Abstract

We study a dynamic principal-agent problem where social capital is an important part of the system of incentives. In each period the firm chooses an incentive intensity, and its employees allocate effort between individual and cooperative tasks. Cooperative tasks are - within bounds - more productive than individual tasks, but employees are not monetarily rewarded for them. Rather, and consistent with recent work in experimental economics, employees allocate effort to cooperative tasks because they derive utility from cooperation. The utility from cooperation is endogenously determined, and depends on how much others have cooperated in the past and on the firm's incentive intensity. Consequently, the cooperativeness of the workforce, which we also call the firm's "social capital," follows a dynamic process where the incentive intensity acts as a control variable. We show that the optimal choice of incentives can create cultural differences across firms.

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1. Introduction

The results of recent experimental work on how much subjects contribute to a public good are at odds with the predictions of economic theory. The public-good game studied in these experiments is a multi-person prisoner’s dilemma, or a “social dilemma,” where egotistical subjects are predicted to contribute nothing. Yet, the experiments show sizable and significant contributions (see Section 2), which are robust to numerous designs of the experiment. One way to rectify the divergence between theory and experiments is that subjects possess non-selfish preferences, i.e., that they derive utility from the act of giving. The objective of this paper is to integrate this hypothesis into the theory of the firm, in particular the theory of optimal incentives.

The reason a social dilemma figures in our theory of the firm is that workers are engaged in multi-task production. One task, which we call individual, is relatively easy to measure, whereas the other task, which we call cooperative (e.g., “helping others”), is hard to measure. In such an environment, if a worker’s payoff depends on his measured output and if he is an egoist he has little reason to cooperate. Yet, the cooperative task - at least within bounds - may contribute more to the firm’s output than the individual task. Therefore, a worker’s allocation of effort between the two tasks exhibits a social dilemma.

A simple example is shiftwork in a factory: a worker can focus on the number of units his machine produces (individual task), or he might take time away from production to make sure his machine is properly maintained (cooperative task.) In that case the machine is less likely to break down when another worker succeeds him, but he may not get credit for that if the firm has trouble rewarding workers based on machine down time.1

The design of incentives in a multi-task environment is the subject of an important paper by Holmstrom and Milgrom (1991), and in many respects the model we develop in this paper is a special case of theirs. However, Holmstrom and Milgrom consider a static framework where workers’ preferences are fixed and given. By contrast, when the firm designs incentives in our dynamic model it tries to

1The literature on “organizational behavior” addresses the importance of hard to measure cooperative effort: “every factory, office, or bureau depends daily on a myriad of acts of cooperation, helpfulness, suggestions, gestures of goodwill, altruism, and other instances of what we might call citizenship behavior.... Furthermore, much of what we call citizenship behavior is not easily governed by individual incentive schemes, because such behavior is often subtle, difficult to measure, [and] may contribute more to others’ performance than one’s own” (quoted from Smith et al. 1983, but see also Organ 1988 and Deckop et al. 1999).
of cooperation affect the sense of guilt employees feel if they do not cooperate, which in turn affects the level of current cooperation. We associate the level of cooperation with the firm’s stock of social capital.

Second, we provide a novel perspective on the optimal incentive intensity. If the firm sets a high incentive intensity, the lure of monetary rewards looms large relative to the employees’ sense of guilt and cooperation levels fall. Conversely, a low incentive intensity encourages cooperation. Thus, the incentive intensity controls the firm’s stock of social capital. The optimal incentive intensity is determined by an intertemporal trade-off. As the incentive intensity increases, total employee effort increases but the firm’s stock of social capital falls.\(^3\)

Third, we are able to explore differences in corporate cultures, specifically the extent to which firms might vary in the cooperativeness of their culture. An important attribute of culture is that it tends to be self-reinforcing due to positive feedback and, hence, multiple cultures are possible. In our context, this means that highly cooperative workforces tend to stay that way while less cooperative workforces remain as such. Surprisingly, perhaps, we show that the multiplicity of cultures comes about (for some parameter values) only when the firm is varying its incentive intensity over time. Therefore, rather than undoing the effect of positive feedbacks in the model, the firm is reinforcing them.

The paper proceeds as follows. Section 2 summarizes the evidence from the experimental literature relevant to our model. Section 3 presents and discusses our model. Section 4 characterizes employee behavior and defines social capital, while Section 5 introduces the concept of corporate culture. Section 6 derives the firm’s objective function. Section 7 solves the firm’s dynamic maximization problem. Section 8 contains the main results and discusses the implications for cultural differences. Section 9 discusses the empirical implications of our results. Section 10 concludes. Some of the results are proven in the Appendix.

## 2. Experimental Support

Our approach to social dilemmas in organizations is motivated by a large experimental economics literature on behavior in social dilemmas and other games.\(^4\)

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\(^3\)Our formalization of social capital has strong parallels to capital theory where physical assets can be gradually accumulated by sacrificing short-run consumption. Here, the firm can sacrifice short-run profit to grow social capital by lowering the incentive intensity.

\(^4\)We focus on the results from public goods games. However, non-selfish behavior is also prevalent in trust games (e.g., Berg et al. 1995) and dictator games (e.g., Hoffman et al, 1994).
of contributions, i.e., the decay observed in the experiments.\textsuperscript{5} Fehr and Gachter (2000) summarize experimental work documenting the existence of such reciprocity. Consistent with our approach, they conclude that “the often observed decay of cooperation in a repeated public-goods game can be explained as a reaction to other players’ contributions.”

- **Anonymity**: Most of the public good experiments are conducted under conditions of neutrality and anonymity with the intention of filtering out “social” and “cultural” factors. This raises the obvious question whether one can extrapolate the results of laboratory experiments to behavior in real firms where interactions among workers are far from anonymous. Gachter and Fehr (1999) and Spagnolo (1999) conduct experiments where they vary the degree of anonymity and show, not surprisingly, that cooperation is actually stronger when subjects know each other. More strikingly, perhaps, Bohnet and Frey (1999) show that subjects become more cooperative merely when they are shown a picture of the individual with whom they are playing a prisoners’ dilemma even while they themselves remain anonymous. Given that workers interact non-anonymously in firms these result strengthen the case for assuming that workers possess cooperative preferences.

- **Crowding Out**: Suppose that there are noisy measures of employee cooperation. Is it not in the firm’s interest to use these to some extent? There is a counter argument due to social psychologists that the use of extrinsic motivators can undermine, or “crowd out,” intrinsic motivation; see Kreps (1997). This argument has been tested by experimental economists and is receiving support; see Frey and Oberholtzer-Gee (1997), Frey and Jegen (forthcoming), and Gneezy and Rustichini (2000). Given that our model is predicated on individuals being intrinsically motivated to cooperate, this offers support for not including explicit incentives for cooperation.\textsuperscript{6}

\textsuperscript{5}One particularly interesting paper on such reciprocity is Fischbacher et al. (2000) in which an experiment is run that elicits subjects’ willingness to cooperate as a function of group cooperation levels (subjects fill out a table as part of playing a public goods game.) They find that half of their subjects are conditionally cooperative in that their cooperation levels increase in the cooperation of others, whereas 30 percent are free riders who contribute little or nothing.

\textsuperscript{6}Moreover, the social psychologists Goranson and Berkowitz (1966) find that subjects are less likely to reciprocate cooperation when they know that the giver is being paid for his cooperation. Thus, even if there are explicit incentive for some cooperative tasks, the impact on cooperation in other tasks might be muted.
cooperative effort falls to zero. The total output of a worker is:

\[ Q = ae_I + e_C. \]

While \( Q \) is total output, the firm is unable to observe it. Instead the firm observes a proxy of \( Q \), call it \( \hat{Q} \), which is:

\[ \hat{Q} = ae_I + [e_C + \int_0^1 e_C(i)di]/2. \]

The difference between \( Q \) and \( \hat{Q} \) - or between “true” and “measured” output - comes about because a worker’s cooperative effort helps (i.e., raises the measured output of) his co-workers, and the firm is unable to give him full credit for this help. Symmetrically, the worker gets “undue credit,” \( \frac{1}{2} \int_0^1 e_C(i)di \), due to the help of others. The particular formula we use here assumes that for every hour the worker helps his co-workers he gets only 1/2 an hour credit.

We assume \( 1/2 < a < 1 \). This implies that cooperative effort is more productive than individual effort for \( 0 \leq e_C \leq h \), and vice versa for \( e_C > h \). Hence, in a world of full observability, the firm and other workers are better off if \( e_C = h \). However, if a worker’s pay is based on \( \hat{Q} \) and if the worker is selfish, he chooses \( e_C = 0 \). Thus, the allocation of effort in this model is subject to a social dilemma.

3.2. Workers’ Preferences

A worker’s utility depends on his wage, the disutility of total effort, and the disutility of defecting. A worker is said to “defect” if his choice of cooperative effort falls short of the “ideal” \( e_C = h \). A worker’s overall utility is:

\[ U = W - C(e) - (h - e_C)g, \]

where \( W \) is the wage, \( C(e) \) is the disutility of total effort, and \( (h - e_C)g \) is the disutility of defecting. If \( e_C \geq h \), the last term is 0, i.e., the worker does not get extra utility by choosing \( e_C \) above the ideal level. The disutility from defecting

\[ ^7 \text{For example, it may be efficient to have an assembly-line worker take one hour a day to maintain the equipment he works on. More than that is a waste of time. More generally, one might entertain a general multi-task production function with diminishing returns for both tasks.} \]

\[ ^8 \text{Equivalently, the true marginal productivity of } e_C \text{ is 1, whereas the measured marginal productivity is only } 1/2. \]
where $p$ is the price of output and $E$ is expectation as we go across workers.\footnote{While we have assumed that the firm does not observe the $Q$ of individual workers, it might still observe total output. Nonetheless, the observation of total output gives no information about the output of an individual worker given that there is a continuum of workers.} We assume that $p > (1 + r)/(a - 1/2)$, so that the price is high relative to the willingness to cooperate.\footnote{This simplifies the analysis by assuring that with the first-best incentive intensity, $w = p$, there is no cooperation; see Lemma 7.3.}

The firm maximizes profits by selecting a compensation system in each period. Specifically, we assume that wages are a linear function of a worker's measured output, $W = b + wQ$. We refer to $b$ as the base wage and to $w$ as the incentive intensity. The compensation system satisfies an individual rationality constraint: In each period, each worker must be given a level of utility of at least $u$, which we normalize to be 0. The firm starts with an initial level of cooperation $z_0$.

### 3.4. Discussion of Assumptions

1. Based on the above formulation, a worker's marginal utility from cooperating, $\frac{\partial U}{\partial e_C}$, is $g$, which depend on last period's $z$. Thus, greater cooperation last period implies greater marginal utility to cooperate this period. This is one factor driving the dynamic in our model. There are other and, probably, just as natural ways to achieve the same effect. For example, we could specify a quadratic loss from deviating from the "cooperative norm" $h$. Namely, specify cooperative utility as $-g(zh - e_C)^2$, where $g$ is independent of $z$. In this case the marginal utility from cooperation, $2g(zh - e_C)$, would still be increasing in $z$. On the other hand, if the loss from not cooperating were linear, say $g(zh - e_C)$, the marginal utility from cooperation would be independent of $z$, and nothing like what we derive below would go through. So the critical assumption is that there be a positive feedback from last period's cooperation to this period's cooperation.

2. The firm is restricted to the use of a single, linear compensation system for the whole workforce. The firm could do better with forcing contracts. The obvious reason for using linear incentives is tractability and commonality of use in real life. Alternatively, linear incentives might be the result of a more complex environment, including noise, risk aversion and dynamic production, as is shown in Holmstrom and Milgrom (1987). To focus on the issue of immediate interest, we take linear incentives for granted here, and do not model how they come about.
\[ z' = f(w, z) = \begin{cases} 
0 & \text{if } wd > rz + 1, \\
1 + rz - wd & \text{if } rz < wd < rz + 1, \\
1 & \text{if } wd < rz.
\end{cases} \] (4.2)

**Proof.** According to Lemma 4.1 a worker chooses \( e_C = h \) if and only if \( q > wd \). Hence, the measure of workers choosing \( e_C = h \) is \( \Pr(g > wd \mid z) \). The result follows now since \( g \) is uniformly distributed over the interval \([rz, rz + 1]\); see discussion following equation (3.2).

For the analysis in the remainder of the paper, employee behavior is fully summarized by the increasing effort supply function \( e^*(w) \), and the law-of-motion of social capital, \( f(w, z) \).

## 5. From Social Capital to Corporate Culture

We have just established that the firm’s stock of social capital evolves over time, partly driven by incentives and partly driven by its own “internal dynamics.” In this section we fix the incentives and consider the internal dynamics of social capital. This will serve as a benchmark against which to compare the evolution of social capital under an endogenously determined \( w \).

Of particular interest is a steady state of social capital under the law-of-motion (4.2). Namely, we fix the incentive intensity at some value, \( \bar{w} \), and seek a value of \( z \), denote it by \( \bar{z} \), so that \( \bar{z} = f(\bar{w}, \bar{z}) \). We are interested in steady states because we equate them with a firm’s corporate culture. This name is motivated by the idea that culture is a stable, or self-reproducing, pattern of behavior in a group (in this case the firm.)

**Proposition 5.1.** (i) For \( r < 1 \) and a fixed \( w \), there is a unique steady state. (ii) For \( r \geq 1 \), and a fixed \( w \in [1/d, r/d] \) there are multiple steady states.

**Proof.** (i) Since \( f \) is continuous with domain = range = \([0, 1]\), there must be at least one steady state. Suppose now \( r < 1 \). Given the specific functional form for \( f \) given in Lemma 4.2 there are three possible steady states: 0, 1, and \((1 - wd)/(1 - r)\). We show uniqueness for each of four possible regions of \( w \). First, if \( wd \geq 1 + r \), then \( f(w, z) \equiv 0 \), so \( z = 0 \) is the only steady state. Second, if \( wd \in [1, 1 + r) \), then \( f(w, z) < 1 \) and \((1 - wd)/(1 - r) \leq 0 \). So the unique steady state is \( z = 0 \). Third, if \( wd \in (r, 1) \), then \( f(w, z) \in (0, 1) \), and only the interior steady state, \((1 - wd)/(1 - r)\), is possible. Finally, if \( wd \leq r \), then \( f(w, z) > 0 \).
Proof. Recall that the one period profit is \( pE[Q] - E[W] \). The expected output of an employee is
\[
E[Q] = ae^* + (1 - a)hz'.
\]
The first term, \( ae^* \), is the output if all effort is put into individual production. The second term is the increase in output from the \( hz' \) units of effort that are put into cooperative production. All that remains is to find an expression for \( E[W] \).

The expected wage is \( E[W] = b + wE[Q] \). We can eliminate the base wage, \( b \), from this expression as follows. The utility for an employee who cooperates is
\[
U_C = b + w( ae^* + hz'/2 - hd) - C(e^*),
\]
whereas the utility for one who defects is
\[
U_D = b + w( ae^* + hz'/2) - hg - C(e^*).
\]
An employee decides to defect whenever \( U_D > U_C \), i.e., whenever \( hg < hw_d \).

Therefore, the utility for a defector is higher than the utility for a cooperator. Thus, if the firm tries to maximize profits and if \( z' > 0 \), the individual rationality constraint on cooperators is binding, while the individual rationality constraint on defectors is not, i.e., \( U_C = 0 \). This yields
\[
b = C(e^*) - w( ae^* + hz'/2 - hd).
\]
The expected wage is then
\[
E[W] = b + w[z'Q_C + (1 - z')Q_D] = C(e^*) + wd_h(1 - z'),
\]
where \( Q_C \) (\( Q_D \)) is the performance measure of cooperators (defectors). On the other hand, if \( z' = 0 \), there is a discontinuity in the expected wage: The firm no longer has to satisfy \( U_C \geq 0 \) because there are no cooperators. Now it can set \( U_D = 0 \) and the expected wage is then \( E[W] = C(e^*) \). Combining the expressions for \( E[Q] \) and \( E[W] \) and substituting the expression for \( e^* \) from Lemma 4.1 gives the result. The rest of the Lemma, i.e., properties (ii)-(iv), follow from equations (6.1) and (6.2).

The first two terms in \( \pi(w, z) \) are the profit if effort is entirely dedicated to the individual task. The third term is the increase in profit that comes from the \( z' \) workers that cooperate. The final \( R(w, z) \) term reflects the rents that go to defectors.\(^\text{14}\)

\(^{14}\)Rents are paid to defectors (those choosing \( e_C = 0 \)) because the incentive system is calibrated to satisfy the participation constraint of those who cooperate and who consequently have lower measured output.
Lemma 7.2. Consider $z_1, z_2, z'_1, z'_2$, so that $z_i \in [0,1]$, $z_1 < z_2$ and $z'_i \in \zeta(z_i)$, for $i = 1, 2$. Then $z'_1 \leq z'_2$, and if $z_1$ or $z_2 \in (0,1)$, then $z'_1 < z'_2$.

Proof. In the Appendix.

With monotonicity in $\zeta$, there is a “critical mass,” $\bar{z}$, of social capital such that when $z_0 > \bar{z}$ the level of social capital does not go to zero. In particular, we can define

$$\bar{z} = \inf\{z > 0 \mid z' \geq z \text{ for some } z' \in \zeta(z)\}.$$  

If $z' < z$ for all $z' \in \zeta(z)$ and all $z \in [0,1]$, we set $\bar{z} = 1$. We now characterize the dynamic depending on whether the initial stock of social capital $z_0$ is above or below $\bar{z}$. We say that $z_s$ is a **steady state** if it satisfies $z_s \in \zeta(z_s)$, which is equivalent to $z_s = f(w_s, z_s)$ for some $w_s \in \omega(z_s)$.

Lemma 7.3. (i) For any $z_0 < \bar{z}$ (if there are such $z_0$'s), any optimal sequence $(z_t)_{t=0}^{\infty}$ converges to the steady state $z_s = 0$. (ii) At that steady state $\omega(z_s) = p$.

Proof. (i) From Lemma 7.2 and the definition of $\bar{z}$, it follows that $z_{t+1} < z_t$ for all $t$. Therefore, the sequence $(z_t)_{t=0}^{\infty}$ must converge to a limit $z^*$ and, by the upper-semi-continuity of $\zeta$, $z^* \in \zeta(z^*)$. Therefore $z^*$ is a steady-state. Assume $z^* > 0$. Then this contradicts the definition of $\bar{z}$ since $0 < z^* < \bar{z}$. Therefore $z^* = 0$.

(ii) Starting from $z_0 = 0$, the firm follows the optimal path $z_t = 0$, i.e., it does not invest in culture. But then it maximizes its static profit by choosing $w = p$. ■

Consider now $z_0 > \bar{z}$ (if there are such $z_0$'s) and let $(z) = (z_t)_{t=0}^{\infty}$ be an optimal sequence starting at $z_0$. Then, by the definition of $\bar{z}$, $z_t \geq z_0 > 0$. So we can restrict the maximization program $(P_0)$ to sequences $(z)$ which satisfy this constraint. But, under this constraint, the period payoff is strictly concave and continuously differentiable. Consequently we have:

Lemma 7.4. Consider $z_0 > \bar{z}$. Then, (i) the sequence program $(P_0)$ has the same value and the same set of maximizers as the following dynamic programming program (where $z$ is restricted to the domain $z > \bar{z}$):

$$V(z) = \max_{0 \leq w \leq p} \{\pi(w, z) + \delta V(f(w, z))\}. \quad (7.2)$$

(ii) There exists a unique, increasing value function, $V$. (iii) The policy function, $\omega(z)$, is increasing in $z$. (iv) The stock of social capital converges to some steady state $z_s \in [\bar{z}, 1]$. (v) There is at most one positive steady state.
because workers are already cooperative. Hence, a firm starting with a high stock of social capital wants to keep it high while a firm with a low stock depletes it completely.

Figure 8.1 plots $\gamma'$ as a function of $\gamma$ for the parameters values $\delta = 0$, $p = 7$, $a = 0.7$, $h = 1$, $c = 4$, and $r = 0.4$. The figure illustrates the existence of multiple steady states: For $\gamma_0$ below $\bar{\gamma} = 0.42$, the culture converges to $\gamma_s = 0$. For $\gamma_0$ above 0.42, the culture converges to $\gamma_s = 0.84$. When the $\gamma - \gamma'$ curve lies above the 45° line, social capital increases over time. Conversely, when the curve is below the 45° line, the firm's social capital decreases.\(^{16}\) It is also possible to find parameter values for which there is a unique steady state with either $\gamma_s = 0$ or $\gamma_s > 0$.

We find that cultural differences can come about purely because of differences in initial conditions $\gamma_0$. Firms that are fortunate enough to attract cooperative workers when they are first established develop cooperative culture, whereas firms that attract noncooperative workers end up with a noncooperative culture. In that sense the model exhibits history dependence.\(^{17}\)

Other forces that shape cultural differences across firms are fundamentals of the model, e.g., the production technology or workers’ preferences. To explore

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\(^{16}\)For a non myopic firm (i.e. $\delta > 0$), the $\gamma - \gamma'$ curve shifts up so that the positive steady state is higher and has a larger basin of attraction; see Proposition 8.2.

\(^{17}\)There is some tension between our assumption of a continuum of workers and the interpretation that firms vary in initial conditions. History dependence could also arise if the model is extended to allow for a stochastic element to the law of motion of social capital. Such shocks could arise from variety of factors such as good or bad news about the firm’s financial performance.
Proposition 8.2 shows that the incentive intensity in our model is increasing in \( p \) and decreasing in \( c \), as in the standard theory. However, we also find that optimal incentives depend on the importance of cooperative production, \( h \), the strength of reciprocity, \( r \), and the discount factor, \( \delta \). Thus, for example, we expect stronger incentive intensity in firms, or activities within a firm, where output is strongly based on individual effort (e.g., sales personnel) as compared to firms where output strongly depends on cooperative effort (e.g., engineers and scientists in a research facility.)

Another important feature of our theory is that a firm that increases \( w \) from its optimal level will see its short-run profit increase at the expense of long-run profits. This is consistent with claims made by some management scholars that the use of incentives in organizations can be short-sighted.\(^{19}\) Thus we expect firms in financial distress to increase the incentive intensity in order to shift profits from the future into the present.

A further implication of our theory is that incentives and profits vary over time, without a corresponding variation in any of the fundamentals. In Figure 8.1, a firm with \( z_0 > z_s = 0.84 \) decreases its incentive intensity over time as \( z_t \rightarrow z_s \). Conversely, a firm with \( z_0 \in (0.42, 0.84) \) increases its incentive intensity over time.\(^{20}\)

Our theory predicts cross-sectional variation in incentives and profitability across firms, holding their production technology and other fundamentals constant. This variation is due to cultural differences, which come about because of historical factors. Moreover, there is a negative correlation between incentives and profits: A firm with a cooperative culture, \( z_s > 0 \), has higher profits and a lower incentive intensity than a firm with a noncooperative culture, \( z_s = 0 \).

Our finding of culture-based profit differences is consistent with the business-strategy literature which emphasizes that performance differences across firms, even those operating in the same industry, can be attributed to corporate culture. This literature explains the sustainability of these performance differences based on the inimitability of some cultures (Barney, 1986, Besanko et al. 2000). In contrast, in our model the imitation of corporate culture (or the lack thereof) is an economic decision: A firm with an uncooperative culture could imitate a more cooperative culture, but the cost of building up the required stock of social capital

\(^{19}\)For example, Kohn (1993) claims “rewards buy temporary compliance, so it looks like the problems are solved. It’s harder to spot the harm they cause in the long-term.”

\(^{20}\)Of course, the change in \( w_t \) dies down over time. However, one could add stochastic shocks to the model to explain firms that periodically change their incentive intensity.
A. Proofs of Propositions

Proof of Lemma 7.1

A potential technical problem with program (P) is that the per-period payoff function, \( \pi(w, z) \), has a discontinuity at \( w = (rz + 1)/d \); see Lemma 6.1. Fortunately, our program is equivalent to one with a continuous per-period return function, which we establish via the following 2 Claims.

Claim A1 There is no loss to maximizing \((P)\) over the domain 
\( w_t \in [rz/d, (rz + 1)/d] \cup \{p\} \).

Proof: Let \((w) = (w_t)_{t=1}^{\infty}\) be a candidate for a solution to \((P)\). Then if one of the \(w_t\)'s is in the range \(((rz + 1)/d, p)\), we can replace it with \(p\). This will give a higher payoff in the current period and will result in the same \(z_t = 0\). Hence it will generate a higher value for the objective. Likewise, if one of the \(w_t\)'s is in the range \([0, rz/d]\) we can replace it by \(rz/d\). Consequently, we can maximize \((P)\) over the domain \([rz/d, (rz + 1)/d] \cup \{p\}\).

Claim A2 Let \(\bar{\pi}\) be a continuous function which equals \(\pi\) on \([0, (rz + 1)/d] \cup \{p\}\) and lies below \(\pi\) on \(((rz + 1)/d, p)\). Then the set of solutions to \((P)\) coincides with the set of solutions to \((P)\) when \(\bar{\pi}\) replaces \(\pi\).

Proof: Since \(\pi \geq \bar{\pi}\), the value of the objective for any \((w)\) is no less under \(\pi\) than it is under \(\bar{\pi}\). Let us call the maximization program under \(\bar{\pi}\), \((\bar{P})\). Consider a solution \((w^*)\) to \((P)\). Then, by Claim A1 we can assume that none of the \(w_t^*\)'s is in the interval \(((rz + 1)/d, p)\). Therefore, the value of the objective in \((\bar{P})\) at \((w^*)\) is the same. So \((w^*)\) is a solution to \((\bar{P})\). Conversely, assume that we have a solution, \((\bar{w}^*)\), to \((\bar{P})\). If it was not a solution to \((P)\) then we could find another sequence, say \((\bar{w})\), which makes the objective in \((P)\) higher and, by Claim A1, none of the \(w_t\)'s is in \(((rz + 1)/d, p)\). But then the value of the objective in \((\bar{P})\) is higher at \((\bar{w})\) than it was at \((\bar{w}^*)\), contradicting the optimality of \((\bar{w}^*)\).

Therefore, we might as well seek and characterize solutions to the program with \(\bar{\pi}\) since it has the same set of solutions as the original program. For economy of notation, let us continue to refer to the new program as \((P)\) and the new period payoff as \(\pi\). Now the period payoff is bounded and continuous, and we proceed with the proof of Lemma 7.1.

\((P)\) is written as the maximization of \(J(w) = J(w_1, w_2, ...)\) over the domain \([0, p]^{\infty}\). This domain is compact under the topology of weak convergence (Ty-
affect workers' preferences so that workers become more cooperative.

Indeed when a cooperative task is hard to measure and reward, an "obvious" solution is to affect workers' preferences so they derive utility from the act of cooperating. Then, workers choose to cooperate on their own volition, although their cooperative efforts are not monetarily rewarded. And that, given the importance of cooperation in the production function, works to the firm's benefit. We say that a firm has built "social capital," if it manages to affect workers' preferences in this way. In our model, when the firm chooses incentives it considers the effect they have on its stock of social capital.

More specifically, our model is based on the following hypotheses. First, workers allocate their effort between an individual task and a cooperative task. Effort devoted to the individual task is more effective at increasing an employee's measured output while effort devoted to the cooperative task increases the measured output of co-workers so that workers face a social dilemma. Second, in addition to deriving utility from monetary rewards employees have a disutility ("guilt") from not cooperating. Third, the strength of these feelings of guilt increase in past levels of cooperation in the organization and hence varies over time. As discussed in Section 2, the experimental-economics literature strongly supports the second and third hypotheses, while the first hypothesis is how we capture multi-task production in the firm.

With these three hypotheses we are able to deliver the following ideas. First, we make explicit the dynamics underlying social capital. In particular, the level of cooperation among the firm's workforce follows a dynamic process: Prior levels

2The term social capital is often attributed to the sociologist Coleman (1988), who builds on Granovetter's (1985) argument that social structure has important effects on economic action. The concept came to prominence with the work of the political scientist Putnam (1993, 1995), who defines it as follows: "Social capital refers to features of social organization such as networks, norms and social trust that facilitate coordination and cooperation for mutual benefit. For a variety of reasons, life is easier in a community blessed with a substantial stock of social capital. In the first place, networks of civic engagement foster sturdy norms of generalized reciprocity and encourage the emergence of social trust. Such networks facilitate coordination and communication, amplify reputations, and thus allow dilemmas of collective action to be resolved... At the same time, networks of civic engagement embody past success at collaboration, which can serve as a cultural template for future collaboration. Finally, dense networks of interaction probably broaden the participants' sense of self, developing the 'I' into the 'we,' or (in the language of rational-choice theorists) enhancing the participants' 'taste' for collective benefits." (Putnam 1995). He has also written that stocks of social capital "tend to be self-reinforcing and cumulative" (Putnam, 1993). A recent contribution in the economics literature is the one by Glaeser et al. (2000).
We discuss the hypotheses, the modeling approaches and the implications of our model in light of the experimental literature.

- **High Level of Cooperation:** The classical result in experiments on public-good contributions is that subjects contribute as much as 50% of their endowment, although contributing zero is a dominant action; see surveys by Ledyard (1995), Andreoni and Croson (forthcoming) and Keser (2000). In a notable contribution, Andreoni (1995) designed an experiment in which he tries to distinguish between contribution due to "errors" or "confusion" and contributions due to non-selfish preferences. His main conclusion is that much of the contribution is due to non-selfish preferences.

- **Warm Glow or Altruism:** The non-selfish motive for contributing might be altruism (see Rabin (1998)), whereby a subject derives utility from the utility of others; or, it might be "warm glow" whereby a subject derives utility from the very act of giving independent of the utility that this delivers to others. By randomly varying the returns from private consumption in a public goods game, Palfrey and Prisbey (1997) find significant evidence of warm-glow effects while statistically rejecting altruism effects. We assume a form of warm-glow preferences.

- **Heterogeneity:** The amount contributed by subjects varies a great deal. In particular, many subjects contribute nothing while others make large contributions. Andreoni (1995) and Palfrey and Prisbey (1997) have documented the degree to which contributions vary across subjects. We incorporate such heterogeneity into our model.

- **Variation Over Time:** Another classical result is that the amount contributed decreases, or "decays," significantly over time; see the surveys by Ledyard (1995), Andreoni and Croson (forthcoming) and Keser (2000).

- **Reciprocity:** One force behind the decay of contributions is that initially generous subjects get disillusioned by the stinginess of others, and retaliate by reducing their own contributions. This sets in motion a downward spiral.

Moreover there is a prior literature in social psychology documenting cooperation in public-goods games (Dawes, 1980).
• Reputations: Our model rules out the reputational motive to cooperate stressed in the theory of repeated games, e.g., Kreps et al. (1982). The experimental literature finds that many subjects do not, in fact, play in accordance with theories of reputation in repeated games. For example, Andreoni (1988) finds that there is less cooperation when players play a repeated social dilemma than when players are rematched after each stage-game. Andreoni and Croson (forthcoming) summarizes further findings along these lines.

• Multiplicity of Cultures: One focus of our analysis is the source of cultural differences across firms. While we know of no experimental studies that look at behavioral differences across firms, experiments do show that levels of cooperation vary across countries (Weimann (1994) and Burlando and Hey (1997).) Ockenfels and Weimann (1999) are able to avoid many of the problems usually inherent in cross-cultural studies (e.g., differences in language) by using subjects from former East and West Germany. They find that “cooperation and solidarity behavior seem to depend strongly on different culture-specific norms resulting from opposing economic and social histories in the two parts of Germany.”

3. The Model

The firm employs a continuum of risk-neutral workers. The size of its workforce is fixed and normalized to 1. Workers are indexed by \( i \in [0,1] \). The firm operates over an infinite number of discrete time periods, \( t = 1, 2, ... \)

3.1. The Production and Monitoring Technology

Each period each worker makes two decisions: How much total effort to exert, \( e \), and how to allocate it between individual effort, \( e_I \), and cooperative effort, \( e_C \). Each worker’s decisions satisfy \( e = e_I + e_C \), \( e_I, e_C \geq 0 \).

The output from individual effort is \( ae_I \), whereas the output from cooperative effort is \( \min(e_C, h) \), where \( h \) is the point at which the marginal productivity of
is the product of the extent to which the worker defects, \( h - e_C \), and the guilt he suffers per unit of defection, \( g \).

The parameter \( g \) varies across workers and over time, and depends on last period’s level of cooperation, denoted \( z \), and on a worker’s predisposition to cooperate, denoted \( \gamma \). \( z \) is defined as \( z = \frac{1}{h} \int_0^1 e_C(z)dz \), i.e., as the normalized average cooperative effort, where “average” means we take the average across workers, and “normalize” means we divide by \( h \). Because of the normalization, \( z \) is a number between 0 and 1, which simplifies some of the expressions below. \( \gamma \) is a random draw from a uniform distribution on \([0, 1]\), which is i.i.d. across workers.

The dependence of \( g \) on \( z \) and \( \gamma \) is specified via:

\[
g = rz + \gamma,
\]

where \( r > 0 \) is a reciprocity parameter, capturing the extent to which last period’s cooperation raises the worker’s taste for cooperation. Based on these assumptions and given last period’s value of \( z \), \( g \) is uniformly distributed over the interval \([rz, rz + 1]\). Thus, \( g \) varies across workers, i.e., it exhibits heterogeneity. Further, \( g \) varies (potentially) over time as \( z \) changes.

We assume a quadratic cost of effort:

\[
C(e) = \begin{cases} 
0 & \text{if } e \leq \bar{e}, \\
c(e - \bar{e})^2 / 2 & \text{otherwise},
\end{cases}
\]

where \( \bar{e} \) is a threshold beyond which workers start to experience disutility of effort. To simplify the analysis we assume \( \bar{e} > h \), i.e., workers can choose the maximum level of cooperative effort without feeling any direct disutility.

### 3.3. The Firm’s Problem

The firm seeks to maximize the discounted sum of profits given its discount factor \( \delta \). The profit in one period is

\[
\pi = pE[Q] - E[W],
\]

---

\(^9\)In addition to reciprocity, the parameter \( r \) captures the importance of “socialization” where workers over time come to value the behavior they see in their environment. We emphasize the reciprocity interpretation in the exposition because it is strongly supported by the experimental literature.

\(^{10}\)This assumption allows us to separate an employee’s decisions: First the employee chooses total effort, \( e \), then he decides how to allocate it between \( e_I \) and \( e_C \); see Lemma 4.1.
4. Employee Behavior and Social Capital

We begin by characterizing the behavior of workers. Since there is a continuum of workers, a single worker has no effect on the behavior of other workers or the firm. Thus, in each period, workers choose \( e_I \) and \( e_C \) myopically. The objective of each worker is formed by substituting \( W = b + w\tilde{Q}, \tilde{Q} = ae_I + (e_C + z')/2 \) and \( e_I = e - e_C \) into the definition of a worker’s utility, (3.1), and eliminating constants (terms over which the worker has no control.) This gives the following maximization problem:

\[
\max_{e, e_C} \{ wae - C^e(e) + (g - wd)e_C \},
\]

where \( d \equiv a - 1/2 \) is the physical cost of shifting effort from individual to cooperative production, whereas \( wd \) is the monetary cost. Now we can maximize (4.1), which gives:

**Lemma 4.1.** (i) The optimal total effort of a worker satisfies the first-order condition \( C'(e^*) = aw \), which gives \( e^*(w) = aw/c + \bar{e} \). (ii) If \( g > wd \), the worker fully cooperates, \( e_C = h \), while if \( g < wd \), the worker does not cooperate at all, \( e_C = 0 \).

**Proof.** (i) The objective in (4.1) is concave in \( e \), so the usual first order condition applies to \( e^* \). The expression \( e^* = aw/c + \bar{e} \) is obtained by substituting from (3.3) into the first order conditions. (ii) From (4.1) the objective is linear in \( e_C \), so the optimal \( e_C \) is at a corner with \( e_C = 0 \) or \( e_C = h \), depending on whether the coefficient \( g - wd \) is negative or positive.

Therefore, as in standard Principal-Agent theory, total effort is increasing in the incentive intensity \( w \). On the other hand, cooperative effort decreases in \( w \). Cooperative effort also depends on \( g \) which in turn depends on last period’s \( z \): The higher is last period’s \( z \), the more likely is \( g \) is to exceed \( wd \), and the more likely is the worker to cooperate.

Since cooperation is valuable to the firm, since \( z \) fosters cooperation and since \( z \) tends to persist from one period to the next (see next Lemma) we call \( z \) the firm’s social capital. The next result shows that social capital follows a dynamic process.

**Lemma 4.2.** Social capital in the firm evolves according to
and \((1 - wd)/(1 - r) \geq 1\). So the only steady state is \(z = 1\). (ii) Suppose \(r \geq 1\) and \(w \in [1/d, r/d]\). Then \(f(w,0) = 0\) and \(f(w,1) = 1\). Therefore both 0 and 1 are steady states. ■

Therefore, when incentives are held fixed, the existence of multiple cultures depends on the parameter \(r\). If reciprocity is weak, i.e., if \(r\) is small, multiple cultures are not possible. Conversely, if reciprocity is strong then the "internal feedback" is sufficient - on its own - to produce multiple cultures.

Corporate culture differs from the culture of other groups in that there is an entity, the firm, which may want to influence the evolution of its culture in order to increase its profits. In our model, the firm may do this by altering its incentive intensity and, thereby, altering its corporate culture. Accordingly, in the rest of the paper we determine the firm’s culture given that the firm is optimally adjusting its incentive intensity over time.\(^\text{13}\) We consider this under the further assumption that \(r < 1\). In this case, as Proposition 5.1 shows, there is necessarily a unique culture under fixed incentives. This will no longer be the case under endogenously determined incentives.

6. The Firm’s Objective

To characterize the optimal incentive intensity, we need the firm’s objective function. Substituting the expressions in Lemma 4.1 into the firm’s one period profit function (3.4) gives us the following result:

**Lemma 6.1.** (i) The firm’s one period payoff is the following function of \(w\) and \(z\).

\[
\pi(w, z) = aep + \frac{a^2}{e}w(p - \frac{w}{2}) + p(1 - a)hz' - R(w, z),
\]

where:

\[
R(w, z) = \begin{cases} 
wdh(1 - z') & \text{if } z' > 0 \\
0 & \text{if } z' = 0
\end{cases},
\]

and \(z' = f(w, z)\). (ii) \(\pi(w, z)\) is discontinuous at any \((w, z)\) for which \(w = (rz + 1)/d\). (iii) Fixing \(z\), \(\pi\) is single-peaked both to the right and to the left of the discontinuity, with one local maximum at \(w = p > (rz + 1)/d\). (iv) Fixing \(w\), \(\pi\) is nondecreasing in \(z\), except where it is discontinuous.

\(^{13}\)Formally, we are interested in steady states \(z_s = f(w_s, z_s)\) for some \(w_s \in \omega(z_s)\) where \(\omega(\cdot)\) is the optimal policy correspondence; see Section 7.
Now $w$ not only determines the one-period payoff but it also affects the evolution of the firm's stock of social capital. The optimal incentive intensity must incorporate these dynamic effects. Formally, the firm solves a dynamic program, which we formulate as:

$$(P) \ V(z) = \max_{(w_t)} \left\{ J(w_1, w_2, \ldots) \right\}$$

$$= \max_{(w_t)} \left\{ \sum_{t=1}^{\infty} \delta^{t-1} \pi(w_t, z_{t-1}) \right\} \text{ s.t. } z_t = f(w_t, z_{t-1}) \text{ and } z_0 = z.$$

This program is similar to capital theory in that the firm can build up, or deplete, its stock of social capital over time. Investments in social capital are made by reducing the incentive intensity below the level that maximizes the one-period profit. The following section analyzes program (6.3).

7. Analysis of the Dynamic Program

The first result concerning $(P)$ is the following.

**Lemma 7.1.** There exists a solution to program $(P)$.

**Proof.** In the Appendix.

It is sometimes convenient to transform the program $(P)$ so that maximization is with respect to next period's social capital, $z'$, rather than this period $w$:

$$(P^0) \ \max_{(z_t)} \left\{ J^0(z_1, z_2, \ldots) \right\} = \max_{(z_t)} \left\{ \sum_{t=1}^{\infty} \delta^{t-1} \pi^0(z_{t-1}, z_t) \right\},$$

where

$$\pi^0(z, z') \equiv \pi(q(z, z'), z) \text{ and } q(z, z') \equiv \frac{1 + rz - z'}{d}. \quad (7.1)$$

Let $\omega(z)$ and $\zeta(z)$ be the policy correspondences of $(P)$ and $(P^0)$, which, by Lemma 7.1, are not empty. The next Lemma is key to our results.

15 Our problem is slightly different from a standard growth problem in that the one-period payoff is not necessarily maximized by no investment, i.e., at $w = p$. Lowering $w$ to induce cooperation may increase the one-period profit because cooperative effort is more productive than individual effort. Relatedly, the period payoff is double peaked in $w$, whereas in capital theory it would be monotonically increasing.
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by

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**Proof.** In the Appendix.

When there is convergence to an interior steady state the evolution of social capital is characterized by a second-order difference equation.

**Lemma 7.5.** Consider an optimal sequence \((z_t)_{t=0}^{\infty}\) such that \(z_t \in (0, 1)\) for all \(t\). Then:

\[
[hcd^2 r + a^2 r]z_{t-1} - [2hcd^2 + a^2 + \delta a^2 r^2]z_t + [\delta a^2 r + \delta hcd^2 r]z_{t+1} =
\]

\[
a^2(1 - \delta r)(pd - 1) + \delta hcd^2 r - h(1 - a)cd^2 p - 2hcd^2.
\]  

(7.3)

**Proof.** In the Appendix.

### 8. Optimal Incentives and Cultural Differences

We find that endogenizing the incentive intensity can increase the scope for cultural differences.

**Proposition 8.1.** (i) For any set of parameter values and for any initial condition, \(z_0\), the firm's social capital converges to a steady state \(z_s\). (ii) There are either one or two steady states for a given set of parameters. (iii) If there are two steady states there exists a \(\bar{z} \in [0, 1]\) so that if \(z_0 < \bar{z}\), there is convergence to a steady state with \(z_s = 0\) and \(w_s = p\), while if \(z_0 > \bar{z}\), there is convergence to a steady state with \(z_s \geq \bar{z}\) and \(w_s < p\). (iv) If there are two steady states, the flow profit at \(z_s > \bar{z}\) is higher than the flow profit at \(z_s = 0\).

**Proof.** The convergence results follow from Lemmas 7.3 and 7.4. The possibility of multiple steady states is demonstrated by Figure 8.1. The profit ordering, for \(z_s > \bar{z}\), follows from Lemma 7.4. 

It is instructive to compare this result to Proposition 5.1. With \(r < 1\), reciprocity - on its own - is not sufficient to produce multiple cultures. On the other hand, when \(w\) is endogenized we might very well get multiple cultures. Therefore, the firm's incentive policy reinforces the positive feedbacks in the model, and can bring about multiple cultures. When \(z\) is low, it is costly to induce cooperation: Because workers are not cooperative to begin with, the firm must set a low incentive intensity to get them to cooperate, and that reduces its current profit considerably. Conversely, when \(z\) is high, it is less costly to induce cooperation.
these forces, we look at the comparative statics of the interior steady state.

**Proposition 8.2.** *When it exists the interior steady state is*

\[ z_s = \frac{hcd^2[2 + (1 - a)p - \delta r] - a^2(1 - \delta r)(pd - 1)}{hcd^2[2 - r(1 + \delta)] + a^2(1 - r)(1 - \delta r)}. \]

\( z_s \) is increasing in \( h, \delta, r \) and \( c \) and is decreasing in \( p \). The steady state incentive intensity is \( w_s = [1 - z_s(1 - r)]/d \) and is decreasing in \( h, \delta \) and \( c \) while it is increasing in \( p \), and, for \( r \) sufficiently close to 1, \( w_s \) increases in \( r \).

**Proof.** In the Appendix.

We find that corporate cultures are more cooperative the more important is cooperative production, \( h \), the more patient are firms, \( \delta \), the greater the extent of reciprocity, \( r \), the less responsive is total effort to incentives, \( c \), and the less productive is each worker, \( p \).\(^{18}\) The logic behind the comparative static results is straightforward: As \( p \) increases, the firm wants more total effort, which calls for a higher \( w \) which lowers \( z \). As \( c \) increases, \( w \) is less effective at increasing total effort. Hence, the firm chooses a smaller \( w \) and \( z \) is larger. As \( h \) increases, cooperative production is more important and social capital is more valuable. Hence, the firm wants a higher \( z \) which requires a lower \( w \). As \( \delta \) increases, the firm puts a higher weight on future profits which, again, leads to a higher \( z \) and a lower \( w \). For a fixed \( w \), the steady state level of cooperation is increasing in \( r \). While \( w \) may be increasing or decreasing in \( r \), an increase in \( w \) is not sufficient to offset the direct positive effect of \( r \) on \( z \).

**9. Empirical Implications our Theory**

In standard Principal-Agent theory, the optimal incentive intensity trades-off costly risk bearing by employees and increases in employee effort (Milgrom and Roberts, 1992). If so, the strength of incentives should be negatively correlated with the degree of uncertainty. Prendergast (2000) reviews empirical tests of this theory, and concludes that the data seem to suggest a positive correlation! We develop an alternative approach, which produces a different set of empirical implications.

\(^{18}\)The comparative statics with respect to \( \alpha \) could go either way.
chonoff theorem.) Also, if we let \( J_0 = 2 \max \pi(w, z) / (1 - \delta) \), where \( w \) ranges over \([0, p]\) and \( z \) over \([0, 1]\), then \( J_0 < \infty \) (by the boundedness of \( \pi \).) Now let \( w^n \to w^\infty \) weakly. Then, for any \( T \),

\[
|J(w^n) - J(w^\infty)| \leq \delta^T J_0 + \max_{1 \leq t \leq T - 1} \{|\pi(w^n_t, z^n_t) - \pi(w^\infty_t, z^\infty_t)|\}.
\]

Since \( J_0 < \infty \), we can choose a \( T \) large enough to make the first term less than \( \varepsilon / 2 \). Then, given this \( T \) and the continuity of \( \pi \), we can choose an \( n \) large enough that the second term is also less than \( \varepsilon / 2 \). Therefore \( J \) is continuous under the topology of weak convergence. So it must have a maximum over \([0, p]^\infty\). ■

**Proof of Lemma 7.2**

Assume \( z'_2 < z'_1 \). Then, since \( z'_1 \) is optimal at \( z_1 \), we must have: \( \pi^0(z_1, z'_1) + \delta V(z'_1) \geq \pi^0(z_1, z'_2) + \delta V(z'_2) \), or

\[
\delta[V(z'_1) - V(z'_2)] \geq \pi^0(z_1, z'_2) - \pi^0(z_1, z'_1).
\]

We will now show that \( \pi^0(z_1, z'_2) - \pi^0(z_1, z'_1) > \pi^0(z_2, z'_2) - \pi^0(z_2, z'_1) \). This together with above inequality shows that \( z'_2 \) cannot be optimal at \( z_2 \). There are two cases to consider.

(a) \( z'_2 = 0 \). Then, from (7.1), \( \pi^0(z_1, z'_2) = \pi^0(z_2, z'_2) \) and \( \pi^0(z_2, z'_1) > \pi^0(z_1, z'_1) \). So the desired inequality is established.

(b) \( z'_2 > 0 \). Then

\[
\pi^0(z_1, z'_2) = -hq(z_1, z'_2)d(1 - z'_2) + hp(1 - a)z'_2 + I(q(z_1, z'_2)), \ i, j = 1, 2;
\]

where \( I(w) \equiv a^2 \frac{w(p - w)}{2} + a \varepsilon p \). Therefore:

\[
\pi^0(z_1, z'_2) - \pi^0(z_1, z'_1) = -h(q(z_1, z'_1)d(1 - z'_1) - hq(z_1, z'_1)d(1 - z'_2) + hp(1 - a)(z'_2 - z'_1) + I(q(z_1, z'_2)) - I(q(z_1, z'_1))
\]

and

\[
\pi^0(z_2, z'_2) - \pi^0(z_2, z'_1) = -h(q(z_2, z'_2)d(1 - z'_1) - hq(z_2, z'_2)d(1 - z'_2) + hp(1 - a)(z'_2 - z'_1) + I(q(z_2, z'_2)) - I(q(z_2, z'_1)).
\]

The term, \( hp(1 - a)(z'_2 - z'_1) \) is common and hence it will cancel. So it suffices to show:
10. Conclusion

After acknowledging at the beginning of *Economics, Organizations and Management* that “important features of many organizations can best be understood in terms of deliberate attempts to change preferences of individual participants,” Milgrom and Roberts follow the bulk of the formal literature on organizations and proceed “as if people were entirely motivated by narrow, selfish concerns” (p. 42). Our paper offers a formal approach to how business practices can be understood as attempts to change preferences. By moving beyond the approach where preferences are fixed and given, we are able to formalize both social capital and corporate culture, to explore the sources of cultural differences across firms and to show how such differences introduce variation in firms’ profits. We also offer a novel perspective on the optimal incentive intensity.

Our paper contributes to a theory of the firm in which the firm is fundamentally a collection of processes that build up specialized assets over time. This view, which can be found in Prescott and Visscher (1980), has not figured prominently in economic theory despite its prominence in the management literature (Dierickx and Cool, 1994), which emphasizes the importance of firm’s stocks of not just social capital, but human resources and organizational learning as well. One can study a variety of policies based on their impact on the accumulation of such assets, as we have done with the incentive intensity and social capital. For example, Athey et al. (2000) show how promotion policies influence the evolution of a firm’s stock of management talent. More remains to be done even in the context of social capital. In particular, one could study policies affecting employee turnover (hiring and firing) since the preferences of those who leave and join the firm also affect the evolution of the firm’s social capital.\(^\text{21}\)

\(^{21}\)We already have one observation on this topic: Because defectors earn rents in firms with cooperative cultures, our theory suggests that agents with low inherent feelings of guilt (low \(\gamma\)) are especially attracted to firms with cooperative cultures. In this case, firms with highly cooperative cultures must take special measures to screen out applicants with a low sense of social responsibility and to retain those with a high sense of social responsibility. Otherwise, they may see their social capital erode over time.
\begin{equation}
\begin{aligned}
&\text{(i) } \mathcal{H}(z_1, z_1')d(1-z_1') - \mathcal{H}(z_2, z_2')d(1-z_2') \geq \mathcal{H}(z_2, z_1')d(1-z_1') - \mathcal{H}(z_2, z_2')d(1-z_2').
\end{aligned}
\end{equation}
and
\begin{equation}
\begin{aligned}
&\text{(ii) } I(q(z_1, z_1')) - I(q(z_1, z_1')) = I(q(z_2, z_2')) - I(q(z_2, z_1')).
\end{aligned}
\end{equation}

But (i) is equivalent to
\begin{equation}
\begin{aligned}
&\mathcal{H}(1-z_2)[q(z_2, z_1') - q(z_1, z_1')] \geq \mathcal{H}(1-z_1)[q(z_2, z_1') - q(z_1, z_1')],
\end{aligned}
\end{equation}
which holds because \(q(z_2, z_1') - q(z_1, z_1') = q(z_2, z_1') - q(z_1, z_1')\) (see (7.1)), and because \(z_1' \geq z_2\). (ii) holds because \(I(\cdot)\) is quadratic, the differences \(q(z_1, z_1') - q(z_2, z_1')\) are equal, and \(q(z_1, z_1') < q(z_2, z_1')\).

We now show that \(\zeta(\cdot)\) is strictly monotonic whenever \(z_1\) or \(z_2'\) is in \((0, 1)\). Assume \(z_1' \in (0, 1)\) and let \(z_2' \in \zeta(z_1')\). Then, \(z_1' \in \arg \max \{\pi^0(z_1, r) + \delta\pi^0(r, z_1') + \delta^2\mathcal{V}(z_1')\}\). But \(z_1'\) is interior and, hence, must satisfy the first order condition \(\pi^0_2(z_1, z_1') + \delta\pi^0_0(z_1', z_1') = 0\), where \(\pi^0_0\) is the partial derivative of \(\pi^0\) with respect to the \(i^{th}\) variable. Since \(z_1 < z_2\), \(\pi^0_2(z_1, z_1') < \pi^0_2(z_2, z_1')\) and, thus, \(\pi^0_2(z_2, z_1') + \delta\pi^0_0(z_1', z_1') > 0\). Therefore, we can find a \(\bar{z}_1 > z_1\) so that \(\pi^0(z_2, \bar{z}_1) + \delta\pi^0(\bar{z}_1, z_1') + \delta^2\mathcal{V}(z_1') > \pi^0(z_2, z_1') + \delta\pi^0(z_1, z_1') + \delta^2\mathcal{V}(z_1')\). This shows that \(z_1' \notin \zeta(z_2')\). Since weak monotonicity of \(\zeta\) has already been established, we must have \(z_2 > z_1\). A similar argument works for \(0 < z_2' < 1\). •

**Proof of Lemma 7.4**

By Claims A1 and A2, \(\pi\) can be replaced - without affecting the solution - by another period payoff, which we continue to call \(\pi\) and which is continuous. Equivalence between (6.3) and (7.2) follows from \(\pi\) bounded and continuous and all \(z_i > \bar{z}\). Furthermore, since \(\pi\) is strictly concave, so is \(\mathcal{V}\) over \((\bar{z}, 1)\). Uniqueness of the maximizing wage sequence follows from the strict concavity of \(\pi\) and from theorem 4.8 in [23]. The differentiability of \(\pi\) and theorem 4.10 of [23] imply that \(\mathcal{V}\) is continuously differentiable at any \(z\) at which \(\zeta(z) < 1\). Assume \(\zeta(z) = 1\) for some \(z < 1\) and let \(z_* = \inf \{z \mid \zeta(z) = 1\}\). Then, by Lemma 4.1, \(\zeta(z) = 1\) and \(\omega(z) = rz/d\) for any \(z > z_*\). Thus \(\mathcal{V}(z) = \pi(rz/d, z) + \delta\mathcal{V}(1)\). Therefore \(\mathcal{V}\) is differentiable for all \(z > z_*\) and all \(\bar{z} < z < z_*\). So the only possible non-differentiability is at \(z_*\). The strict monotonicity of \(\mathcal{V}\) follows from the fact that \(z > \bar{z} > 0\) which implies \(\omega(z) \in (rz/d, (rz + 1)/d)\), a range over which \(\pi\) is strictly monotonic.

Consider now the maximization programs on the RHS of (5.1) at \(z_1\) and \(z_2\), \(\bar{z} < z_1 < z_2 \leq 1\) and let \(w^*_i = \omega(z_i)\). Then we must have \(\partial^- \{\pi(w^*_1, z_1) + \delta\mathcal{V}(f(w^*_1, z_1))\} \geq 0\), where \(\partial^-\) denotes the left-hand derivative. But, since \(z_2 > z_1\),
A < 0 whenever \( z_s < 1 \). To differentiate \( z_s \) with respect to \( \delta \) form \( z_s = \frac{A + B\delta}{C + D\delta} \). Then \( \text{sign}\{\partial z_s/\partial \delta\} = \text{sign}\{B(C + D\delta) - D(A + B\delta)\} \). This is positive since \( C + D\delta > A + B\delta \) and \( B \equiv -rhcd^2 + ra^2(pd - 1) > -rhcd^2 - r(1 - r)a^2 \equiv D \). To differentiate \( z_s \) with respect to \( \delta \) form \( z_s = \frac{(1-\delta r)(hcd^2 - a^2(pd - 1)) + hcd^2(1 - (1 - a)p)}{(1-\delta r)(a^2(1-r) + hcd^2) + hcd^2(1-r)} \), and let \( \dot{z} = \frac{(1-\delta r)(hcd^2 - a^2(pd - 1)) + hcd^2(1 - (1 - a)p)}{(1-\delta r)(a^2(1-r) + hcd^2) + hcd^2(1-r)} \). Then, \( \partial z_s/\partial h \geq \partial \dot{z}/\partial h > 0 \).

Since \( z_s = f(w_s, z_s) \), we have \( w_s = \frac{[1 - (1 - r)z_s]}{d}. \) Hence, the comparative statics for \( w_s \) with respect to \( c, h, \delta \) and \( p \) are the opposite of those for \( z_s \). ■

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\[ \frac{\partial}{\partial w} \{ \pi(w^*_1, z_1) + \delta V(f(w^*_1, z_1)) \} < \frac{\partial}{\partial w} \{ \pi(w^*_1, z_2) + \delta V(f(w^*_1, z_2)) \}. \]

So, given that the objective is concave, the maximizer \( w^*_1 \) must be \( \geq w^*_1 \).

(v) Assume there are two, \( 0 < z^1 < z^2 \). Then \( z^1 \geq \bar{z} \), otherwise \( z^1 < \bar{z} \) and, by Lemma 4.2, the sequence \( (z_t)_{t=0}^{\infty} \) for which \( z_0 = z^1 \) converges to 0—contrary to the assumption that \( z^1 \) is a positive steady state. Now, according to (7.1), the wages which sustain these steady states are \( w^i = \left[ 1 - (1-r)z^i \right]/d \), so \( w^1 > w^2 \). However, by Lemma 4.2, \( w^1 \leq w^2 \). Since these inequalities cannot hold simultaneously, this contradicts the existence of 2 positive steady states.

**Proof of Lemma 7.5**

A necessary condition for an optimal and interior \( z_t \) is:

\[ \frac{\partial J^0(z_1, z_2, \ldots)}{\partial z_t} = \delta^{t-1} \pi^0_2(z_{t-1}, z_t) + \delta^{t-1} \pi^0_1(z_t, z_{t+1}) = 0. \]

Or, after cancellation of \( \delta^{t-1} \),

\[ \pi^0_2(z_{t-1}, z_t) + \delta \pi^0_1(z_t, z_{t+1}) = 0. \] (A.1)

Computing the partial derivatives of \( \pi^0 \) we have:

\[ \pi^0_1(z_t, z_{t+1}) = \frac{r}{d} \left[ \frac{a^2}{c} (p - w_t) - hw(1 - z_{t+1}) \right], \]

\[ \pi^0_2 = \left( \frac{-1}{d} \right) \left[ \frac{a^2}{c} (p - w_t) - hw(1 - z_{t+1}) \right] + h(1 - a)p + hzw_t, \]

where \( w_t = (1 + rz_t - z_{t+1})/d \). Substituting this into (A.1) gives the desired equation.

**Proof of Proposition 8.2**

To find the expression for \( z_s \), set \( z_{t-1} = z_t = z_{t+1} = z_s \) in equation (7.3) and solve for \( z_s \). To find the expression for \( w_s \) use the relationship \( z_s = f(w_s, z_s) \). To differentiate \( z_s \) with respect to \( h \), form \( z_s = \frac{Ah + B}{Ch + D} \). Then \( \text{sign}\{\partial z_s/\partial h\} = \text{sign}\{A(Ch + D) - C(Ah + B)\} \). This is positive since \( Ch + D > Ah + B \) whenever \( z_s < 1 \), and \( A \equiv 2 + (1 - a)p - \delta r > 2 - (1 + r)\delta \equiv C \). Likewise \( \partial z_s/\partial c > 0 \) since \( c \) and \( h \) are equivalent in the expression for \( z_s \). To differentiate \( z_s \) with respect to \( p \), form \( z_s = \frac{Ap + B}{D} \). Since \( B > D \), it must be that