Foreign Direct Investment and Exports with Growing Demand

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February 9, 2001

Abstract

We explore entry into a foreign market with uncertain demand growth. A multinational can serve the foreign demand by two modes, or by a combination thereof: it can export its product, or it can create productive capacity via Foreign Direct Investment. The advantage of FDI is that it allows lower marginal cost than exports. The disadvantage is that FDI is irreversible and, hence, entails the risk of creating under-utilized capacity in case the market turns out to be smaller than expected. The presence of demand uncertainty and irreversibility gives rise to an interior solution, whereby the multinational does - under certain conditions - both exports and FDI. We argue that this feature is consistent with observed behavior of multinationals, yet it has not arisen in previous theoretical formulations.

Keywords: Foreign Direct Investment, Entry, Exports, New Markets

JEL classi cation: D8, D92, F2

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1 Introduction

When a multinational decides to establish new business in a foreign market, it typically starts out by exporting its product. Then, depending on its experience, it may open production facilities in the foreign market - do Foreign Direct Investment (FDI for short) - and start satisfying some of the local demand from these facilities. All along the multinational may continue to export its product and may gradually expand its local production facilities. Vernon's (1966) celebrated "product cycle" paper is probably the first to have drawn attention to such patterns, and a vast theoretical and empirical literature followed. In this paper we attempt a dynamic modeling of this phenomenon, with an emphasis on demand uncertainty and irreversibility of investments. Our aim is to generate the time-paths of exports and FDI, and relate them to observed behavior of multinationals and to economic fundamentals. Our setting is as follows.

Consider a foreign market where demand is growing stochastically over time. This market can be served through exports from an existing facility (already established in some home market), through investment in the foreign market, or a combination of the two. Serving the market through FDI is less costly than exports if demand is sufficiently high. This is due to lower transportation costs, to lower taxes, or to labor and materials being relatively inexpensive in the foreign market. However, FDI (by definition) requires investment, which becomes irreversible as soon as resources are sunk. Hence, if demand continues to grow FDI justifies itself. Otherwise, the multinational is better off exporting the product, rather than wasting resources on under-utilized capacity. Given this trade-off the multinational picks - at each point in time - an optimal combination of exports and FDI.

Some qualitative features of the optimal solution, as it emerges from our analysis, are as follows. Since the fixed cost of entry represents an irreversible investment, the seller will typically wait and enter the market only when the demand has reached a sufficiently high level. The length of the wait is, of course, affected by the relative costs of exporting and FDI, as well as by the prospects for future demand growth. In the pre-entry stage, exports are increasing as demand is growing. Once the seller enters the market, the initial investment is relatively high. Then, the seller adds to invested capacity as demand grows over time. In this post-entry stage, and depending on the parameters, the seller may choose to use FDI only, or to use a combination of FDI and exports.

These patterns are consistent with several "stylized facts" of the international trade and investments literature. In particular, FDI typically involves some initial delay, is not completed in one-shot and, most importantly, often follows a period of exports.[1] Moreover, the use of exports and FDI varies across markets and firms, both with respect to the timing and the levels chosen. Our framework allows a systematic study of how demand and technology conditions may lead to such differences.[2]

One of our key results is that, under some conditions, a seller finds it optimal to have both FDI and exports. Each mode of serving the market plays a different role, and the roles are complementary: Exports are used to explore future demand, and FDI is used to supply the product more economically once demand is known to exist with certainty. This result relates to a number of empirical findings, which argue that exports and FDI are complements, in the sense that a higher level of exports may be associated with a higher level of FDI.[3]

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[1] This is especially true in the case of substantial demand uncertainty - see, e.g., a discussion of the recent case of entry into the China beer market (INSEAD, 1998). Similar issues in China have been recently raised in the case of the automobile market (see *Time*, May 22, 2000.) Some companies like GM have chosen to invest early in the market, while others like Ford and DaimlerChrysler have chosen to initially reach the market via exports; interestingly, after the initial exports period, Ford has recently announced a decision to build a plant (see *Automotive News,* July 10, 2000.) There are numerous other cases where demand growth is potentially great but uncertain and where investments involve a significant irreversible component. Nicholas *et al.* (1994) provide survey data where 69% of firms indicate that they first exported to Australia before making their investment. Tse, Pan and Au (1997) find that 35% of total foreign operations in China from 1979 through 1993 correspond to exports. This percentage varies substantially, from a low of 17.7% for non-Japanese Asian to a high of 45% for West European operations. While these are aggregate data and do not indicate time trends, they show that, at any given time, both exports and investments are significant.

[2] In our framework, higher import tariffs or transportation costs increase the unit cost of exporting relative to the cost of sales via capacity invested in the foreign country and, thus, lead to lower exports and higher FDI. A higher probability of demand growth may lead to *lower* exports (via an increase in FDI.)

[3] The literature has found empirical evidence that exports and FDI may be not only "substitutes" but also "complements." This terminology is supposed to capture whether a high level of exports is associated with a high or a low level of FDI. See, e.g., the recent work of Clausing (2000), Head and Ries (2000), and Blonigen (2001) - related earlier studies have been done by Lipsey and Weiss (1984) and Yamawaki (1991). These analyses are typically at a level of a firm that trades multiple products or at a sector level. In our analysis, there is a single product. Our results show that a positive demand shock may lead to both higher exports and higher FDI. So, the two variables may appear as "complements" in the data.

Other corroborating evidence suggesting that exports and FDI are used in a complementary way comes from foreign automobile sales in the U.S.. Several Japanese as well as European car companies both export and use their domestic production capacity. Figure 1A presents the pattern of Toyota car sales over the last few years. As demand increases, invested capacity increases as well, but exports remain at a high level.[4] Furthermore, as Figure 1B shows, FDI and exports may move together when goes from the company level to the model level. Toyota responded to the recent growth in the demand for its Camry model by both increasing its exports *and* by increasing its U.S. production capacity.[5] While a number of factors, including political, may be contributing to such a pattern, it appears that the different roles of FDI and exports highlighted by our analysis are related to the dynamics in this case.

As stated in the opening paragraph, the choice of exports versus FDI has been the subject of numerous studies. An early contribution is Caves (1971), who emphasizes scale economies and other cost factors. Subsequent contributions, which formalize these factors, include Buckley and Casson (1981), Smith (1987) and Horstmann and Markusen (1987), (1996).[6] These studies consider the decision to do FDI as driven by a trade-off between incurring an extra xed cost (to establish a foreign production facility) and economizing on variable cost (no tariffs or transportation costs.) The timing and / or the magnitude of FDI turns then on issues such as market size, pre-emption of local competitors, or minimizing agency costs. However, in all of these models once a multinational decides to do FDI, it no longer exports.[7] By contrast, in our model multinationals do (sometimes) both. This is due to

[4] If imports of Lexus (a subsidiary) are included in the data, the Toyota imports to the U.S. in recent years are even higher.

[5] Toyota produces Camry cars both in Georgetown, Kentucky and in Tsutsumi, Japan.

[6] Kogut and Kulatilaka (1994) analyze a multinational rm s network as having an option value under uncertainty, while Saggi (1998) studies a two-period model where initial exporting can be used to gather information about the demand, with the option to invest in the second period.

[7] This follows from the nature of the production technology in these models, i.e., xed cost plus constant marginal cost or, more generally, increasing returns to scale.

demand uncertainty and irreversibility,[8] which make diversi cation optimal.[9]

Another literature to which our paper is naturally related is the one on investments under uncertainty; see, e.g., Dixit and Pindyck (1994). In comparison with that literature the contribution here is that we introduce several types of investments with varying degrees of irreversibilities. Given that, the seller is able to choose an optimal degree of irreversibility (or commitment) by a judicious combination of these investments. Thereby, our model can generate predictions regarding how these investments are combined and how that relates to economic fundamentals.[10]

To focus on the main questions of our paper and avoid excessively complicating the presentation, we have simpli ed several aspects of the problem. In particular, we study the choice between exports and FDI only, whereas in reality there are additional ways to serve a market, such as licensing or joint ventures. Incorporating these alternative ways of serving the market makes the problem more continuous with respect to the degree of commitment available to the seller. For instance, licensing may be construed as an intermediate degree of commitment between exports and FDI. Additionally, we abstract from agency and contracting issues which are indeed a primary reason for doing FDI, rather than delegating production to a local agent.[11] We simply focus here on demand uncertainty, and the

[8] Other than demand uncertainty and irreversibility we maintain the same technological assumptions as in previous models.

[9] Needless to say, there are other ways to attain this feature, e.g., by incorporating an increasing marginal cost curve, implying that division of production between home facility and foreign subsidiary is more economical. However, it seems more natural to assume uncertainty and learning in a new market. And, besides, achieving more economical production can be attained by operating multiple plants in the foreign market, rather than by exporting.

[10] To be more accurate the model of investments under uncertainty already accommodates several capital goods and, hence, different types of investments. The novelty here, though, is that we provide a concrete parametrization of these investments, which corresponds to the different properties of exports and FDI. And that, in turn, enables us to be more concrete about the different roles they play and the relationship between them.

[11] For example, see the work of Dunning (1977), (1981) on FDI decisions, that puts internalization at the center of the argument, and the analysis in Ethier (1986). See also the review and discussion in Markusen (1995).

real side of the entry problem. In addition, we abstract from strategic considerations and analyze the optimal entry strategy of a single seller.

The remainder of the paper is organized as follows. Section 2 presents the model and introduces the basic notation. Section 3 sets up the dynamic programming problem and characterizes the timing of entry (the initial investment) into the market. Section 4 derives the optimal exports path in the pre-entry stage. Section 5 characterizes the optimal exports and investments paths in the post-entry stage. Section 6 discusses the comparative statics and the qualitative features of the solution. Section 7 presents a parametric example, and provides numerical illustrations of the solution. Section 8 contains concluding remarks. Some proofs are relegated to the Appendix.

2 The model

Time is discrete and the horizon is in nite.

One-consumer demand. Let $p = D(q)$ denote the one-period, one-consumer inverse demand function. We assume that the one-consumer revenue function $R(q) \equiv qD(q)$ is strictly concave. Further, by rescaling units, we assume that $R(q)$ is maximized at $q = 1$, that $D(1) = 1$ and, consequently, that $R(1) = 1$. Thus, $q = 1$ is the maximizer of monopoly pro ts when marginal cost equals zero.

Demand dynamics. The number of existing consumers at the beginning of a generic period is denoted by $A \geq 0$. Then, in a period with A consumers and sales q, the inverse demand is $p = D(q/A)$ and the revenue is $AR(q/A)$. The initial A is given. Then, A increases over time, or stays put, as a result of new consumer arrival; A never decreases.

At some point, new consumers stop arriving - demand stops growing - and from that point onwards the market size stabilizes. In particular, at the beginning of each period the arrival of consumers stops with probability s.[12]

[12] While the analysis applies under a broader set of circumstances (in particular, with a variable probability s), to keep the presentation simple we proceed under the assumption of a constant s. Clearly, s represents uncertainty about the fact that demand will continue growing in the future: at some point, demand growth will stop, for example, because the foreign market has been saturated, but the seller is uncertain about when this will happen.

If the market does not stop growing, a new customers arrive, where a is a random variable with probability density function $f(a)$, and cumulative distribution function $F(a)$. F is continuously differentiable, and its support is contained in $(0, \infty)$. Then, as long as demand is growing, the dynamic on A is represented by $A' = A + a$, where A' is the number of consumers as of the start of the following period, and a is the inter-period arrival of new consumers.[13]

Production and sales. The seller s discount factor is $\delta \in (0, 1)$. The seller can serve the market either through exports or by installing productive capacity through FDI. The payoff-relevant and observable variables at the end of a generic period are the number of consumers, A, and the productive capacity, X. When productive capacity is X the seller can produce up to X units per period at zero variable cost. A and X are the state variables of the system.[14]

At the end of a generic period, the seller is making the following decisions. If some capacity has already been installed in the foreign market, $X > 0$, the seller is choosing the levels of addition to capacity, denoted by x, and exports, denoted by y. If capacity has not been installed, the seller decides whether to install some capacity in the current period or not. In the former case, the seller chooses x and y as above. In the latter, the seller chooses y only. These decision are made prior to next period s demand realization, and the costs associated with them are non-recoverable.[15]

Thus, given X, the capacity as of the start of the next period is $X' = X + x$. The number

[13] Our focus is on new markets where demand is growing stochastically, which is why we assume A can only increase. Allowing for the possibility that the market shrinks would require a signi cant modi cation of the model, but we expect the basic implications of the analysis to remain valid.

[14] We assume that exports not sold in a given period cannot be stored and brought to the market again in the subsequent period.

[15] In some markets, it may be possible to have exports after the seller has observed the demand. However, waiting for demand uncertainty to be resolved and then deciding on the level of exports is typically infeasible or very costly (partly, because it may involve signi cant delay.) Accordingly, export decisions are made under uncertainty. Our formulation assumes that it is prohibitively costly to wait and supply the market after the demand has been revealed. If waiting and exporting when demand uncertainty is resolved is a reasonable alternative, then our model has to be modi ed in a straightforward way and an additional trade-off would appear.

of units the seller can sell as of the start of the next period is the sum of productive capacity and exports: $X' + y$.[16]

The cost of adding x units of productive capacity is kx, while the cost of exporting y units is cy. The cost of entry into the market, paid the rst time the seller does FDI, is $e \geq 0$. We assume that the following parameter restrictions hold:

$$D(0) > c > (1 - \delta)k. \tag{1}$$

The rst inequality says that the market is viable, and the second inequality says that, in the absence of demand uncertainty, it is less expensive to serve the market through FDI as compared to exports.[17]

Let q_k be the maximizer of $R(q) - (1 - \delta)kq$ and let $\pi_k \equiv R(q_k) - (1 - \delta)kq_k$. Also let q_c be the maximizer of $R(q) - cq$ and let $\pi_c \equiv R(q_c) - cq_c$. From the assumptions and parameter restrictions above, it follows that $q_c < q_k < 1$ and that $\pi_c < \pi_k < 1$.

Therefore, the seller makes two decisions within a period. First, at the end of a period he observes A and chooses whether to invest in the foreign market for the rst time (if he has not done so already), along with the levels of x and y. Then, at the beginning of the next period, he observes A', at which point he chooses whether to sell all of $X' + y$ or only part of it. At that point he is a seller with zero variable cost. Therefore, according to our normalization, he sells $\min\{X' + y, A'\}$.[18]

3 Value function and characterization of entry

We set up now the dynamic programming problem facing the seller and determine the point of rst entry into the foreign market.

[16] We assume, for simplicity, that invested capacity does not depreciate over time.

[17] Underlying reasons may be a high transportation cost associated with exports and a relatively lower foreign labor cost. If we had $c < (1 - \delta)k$, then the monopolist would never do FDI.

[18] The model implicitly assumes that the seller does not face a binding capacity constraint at the home country production. If there was a binding constraint on how many units can be exported, the model would have to be modi ed accordingly. While some of the derivations (including that of the critical level of demand that triggers FDI) would have to be adjusted, our qualitative results would remain valid.

The post-entry problem. Consider a point in time where the seller has already paid the entry fee. Call the value function over this domain $v(X, A)$.

Case I: Consider the beginning of a generic period in which demand did not stop growing. Recall that in this case the seller sells $\min\{X + y, A\}$.

Then if $X+y > A$, the seller collects revenue A in the current period and the continuation payoff is $v(X, A)$; hence in this case the total value is $A + \delta v(X, A)$.

On the other hand, if $X+y < A$, the seller collects revenue $AR((X+y)/A)$ in the current period,[19] and the total value is $AR((X + y)/A) + \delta v(X, A)$.

Case II: Consider now the beginning of a generic period in which demand stopped growing and the seller nds itself with X and y, and with A consumers. Let us call the seller s terminal value $H(X, y, A)$. This value is as follows:[20]

- if $X + y \geq A$

$$H(X, y, A) = \begin{cases} \frac{A}{1-\delta} & \text{if } X \geq A \\ A + \frac{\delta}{1-\delta} AR(\frac{X}{A}) & \text{if } A > X \geq Aq_k \\ A + \delta[kX + \frac{A\pi_k}{1-\delta}] & \text{if } Aq_k > X, \end{cases} \tag{2}$$

- and, if $X + y < A$

$$H(X, y, A) = \begin{cases} AR(\frac{X+y}{A}) + \frac{\delta}{1-\delta} AR(\frac{X}{A}) & \text{if } A > X \geq Aq_k \\ AR(\frac{X+y}{A}) + \delta[kX + \frac{A\pi_k}{1-\delta}] & \text{if } Aq_k > X. \end{cases} \tag{3}$$

So, between cases I and II, the post-FDI value function is:

$$v(X, A) = \max_{(x,y)\in\Re^2_+} \{-kx - cy + sH(X + x, y, A)+ \tag{4}$$

[19] In this case the monopolist sells $X + y$ at the price that the market will bear for it, $D((X + y)/A)$, so his revenue is $AR((X + y)/A)$.

[20] The current-period payoff is A, when the quantity available for sale, $X + y$, is at least A, and is equal to $AR((X+y)/A)$ otherwise. The payoff in each of the following periods depends on the invested capacity, X. If $X \geq Aq_k$, the seller never invests again and the per-period future payoff is either A or $AR(X/A)$, depending of whether X exceeds A or not. If $X < Aq_k$, the seller invests in the following period the quantity required to bring the level of total capacity to Aq_k.

$$(1-s)[\int_0^{X+x+y-A}(A+a)f(a)da+\int_{X+x+y-A}^{\infty}(A+a)R(\frac{X+x+y}{A+a})f(a)da$$

$$+\delta\int_0^{\infty}v(X+x,A+a)f(a)da]\}$$

$$\equiv \underset{(x,y)\in\Re^2_+}{Max}\{\widehat{\phi}(x,y,X,A)+(1-s)\delta\int_0^{\infty}v(X+x,A+a)f(a)da\}, \qquad (5)$$

where $\widehat{\phi}$ is the (unmaximized) period payoff, and $(1-s)\delta\int_0^{\infty}v(X+x,A+a)f(a)da$ is the continuation payoff.

Since the marginal cost of investments is constant, v satis es the following:

$$v(X+\overline{x},A)=v(X,A)+k\overline{x}, \qquad (6)$$

for any $\overline{x}$ that is no bigger than the right-hand-side maximizer of (4).

The pre-entry problem. Now consider the pre-entry stage, that is, when $X=0$. Call the value function over this domain $u(A)$.

Assume rst that the seller chooses *exports only.* Then, if demand stops, he gets[21]

$$G(y,A)=\begin{cases} A+\delta\max\{\frac{A\pi_c}{1-\delta},-e+\frac{A\pi_k}{1-\delta}\} & \text{if } y\geq A \\ AR(y/A)+\delta\max\{\frac{A\pi_c}{1-\delta},-e+\frac{A\pi_k}{1-\delta}\} & \text{if } y<A. \end{cases}$$

Therefore the expected payoff - predicated on exports only - is equal to

$$\max_{y\in\Re_+}\{-cy+sG(y,A)+ \qquad (7)$$

$$(1-s)[\int_0^{y-A}(A+a)f(a)da+\int_{y-A}^{\infty}(A+a)R(\frac{y}{A+a})f(a)da+\delta\int_0^{\infty}u(A+a)f(a)da]\}$$

$$\equiv \underset{y\in\Re_+}{Max}\{\psi(y,A)+(1-s)\delta\int_0^{\infty}u(A+a)f(a)da\},$$

[21] This terminal value is calculated as follows. Once demand growth stops, the seller collects the current period payoff (A or $AR(y/A)$ depending on the current level of exports) and then chooses whether to cover future demand via exports (if $A\pi_c/(1-\delta)>-e+A\pi_k/(1-\delta)$) or via FDI (otherwise.)

where ψ is the (unmaximized) period payoff.

On the other hand, if the seller *installs capacity* his payoff is:

$$-e + v(0, A) = -e + \phi(x^*(A), y^*(A), A) + (1-s)\delta \int_0^\infty v(x^*(A), A+a) f(a) da, \quad (8)$$

where $\phi(x, y, A) \equiv \widehat{\phi}(x, y, 0, A)$ and $x^*(A), y^*(A)$ are the maximizers of $v(0, A)$. Using (6), this is equal to

$$= -e + \phi(x^*(A), y^*(A), A) + (1-s)\delta \int_0^\infty [v(0, A+a) + kx^*(A)] f(a) da. \quad (9)$$

Also, since the monopolist pays the entry fee his initial investment is positive (for, otherwise, he could have waited and saved himself the interest cost on e), and is no less than Aq_k.

Consider now a point of indifference, a value of A such that the seller is indifferent between paying the entry fee and continuing to serve the market exclusively through exports. Then, if the seller does not do FDI this period and demand continues to grow, he will certainly do it next period (the formal argument is in the appendix.) Thus, $u(A+a) = -e + v(0, A+a)$. So, plugging this into the RHS of (7), the expected payoff under exports only is

$$\psi(\bar{y}(A), A) + (1-s)\delta \int_0^\infty [-e + v(0, A+a)] f(a) da, \quad (10)$$

where $\bar{y}(A)$ is the maximizer under exports only. On the other hand, if the seller does FDI, his payoff is given by (9). At the point of indifference (9) and (10) are equal. Thus, after some manipulations,

$$[1 - (1-s)\delta]e = (1-s)\delta k x^*(A) - \psi(\bar{y}(A), A) + \phi(x^*(A), y^*(A), A). \quad (11)$$

We can now state our rst result.

Proposition 1 *(i) The RHS of (11) is strictly increasing in A. Thus: (ii) Entry obeys a cutoff-value rule. Either the RHS of (11) is no less than its LHS for all $A \geq 0$, in which case the seller does FDI from the very start. Otherwise, there exists a unique $A^* \geq 0$ so that (11) is satis ed. The seller only exports if $A < A^*$ and does FDI and, possibly, exports as soon as $A \geq A^*$.*

(iii) A^* *satis es*

$$A^* < \bar{A} \equiv \frac{e(1-\delta)}{\pi_k - \pi_c}. \tag{12}$$

(iv) A^* *is increasing in* e*.*

Proof. In the Appendix.

The reason why it is optimal for the seller to enter the market at a level of demand lower than $\bar{A}$ is that demand can only increase. Indeed, if we had $s = 1$, so that the market will never grow beyond the current level, then we have $A^* = \bar{A}$ since nothing is going to change in the future. But as long as $s < 1$, even if in the current period A is below $\bar{A}$, it is possible that demand will grow and exceed, in some period, the level $\bar{A}$. At that point, the seller should certainly enter the market - but, *ex-post*, he would then regret the fact that he had not entered somewhat earlier, because the cost of serving the market is lower with local production than with exports. Thus, by entering the market earlier, the seller collects some of these cost savings (a formal argument is found in the Appendix.)

4 Pre-entry behavior: exports only

Before we proceed to the characterization of the post-entry optimal behavior, we derive the optimal pre-entry export path. This can be found by maximizing the RHS of (7). Since u does not depend on y, this is a static maximization program. Hence, equating the derivative with respect to y to zero, we nd:

Proposition 2 *(i) If* $A > 0$ *or if* $(1-s)D(0) > c$ *the optimal level of exports in the pre-entry stage is positive, and can be found by solving :*

$$c = sG_1(y, A) + (1-s)\int_{y-A}^{\infty} R'(\frac{y}{A+a})f(a)da, \tag{13}$$

where

$$G_1(y, A) = \begin{cases} 0 & \text{if } y \geq A \\ R'(y/A) & \text{if } y < A \end{cases}$$

is the derivative of $G(y, A)$ *with respect to* y*. Furthermore, the quantity exported is at least* Aq_c*.*

(ii) Otherwise, the LHS of (13) is no less than the RHS, and exports are zero. ■

Further, we see that the optimal exports level is increasing in the current level of A, and decreasing in s. Therefore, in the pre-entry stage, the level of exports is increasing over time, as A increases.

5 Post-entry behavior

We proceed now to characterize the optimal path of exports and FDI in the post-entry stage.

5.1 Optimality conditions for an interior solution

Assuming an interior optimum, $x, y > 0$ for all t and all sample paths $(a_t)_{t=1}^{\infty}$, we can characterize it via:

$$\begin{aligned} k &= sH_1(X+x,y,A) + (1-s)\int_{X+x+y-A}^{\infty} R'(\frac{X+x+y}{A+a})f(a)da + \\ & (1-s)\delta \int v_1(X+x,A+a)f(a)da \end{aligned} \tag{14}$$

and

$$c = sH_2(X+x,y,A) + (1-s)\int_{X+x+y-A}^{\infty} R'(\frac{X+x+y}{A+a})f(a)da, \tag{15}$$

where (14) and (15) are the rst-order conditions for x and y, respectively, and where H_1 and H_2 are the derivatives of H with respect to its rst and second argument, respectively. Using the envelope theorem, we can differentiate the value function (4) with respect to X, and use the following period s optimality condition (14) to obtain

$$v_1(X+x, A+a) = k.$$

We can now insert the above relation into the rst-order condition (14), eliminating the value function from its RHS. This gives:

$$k[1-(1-s)\delta] = sH_1(X+x,y,A) + (1-s)\int_{X+x+y-A}^{\infty} R'(\frac{X+x+y}{A+a})f(a)da. \tag{16}$$

Note that (15) and (16) are similar: They only have different constants on the LHS and - on the RHS - they have partial derivatives with respect to different variables (x versus y). From the expressions in the next sections the RHS of (15) and (16) are decreasing in x and y (because the revenue function is concave.) Therefore, when we plot (15) and (16) in (x, y)-space the resulting curves are downward sloping (see the gures in the next section.) We refer to the curve corresponding to (15) as $y^o(x)$, and to the curve corresponding to (16) as $x^o(y)$.

5.2 Characterization of the solution

We are going to work with the rst order conditions, (15) and (16). To that end we need the derivatives of H:

If $X + y \geq A$, we have

$$H_1(X, y, A) = \begin{cases} 0 & \text{if } X \geq A \\ \frac{\delta}{1-\delta} R'(\frac{X}{A}) & \text{if } A > X \geq Aq_k \\ \delta k & \text{if } Aq_k > X, \end{cases} \tag{17}$$

and, if $X + y < A$, we have

$$H_1(X, y, A) = \begin{cases} R'(\frac{X+y}{A}) + \frac{\delta}{1-\delta} R'(\frac{X}{A}) & \text{if } A > X \geq Aq_k \\ R'(\frac{X+y}{A}) + \delta k & \text{if } Aq_k > X. \end{cases} \tag{18}$$

Also,

$$H_2(X, y, A) = \begin{cases} 0 & \text{if } X + y \geq A \\ R'(\frac{X+y}{A}) & \text{if } X + y < A. \end{cases} \tag{19}$$

We also need the difference, $H_1 - H_2$, which can be readily computed as

$$H_1(X, y, A) - H_2(X, y, A) = \begin{cases} 0 & \text{if } X \geq A \\ \frac{\delta}{1-\delta} R'(\frac{X}{A}) & \text{if } A > X \geq Aq_k \\ \delta k & \text{if } Aq_k > X. \end{cases} \tag{20}$$

The following Lemma is proven in the Appendix:

Lemma 1 *If we x an x and look at the y points above it (one on $x^o(y)$ and one on $y^o(x)$) the curve corresponding to $x^o(y)$ is steeper than the curve corresponding to $y^o(x)$.*

This Lemma implies $x^o(y)$ and $y^o(x)$ cross at most once. So there are three cases to consider:

Case a: $x^o(y)$ intersects uniquely $y^o(x)$ in $\Re^2_{++}$. In this case we have an interior solution; see Figure 2A.

Case b: $x^o(y)$ is uniformly above $y^o(x)$. In that case, we have a corner solution with $y = 0$ and x where $x^o(y)$ intercepts the horizontal axis; see Figure 2B. Let us call that point $\hat{x} \equiv x^o(0)$.

Case c: $x^o(y)$ is uniformly below $y^o(x)$. In this case, we have a corner solution with $x = 0$ and y where $y^o(x)$ intercepts the vertical axis; see Figure 2C. Let us call that point $\widehat{y} \equiv y^o(0)$.

Let us start with case (a).[22] When an interior solution obtains, it can be calculated as follows. We insert equation (15) into (16) and get:

$$k[1 - (1 - s)\delta] - c = s[H_1(X + x, y, A) - H_2(X + x, y, A)]. \tag{21}$$

Consider $X + x \leq Aq_k$. Then - by (20) - we have $H_1 - H_2 = \delta k$. Therefore, (21) becomes

$$k[1 - (1 - s)\delta] - c = s\delta k \qquad \text{or}$$

$$c = (1 - \delta)k.$$

But we maintain, from (1), the assumption $c > (1 - \delta)k$. So this means that x should be increased to its maximum in this region, i.e., if x is to remain in this region we should set $X + x = Aq_k$.

Consider now an x such that $A > X + x > Aq_k$. Then by (20) $H_1 - H_2 = \frac{\delta}{1-\delta} R'(\frac{X}{A})$, and in this case (21) becomes

$$k[1 - (1 - s)\delta] - c = s\,\frac{\delta}{1 - \delta} R'(\frac{X + x}{A}). \tag{22}$$

[22] The analysis of the next 2 cases is predicated on x^* being positive in the subsequent period. Lemma 2 shows that x^* is indeed positive in all periods after the initial FDI.

The RHS is strictly decreasing in x. When $X + x = Aq_k$, the RHS equals $s\delta k$ which, by assumption (1), is larger than $k[1 - (1 - s)\delta] - c$. So $X + x$ is optimally set above Aq_k. When $X + x = A$, the RHS equals zero, which is below $k[1 - (1 - s)\delta] - c$ (otherwise, as we shall shortly show, we have a corner solution.) Thus the optimizing x must be such that $A > X + x > Aq_k$.[23] Once we obtain x, we can solve for y from (15).

Now, the borderline between cases (a) and (b) is where corner and interior solutions are the same, that is, where $x^o(y)$ intersects $y^o(x)$ on the horizontal axis (see Figure 2D.) Algebraically, cases (a) and (b) are distinguished as follows. Set $y = 0$ in the RHS of (16). Then we get one equation in one unknown:

$$k[1 - (1 - s)\delta] = sH_1(X + x, 0, A) + (1 - s) \int_{X+x-A}^{\infty} R'(\frac{X + x}{A + a}) f(a)da. \tag{23}$$

The solution to this equation is $\hat{x}$. Substitute $\hat{x}$ into the RHS of (15) and also set $y = 0$. If the resulting value at the RHS is exactly equal to c, then the two curves, $x^o(y)$ and $y^o(x)$, intersect on the x-axis. If the RHS is larger than c, we have an interior solution. If it is smaller than c then we have a corner solution (the marginal cost of y exceeds the marginal bene t of y, when $x = \hat{x}$.) After further manipulations of (15) we obtain the following characterization. We have an *interior* solution if

$$k[1 - (1 - s)\delta] - c > s[H_1(X + \hat{x}, 0, A) - H_2(X + \hat{x}, 0, A)], \tag{24}$$

and a *corner* solution ($y = 0$ and $x = \hat{x}$) otherwise.[24] Further, since $H_1 - H_2 \geq 0$ (see equation (20)), we obtain:

- If $c > k[1 - (1 - s)\delta]$ then we have a corner solution.

- If $c < k[1 - (1 - s)\delta]$ then we can have either a corner or an interior solution, as determined by (24).

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[23] Equation (22) can be written as $(k - c) = s\frac{\delta}{1-\delta}R' + (1 - s)\delta k$. The LHS is the (additional) cost of investing 1 unit rather than exporting it. The RHS is the (additional) marginal bene t of this investment. In the event that growth stops (which happens with probability s), the seller has increased capacity by 1 unit and increased revenue by MR (in perpetuity). If growth does not stop, then the seller has saved the cost (k) of investing capacity next period (discounted by δ.)

[24] See the section on comparative statics for how changes in parameters affect the boundary between corner and interior solutions.

Concerning the corner solution, $\hat{x}$, it satisfies $X+\hat{x} > Aq_k$. A value such that $X+\hat{x} \leq Aq_k$ would violate (23), since the RHS would exceed LHS (recall that F puts probability 1 on strictly positive values of a.) However, and unlike case (a), we may have $X + \hat{x} > A$.

What about case (c)? We now show that if the dynamic starts with $X = 0$, this case cannot occur along the optimal path. More precisely, we have:

Lemma 2 *The optimal FDI is positive in all periods following entry (until demand growth stops.)*

Proof. We start by proving the following Claim.

Claim: Consider the first period of entry. Then, if X is the first FDI installment (which must be positive), and y is the first flow of exports (which might be zero), we have:

$$k - (1-s)\delta \int v_1(X+x, A+a)f(a)da - c \leq s[H_1(X,y,A) - H_2(X,y,A)].$$

Proof. Since X is positive (14) is satisfied, and we re-write it as:

$$\begin{aligned} & k - (1-s)\delta \int v_1(X+x, A+a)f(a)da \\ = \quad & sH_1(X+x, y, A) + (1-s) \int_{X+x+y-A}^{\infty} R'(\frac{X+x+y}{A+a})f(a)da. \end{aligned} \tag{25}$$

The maximizing y, on the other hand, may be positive or zero. So the generalized first-order condition for it is:

$$c \geq sH_2(X+x, y, A) + (1-s) \int_{X+x+y-A}^{\infty} R'(\frac{X+x+y}{A+a})f(a)da.$$

Using the second equation to eliminate $(1-s) \int_{X+x+y-A}^{\infty} R'(\frac{X+x+y}{A+a})f(a)da$ from the first equation, proves the Claim. ■

Let us go now to the period after initial FDI. Assume, by way of contradiction, that we have a corner solution with $x^* = 0$. Then, the optimal y in that period, $\widehat{y}$, is either 0 or positive. Assume it is 0. Then in the previous period (the period when FDI is initially installed) (25) is satisfied with $y = 0$. If we replace the A in this equation by next-period s A

(which is bigger with probability 1), the RHS exceeds the LHS so $x^* = \widehat{y} = 0$ is impossible. The other possibility is $\widehat{y} > 0$. Then (15) is satis ed, which we re-write as:

$$c = sH_2(X, \widehat{y}, A) + (1-s) \int_{X+x+y-A}^{\infty} R'(\frac{X+\widehat{y}}{A+a}) f(a)da.$$

Also, since $x^* = 0$, we have

$$k - (1-s)\delta \int v_1(X, A+a) f(a)da > sH_1(X, \widehat{y}, A) + (1-s) \int_{X+y-A}^{\infty} R'(\frac{X+\widehat{y}}{A+a}) f(a)da,$$

where A is understood as the period-after-entry A. We can now use the rst equation to eliminate $(1-s) \int_{X+y-A}^{\infty} R'(\frac{X+\widehat{y}}{A+a}) f(a)da$ from the second equation. This gives us

$$k - (1-s)\delta \int v_1(X, A+a) f(a)da - c > s[H_1(X, \widehat{y}, A) - H_2(X, \widehat{y}, A)].$$

However, this is impossible because, by the above Claim, the reverse inequality holds for a smaller A, because $H_1 - H_2$ is increasing in A and because $H_1 - H_2$ is independent of y.

More generally, what we have shown is that if x^* is positive in some period, it must again be positive in the next period for any inter-period demand realization. Since the initial FDI is positive this implies FDIs are positive in *all* time periods. ■

The intuition is that optimal capacity increases with market size. Hence, since market size increases over time, so does capacity.[25]

In conclusion we have:

Proposition 3 *Consider the post-entry stage. Then (i) FDIs are positive in each and every period, and are found as follows. (ii) If (24) holds, we have an interior solution. The optimal x is found from (22). The optimal y is determined then by equation (15), once we substitute the optimal x. (iii) If (24) fails to hold the optimal solution is $y = 0$, and the optimal x is found from (23).*

[25] Note that, after the initial entry, the cost of investment is linear so there is no reason for the seller to concentrate his investments in some periods without investing in other. Such behavior may be optimal if there was a xed cost to be paid every time there is an investment.

Thus, we have two types of behavior in the post-entry stage. If FDI is not very costly, exports are zero and the market is entirely served by FDI. On the other hand, if FDI is costly, the seller does both: he uses y to explore new demand and uses x to (more than) cover the known demand. The exact combination of exports and FDI depends on all the parameters, and we esh out this dependence in the next section. For the time being, we note that the seller always chooses x so that $X + x$ exceeds Aq_k, i.e., he invests in excess of guaranteed demand. Moreover, in the case when the seller uses exports as well, the investment is such that $X + x$ does not exceed A, while in the case when there are no exports, $X + x$ may exceed A (if the seller is optimistic enough about future demand growth.)

The following remarks are in order. First, $k - (1 - s)\delta k$ is the one-period risk-adjusted cost of serving the market through FDI (the risk being that the marginal FDI causes over-capacity). Hence, $k < c + (1 - s)\delta k$ means the cost of serving the market through exports exceeds the cost of serving it through FDI. This explains why exports are not used at all in this case. Second, the problem is solved recursively: rst we nd the optimal x and then we nd the optimal y (as opposed to simultaneously solving 2 equations in 2 unknowns.) Third, if the seller follows the optimal policy, as per Proposition 3, he does FDI each period the demand is still growing (following the initial entry.) So his investments in the foreign country grow gradually.

6 Comparative statics and qualitative features of the dynamics

6.1 Comparative statics

We now consider how the solution depends on the parameters of the model, speci cally on c, k, δ, s, and the current number of consumers A. We do this for a xed value of X, i.e., we consider a generic period and determine how that period s x and y change when each of the parameters change. The results are summarized in the following Proposition, which we prove in the Appendix.

Proposition 4 *For a given level of* X, *the optimal solution responds to changes in the values of the parameters as follows.*

(i) An increase in k decreases x and increases y, if there is an interior solution; it decreases x if we have a corner solution; and it tends to move the solution from a corner to an interior point.

(ii) An increase in c increases x and decreases y, if there is an interior solution; it leaves x unchanged if there is a corner solution; and it tends to move the solution from an interior point to a corner.

(iii) An increase in s decreases x, if there is an interior solution, while its effect on y is ambiguous; it decreases x if we have a corner solution; and it tends to move the solution from a corner to an interior point.

(iv) An increase in δ increases x and decreases y, if there is an interior solution; it increases x if there is a corner solution; and it tends to move the solution from an interior point to a corner.

(v) An increase in A increases x, if there is an interior solution, while its effect on y is ambiguous; it increases x if there is a corner solution; and it either tends to move the solution from an interior point to a corner (if $A > X + \hat{x} \geq Aq_k$) or it leaves the boundary unaffected (otherwise.)

This result conforms with intuition. In particular, the ratio of exports to investment increases, y/x, when k increases or when c or δ decreases.

Note that in cases *(i)*, *(ii)* and *(iv)* a parametric change results in x and y moving in opposite directions. However, in cases *(iii)* and *(v),* with respect to the effect of either an increase in s or a decrease in A on y, there are two opposing effects. There is a direct tendency to decrease y (because the market becomes less attractive), but this may be offset by an indirect effect (the seller decreases x and may wish to substitute y for x). Thus, a parametric change in these cases may result in x and y moving in the same direction. Thus, if one is to interpret our comparative statics results as shedding light on FDI and exports being substitutes or complements, then our model predicts they can be either (depending on which underlying parameter is perturbed and what the primitive data of the problem are.)

6.2 Discussion of the dynamics

A number of qualitative features of the solution can be discussed at this point - these features are also illustrated in the following section, where the solution for a particular example is explicitly calculated. The main features of the solution are as follows.

First, due to the existence of the xed cost of entry, e, the seller typically does not do FDI right away. Instead, he waits until A has become large enough and only then decides to invest in the market. In the meantime, he is serving the market via exports. In fact, his exports are growing over time as the demand (as measured by A) is growing. At the time of entry, the level of his investment is large, as a consequence of the accumulated demand. The waiting period for the initial entry is longer when each of the entry cost, e, the cost of investment, k, and the probability that growth stops, s, are large and when the cost of exports, c, is small. Of course, if e, k and s are small and c is large, or if the expected value of a is large, the seller may choose to serve the market via FDI right away and skip the initial stage of exports-only.

In the post-entry stage, and following the large initial investment, the seller s investments grow gradually. The seller adds to the invested capacity over time as demand grows. Since demand growth is stochastic, the level of these additional investments are sometimes high and sometimes low. Investments and exports stop once demand stops growing.

Moreover, in the post-entry stage there are two possible regimes (and it is possible that the seller switches from one to the other.) The seller may choose to either only serve the market via FDI or to supplement it with exports. The critical factor here is whether - depending on the cost parameters - the seller wishes to position himself for the following period s demand by adding some new capacity or by exporting. When the cost of FDI or the probability that demand growth will stop are high, the seller prefers to use exports to explore the new demand, and to do FDI only after the level of demand has been established with certainty. In this case, the seller is using both exports and FDI, but each play a different role. Otherwise, when the cost of FDI is relatively low, there is no need to use exports and the seller chooses to only have FDI as a way of serving the market.

Finally, the structure of the dynamics is such that a long period of wait before the initial FDI is associated with a relatively low FDI and high exports in the post entry stage. Thus, in cases were a seller waits a long time to enter the market, we expect a large initial investment but we also expect subsequent investments to be low (relative to exports.)

7 A parametric example: linear demand

To illustrate our approach, we now provide the solution for the particular case of linear demand. Speci cally, let $D(q) = 2 - q$. Then $R(q) = q(2 - q) = 2q - q^2$ is maximized at $q = 1$, with $R(1) = 1$.

Equation (1) translates into $2 > c > (1-\delta)k$. Now q_k is the maximizer of $2q-q^2-(1-\delta)kq$ or $q_k = [2 - (1 - \delta)k]/2$, while q_c is the maximizer of $2q - q^2 - cq$ or $q_c = (2 - c)/2$. Then $\pi_k \equiv R(q_k) - (1 - \delta)kq_k = [2 - (1 - \delta)k]^2/4$ and $\pi_c \equiv R(q_c) - cq_c = (2 - c)^2/4$. With respect to the terminal values, we obtain:

If $X + y > A$

$$H_1(X, y, A) = \begin{cases} 0 & \text{if } X \geq A \\ \frac{\delta}{1-\delta}\frac{2(A-X)}{A} & \text{if } A > X \geq q_k A \\ \delta k & \text{if } q_k A > X, \end{cases}$$

and if $X + y < A$

$$H_1(X, y, A) = \begin{cases} \frac{2(A-X-y)}{A} + \frac{\delta}{1-\delta}\frac{2(A-X)}{A} & \text{if } A > X \geq q_k A \\ \frac{2(A-X-y)}{A} + \delta k & \text{if } q_k A > X. \end{cases}$$

We also have

$$H_2(X, y, A) = \begin{cases} \frac{2(A-X-y)}{A} & \text{if } X + y \leq A \\ 0 & \text{if } X + y > A. \end{cases} \tag{26}$$

In addition, equation (22) becomes

$$k[1 - (1 - s)\delta] - c = s\,\frac{\delta}{1-\delta}\,\frac{2(A - X - x)}{A}, \tag{27}$$

or, solving for x,[26]

[26] Note that $(X + x)/A = 1 - B$ where $B \equiv (1 - \delta)/2s\delta[k(1 - (1 - s)\delta) - c] < 1$, and thus for linear demand - and when we have an interior solution ($y > 0$) - the ratio of total capacity to A is constant.

$$x = \frac{2s\delta(A-X) + [c-k+\delta k(1-s)]\,(1-\delta)\,A}{2s\delta}. \tag{28}$$

Further, equation (23) becomes

$$\begin{aligned} k[1-(1-s)\delta] &= sH_1(X+x,0,A) + \\ & 2(1-s)\left\{[1-F(X+x-A)] - (X+x)\int_{X+x-A}^{\infty}\frac{f(a)}{A+a}da\right\}. \end{aligned} \tag{29}$$

7.1 Explicit solution with uniform f

In addition to linear demand, suppose now that f is uniform on some interval $[0,\bar{a}]$. Then $f(a) = 1/\bar{a}$ and $1-F(a) = (\bar{a}-a)/\bar{a}$. In this case, equation (15) becomes

$$\begin{aligned} c &= sH_2(X+x,y,A) + \\ & \frac{2(1-s)}{\bar{a}}[(\bar{a}-(X+x+y-A)) + (X+x+y)\ln\frac{X+x+y}{A+\bar{a}}], \end{aligned} \tag{30}$$

where, from (26), we have

$$H_2(X+x,y,A) = \begin{cases} \frac{2(A-X-x-y)}{A} & \text{if } X+x+y \leq A \\ 0 & \text{if } X+x+y > A. \end{cases} \tag{31}$$

In addition, equation (29) becomes

$$\begin{aligned} k[1-(1-s)\delta] &= sH_1(X+x,0,A) + \\ & \frac{2(1-s)}{\bar{a}}[(\bar{a}-(X+x-A)) + (X+x)\ln\frac{X+x}{A+\bar{a}}], \end{aligned} \tag{32}$$

where

$$H_1(X+x,0,A) = \begin{cases} 0 & \text{if } X+x \geq A \\ \frac{1}{1-\delta}\frac{2(A-X-x)}{A} & \text{if } A > X+x \geq q_k A \\ \frac{2(A-X-x)}{A} + \delta k & \text{if } q_k A > X+x. \end{cases} \tag{33}$$

To nd the range of parameters for which we have a corner or an interior solution we proceed as follows. Call $\hat{x}$ the solution to (32). We have an interior solution if

$$k[1-(1-s)\delta]-c > s[H_1(X+\hat{x},0,A)-H_2(X+\hat{x},0,A)] \quad (34)$$

and a corner solution otherwise, where

$$H_1(X+\hat{x},0,A)-H_2(X+\hat{x},0,A) = \begin{cases} 0 & \text{if } X+\hat{x} \geq A \\ \frac{\delta}{1-\delta}\frac{2(A-X-\hat{x})}{A} & \text{if } A > X+\hat{x} \geq Aq_k \\ \delta k & \text{if } Aq_k > X+\hat{x}. \end{cases}$$

We now characterize the post-entry optimal solution as follows:

Proposition 5 *Suppose that $D(q) = 2-q$ and that f is uniform on $[0,\bar{a}]$. The post-entry optimal solution is as follows. If (34) holds, the optimal x is given by (28); this x is then substituted into (30) to nd the optimal y. If (34) fails to hold (and, in particular, if $k < c+(1-s)\delta k$) then $y=0$ and the optimal x is the solution to (32).*

Further, we rewrite (13) for the case of this example. In the *pre-entry* stage, optimal exports can be found by solving

$$c = sG_1(y,A) + \frac{2(1-s)}{\bar{a}}[(A+\bar{a}-y)+y\ln(\frac{y}{A+\bar{a}})], \quad (35)$$

where

$$G_1(y,A) = \begin{cases} 0 & \text{if } y \geq A \\ 2(A-y)/A & \text{if } y < A. \end{cases}$$

Finally, the threshold A^* for initial entry into the market is calculated as follows. We work with equation (11). There are two cases to consider, depending on whether at the value A^* we would have a corner solution $\hat{x}$ with $y=0$ or an interior one (if we were at the post-entry stage.) Suppose rst there is a corner solution. Then $x^*(A)$ is calculated from (32) while $\bar{y}(A)$ is calculated from (35). Substituting these values into (11), we obtain one equation in one unknown, A. Note also that equation (11), in the case examined here, becomes (writing $\hat{x}$ for $x^*(A)$ and $\bar{y}$ for $\bar{y}(A)$):

$$[1-(1-s)\delta]e = [(1-s)\delta-1]k\hat{x} + c\bar{y} + s[H(\hat{x},0,A)-G(\bar{y},A)]+$$

$$\frac{(1-s)[\hat{x}^2-\bar{y}^2]}{2\bar{a}}+(1-s)\frac{\hat{x}}{\bar{a}}[2(A+\bar{a}-\hat{x})+\hat{x}\ln(\frac{\hat{x}}{A+\bar{a}})]- \qquad (36)$$
$$(1-s)\frac{\bar{y}}{\bar{a}}[2(A+\bar{a}-\bar{y})+\bar{y}\ln(\frac{\bar{y}}{A+\bar{a}})].$$

Suppose now that we have an interior solution. Then $x^*(A)$ is calculated from (28) and $y^*(A)$ is calculated from (30), while $\bar{y}(A)=\bar{y}$ is again calculated from (35). Substituting these values into (11), we have one equation in one unknown, A. Note that, in this case, we have $x^*(A)+y^*(A)=\bar{y}(A)$ (the proof of this claim can be found as part of the proof of Proposition 1 in the Appendix.) Then, in this case, equation (11) can be simpli ed as

$$[1-(1-s)\delta]e=[(1-s)\delta-1]kx^*+c(\bar{y}-y^*)+s[H(x^*,y^*,A)-G(\bar{y},A)]. \qquad (37)$$

Depending on whether at the critical value A^* the investment path involves a corner or an interior solution, either (36) or (37) holds and can be solved uniquely to give the value A^*.

7.2 Numerical results

We provide a completely solved numerical example in the case of linear demand and uniform density. We chose the following parameter values: $e=12$, $c=1.5, k=10$, $\delta=0.9$, $\bar{a}=1$. Then, a direct calculation shows that $q_k=0.5$, and the value of s that gives $k[1-(1-s)\delta]=c$ is $s\approx 0.0555$. Thus, as per Proposition 3, exports in the post-entry stage are zero for any $s<0.0555$, whereas for $s>0.0555$ exports are zero when s is small enough, but become positive as s is increased. Therefore, to make the calculations interesting, we have chosen s values (see below) where exports are potentially positive, i.e., s is always chosen to exceed 0.0555. Regarding the construction of the A sequence, we obtain a value of a in every period via a random-number generator.

We rst assume that $s=0.1$. The optimal pre-entry sequence (exports only) and post-entry sequences (FDI and exports) can then be calculated as described in the previous section. The critical value of A for entry into the market can be calculated to be $A^*\approx 4.4257$. Figures 3A and 3B present the optimal solution graphically for a particular realization of the A sequence. For $A<A^*$ we have only exports and their level is increasing, as A is increasing over time. In the post-entry stage ($A>A^*$) we see that the seller nds it optimal to proceed without exports (we have only FDI.)

We have also calculated examples for other parameter values to illustrate how the solution varies with the parameters. In particular, as s increases from the level of 0.1 we observe two major changes. First, the critical number of consumers required for entry, A^*, increases. So entry takes place later, and when it does it is on a larger scale. Second, higher values of s imply that, in the post-entry stage, the seller finds it optimal to have both investment and exports.

Finally, we calculated a hybrid example where s increases with A. This reflects the scenario where the probability of new consumer arrival diminishes (continuously) as the size of assured market increases. Since our solution is derived recursively, our analysis applies to this scenario as long as s is sufficiently small.[27] We have chosen $s = 0.1$ for $A \in [0, 7)$, $s = 0.0666A - 0.3666$ for $A \in [7, 10)$, and $s = 0.3$ for $A \in [10, \infty)$. The critical value A^* remains unchanged at the level calculated above ($A^* \approx 4.4257$). To make the comparison with the previous 2 examples transparent, we use the same realization of the A sequence as before. Table 1 presents the optimal solution. Figures 4A and 4B present the solution graphically. The time-paths of exports and FDI are as follows. There is an initial stage with only exports in which their level is increasing over time. Once demand reaches a level that exceeds A^*, the seller enters the market via FDI and the level of this initial FDI is high. Subsequently, there is a stage where the seller adds to the productive capacity by doing additional FDI, but there are no exports. Finally, there is a stage where the seller does both exports and FDI.

8 Concluding remarks and extensions

This paper characterizes optimal entry into a new market. The key features of the model are (i) stochastic demand growth and (ii) the availability of two instruments to serve the market:

[27] When s depends on A and is sufficiently close to 1 (for some value of A), the seller may no longer choose a positive x in each and every period after entry. The intuition being that the risk of overshooting the market biases the solution towards exports only. Thus, Lemma 2 no longer holds, and case (c) (see discussion preceding the statement of Lemma 2) becomes a viable possibility. Once the derivations preceding the statement of Proposition 3 are adjusted to accommodate this possibility, the analysis proceeds along the same lines. In the example we consider here s is uniformly below 1, so this issue does not arise, and the analysis in the text goes through verbatim.

investing in building capacity (which would be preferable if the demand were - in retrospect - large enough) and exports (which would be preferable if the demand were - in retrospect - small.) Viewed somewhat more generally, our model explores the dynamics between a short-run and a long-run investment, or of the choice over time between a technology with lower marginal cost (FDI) and one with lower xed cost (exports.)

Several extensions of our analysis may make it more directly applicable to particular situations. First, in addition to the initial entry cost, there may be a xed cost to be paid every time the rm decides to add capacity through FDI. This variation of the model is expected to modify the results by making the FDI path more lumpy : as a result of the xed cost, the seller may want to wait a few periods until he invests and then invest at a higher level. This behavior is directly analogous to the initial phase in our formulation, where because of the entry cost, the seller waits a few periods until demand reaches a certain threshold and only then does FDI.

Second, we have focused on a seller with an existing production facility (at the home or source country) that enters a new (target) market. We have not characterized the solution to the more general problem of a seller with (potentially) multiple production facilities that can supply multiple markets. In particular, in our model we do not explore the possibility that, as demand in the new market grows, the seller may wish to move all production there, and reverse-export from the target back to the source country. In addition, the model has been solved under the assumption that there are no binding capacity constraints for the level of exports. This is a natural assumption for our analysis, since - as explained above - we focus on the choice between a short-run and a long-run investment. Generalizing the model to allow for capacity constraints at home is not expected to qualitatively change our main results: As long as the start-up cost, e, has been paid at home but not at the new market there will be a delay until FDI is started and once it starts there will be optimal mixing between FDI and exports.

Third, the entry considerations examined in this paper can also be studied within a strategic framework. In particular, one may explore entry by oligopolists into a market with growing and stochastic demand. Strategic considerations could distort the optimal choice between exports and FDI. We expect rms to do FDI quicker and at a higher level than otherwise, in order to obtain a stronger position and deter their rivals from investing in the following periods.[28]

Finally, there are aspects of the entry problem that have not been captured in our model and may further enrich the dynamics. For example, the seller may be able to learn more about the demand by penetrating the market faster (e.g. Rob, 1991); there may be scope for experimentation and strategic pricing (e.g. Bergemann and Välimäki, 1996); or entry itself may cause the demand to grow over time as a result of consumers learning (e.g. Vettas, 1998).

[28] This effect is similar to building capacity for strategic reasons, as in Dixit (1980). For related ideas and the role of uncertainty see, e.g., Maggi (1996). For some strategic issues related to multinational activity see Horstmann and Markusen (1987).

9 Appendix

9.1 Appendix 1

In this Appendix we prove Proposition 1.

To prove (i) of Proposition 1, the following Lemma is needed.

Lemma A1: *$x^*(A) + y^*(A) \geq \overline{y}(A)$, with equality if and only if $y^*(A) > 0$.*

Proof. To alleviate the notation we use x, y and $\overline{y}$ instead of $x^*(A), y^*(A)$ and $\overline{y}(A)$.

In proving the Lemma, we use the optimality conditions in Propositions 2 and 3, which are derived without reliance on Lemma A1 that we are proving here.

The FOC for $\overline{y}$ is:

$$c = sG_1(\overline{y}, A) + (1-s)\int_{\overline{y}-A}^{\infty} R'(\frac{\overline{y}}{A+a})f(a)da,$$

where

$$G_1(\overline{y}, A) = \begin{cases} 0 & \text{if } \overline{y} \geq A \\ R'(\frac{\overline{y}}{A}) & \text{if } \overline{y} < A. \end{cases}$$

We now distinguish the following cases:

- If $c > k[1-(1-s)\delta]$, then $y = 0$ and x is determined via equation (23) once we substitute $X = 0$:

$$k[1-(1-s)\delta] = sH_1(x, 0, A) + (1-s)\int_{x-A}^{\infty} R'(\frac{x}{A+a})f(a)da.$$

Note that the RHS in the two rst-order conditions above is decreasing in its argument. Since the RHS is bigger (because $H_1 > G_1$), while the LHS is smaller (because, in this case, $c > k-(1-s)\delta k$) we have $x > \overline{y}$.

- If $k > c + (1-s)\delta k$, then there are two possibilities: an interior solution or a corner solution (with $y = 0$.) If we have an interior solution, then $x + y$ satis es

$$c = sH_2(x, y, A) + (1-s)\int_{x+y-A}^{\infty} R'(\frac{x+y}{A+a})f(a)da.$$

We further see that $H_2 = G_1$, which implies that the two rst-order conditions are identical and, hence, in this case, we have $x + y = \overline{y}$.

- The nal case is when $k > c + (1-s)\delta k$ and we have a corner solution, $y = 0$. The optimal x is again determined via (23):

$$k[1-(1-s)\delta] = sH_1(x, 0, A) + (1-s) \int_{x-A}^{\infty} R'(\frac{x}{A+a}) f(a) da.$$

Compared to the FOC for $\overline{y}$, both the RHS and the LHS of the equation above are larger. From equation (24) and the fact that $H_2 = G_1$ (and keeping in mind that we explore the case $X = 0$), we know that we have a corner solution if and only if

$$k[1-(1-s)\delta] - c < s[H_1(\hat{x}, 0, A) - G_1(\hat{x}, A)]. \tag{A1}$$

Arguing by contradiction, suppose that in the corner solution ($y = 0$) we have ($\hat{x} =$) $x \leq \overline{y}$. The FOC under exports only holds with equality for $y = \overline{y}$:

$$c = sG_1(\overline{y}, A) + (1-s) \int_{\overline{y}-A}^{\infty} R'(\frac{\overline{y}}{A+a}) f(a) da.$$

Since the RHS is decreasing in its argument and we assume $\hat{x} \leq \overline{y}$, we have at $\hat{x}$:

$$c \leq sG_1(\hat{x}, A) + (1-s) \int_{\hat{x}-A}^{\infty} R'(\frac{\hat{x}}{A+a}) f(a) da.$$

Further, the FOC for x holds at $\hat{x}$:

$$k[1-(1-s)\delta] = sH_1(\hat{x}, 0, A) + (1-s) \int_{\hat{x}-A}^{\infty} R'(\frac{\hat{x}}{A+a}) f(a) da.$$

If we subtract the two above expressions, we obtain

$$k[1-(1-s)\delta] - c \geq s[H_1(\hat{x}, 0, A) - G_1(\hat{x}, A)],$$

which contradicts (A1). We conclude that, in this case, we have ($\hat{x} =$) $x > \overline{y}$. ■

We proceed now to prove that the RHS of (11) is increasing in A, as stated in Proposition 1:

Let us rewrite the value equation at $X = 0$:

$$v(0, A) = \underset{(x,y)\in\Re^2_+}{Max} \{\phi(x, y, A) + \delta(1 - s) \int\limits_0^\infty v(x, A + a)f(a)da]\}.$$

The rst-order conditions, with respect to x and y, are:

$$\begin{aligned} \phi_1 + \delta(1 - s) \int\limits_0^\infty v_1(x, A + a)f(a)da &= 0, \\ \phi_2 &= 0. \end{aligned}$$

But $v_1 = k$ (this expression presumes that $x > 0$ along the optimal path - it is shown in the paper that this is indeed the case.) Thus, $\phi_1 = -\delta(1 - s)k$. Therefore,

$$\phi_1 \frac{\partial x^*}{\partial A} + \phi_2 \frac{\partial y^*}{\partial A} + \phi_3 = -\delta(1 - s)k \frac{\partial x^*}{\partial A} + 0 + \phi_3.$$

We also have

$$\psi_1 \frac{\partial y^*}{\partial A} + \psi_2 = \psi_2.$$

Therefore,

$$\frac{d}{dA}\{(1 - s)\delta k x^*(A) - \psi(\bar{y}(A), A) + \phi(x^*(A), y^*(A), A)\} = \phi_3 - \psi_2.$$

It remains to show that $\phi_3 > \psi_2$. We have:

$$\begin{aligned} \phi_3 &= sH_3(x, y, A) + \\ &\quad (1 - s)[F(x + y - A) + \int\limits_{x+y-A}^\infty [R(\frac{x + y}{A + a}) - \frac{x + y}{A + a} R'(\frac{x + y}{A + a})]f(a)da], \end{aligned} \tag{A2}$$

and

$$\begin{aligned} \psi_2 &= sG_2(\overline{y}, A) + \\ &\quad (1 - s)[F(\overline{y} - A) + \int\limits_{\overline{y}-A}^\infty [R(\frac{\overline{y}}{A + a}) - \frac{\overline{y}}{A + a} R'(\frac{\overline{y}}{A + a})]f(a)da]. \end{aligned} \tag{A3}$$

We will show that:

$$H_3(x, y, A) \geq G_2(\overline{y}, A),$$

that

$$F(x+y-A) + \int_{x+y-A}^{\infty} [R(\frac{x+y}{A+a}) - \frac{x+y}{A+a}R'(\frac{x+y}{A+a})]f(a)da \geq$$

$$F(\overline{y}-A) + \int_{\overline{y}-A}^{\infty} [R(\frac{\overline{y}}{A+a}) - \frac{\overline{y}}{A+a}R'(\frac{\overline{y}}{A+a})]f(a)da,$$

and that at least one of the above inequalities is strict.

First note that the derivatives of the terminal values with respect to A are as follows:

$$G_2(\overline{y}, A) = \begin{cases} 1 + \frac{\delta}{1-\delta}\lambda & \text{if } \overline{y} \geq A \\ R(\frac{\overline{y}}{A}) - \frac{\overline{y}}{A}R'(\frac{\overline{y}}{A}) + \frac{\delta}{1-\delta}\lambda & \text{if } \overline{y} < A, \end{cases}$$

where

$$\lambda \equiv \begin{cases} \pi_c & \text{if } \frac{A\pi_c}{1-\delta} \geq -e + \frac{A\pi_k}{1-\delta} \\ \pi_k & \text{if } \frac{A\pi_c}{1-\delta} < -e + \frac{A\pi_k}{1-\delta}. \end{cases}$$

The above derivative is de ned for any $A \neq \bar{A} \equiv e(1-\delta)/(\pi_k - \pi_c)$ ($G(\overline{y}, A)$ is not differentiable at $A = \bar{A}$). Note, however, that we are now studying the pre-entry case and, as shown in part (iii) of Proposition 1, this means that the relevant A is lower than $\bar{A}$. Hence, we restrict attention to $A < \bar{A}$, and we have:

$$G_2(\overline{y}, A) = \begin{cases} 1 + \frac{\delta}{1-\delta}\pi_c & \text{if } \overline{y} \geq A \\ R(\frac{\overline{y}}{A}) - \frac{\overline{y}}{A}R'(\frac{\overline{y}}{A}) + \frac{\delta}{1-\delta}\pi_c & \text{if } \overline{y} < A. \end{cases}$$

We also have, for $x + y > A$:

$$H_3(x, y, A) = \begin{cases} \frac{1}{1-\delta} & \text{if } x > A \\ 1 + \frac{\delta}{1-\delta}[R(\frac{x}{A}) - \frac{x}{A}R'(\frac{x}{A})] & \text{if } A > x \geq Aq_k \\ 1 + \frac{\delta}{1-\delta}\pi_k & \text{if } Aq_k > x. \end{cases}$$

And for $x + y < A$:

$$H_3(x, y, A) = \begin{cases} R(\frac{x+y}{A}) - \frac{x}{A}R'(\frac{x+y}{A}) + \frac{\delta}{1-\delta}[R(\frac{x}{A}) - \frac{x}{A}R'(\frac{x}{A})] & \text{if } A > x \geq Aq_k \\ R(\frac{x+y}{A}) - \frac{x}{A}R'(\frac{x+y}{A}) + \frac{\delta}{1-\delta}\pi_k & \text{if } Aq_k > x. \end{cases}$$

Also note the following property that will be used in the remainder of this proof. The function $R(q) - qR'(q)$ is increasing, in particular, taking value 0 when $q = 0$ and value 1 when $q = 1$. This follows from the concavity of $R(q)$, and the fact that $R(q)$ is maximized at $q = 1$ (in particular, we have $\partial[R(q) - qR'(q)]/\partial q = -qR''(q) > 0$).

From Lemma A1, we know that $x + y \geq \bar{y}$. We now need to distinguish two cases:

- First consider the case $x + y = \bar{y}$.

In this case, the second terms at the RHS of (A2) and (A3) are obviously equal. So we need to show that $H_3(x, y, A) > G_2(\overline{y}, A)$. First note that $\pi_c < \pi_k < 1$. Note also that $H_3(x, y, A)$ is increasing in x. Thus, when we show below that $H_3(x, y, A) > G_2(\overline{y}, A)$ holds for $x < Aq_k$ we also know that it holds for any x. There are two subcases to examine. If $\overline{y} \geq A$ then (for $x < Aq_k$) we have $H_3(x, y, A) = 1+\frac{\delta}{1-\delta}\pi_k > 1+\frac{\delta}{1-\delta}\pi_c = G_2(\overline{y}, A)$. If $\overline{y} < A$ then (for $x < Aq_k$) we have $H_3(x, y, A) = R(\frac{x+y}{A}) - \frac{x+y}{A}R'(\frac{x+y}{A}) + \frac{\delta}{1-\delta}\pi_k > R(\frac{\overline{y}}{A}) - \frac{\overline{y}}{A}R'(\frac{\overline{y}}{A}) + \frac{\delta}{1-\delta}\pi_c = G_2(\overline{y}, A)$.

- Now consider the case $x + y > \bar{y}$.

In this case, the term multiplied by $(1 - s)$ at the RHS of (A2) is strictly higher that the corresponding term of (A3). The proof is as follows:

$$F(x + y - A) + \int_{x+y-A}^{\infty} [R(\frac{x + y}{A + a}) - \frac{x + y}{A + a}R'(\frac{x + y}{A + a})]f(a)da =$$

$$F(\bar{y} - A) + [F(x + y - A) - F(\bar{y} - A)] + \int_{x+y-A}^{\infty} [R(\frac{x + y}{A + a}) - \frac{x + y}{A + a}R'(\frac{x + y}{A + a})]f(a)da =$$

$$F(\bar{y} - A) + \int_{\bar{y}-A}^{x+y-A} 1 \cdot f(a)da + \int_{x+y-A}^{\infty} [R(\frac{x + y}{A + a}) - \frac{x + y}{A + a}R'(\frac{x + y}{A + a})]f(a)da >$$

$$F(\bar{y}-A)+\int_{\bar{y}-A}^{x+y-A} [R(\frac{\overline{y}}{A + a})-\frac{\overline{y}}{A + a}R'(\frac{\overline{y}}{A + a})]f(a)da+\int_{x+y-A}^{\infty} [R(\frac{\overline{y}}{A + a})-\frac{\overline{y}}{A + a}R'(\frac{\overline{y}}{A + a})]f(a)da =$$

$$F(\overline{y} - A) + \int_{\overline{y}-A}^{\infty} [R(\frac{\overline{y}}{A + a}) - \frac{\overline{y}}{A + a}R'(\frac{\overline{y}}{A + a})]f(a)da,$$

where the inequality follows from the fact that $R(q) - qR'(q)$ is less than 1 and increasing in q, for $q \in [0, 1]$.

Finally, in this case we have $H_3(x, y, A) \geq G_2(\overline{y}, A)$. The proof follows the same steps as the proof of $H_3(x, y, A) > G_2(\overline{y}, A)$ in the $x + y = \bar{y}$ case above (and, again, the property that $R(q) - qR'(q)$ increasing in q is used here.) This completes the proof of part (i) of Proposition 1.

Part (ii) of Proposition 1 follows immediately from part (i).

To prove part (iii) of Proposition 1 we rst show that if demand stops growing at $\overline{A}$, the seller is indifferent between exports and FDI. The value under exports forever is $\frac{A\pi_c}{1-\delta}$, whereas the value under immediately doing FDI is $-e + \frac{A\pi_k}{1-\delta}$(once the seller does FDI it is not optimal to do any exports.) Since $\overline{A}$ is de ned by equality between these two, the seller is indifferent. The only remaining possibility is that the seller exports for a nite duration, say t, and then does FDI, and sticks with local production thereafter. However, since the above two terms are equal, that possibility gives the same value for any t.

Now consider $\overline{A}$, and assume demand has not stopped growing. Assume the monopolist does FDI, and chooses $x^{**} = Aq_k$ and a y^{**} for which $x^{**} + y^{**} = \overline{y}$. There are 2 possibilities. Either, demand stops growing in the next period. In this case, and as we have just shown, the seller attains the same value as under $u(\overline{A})$. Or, demand does not stop growing, in which case the seller attains a bigger value. So between these 2 cases the value under (x^{**}, y^{**}) exceeds $u(\overline{A})$. Furthermore, since (x^{**}, y^{**}) is not the optimal choice, $v(0, \overline{A})$ exceeds the value he attains with this choice and, therefore, exceeds $u(\overline{A})$. Therefore, at $\overline{A}$ the seller is better off with FDI and, since $v(0, A) - u(A)$ is increasing in A, he prefers FDI for any $A > \overline{A}$.

To show part (iv) of Proposition 1, observe that the RHS of (11) is increasing in A and constant with respect to e, while the LHS is increasing in e. ■

9.2 Appendix 2

Proof of Lemma 1:

Let us rewrite (15) as

$$c = Z(x, y),$$

and (16) as

$$k[1 - (1 - s)\delta] = W(x, y),$$

where the RHS expressions are de ned in an obvious manner. We then have that the slope of $y^o(x)$ is $dy/dx = -(\partial Z/\partial x)/(\partial Z/\partial y) < 0$ and the slope of $x^o(y)$ is $dy/dx = -(\partial W/\partial x)/(\partial W/\partial y) < 0$. We need to show that

$$(\partial W/\partial x)/(\partial W/\partial y) \geq (\partial Z/\partial x)/(\partial Z/\partial y).$$

Note that the term multiplied by $(1-s)$ is the same in both (15) and (16). We also have that $\frac{\partial W}{\partial x}$, $\frac{\partial W}{\partial y}$, $\frac{\partial Z}{\partial x}$, and $\frac{\partial Z}{\partial y}$ are all negative. Therefore, it suffices to show

$$H_{11} \leq H_{12} = H_{21} = H_{22} \leq 0$$

(where $H_{11} \equiv \partial H_1/\partial x$, and so on), which we prove next.

Direct calculations, using (17) (18) and (19), show that

$$H_{12}(X+x,y,A) = H_{21}(X+x,y,A) = H_{22}(X+x,y,A) = \begin{cases} 0 & \text{if } X+x+y \geq A \\ \frac{1}{A}R''(\frac{X+x+y}{A}) & \text{if } X+x+y < A, \end{cases}$$

and that if $X+x+y \geq A$,

$$H_{11}(X+x,y,A) = \begin{cases} 0 & \text{if } X+x \geq A \\ \frac{\delta}{1-\delta}R''(\frac{X+x}{A})\frac{1}{A} & \text{if } A > X+x \geq Aq_k \\ 0 & \text{if } Aq_k > X+x, \end{cases}$$

and, if $X+x+y < A$,

$$H_{11}(X+x,y,A) = \begin{cases} R''(\frac{X+x+y}{A})\frac{1}{A} + \frac{\delta}{1-\delta}R''(\frac{X+x}{A})\frac{1}{A} & \text{if } A > X+x \geq Aq_k \\ R''(\frac{X+x+y}{A})\frac{1}{A} & \text{if } Aq_k > X+x. \end{cases}$$

A direct comparison shows that indeed we have $H_{11} \leq H_{12} = H_{21} = H_{22} \leq 0$ since R is concave (and therefore $\frac{\delta}{1-\delta}R''(\frac{X+x}{A})\frac{1}{A} < 0$.) ■

9.3 Appendix 3

In this Appendix we prove our comparative statics results, Proposition 4.

In all cases, we proceed as follows. Let b be the parameter that is being studied (that is, b stands for c, k, δ, s, or A.) In the case of an interior solution, we rst obtain $\partial x/\partial b$

from equation (22). Then, we obtain $\partial y/\partial b$ from equation (15) (after we substitute the value $\partial x/\partial b$ from the previous step.) If we have a corner solution ($y = 0$), we calculate $\partial x/\partial b$ ($= \partial \hat{x}/\partial b$) from equation (23). Finally, with respect to how the boundary between an interior and a corner solution changes, when the parameter b changes, we consider (24) with equality and see how the critical value changes with b. In this calculation, since the critical value depends on $\hat{x}$, we employ the value $\partial \hat{x}/\partial b$ from the previous step.

In the calculations described above, an important property used is the concavity of R (and for the formal calculations one should keep in mind our normalization that R is maximized at 1.) The details are as follows.

- *Comparative statics with respect to k.*

Consider rst an interior solution. Then, from (22) we see that, as k increases, the RHS increases. For the equality to be restored, it is required that R' increases which implies a decrease in x. Next, we turn to equation (15). An increase in k only affects (15) through a decrease in x which means that, for the equality to be restored, we should have an increase in y.

Consider now a corner solution. Then, by implicitly differentiating (23), we obtain

$$\frac{\partial \hat{x}}{\partial k} = \frac{[1-(1-s)\delta]}{s\frac{\partial H_1}{\partial \hat{x}} + (1-s)[\partial(\int\limits_{X+\hat{x}-A}^{\infty} R'(\frac{X+\hat{x}}{A+a})f(a)da)/\partial \hat{x}]} < 0,$$

where the numerator is positive and the denominator is negative.

Finally, consider the boundary between interior and corner solutions that, by (23), can be written as

$$N \equiv k[1-(1-s)\delta] - c - s[H_1(X+\hat{x},0,A) - H_2(X+\hat{x},0,A)] = 0.$$

We then have

$$\frac{\partial N}{\partial k} = [1-(1-s)\delta] - s\frac{\partial(H_1 - H_2)}{\partial \widehat{x}}\frac{\partial \widehat{x}}{\partial k},$$

where

$$\frac{\partial(H_1 - H_2)}{\partial \widehat{x}} = \begin{cases} 0 & \text{if } X+\hat{x} \geq A \\ \frac{\delta}{1-\delta}R''(\frac{X+\hat{x}}{A})\frac{1}{A} & \text{if } A > X+\hat{x} \geq Aq_k \\ 0 & \text{if } Aq_k > X+\hat{x} \end{cases}.$$

Remember also that the corner solution satis es $X + \hat{x} > Aq_k$. Now, for the case $X + \hat{x} \geq A$, we have $\partial N/\partial k = 1 - (1-s)\delta > 0$. For the $A > X + \hat{x} \geq Aq_k$ case we have $\partial(H_1 - H_2)/\partial\widehat{x} = \delta(\partial H_1/\partial\widehat{x}) = \frac{\delta}{1-\delta}R''(\frac{X+\hat{x}}{A})\frac{1}{A}$ and we obtain

$$\frac{\partial N}{\partial k} = [1-(1-s)\delta] - s\delta\frac{\partial H_1}{\partial\widehat{x}}\frac{[1-(1-s)\delta]}{s\frac{\partial H_1}{\partial\widehat{x}} + (1-s)[\partial(\int\limits_{X+\hat{x}-A}^{\infty} R'(\frac{X+\hat{x}}{A+a})f(a)da)/\partial\hat{x}]} =$$

$$= [1-(1-s)\delta]\left\{1 - \delta\frac{s\frac{\partial H_1}{\partial\widehat{x}}}{s\frac{\partial H_1}{\partial\widehat{x}} + (1-s)[\partial(\int\limits_{X+\hat{x}-A}^{\infty} R'(\frac{X+\hat{x}}{A+a})f(a)da)/\partial\hat{x}]}\right\} > 0,$$

where both $[1-(1-s)\delta]$ and the expression in brackets are between 0 and 1. Thus, an increase in k tends to move the solution from a corner to an interior point.

- *Comparative statics with respect to c.*

Consider rst an interior solution. From (22) we see that, as c increases, the LHS decreases and, to restore the equality, we require a lower R' and, consequently, a higher x. Now turn to (15). An increase in c increases the LHS and also (through the increase is x) it decreases the RHS. Hence it is required that we have an increase in R', and hence a decrease in y.

Concerning a corner solution, clearly c does not affect (23) or the value of $\widehat{x}$

Consider now the boundary N. Since we have $\partial\widehat{x}/\partial c = 0$, we readily obtain $\partial N/\partial c = -1 < 0$ and hence an increase in the solution moves us from an interior to a corner.

- *Comparative statics with respect to s.*

Consider rst an interior solution. Moving s to the LHS of (22) and differentiating, we obtain

$$\partial\left[\frac{k[1-(1-s)\delta] - c}{s}\right]/\partial s > 0,$$

and thus, for the equality to be restored after an increase in s, we should have (an increase in R' and) a decrease in x. With respect to y, note that s enters (15) both directly and through x, with the two effects moving in opposite directions. Further manipulation shows that $\partial y/\partial s$ has ambiguous sign.

Consider now a corner solution. We obtain (from 23)

$$\frac{\partial \widehat{x}}{\partial s} = -\frac{H_1 - \int\limits_{X+\hat{x}-A}^{\infty} R'(\frac{X+\hat{x}}{A+a})f(a)da) - \delta k}{s\frac{\partial H_1}{\partial \widehat{x}} + (1-s)[\partial(\int\limits_{X+\hat{x}-A}^{\infty} R'(\frac{X+\hat{x}}{A+a})f(a)da)/\partial \hat{x}]} < 0,$$

where both the numerator and the denominator are negative. While to see the sign of the denominator is immediate (given the concavity of R), the following argument can be used to establish the sign of the numerator. Recall that $\hat{x}$ solves (23). This equation can be rewritten as

$$s = [k(1-\delta) - \int\limits_{X+\hat{x}-A}^{\infty} R'(\frac{X+\hat{x}}{A+a})f(a)da]/[H_1(X+\hat{x},0,A) - k\delta - \int\limits_{X+\hat{x}-A}^{\infty} R'(\frac{X+\hat{x}}{A+a})f(a)da].$$

Note that the numerator in the above expression is the same as the numerator in $\partial\widehat{x}/\partial s$. Now, since $s \in [0,1]$, both the numerator and the denominator in the above expression have the same sign. Moreover, note that $H_1(X+\hat{x},0,A) < k$. Then, arguing by contradiction, if the numerator and the denominator were positive then we would have $s > 1$, a contradiction. Thus the numerator (both in the above expression, as well as in $\partial\widehat{x}/\partial s$) is negative.

Turning now to the boundary, we have $\partial N/\partial s = \delta k - (H_1 - H_2) = \delta k > 0$ when $X + \hat{x} \geq A$, whereas when $A > X + \hat{x} \geq Aq_k$ we have

$$\frac{\partial N}{\partial s} = \delta k - (H_1 - H_2) + \frac{s\frac{\partial H_1}{\partial \widehat{x}}[H_1 - \int\limits_{X+\hat{x}-A}^{\infty} R'(\frac{X+\hat{x}}{A+a})f(a)da - \delta k]}{s\frac{\partial H_1}{\partial \widehat{x}} + (1-s)[\partial(\int\limits_{X+\hat{x}-A}^{\infty} R'(\frac{X+\hat{x}}{A+a})f(a)da)/\partial \hat{x}]} >$$

$$[H_1 - \int\limits_{X+\hat{x}-A}^{\infty} R'(\frac{X+\hat{x}}{A+a})f(a)da - \delta k]\left\{\frac{s\frac{\partial H_1}{\partial \widehat{x}}}{s\frac{\partial H_1}{\partial \widehat{x}} + (1-s)[\partial(\int\limits_{X+\hat{x}-A}^{\infty} R'(\frac{X+\hat{x}}{A+a})f(a)da)/\partial \hat{x}]} - 1\right\} > 0$$

where the rst inequality is true because (remembering that in this case $A > X + \hat{x}$) the concavity of R implies

$$H_2 = R'(\frac{X+\hat{x}}{A}) > \int\limits_{X+\hat{x}-A}^{\infty} R'(\frac{X+\hat{x}}{A+a})f(a)da,$$

and, for the second inequality, we have shown earlier that the rst factor is negative while the second factor is clearly negative, as well.

Thus, an increase in s tends to move the solution from a corner to an interior point.

- *Comparative statics with respect to* δ*.*

Consider rst an interior solution. An increase in δ decreases the LHS of (22) and increases its RHS. Thus, for the equality to be restored following an increase in s, we should have (a decrease in $R^{'}$ and) an increase in x. With respect to y, note that δ enters (15) only through x, therefore an increase in δ would decrease y.

Consider now a corner solution. We obtain (from 23)

$$\frac{\partial \widehat{x}}{\partial \delta} = -\frac{(1-s)k + s\frac{\partial H_1}{\partial \delta}}{s\frac{\partial H_1}{\partial \widehat{x}} + (1-s)[\partial(\int\limits_{X+\hat{x}-A}^{\infty} R'(\frac{X+\hat{x}}{A+a})f(a)da)/\partial \hat{x}]} > 0,$$

where the numerator is positive and the denominator is negative.

Concerning the boundary, we have $\partial N/\partial \delta = -(1-s)k < 0$ when $X + \hat{x} \geq A$, whereas when $A > X + \hat{x} \geq Aq_k$ we have

$$\frac{\partial N}{\partial \delta} = -(1-s)k - s\frac{\partial (H_1 - H_2)}{\partial \widehat{x}}\frac{\partial \widehat{x}}{\partial \delta} - s\frac{\partial (H_1 - H_2)}{\partial \delta} =$$

$$-(1-s)k + s\delta\frac{\partial H_1}{\partial \widehat{x}}\frac{(1-s)k + s\frac{\partial H_1}{\partial \delta}}{s\frac{\partial H_1}{\partial \widehat{x}} + (1-s)[\partial(\int\limits_{X+\hat{x}-A}^{\infty} R'(\frac{X+\hat{x}}{A+a})f(a)da)/\partial \hat{x}]} - s\frac{\partial H_1}{\partial \delta} =$$

$$= [(1-s)k + s\frac{\partial H_1}{\partial \delta}][\frac{\delta s\frac{\partial H_1}{\partial \widehat{x}}}{s\frac{\partial H_1}{\partial \widehat{x}} + (1-s)[\partial(\int\limits_{X+\hat{x}-A}^{\infty} R'(\frac{X+\hat{x}}{A+a})f(a)da)/\partial \hat{x}]} - 1] < 0.$$

Note that, to determine the sign of the above expression, we have

$$\frac{\partial (H_1 - H_2)}{\partial \widehat{x}} = \delta\frac{\partial H_1}{\partial \widehat{x}} = \frac{\delta}{1-\delta}R^{''}\frac{1}{A} < 0,$$

$$\frac{\partial (H_1 - H_2)}{\partial \delta} = \frac{\partial H_1}{\partial \delta} = \frac{1}{(1-\delta)^2}R^{'} > 0.$$

Thus, an increase in δ tends to move the solution from an interior point to a corner.

- *Comparative statics with respect to A.*

Finally, we consider how the solution (for a given X) changes if we have a higher A. Concerning an interior solution, an increase in A tends to increase R' and, for the equality to be restored, we should have an increase in x. With respect to y, note that A enters (15) both directly and through x, with the two effects moving in opposite directions. Further manipulation shows that $\partial y/\partial A$ has ambiguous sign.

Consider now a corner solution. We obtain (from 23)

$$\frac{\partial \widehat{x}}{\partial A} = -\frac{s\frac{\partial H_1}{\partial A} + (1-s)[\partial(\int\limits_{X+\hat{x}-A}^{\infty} R'(\frac{X+\hat{x}}{A+a})f(a)da)/\partial A]}{s\frac{\partial H_1}{\partial \widehat{x}} + (1-s)[\partial(\int\limits_{X+\hat{x}-A}^{\infty} R'(\frac{X+\hat{x}}{A+a})f(a)da)/\partial \hat{x}]} > 0,$$

where the numerator is positive and the denominator is negative.

Turning now to the boundary, we have $\partial N/\partial A = 0$ in the $X + \hat{x} \geq A$ case (since N is affected by A only through $H_1 - H_2$). We now turn to the $A > X + \hat{x} \geq Aq_k$ case. We have:

$$\frac{\partial N}{\partial A} < 0 \Leftrightarrow \frac{\partial(H_1 - H_2)}{\partial A} > 0 \Leftrightarrow \frac{\partial[\frac{\delta}{1-\delta}R'(\frac{X+\hat{x}}{A})]}{\partial A} > 0 \Leftrightarrow \frac{\partial(\frac{X+\hat{x}}{A})}{\partial A} < 0 \Leftrightarrow$$

$$\frac{\partial \hat{x}}{\partial A} < \frac{X+\hat{x}}{A}.$$

Now, using $\partial\widehat{x}/\partial A$ that we have calculated above and rearranging, the above inequality is equivalent to

$$A[s\frac{\partial H_1}{\partial A} + (1-s)\frac{\partial(\int\limits_{X+\hat{x}-A}^{\infty} R'(\frac{X+\hat{x}}{A+a})f(a)da)}{\partial A}] + (X+\hat{x})[s\frac{\partial H_1}{\partial \widehat{x}} + (1-s)\frac{\partial(\int\limits_{X+\hat{x}-A}^{\infty} R'(\frac{X+\hat{x}}{A+a})f(a)da)}{\partial \hat{x}}] < 0$$

or

$$s[A\frac{\partial H_1}{\partial A} + (X+\hat{x})\frac{\partial H_1}{\partial \widehat{x}}] + (1-s)[A\frac{\partial(\int\limits_{X+\hat{x}-A}^{\infty} R'(\frac{X+\hat{x}}{A+a})f(a)da)}{\partial A} + (X+\hat{x})\frac{\partial(\int\limits_{X+\hat{x}-A}^{\infty} R'(\frac{X+\hat{x}}{A+a})f(a)da)}{\partial \hat{x}}] < 0.$$

We now have

$$\frac{\partial H_1}{\partial A} = -\frac{\delta}{1-\delta}R''(\frac{X+\hat{x}}{A})\frac{X+\hat{x}}{A^2} \qquad \text{and} \qquad \frac{\partial H_1}{\partial \widehat{x}} = \frac{\delta}{1-\delta}R''(\frac{X+\hat{x}}{A})\frac{1}{A},$$

and direct calculation shows that the rst term in the above inequality is zero. The second term of the inequality is equal to

$$(1-s)(X+\hat{x}) \int_{X+\hat{x}-A}^{\infty} R''(\frac{X+\hat{x}}{A+a}) \frac{a}{(A+a)^2} f(a)da < 0.$$

Thus, in the $A > X + \hat{x} \geq Aq_k$ case, we have $\partial N/\partial A < 0$, that is, an increase in A tends to move the solution from an interior point to a corner. ■

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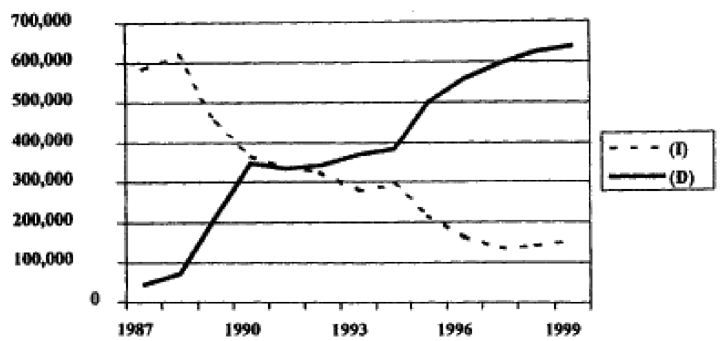
10 Table and Figures

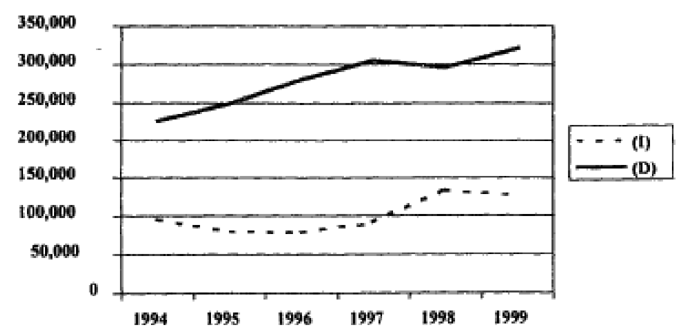
A	s	x	y	$X+x$
0.5775	0.1000	0	0.2998	0
1.4821	0.1000	0	0.7951	0
2.3011	0.1000	0	1.3047	0
3.1644	0.1000	0	1.8812	0
3.2873	0.1000	0	1.9658	0
4.1165	0.1000	0	2.5496	0
4.5182	0.1000	3.0090	0	3.0090
4.8528	0.1000	0.2439	0	3.2529
5.4265	0.1000	0.4231	0	3.6760
5.5333	0.1000	0.0793	0	3.7553
5.7757	0.1000	0.1809	0	3.9362
6.2255	0.1000	0.3380	0	4.2741
6.7953	0.1000	0.4324	0	4.7065
7.4506	0.1296	0.3940	0	5.1005
7.6841	0.1452	0.1231	0	5.2236
7.8683	0.1574	0.0973	0	5.3208
8.7474	0.2160	0.1779	0.4359	5.4987
9.0131	0.2337	0.0792	0.5405	5.5780
9.6023	0.2729	0.2005	0.7439	5.7785
10.4892	0.3000	0.4373	0.9595	6.2158
10.8881	0.3000	0.2364	1.0383	6.4522
11.6944	0.3000	0.4778	1.2018	6.9300
12.5523	0.3000	0.5084	1.3812	7.4384
13.0291	0.3000	0.2825	1.4832	7.7209
13.1891	0.3000	0.0948	1.5177	7.8157

Table for the example with variable s and $D(q) = 2 - q$, $e = 12$, $c = 1.5$, $k = 10$, $\delta = 0.9$ and f uniform on $[0, 1]$ - Flow and Stock.



Source: Automotive News

FIGURE 1A: Toyota car sales in the U.S.: Imports (I) and Domestically produced (D).



Source: Automotive News

FIGURE 1B: Toyota car sales of Camry model in the U.S.: Imports (I) and Domestically produced (D).

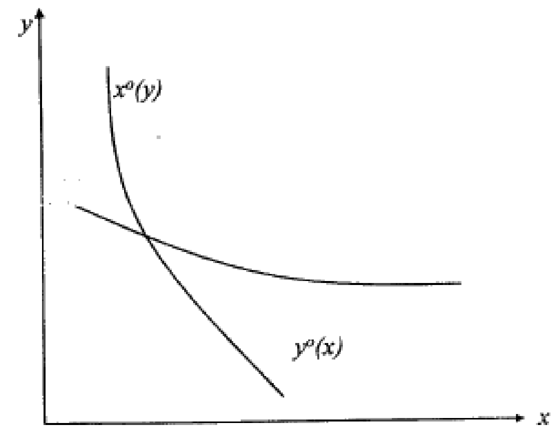


Fig. 2A

Interior Solution: x°(y) intersects y°(x) in R^2_{++}.

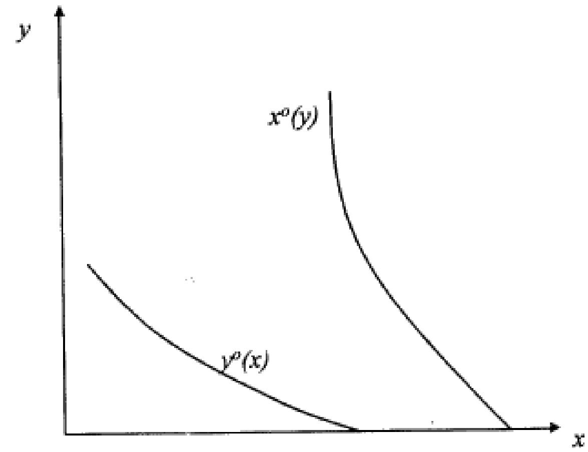


Fig. 2B

x°(y) uniformly above y°(x).

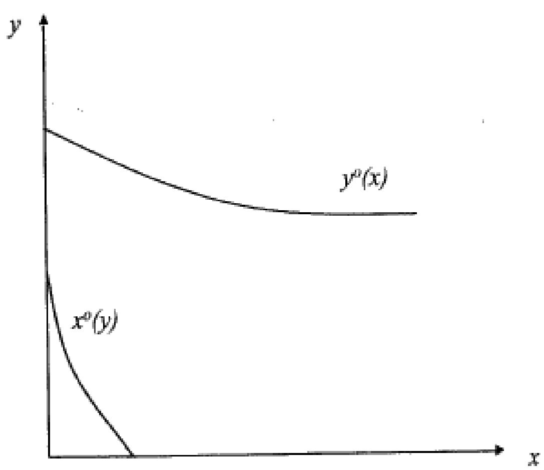


Fig. 2C

x°(y) uniformly below y°(x).

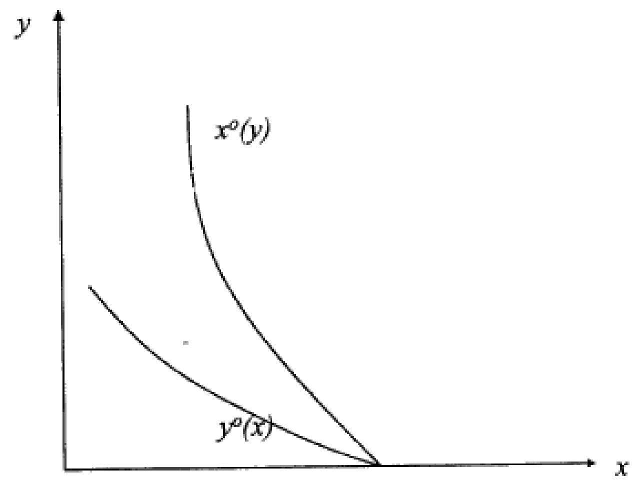


Fig. 2D

Borderline in (x,y)-space.

FIGURE 2: Determiniation of optimal exports and investments.

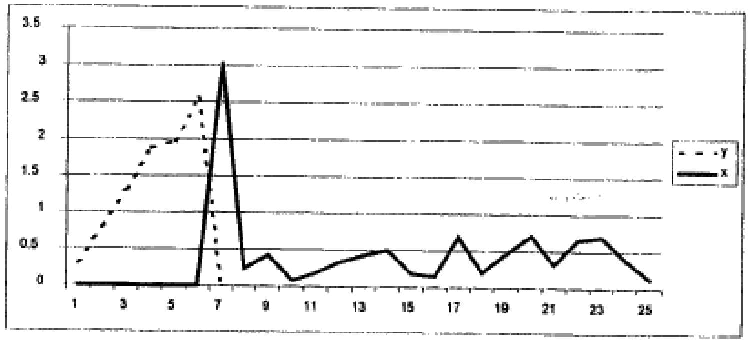


FIGURE 3A: Flow

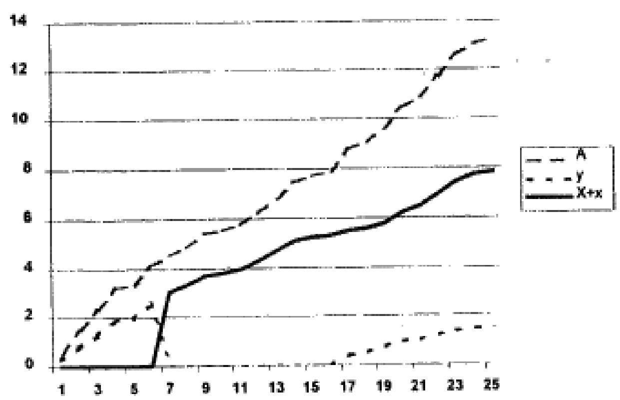


FIGURE 3B: Stock

FIGURE 3: Example with $D(q) = 2 - q$, $e = 12$, $c = 1.5$, $k = 10$, $\delta = 0.9$, $s = 0.1$, and f uniform on $[0, 1]$.

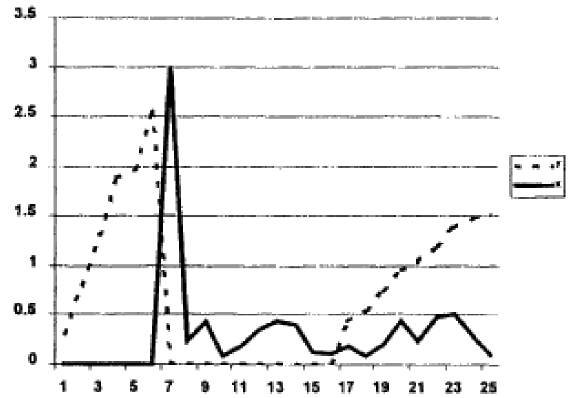


FIGURE 4A: Flow

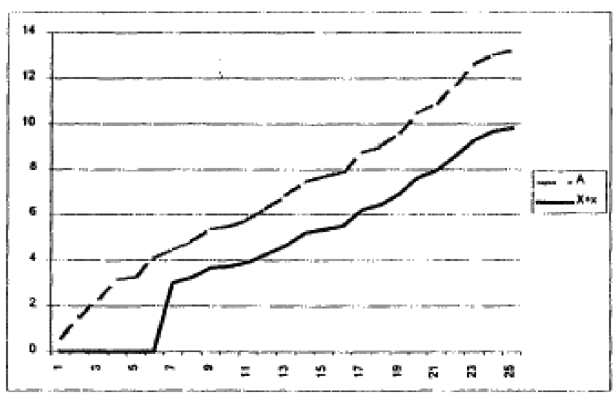


FIGURE 4B: Stock

FIGURE 4: Example with variable (increasing) s and $D(q) = 2 - q$, $e = 12$, $c = 1.5$,
$k = 10$, $\delta = 0.9$, and f uniform on $[0, 1]$.