank lending is important in the quest to restore confidence (information-insensitivity). Bernanke (2009b): “Releasing the names of [the borrowing] institutions in real-time, in the midst of the financial crisis, would have seriously undermined the effectiveness of the emergency lending and the confidence of investors and borrowers” (p. 1). Transparency would inhibit the desired information externality, which is at the root of ending a financial crisis.

Borrowers from the central bank do not want to reveal that they borrowed due to stigma. Stigma is costly to a borrower if their borrowing is revealed. So, on the one hand, the identity of borrowers needs to be kept secret but, on the other hand, stigma has an important role as a threat. It keeps borrowers from revealing that they now have good collateral (a U.S. Treasury bill, for example) which could result in more favorable lending terms. Borrowers must not want to make such a revelation because it entails a future cost. So, stigma works in this sense, but it is not observed in equilibrium. The central bank would like to avoid public stigma, while desiring it to be a real cost off-equilibrium if a bank’s borrowing is revealed.

Firm/banks cannot finance using repo during a crisis because that does not allow them to pool their borrowings from the central bank with the rest of their portfolio. The suspect assets, asset-backed securities in the recent crisis, must be financed by claims on portfolios. In the model this occurs when the firm/banks become regulated. Haircuts on collateral serve to determine the amount of lending the central bank will do. This can involve a trade-off if the lending has to be financed by distortionary taxation. In our setting central bank haircuts are unrelated to the quality of collateral, i.e., to expected losses on the collateral. Indeed, in a financial crisis, from the beginning of central banking, the concern has not been the quality of the collateral, which likely cannot be determined in a crisis in any case. For example, Jeremiah Harman, a Director of the Bank of England, speaking of the Panic of 1832 in England, said that the Bank lent money “by every possible means and in modes we have never adopted before; we took in stock on security, we purchased Exchequer bills, we made advances
on Exchequer bills, we not only discounted outright, but we made advances on the deposit of bills of exchange to an immense amount, in short by every possible means consistent with the safety of the Bank, and we were not on some occasions over nice” (Hawtrey (1932), p. 187). Our model is consistent with this. The central bank wants to supply a sufficient amount of liquidity to recreate confidence, but may be constrained in this because of concerns about its “safety,” which means fiscal constraints (or political constraints).

Hawtrey (1932) also observed in his book The Art of Central Banking “that the facilities offered by the central bank as the lender of last resort may be abused by banks whose position has become impaired” (p. 191). In our setting this is the moral hazard that the unproductive firms will borrow from the central bank. Aside from the practical problem of determining which banks are “solvent” and which are not, in our setting it is beneficial to lend to the unproductive firms because that makes the benefit of producing information lower, allowing less central bank lending to create the required perceived average collateral quality.

In a crisis, the goal of the central bank is to prevent information from being produced about backing portfolios. This corresponds to attempting to maintain the opacity of bank portfolios or of asset-backed securities. It can accomplish this only by improving the mix of bonds in firm/banks portfolios, without revealing which particular banks have borrowed at the discount window or other lending facility. In effect, financial crises are ended when market participants believe that the asset quality backing financial claims (short-term bank debt) is expected to be of sufficient quality that there is no need to produce information to verify that. This can happen if the central bank can credibly and secretly inject good collateral into the economy to make these expectations rational.
References


