Is Bigger Better? Investing in Reputation

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Abstract

We develop a model in which the value of a firm's reputation for quality increases gradually over time. In our model, a firm's ability to deliver high quality at any given period depends on how much it invests in quality. This investment is the firm's private information. Also, a firm's current quality is unobservable. Thus the only observable is a firm's past performance - the realized quality of the products it delivered. We assume that information about a firm's past performance diffuses only gradually in the market. Thus, the longer a firm has been delivering high-quality products, the larger the number of potential customers which are aware of it. We show that in equilibrium, the firm's investment in quality increases over time, as its reputation - the number of consumers who are aware of its history - increases. This is because the greater its reputation, the more it has to lose from tarnishing it by under-investing and, conversely, the more it has to gain from maintaining it. This is recognized by rational consumers. Therefore, older - and hence larger firms - command higher prices as quality premia. This in turn feeds back into firms' investment incentives: the fact that they are able to command higher prices motivates older and larger firms to invest

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still more. So the older and larger a firm is, the more valuable an asset its reputation is.

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1. Introduction

A firm's reputation is often its most valuable asset. For example, if a corporate giant like Coca Cola, McDonald's or Nike were stripped of its name - and the reputational resources associated with it - its value would be reduced to only a small fraction of what it is today. The importance of a firm's name and reputation for its balance sheet dictates that considerable managerial resources be devoted to establishing, maintaining and enhancing the value of the firm's name and reputation. The goal of this paper is to develop a modeling framework in which a firm perceives its reputation as a capital asset whose value is established, maintained and enhanced through a process of active and continuous investment.

Consider a market for a product or service whose quality is unobservable at the time of purchase. Consequently, consumers' purchasing decisions are based on what they know about a firm's past performance - the realized quality of the products it delivered in the past. Our model has two main ingredients. First, we assume that the ability to produce high-quality products requires continuous investment in quality. Second, we assume that information about past performance diffuses only gradually in the market. Hence, reputation formation is a gradual process.

Because building a reputation takes time, an older and more established firm, with a longer observable track record of success, has more to gain from maintaining its reputation for quality and, conversely, has more to lose from tarnishing it. It therefore invests more and hence delivers higher expected quality than a younger firm. Consequently, consumers associate market tenure (firm age) with quality and are willing to pay older firms more. And the fact that an older firm commands higher prices further increases its incentive to invest.

The association between market tenure and/or firm size and perceived quality predicted by our model seems to fit the observation that producers of high-quality products with a long history in the market tend to emphasize this characteristic in their advertising. For example, the New York Times heralds the year in which it was founded on its front page and a quality beer like Stella Artois has the year in which the brand was established on the label. Similarly, advertising often seems to signal quality through market share. For example, the Hertz ad: "We're number one."

Our account of reputation formation contrasts with the standard reputation paradigm, e.g., Klein and Leffler (1981) or Shapiro (1983), in which establishing a reputation is an instantaneous event rather than a dynamic process. A more closely related modeling approach is Diamond's (1989) credit market model in which borrowers (who have not previously defaulted) have a lower probability of default, and consequently pay lower interest charges, the longer their credit history. Thus the appreciation of the value of reputational assets over time is a theme shared by both models. However, both the logic and some empirical implications are very different. Here, the value of reputation appreciates over time because information diffuses only gradually. By contrast, in Diamond's model the role of credit history is to enable lenders to statistically distinguish between bad and good borrower "types," à la Kreps and Wilson (1983). More significantly, in Diamond's model, reputation formation is a passive process; borrowers whose earlier risky investments turn out favorably adopt a more conservative investment stance. By contrast, our analysis emphasizes the role of proactive investment in reputation formation. Finally, Diamond does not consider the link between reputation and firm size, whereas in our model better reputation is associated with a larger size. A recent paper on reputation is Tadelis (1999). Tadelis shows that a firm's name, which stands for the firm's reputation, is a tradeable asset, which commands a positive price when the firm changes ownership. However, Tadelis does not consider a process of establishing and enhancing reputation, which is our focus here.

The rest of the paper is organized as follows. In the next Section we set up the model, and in Section 3 we analyze it. Section 4 discusses the equilibria we derive, and Section 5 discusses our underlying assumptions.

2. The Model

Time is discrete and the horizon is infinite. There is a continuum of firms and consumers. The measure of consumers is one, while the measure of firms is endogenous, and yet to be determined.

There are two product-quality levels, high and low. A consumer's utility from one unit of the high-quality product is $V > 0$, and her utility from one unit of the low-quality product is zero. Each consumer lives one period and demands either one or zero units.

Firms are differentiated with respect to their ability to produce a high-quality product. A low-ability firm only produces a low-quality product, while a high-ability firm only produces a high-quality product.[1] A low-ability firm can not become high-ability. But, a high-ability firm can deteriorate and become low-ability. For example, the firm may lose key management or key employees, or allow its 'corporate culture' to deteriorate. And once a firm becomes low-ability, it can not become high-ability again. A high-ability firm can reduce the probability of becoming low-ability by investing in ability. Specifically, at the beginning of each period, a high-ability firm invests $x \in [0, \overline{x}]$, where $\overline{x} < \infty$. It then remains high-ability and produces a high-quality product that period with probability $f(x)$; it becomes a low-ability firm and produces a low-quality product with probability $1 - f(x)$. $f(\cdot)$ is strictly concave, strictly increasing and takes values in $[0, 1)$. In particular, $f(\overline{x}) < 1$; thus, although a high-ability firm can reduce the likelihood of becoming low-ability, it cannot eliminate this possibility altogether.[2] Apart

[1] Thus the informational issue is adverse selection rather than moral hazard

[2] This specification assumes that the effectiveness of investment in remaining high-ability is independent of the length of time, τ, during which the firm has been high-ability. If f increases in τ, which is more natural in some applications, the result we derive below still holds. We

from x, which is independent of the scale of the firm's output, the firm's variable cost of production is zero (it can be any constant $< V$).

The same is true for a new entrant; if it invests x, it is "born" as a high-ability firm with probability $f(x)$ and as a low-ability firm with probability $1 - f(x)$. To be operative at any period, a firm must pay a non-recoverable fixed cost of $F > 0$ at the beginning of the period. This cost can be saved by exiting.

A firm's investment, x, is its own private information. The quality which is about to be realized as a result of investing x is known neither to the firm nor to consumers. However, once the product is purchased (if at all) and used its realized quality, which is the same for all units sold, becomes known to the firm and to consumers who bought this product, also referred to as the firm's customers. Thus, last period's quality is observed by the firm and its customers. Last period's product-quality can also be ascertained by consumers who find the firm by searching, where the meaning of "searching" is explained below.

Because a firm's *current* quality and its investment are unobservable, we define a **firm's type** by the pair (t, q), where:

(i) q is its realized product-quality at the *preceding* period. $q = low$ if the firm delivered a low-quality product last period, in which case the firm is known (to itself, last period's customers and consumers who search the firm) to be a low-ability firm from this point onwards. $q = high$ if the firm delivered a high-quality product last period.

And,

(ii) t is the firm's age - the time elapsed since the firm entered the market.

The reason age is relevant is that, as shall be seen below, it determines the firm's customer base - the number of customers to which the firm has access. And that variable determines the firm's investment level, and hence its expected quality, as discussed below.

discuss this possibility in Section 5.

At each period a new generation of risk-neutral consumers of measure 1 enters the market. Each consumer lives one period. When they enter the market, consumers know only the distribution of firm types but not which firm is what type. Upon entering the market, a new consumer costlessly learns the type of one randomly selected firm with probability $1-\delta$, and with probability δ does not learn about any firm. For example, a new consumer meets (with probability $1-\delta$) an old consumer of the previous generation who tells her the type of firm from which she bought at the preceding period.[3] We call this firm (if any) the new consumer's reputation firm, and if the consumer buys from this firm we call her a **reputation customer**.

A consumer who has a reputation firm can either buy from this firm, or she can search a randomly selected firm. In the latter case the consumer is called a **search customer**. We assume that when a consumer finds a firm by searching, she learns the firm's type. A consumer who does not have a reputation firm is necessarily a search customer. Thus, whether the consumer is a reputation customer or a search customer she knows the firm's type and can base its purchase decision on this information. For simplicity, we assume that, because of high search costs or time constraints, a consumer can only search once. Also, if a consumer searches, she must either buy from its search firm or leave the market without buying - she can not go back to her reputation firm (if she has one) after searching. In this environment consumers search either if they have no reputation firm, or, if the average surplus from searching exceeds the surplus they get from their reputation firm. We assume searching consumers are divided uniformly across firms, i.e., each firm receives the same number of search customers.

When a consumer is matched with a firm, the two are in a short-term bilateral monopoly situation. It is natural in this situation to assume that prices are

[3] $1-\delta$ measures how quickly information diffuses in the market, or, alternatively, δ measure how quickly information "decays". When $1-\delta$ is large, information diffuses quickly and, as a result, a successful firm is able to quickly build up a clentele base (see below).

determined through bargaining. Accordingly, if the consumer and the firm believe the expected quality of the firm's product to be Q, the consumer pays γQ and gets a consumer surplus of $(1-\gamma)Q$, where γ is an exogenously fixed parameter $\in (0,1)$. γ represents the firm's bargaining skill; the bigger is γ, the larger is the fraction of the trading surplus that the firm captures.[4]

Recall that consumers only observe a firm's type, not its actual investment. We denote consumers' belief about a firm's investment level, as a function of its type, as $x^e(t,q)$, where e stands for expectation. This fully determines consumers' belief about the expected quality of this firm's product, $f(x^e(t,q))V$.

A consumer's decision rule is characterized by an acceptance set, A, such that a consumer buys from her reputation firm (if she has one) of type (t,q) if and only if $(t,q) \in A$, and searches otherwise.

A firm of type (t,q)'s decision rule consists of: (i) an exit rule which determines whether the firm remains operative by paying F or exits the market. If a firm exits the market its outside option is worth zero. And, (ii) if the firm remains operative, an investment rule, $x(t,q)$, which determines how much it invests in remaining high-ability.

The number of firms in the market is determined by free entry. This requires that a new firm's (maximized) expected discounted profit from entering the market, i.e., becoming operative by paying F is zero.

We seek a **rational-expectations steady-state equilibrium**. Such an equilibrium is characterized by:

- The measure of new entrants per period, e.

[4] A concrete interpretation is that, upon meeting, the firm or the consumer is randomly chosen to make a take-it-or leave-it offer to the other, and that γ is the probability with which the firm is chosen to make the offer. Under this interpretation γQ is the ex-ante (i.e., before the identity of the proposer is known) expected surplus of the firm. γ is a measure of bargaining skill in this situation because whoever makes the offer extracts the full surplus, and γ measures how likely the firm is to seize this opportunity.

- An investment rule - $x(t, q)$ - and an exit rule for a firm of each type.
- An acceptance set, A, for consumers and consumers' investment expectations, $x^e(t, q)$.

Such that:

1. Firms decisions maximize future discounted profits, and consumers' decisions maximize utility.
2. Consumers' expectations are realized; $x^e(t, q) = x(t, q)$.
3. Entrants earn zero expected profit, ex ante.
4. There is a constant measure of firms of each type.

3. Analysis

We first prove that in *any* equilibrium, older firms charge higher prices, invest more in remaining high-ability, deliver, on average, higher-quality products, have more customers and enjoy higher profits. After we establish that, we prove the non-vacuousness of these properties, i.e., we prove the existence of an equilibrium. We start out with the following result.

Lemma 3.1. *In any equilibrium, (t, low)-firms exit at once for any t.*

Proof. Since all potential buyers know a firm's type before buying from it, they pay zero for the product of a low-ability firm. Hence low-ability firms earn a period profit of $-F$ while in the market and should optimally exit. ■

Since, by the preceding Lemma, only $(t, high)$-firms are operative, a firm's type will be denoted only by its age, t. Likewise from this point onwards we abbreviate and write x_t and x_t^e.

Let ν be the measure of firms of type $t \in A$. Then there are $(1-\delta)\nu$ reputation consumers and $Y \equiv (1 - \nu)(1 - \delta) + \delta$ searching consumers. The measure of searching consumers per firm is $y \equiv Y/n$.

In equilibrium firms know x_t^e and, hence, the expected utility to a consumer from a unit sold by a firm of type t, $f(x_t^e)V$, is common knowledge between the firm and its potential customers. Thus the price a type-t firm receives for each unit it sells is $p_t = \gamma f(x_t^e)V$. This price is independent of the firm's actual investment x_t; it depends solely on what consumers' *believe* its investment to be, x_t^e.

A firm does not have access to all consumers. It can only sell to consumers who learn about it either by reputation or by search. We denote by z_t the number of customers to which a type-t firm has access and refer to this variable as the firm's **customer base**. Note that for $t \in A$, $z_t = (1-\delta)z_{t-1} + y$, and for $t \notin A$, $z_t = y$.

Let R_t be the maximized value of a firm of type-t. Each of its customers pays p_t, giving a period profit of $-F - x_t + z_t p_t$. With probability $f(x_t)$ the firm produces a high-quality product this period and hence remains operative next period, and with the complementary probability it produces a low-quality product and exits. Hence, R_t satisfies the following recursion:

$$R_t = \max\{0, -F + z_t p_t + max_x[-x + \beta f(x) R_{t+1}]\}, \tag{3.1}$$

where $\beta \in (0,1)$ is the discount factor. By the usual dynamic programming arguments, (3.1) has a unique solution. And, because f is strictly concave, the sequence of maximizers, $(x_t)_{t=1}^{\infty}$, is unique, too.

The following properties of (3.1) are used in the sequel. The proof, which is straightforward, is omitted.

Lemma 3.2. *(i) The maximizer on the RHS of (3.1) is strictly increasing in R_{t+1} and independent of z_t and p_t. Conversely, if $x_t > x_{t'}$ then $R_{t+1} > R_{t'+1}$. (ii) A firm invests zero, $x_{t-1} = 0$, if its future discounted profit, R_t, is zero. (iii) If $R_{t+1} \geq R_t$ and $z_t p_t \geq z_{t-1} p_{t-1}$, with at least one of these inequalities holding strictly, then $R_t > R_{t-1}$.*

The following proposition argues that in any equilibrium, a firm's investment in quality is strictly increasing with age. A sequence x_t is said to be **strictly**

increasing if $x_t > x_{t-1}$ for all t; it is said to be **weakly increasing** if $x_t \geq x_{t-1}$ for all t, with at least one inequality being strict. Analogous definitions apply when "increasing" is replaced by "decreasing."

Proposition 3.3. *In any steady-state equilibrium investment and, hence, continuation profits are strictly increasing with age.*

Proof. The proof is executed in four steps.

Step 1: x_t is not a constant.

Proof of step 1: Suppose x_t is a constant, say $\widehat{x}$. Then, in equilibrium, $x_t^e = \widehat{x}$. Thus each firm offers the same surplus, reputation customers do not search and each firm gets the same price, $p(\widehat{x})$. Thus, $t \in A$ for all t, which implies $z_t > z_{t-1}$. Thus, since the customer base of each firm keeps increasing and since the price it gets is constant, the gross profit, $z_t p_t$, keeps increasing. This implies the discounted profit, R_t, also keeps increasing: $R_{t+1} > R_t$. But this, by the foregoing Lemma, implies $x_t > x_{t-1}$, a contradiction.

Step 2: x_t cannot be weakly decreasing.

Proof of step 2: Suppose x_t is weakly decreasing. Then for large enough t, x_t is below the average x and, a fortiori, below x_1. Therefore there must be a $\bar{t} > 1$ such that, for $t \geq \bar{t}$, $x_t \notin A$. But then firms of age $t \geq \bar{t}$ have only search customers. That is, for $t \geq \bar{t}$, $z_t = y$. Moreover, since by assumption, $x_t^e = x_t$ is weakly decreasing, p_t is weakly decreasing as well. Hence, since both the customer base and the price are weakly decreasing we must have, for all $t \geq \bar{t}$, $R_t \leq R_1 = 0$, where $R_1 = 0$ derives from the free-entry condition. But this implies that firms of age $t \geq \bar{t}$ optimally exit. Consider now a firm of age $\bar{t} - 1$, which is destined to exit the following period. Then, this firm invests zero at $\bar{t} - 1$. And, in a rational expectations equilibrium, consumers know this so the price this firm gets is zero. Therefore firms of age $\bar{t} - 1$ exit at once and $R_{\bar{t}-1} = 0$. And so on. Thus all firms must exit, which cannot be the case in a steady-state equilibrium. Thus x_t cannot be weakly decreasing.

Step 3: x_t is weakly increasing.

Proof of Step 3: By steps 1 and 2, there must be a minimal t so that $x_{t+1} > x_t$ and $x_{t-1} \geq x_t$. But then, by Lemma 3.2, $R_{t+2} > R_{t+1}$ and, since consumers expectations are correct, $p_{t+1} > p_t$ and $z_{t+1} \geq z_t$ (a type-$t+1$ firm offers a higher surplus than a type-t firm and, thus, if consumers accept a firm of type-t they also accept a firm of type-$t+1$, which implies $z_{t+1} \geq z_t$.) Thus, we have $p_{t+1}z_{t+1} > p_t z_t$, which together with $R_{t+2} > R_{t+1}$ implies $R_{t+1} > R_t$. But this, together with Lemma 3.2, implies $x_t > x_{t-1}$. If $t > 1$ this contradicts the minimality of t. Hence, we must have $t = 1$, i.e., $x_2 > x_1$. Assume now x_t is not weakly increasing. Then, there must be a $t > 1$ for which $x_{t+1} < x_t$. But then by the exact same arguments as above (except that all inequalities are reversed), $x_t < x_{t-1}$. And, repeating this argument, we conclude $x_2 < x_1$, which is a contradiction. Hence, x_t must be weakly increasing.

Step 4: x_t is strictly increasing.

Proof of Step 4: Suppose not. Then there exists a t' such that either (i) $x_{t'+1} > x_{t'}$ and $x_{t'} = x_{t'-1}$ or (ii) $x_{t'+1} = x_{t'}$ and $x_{t'} > x_{t'-1}$. Consider (i). In that case, by the exact same argument as in Step 3, $R_{t'+1} > R_{t'}$, which implies $x_{t'} > x_{t'-1}$, a contradiction. Consider (ii). Then either t' and $t'+1 \in A$ or t' and $t'+1 \notin A$. In the first case, for all $t > t'$, $z_t > z_{t-1}$which implies $R_{t'+2} > R_{t'+1}$ which implies $x_{t'+1} > x_{t'}$, a contradiction. In the second case, there exists a $t'' > t'+1$ such that $z_t = z_{t'}$ for $t'' > t \geq t'$ and $z_{t+1} > z_t > z_{t'}$ for $t \geq t''$, i.e., t'' is the smallest $t \in A$. By discounting, this implies that $R_{t'+2} > R_{t'+1}$ which implies $x_{t'+1} > x_{t'}$, a contradiction. This completes the proof. ■

Thus, in any steady-state equilibrium, investment and continuation profits increase with age, which implies that a firm exits only if it becomes low-ability. We turn now to the proof that such an equilibrium exists. To that end, it is useful to introduce the following notation and concepts.

Let us fix the flow of search customers per firm, y, and a sequence of consumers' expectations, $a = (a_t)_{t=1}^{\infty}$, with $a_t \in [0, \overline{x}]$ and with $a_{t+1} > a_t$ for $t = 1, 2, ...$ Since consumers expect older firms to invest more, the surplus associated with buying from older firms is bigger. It follows, then, that consumers accept a type-t firm if and only if t is large enough, i.e., if and only if $t > T(a)$ for some $T(a)$. $T(a)$ is determined as follows.

a induces a distribution over firm types, call it $\mu(a) = (\mu_t(a))_{t=1}^{\infty}$. The measure of 1-year old firms is proportional to 1 and the measure of t-year old firms is proportional to $\prod_{\tau=1}^{t-1} f(a_\tau)$, i.e., $\mu_1(a) = \frac{1}{1+\sum_{t=2}^{\infty}\prod_{\tau=1}^{t-1} f(a_\tau)}$ and $\mu_t(a) = \frac{\prod_{\tau=1}^{t-1} f(a_\tau).}{1+\sum_{t=2}^{\infty}\prod_{\tau=1}^{t-1} f(a_\tau)}$ for $t = 2, 3, ...$ The average surplus that a consumer is looking at if she is to search once under $\mu(a)$ is proportional to $s(a) = \sum_{t=1}^{\infty} \mu_t(a) f(a_t)$. Since $s(a)$ is the average of an increasing sequence, $f(a_\tau)$, there must be an integer $T(a)$ so that $f(a_{T(a)}) < s(a) \leq f(a_{T(a)+1})$.

Given y and a, the firm's objective is written as follows:

$$\underset{(x_t)_{t=1}^{\infty}}{Max} \left\{ \Pi(x_1, x_2, ... \mid y, a) \right\}, \tag{3.2}$$

where

$$\Pi(x_1, x_2, ... \mid y, a) \equiv \sum_{t=1}^{\infty} \beta^{t-1} \prod_{\tau=1}^{t-1} f(x_\tau)[-F - x_t + z_t p_t], \tag{3.3}$$

$p_t = \gamma f(a_t) V$, $z_t = y$, for $t = 1, 2, .., T(a)$ and $z_t = y\frac{1-\delta^{[t+1-T(a)]}}{1-\delta}$ for $t > T(a)$. (3.2) is an alternative way of expressing the recursion, (3.1). Thus, since the maximum to (3.1) is unique, for any (y, a), Π is uniquely maximized by some $b = (b_t)_{t=1}^{\infty}$. Denote this maximizer by $b = g(y, a)$. Or, when y is fixed, $b = g(a)$.

We prove now the existence of an equilibrium in two steps. In the first step, Lemma 3.4, we fix the flow of search customers that each firm gets, y, and prove that consumers' expectations, a, exist, which constitute a fixed point of g: $a = g(a)$. Under these expectations, the investment level which maximizes firms' profits, $g(a)$, coincides with a. This shows that the equilibrium requirements that consumers and firms maximize and that consumers' expectations are realized can be made consistent, i.e., requirements 1 and 2 in the definition of equilibrium are satisfied. This leaves us with the task of satisfying the zero-profit requirement and finding a steady-state distribution over firm types. This is done in the second step, Lemma 3.5.

Lemma 3.4. *Fix y. Then, there exists a sequence $a = (a_t)_{t=1}^{\infty}$ so that the solution to (3.2) satisfies $a = g(a)$.*

Proof. We endow $X \equiv [0, \overline{x}]^{\infty}$ with the topology of weak convergence, which turns it into a convex, compact, linear topological space. Thus, if we show g is continuous, it has a fixed point and we are done (Glicksberg, 1952). Let $(a^n)_{n=1}^{\infty}$ be a convergent sequence in X, and let $b^n = g(a^n)$. Then:

$$\Pi(b^n \mid y, a^n) \geq \Pi(x \mid y, a^n), \text{ for all } x \in X. \tag{3.4}$$

Let b be a limit point of b^n. Then, when we pass to the limit on both sides of (3.4), we get:

$$\Pi(b \mid y, a) \geq \Pi(x \mid y, a), \text{ for all } x \in X. \tag{3.5}$$

Consequently, b is a maximizer of $\Pi(\cdot \mid y, a)$. But, since Π is maximized uniquely, $b = g(a)$, i.e., g is continuous. ■

Lemma 3.4 guarantees, for every y, the existence of an a which is a fixed point of g under y, $a = g(y, a)$. Call this fixed point $\alpha(y)$. Substitute $\alpha(y)$ into the profit function Π and call the resulting function $\pi(y)$:

$$\pi(y) \equiv \Pi(\alpha(y) \mid y, \alpha(y)).$$

$\pi(y)$ is the profit of a new entrant when each firm gets y search customers per period and firms invest $\alpha(y)$. $\pi(y)$ is increasing and continuous in y, goes to zero as y goes to zero, and goes to infinity as y goes to infinity. Thus, there exists a y^* so that $\pi(y^*) = F$. By construction, if each firm gets y^* search customers per period, then new entrants make zero profits. It remains, then, to show that a suitable choice of the entry flow, e, guarantees that each firm gets indeed y^* search customers in each period.

Lemma 3.5. *There exists an entry flow, e^*, so that each firm gets a flow of y^* search customers. e^* along with $\alpha(y^*)$ induce an equilibrium.*

Proof. Assume e new firms enter each period and assume that type-t firms invest x_t. Let n_t be the steady-state measure of firms of age t under these assumptions. Then $n_1 = e$, $n_2 = f(x_1)e$, $n_3 = f(x_2)f(x_1)e$ and so on, which gives $n_t = m_t e$, where $m_t = f(x_{t-1})f(x_{t-2})..f(x_1)$. Let $N = (n_1 + n_2 + ...)e$ be the steady-state measure of all firms in the market (by the definition of m_t and the fact that $f(\cdot)$ is bounded away from 1, $n_1 + n_2 + ...$ converges). Assume we want each firm to receive a flow of y^* search customers, and let $\alpha(y^*)$ be the corresponding fixed point of g (see Lemma 3.4). Let ν be the measure of acceptable firms under $\alpha(y^*)$ and let $Y \equiv (1-\nu)(1-\delta)+\delta$. Given that search consumers divide equally between all firms, the flow of search customers that each firm gets is $y = Y/N = Y/(n_1 + n_2 + ...)e$ (where n_i derive from $\alpha(y^*)$). Thus, there exists a unique e^* which induces y^*, namely, $e^* = Y/(n_1 + n_2 + ...)y^*$. By construction, e^* along with $\alpha(y^*)$ constitute an equilibrium. ■

Taken together the last two Lemmas imply:

Proposition 3.6. *There exists an equilibrium in which investment, price and firm size increase with age.*

4. Discussion of the Equilibrium

The equilibrium described by the preceding proposition divides a firm's life cycle into two phases. In the first phase, firms sell only to randomly arriving search customers. At this stage firms invest in building their reputation but do not yet reap its benefits. They begin to enjoy the fruits of their past investment and attract reputation customers (and thus build up a customer base) only once and if they survive beyond this stage.

During both phases, older firms invest more than younger ones because the value of reputation increases with age.[5] The longer its tenure in the market, the greater the number of potential customers who are aware of a firm's quality history. And since reputation takes time to build, older firms - with a greater investment in this valuable asset - invest more to maintain it than younger ones, whose reputation is not yet as valuable. Consumers, in turn, rationally anticipate this and hence are willing to pay more to older firms, as quality premia. This further increases the returns from reputation and the incentive to invest in it.

5. Discussion of the Assumptions

It is useful to review the role of our assumptions for the results developed above. Our main assumptions are that (i): Consumers are imperfectly informed about firms' types (a consumer knows only the type of its reputation firm and, if he searches, the type of one other randomly selected firm) and (ii): Firms' current expected quality depends on private investment. Suppose that only (ii) holds; i.e., consumers are perfectly and costlessly aware of the type of each firm. In that case, each firm would effectively be a reputation firm of each consumer and so age would not matter. That is, then young and old firms would face identical incentives to invest, hence would invest identically, provide identical expected quality and command the same price. Alternatively, suppose (i) holds but a firm's ability is independent of its investment. In that case, firms' expected quality would be independent of how many consumers know its type. So, again, consumers'

[5] Even at the first stage of its life cycle, when the firm does not attract reputation customers, it invests more the older it is because the time at which the returns from reputation will begin to be realized is nearer at hand.

willingness to pay would be independent of age. Thus, both assumptions are needed to drive the results of the model.

One other of our assumptions bears comment. We assume that a high-ability firm only produces high quality and a low-ability firm always produces low quality. This is somewhat extreme. Realistically, even a high-ability firm might sometimes fail to produce high quality, despite its best efforts. And similarly, even a low-ability firm might sometimes get lucky and provide high quality. For example, not even the best physician cures all his patients and some patients of even a poor physician get better. Similarly, even a bad lawyer wins some cases and even the best one loses some cases. A more realistic formulation might therefore be a probabilistic one: A high-ability firm produces high quality with a higher probability than a low ability one but may on occasion "fail" and produce low quality. Under those conditions, rational (Bayesian) consumers would not infer that a firm is low-ability on the basis of a single failure, but would rather assess a firm's ability on the basis of the relative frequencies of its past successes and failures. We argue that our main result - that older firms command higher prices and invest more - will also obtain under this formulation, with the following modification. Under the probabilistic formulation, it is easier for consumers to distinguish between a high and low-ability firm the older it is (becuase there are a greater number of quality observations on an older firm). Thus, on average, older high-ability firms would command higher prices than young high-ability firms even if investment were independent of age. But this increases the incentive of older high-ability firms to invest. Hence, under the probabilistic formulation, we expect that older high-ability firms will on average command higher prices, and hence invest more, than young ones, not only because they have accumulated more reputation customers but also because age provides more information about inherent ability.

Finally, let us comment on the assumption that the effect of investment on ability is constant, i.e., that the probability of remaining high ability, $f(x)$, depends only on the amount invested and is independent of the firm's age. In some contexts of interest a better assumption is that because of learning by doing, the same investment is more effective the longer the firm has been operative. That is, the $f(\cdot)$ function shifts up after each success. Under such formulation, our result that older firms deliver higher expected quality and command higher prices would only be strengthened. The only difference would be that under those circumstances, it would not necessarily be the case that older firms invest more. If learning from experience is of sufficient importance, older firms might deliver higher expected quality without investing more than newer ones or even by investing less. But in either case older firms deliver higher quality.

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