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"Endogenous Inequality in Integrated Labor Markets with Two-sided Search"

by

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Abstract

We consider a market in which there are two types of workers, "red" and "green," where these labels have no direct payoff implications. Workers can choose to acquire costly skills. Skilled workers must search for firms with a job vacancy, while firms with vacancies also search for unemployed workers. A unique symmetric equilibrium exists in which firms ignore workers' colors. There may also exist an asymmetric equilibrium in which firms only search for green workers, more green than red workers acquire skills, skilled green workers receive higher wage rates than skilled red workers, and the unemployment rate is higher among skilled red than green workers, though there are more unemployed skilled green than red workers. Discrimination between *ex ante* identical individuals thus arises as an equilibrium phenomenon. Our analysis differs from previous models of discrimination in assuming that firms have perfect information about workers with whom they are matched, and strictly prefer to hire minority workers (contingent on meeting a worker), and in generating predictions concerning unemployment as well as wage rates.

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1. Introduction

Bad things happen to good people—two people can appear to be similar in all economically relevant aspects, including skills and opportunities, and yet one of them can fare much better than the other. In its most visible form, this heterogeneity appears as discrimination, in which people hindered by no disadvantage, other than a characteristic that is seemingly irrelevant for any economic purpose, consistently achieve substandard economic outcomes. Moreover, these systematic differences reflect *ex ante* heterogeneity in expected outcomes rather than simply *ex post* bad luck.

The explanation for people with identical skills and education achieving quite different outcomes is often given in the popular maxim: "It's not what you know, it's who you know." Success can depend critically not only on the choices we make in our interactions with others, but also on the identity of those with whom we interact. As a result, people throw tremendous amounts of energy into attempts to associate with the right people. Schools, clubs, neighborhoods, professional organizations, and academic conferences are all chosen because they allow entrance into a desired group.

Theories of statistical discrimination are commonly invoked to explain group-based inequality (see Cain [3] for a survey). Suppose workers' skills are not observable, but that workers have a payoff-irrelevant characteristic, such as being colored red or green, that is observable. If firms can condition their behavior on this characteristic, then the following is an equilibrium: Green workers acquire skills and are hired at a high wage, while red workers do not acquire skills and are hired at a low wage. Red workers choose to be less skilled than green workers, prompting firms to pay them a lower equilibrium wage. Red workers thus have a lower incentive to acquire skills, yielding an equilibrium in which firms and red workers coordinate on a low-skill, low-wage outcome, while firms and green workers coordinate on a high-skill, high-wage outcome.[1] Paradoxically, theories of statistical discrimination yield no economic discrimination (Cain [3]): all workers, including reds, are paid their marginal product. Given skills, color plays no role in explaining wages.

[1] Cornell and Welch [6] show that statistical discrimination can arise, even if workers make no skill-level choice and skill distributions are identical across groups, if employers are better able to screen one group than another.

While statistical discrimination may explain part of what we see, it cannot be the whole story. Theories of statistical discrimination predict that the red labor market clears, while discrimination typically leads to persistent unemployment. Moreover, theories of statistical discrimination focus on an informational friction, arising because firms cannot identify the skill level of workers, that is implausible in many situations. Finally, these theories have a "separability" feature, common to models of multiple equilibria, whose implausibility is discussed below.

In this paper, we focus on search, rather than informational, frictions as an explanation of inequality in outcomes. We examine a dynamic labor model in which newborn workers acquire skills at an idiosyncratic cost. Unemployed skilled workers search for firms with vacancies, while also being a potential target of search on the part of firms with vacancies. Once an unemployed skilled worker meets a firm with a vacancy, they bargain over the division of the surplus.[2] Workers come in two varieties, red and green. A worker's color has no direct effect on payoffs, but can be observed by firms. Because color is payoff-irrelevant, there is always a symmetric equilibrium in which colors are ignored. However, there are also asymmetric equilibria in which fewer red workers acquire skills than green workers, firms search only among skilled green workers, and skilled red workers suffer from lower wage rates and higher unemployment rates than skilled green workers. The payoff-irrelevant characteristic of redness identifies its bearer as a target for discrimination, and reds fare systematically worse than greens.

In our model, red skilled workers have no difficulty in convincing firms of their skills. Moreover, reds have precisely the same opportunity to search firms as greens. In an asymmetric equilibrium, a firm with a vacancy contacted by a searching red worker happily hires that worker, obtaining a skilled worker at a low wage rate. However, since firms seek only green workers, reds have higher unemployment rates and hence lower expected payoffs from market participation. This lower outside option causes worker-firm bargaining to yield lower wage rates for red workers than for identical skilled green workers. This is unambiguous economic discrimination: workers of identical skills receive different wages based on color.

The asymmetric equilibrium we study, in which reds do not acquire skills because employers do not seek reds because reds do not acquire skills, is superficially similar to statistical discrimination, but it is *not* a coordination failure. Statistical discrimination, like other models of coordination failure, yields separable outcomes: There is no interaction between the different groups. In addition to the asymmetric equilibrium in which the reds receive a lower wage, there is

[2]See Osborne and Rubinstein [9] for a survey of the large literature on search and bargaining.

also a *symmetric* equilibrium in which *all* workers receive the same low payoff as do reds in the asymmetric equilibrium (in this sense, the greens are not imposing an externality on the reds). There is also a symmetric equilibrium in which all workers receive the same high payoff as do greens in the asymmetric equilibrium. In our model, there is a unique symmetric equilibrium.[3] Payoffs to workers in an asymmetric equilibrium cannot be replicated in the symmetric equilibrium. Green workers can obtain high payoffs in the asymmetric equilibrium only because red workers obtain low payoffs.

As a result of this externality, an asymmetric equilibrium is more likely to exist in our model if firms search the majority color worker.[4] Minorities are then a natural candidate for discrimination. This contrasts with statistical discrimination, where group size is irrelevant.

There are three key features of our model. The matching pattern, including the meeting probabilities, is endogenous, depending upon firms' choices of which workers to seek and the size of the unemployed worker and vacant firm pools. The workers' choices of whether to acquire skills are also endogenous, depending upon the market values of these skills. Together, these choices allow asymmetries between different groups of *ex ante* identical workers to be generated endogenously. Finally, the terms of trade between matched agents are determined in a bargaining process that reflects market conditions.[5] This builds a natural antidote to discrimination into the model by causing the disadvantaged group of workers to be more attractive to firms because they command a lower wage rate for a given, observable skill level. We identify conditions under which discrimination persists despite this ameliorating effect.

In practice, firms search for workers through formal means, such as advertisements, and through informal networks of contacts and referrals. We interpret a strategy of searching only for greens as the cultivation of a network that involves primarily greens. The importance of firms pursuing potential workers through networks of contacts should not be underestimated. Formal studies emphasizing

[3]Burdett and Smith [2] study a related model with multiple symmetric equilibria, some in which workers acquire high skills and earn high wages but others in which workers acquire low skills and earn low wages. In contrast to our model, firms in their model make nontrivial decisions about vacancies (so that in the high-skill equilibrium there are more vacancies and so higher returns to skills), which allows for a coordination failure.

[4]While our formal analysis is restricted to the case of equal sized groups, it is straightforward but tedious to cover unequal sized groups.

[5]These features distinguish our analysis from that of Sattinger [10], who examines a model in which firms can choose which groups of workers to seek but in which worker groups have exogenously fixed asymmetries, contact rates between firms and workers are insensitive to the sizes of the unemployed worker and vacant firm pools, and the terms of trade are fixed.

such networks are reinforced by the more popular job-search literature.[6] Job-seeking guides routinely emphasize the exploitation of informal contacts. Anecdotes concerning the importance of the "old boy network" are reinforced by popular claims that most jobs are obtained with employers where the new worker already has a personal acquaintance.

The following section introduces the model. Section 3 examines the symmetric equilibrium of the model. Sections 4-6 examine asymmetric equilibria. Section 7 discusses the results and their potential policy implications. Proofs whose arguments are potentially distracting are collected in Section 8.

2. The Model

We consider an economy with a continuum of firms and workers. Firms' and workers' lifetimes are independently distributed according to a Poisson process with death rate δ. Births of new firms and workers also occur at rate δ, so that the size of the populations of firms and workers is constant.[7] The total populations of both workers and firms are normalized to be of measure one. Time is continuous, with interest rate r.

All firms are identical. Each worker has a label, red or green, that has no direct payoff implications. For convenience, we assume that half of the population of workers has a red label and half has a green label. Upon entering the market, or being "born," each worker makes an irrevocable decision either to acquire skills or to eschew skills and enter the unskilled sector of the economy. Workers differ in the opportunity cost of acquiring skill, denoted by $\alpha \geq 0$, where this opportunity cost includes both the direct cost of skill acquisition as well as the foregone value to entering the unskilled labor market. Each worker's opportunity cost α is the realization of a random variable, independent of the worker's color, with continuous cumulative distribution function C. A worker makes the skill acquisition decision knowing his opportunity cost of skill.

Each firm can hire at most one worker. If a firm employs a skilled worker, whether red or green, a flow surplus of x is generated, while a firm hiring an unskilled worker generates a flow surplus of zero.

[6]The importance of personal contacts and referrals in seeking jobs is stressed by Berger [1], Corcoran [5], Holzer [7], Staiger [13], and Wial [14].

[7]Throughout the paper, we assume that the continuum of independent random variables describing firm and worker outcomes yield a market outcome with no aggregate randomness. Since there is a continuum of agents, correctly modelling the underlying economy-wide stochastic process generating the matching requires deep mathematical techniques that we do not make explicit.

A firm currently without an employee is "vacant," while a firm with an employee is "occupied." All meetings between vacant firms and unemployed workers arise from either firm or worker search.[8] Firms can observe workers' colors, and can condition their search activity on colors. Thus, a firm can decide to search only for unemployed green workers, only for unemployed red workers, or to search for both colors of unemployed workers.[9]

Suppose that the firm searches both colors of worker, the skilled worker population is of size H_W, and the unemployment rate of skilled workers is ρ_W. Then the process describing meetings between unemployed skilled workers and firms, generated by a vacant firm's search, follows a Poisson process with rate $\lambda_F \rho_W H_W$. The parameter λ_F captures the intensity of firm search, while $\rho_W H_W$ captures the idea that the firm can more quickly find a member of a large group than of a small one.

If the firm searches only for unemployed green workers and there are $\rho_G H_G$ unemployed skilled green workers,[10] the process describing meetings of the firm and unemployed green workers as a result of firm search follows a Poisson process with rate $2\lambda_F \rho_G H_G$. In this case, there are no meetings between the firm and red workers generated by firm search.[11] Notice that restricting search to one color has the effect of doubling the search intensity on that color, since the firm is then concentrating its search on half as many potentially skilled workers (recall that there are equal numbers of red and green workers), while reducing the search intensity to zero on the other color. We can think of a firm who searches *both* colors of worker as facing two Poisson processes, one generating meetings of the firm and green workers at rate $\lambda_F \rho_G H_G$, and one generating meetings of the firm

[8]Since search is costless and the firm always has the option of declining to hire any worker found, not searching is weakly dominated by searching. We accordingly assume vacant firms always search. A similar comment applies to unemployed workers.

[9]Searching firms cannot distinguish previously employed workers from workers who have never been employed. Notice, however, that once the firm and worker meet, the worker has no difficulty convincing the firm of her skill, making employment history irrelevant for the equilibrium we construct. There may be additional equilibria in which new and previously employed workers are treated differently when bargaining with firms over the division of the surplus created by a match.

[10]We denote the size of the green (red) skilled worker population by H_G (H_R) and the unemployment rate of green (red) skilled workers by ρ_G (ρ_R).

[11]In practice, we would not expect firms to be able to perfectly control the color of workers they contact, though firms can undoubtedly bias their search toward either greens or reds. We could alter the model so that a firm attempting to contact only greens meets some but relatively few reds, or meets only green workers, but at the cost of meeting workers at a lower rate than when searching both colors. This would not affect the symmetric equilibrium we examine and still allows the possibility of an asymmetric equilibrim in which firms are more likely to contact green than red workers.

and red workers at rate $\lambda_F \rho_R H_R$, so that meetings of the firm and *all* workers as a result of firm search follow a Poisson process with rate $\lambda_F \rho_G H_G + \lambda_F \rho_R H_R = \lambda_F \rho_W H_W$ if $\rho_G = \rho_R = \rho_W$. Restricting search to only green workers allows the search efforts formerly dedicated to red workers to be transferred to greens, giving a meeting rate of $\lambda_F \rho_G H_G + \lambda_F \rho_G H_G = 2\lambda_F \rho_G H_G$.

Unemployed skilled workers simultaneously search for vacant firms with intensity λ_W. Since we have assumed that the population of firms is fixed at one, meetings generated by worker search follow a Poisson process with rate $\lambda_W \rho_F$, where ρ_F is the vacancy rate of firms.

Two types of tie can arise in the matching process. The first type of tie is a firm (or worker) being contacted by more than one worker (firm) at the same time. The second type is a worker contacting a firm at the same time as that firm contacts a worker. We can ignore these ties when calculating an agent's value function, because the agent assigns them zero probability. Moreover, we assume that the measure of agents involved in such ties is zero.[12]

After an unemployed skilled worker and a vacant firm make contact, they bargain. We postulate a simple bargaining game: A fair coin determines a proposer, who makes a take-it-or-leave-it wage offer to the responder. In any sequentially-rational equilibrium, the proposer will make an offer that leaves the responder indifferent between accepting and rejecting, and the offer will be accepted (at least on the equilibrium path). A variety of more complicated bargaining conventions suffice for the result and might be more realistic. The essential features of the bargaining process are that each agent captures a share of the surplus that is increasing in the agent's expected value of returning to the search process and decreasing in his rival's expected value of returning to the search process.

We assume that an occupied firm cannot abandon its current worker to bargain with a new worker, nor can a worker abandon a firm to seek a new one. This is a strong assumption in the context of our simple bargaining model, which implicitly requires high transactions costs for dissolving an employment relationship. For example, the worker may have won the initial coin toss and proposed a wage that extracts all the surplus from the firm, leaving the firm anxious for a chance to dismiss the current employee, if it can be done cheaply, and bargain again with a new employee. This assumption is less troubling, and would not hinge on high transactions costs, in a more realistic bargaining process that allowed both the firm and worker to capture sufficient surplus *ex post* as well as *ex ante*.

[12]This is a pervasive, but not trivial, assumption in the search literature, again raising the intracacies of modeling a continuum of random variables.

3. The Symmetric Steady State Equilibrium

In this section we examine a symmetric, steady-state equilibrium in which firms pay no attention to workers' colors. The equilibrium unemployment rates of red and green workers are identical. Any meeting between an unemployed worker and a firm with a vacancy results in the vacancy being filled.

Let V_W denote the value of skills to a worker, or equivalently, the value of entering the market for skilled labor.[13] Since a worker acquires skills if $\alpha < V_W$, the fraction of new workers who become skilled is $C(V_W)$. Let H_W denote the size of the skilled workforce, or equivalently, the proportion of workers who are skilled. In a steady state, the size of the inflow of newly skilled workers (given by the product of the rate at which new workers appear, δ, and the proportion, $C(V_W)$, of new workers acquiring skills) must equal the size of the outflow of existing skilled workers (given by δH_W, the product of the death rate of workers and the size of the skilled labor force), and hence we have the *skilled worker steady state* condition:

$$H_W = C(V_W). \tag{3.1}$$

In a steady state, the rate at which new vacancies are created must match the rate at which they are filled. New vacancies appear at the rate $2\delta(1-\rho_F)$, since $1-\rho_F$ of the firms are currently occupied, and at rate 2δ either a worker dies, creating a vacancy at a previously occupied firm, or an occupied firm dies and is replaced by a new, vacant firm. Vacancies are filled as a result of both firm and worker search. There are ρ_F firms vacant and hence searching, generating meetings at the rate $\rho_F \lambda_F \rho_W H_W$. At the same time, there are $\rho_W H_W$ workers searching, generating additional meetings at the rate $\rho_W H_W \lambda_W \rho_F$. Adding these two sources of filled vacancies, vacancies are filled at the rate $\rho_F \rho_W H_W(\lambda_F + \lambda_W)$. Thus, the *vacancies steady state* condition is

$$2\delta(1-\rho_F) = \rho_F \rho_W H_W(\lambda_F + \lambda_W). \tag{3.2}$$

Flows into and out of unemployment must also balance in a steady state. Newly unemployed workers arrive at the rate $2\delta H_W(1-\rho_W)$ (because there are $H_W(1-\rho_W)$ employed skilled workers, with worker and firm deaths adding unemployed workers at the rate 2δ). Since unemployed workers find jobs at the same rate as vacancies are filled, or $\rho_F \rho_W H_W(\lambda_F + \lambda_W)$, we have (dividing by H_W) the *unemployment steady state* condition:

$$2\delta(1-\rho_W) = \rho_F \rho_W(\lambda_F + \lambda_W). \tag{3.3}$$

[13] In general, these values are functions of time, but our interest in steady-state equilibria allows us to ignore such complications.

Let w be the expected flow payoff of an employed worker and Z_W the steady state value of an employed, skilled worker. To see how the values of an employed and unemployed worker are related, consider temporarily a discrete-time model.[14] Time intervals are of length τ, with the death probability, discount rate, and search intensities given by $\delta\tau$, $r\tau$, and $\lambda_F\tau$ and $\lambda_W\tau$. Consider first the recursive equation determining Z_W. An employed worker earns a flow payoff of $w\tau$ in the current period, survives until the next period with probability $(1-\delta\tau)$, and then loses employment due to firm death with probability $\delta\tau$, for an outcome whose present value is $V_W/(1+r\tau)$, and retains employment with complementary probability, for a present value of $Z_W/(1+r\tau)$. Thus,

$$Z_W = w\tau + (1-\delta\tau)\left(\delta\tau\frac{V_W}{(1+r\tau)} + (1-\delta\tau)\frac{Z_W}{(1+r\tau)}\right).$$

Turning to the equation for V_W, an unemployed worker survives to the next period with probability $(1-\delta\tau)$. If she survives, then with probability $\rho_F(\lambda_F+\lambda_W)\tau$ she is matched with a vacant firm and begins employment, for a present value of $Z_W/(1+r\tau)$. With complementary probability, she is again unemployed, for a present value of $V_W/(1+r\tau)$. Hence,

$$V_W = (1-\delta\tau)\left(\rho_F(\lambda_F+\lambda_W)\tau\frac{Z_W}{(1+r\tau)} + (1-\rho_F(\lambda_F+\lambda_W)\tau)\frac{V_W}{(1+r\tau)}\right).$$

Solving the first equation for Z_W and then taking the limit as τ approaches zero gives an expression that characterizes the continuous-time values of Z_W and V_W:

$$Z_W = \lim_{\tau\to 0}\frac{w\tau(1+r\tau)+(1-\delta\tau)\delta\tau V_W}{1+r\tau-(1-\delta\tau)^2} = \frac{w+\delta V_W}{r+2\delta}.$$

To interpret this equation, we note that the employed worker's value from her current job can be calculated by discounting the flow of w at rate $r+2\delta$, reflecting the discount rate r and the death rates of both partners. The value Z_W consists of this value plus the term $\delta V_W/(r+2\delta)$, capturing the expected present value of being returned to the unemployed pool by surviving a firm death.

Similarly,

$$V_W = \lim_{\tau\to 0}\frac{(1-\delta\tau)\rho_F(\lambda_F+\lambda_W)\tau Z_W}{(1+r\tau)-(1-\delta\tau)(1-\rho_F(\lambda_F+\lambda_W)\tau)} = \frac{\rho_F(\lambda_F+\lambda_W)Z_W}{\rho_F(\lambda_F+\lambda_W)+r+\delta}.$$

[14] An alternative derivation of the value functions that avoids the discrete approximation is in Appendix A.

Analogous calculations for the firms yield

$$Z_F = \frac{f + \delta V_F}{r + 2\delta}$$

and

$$V_F = \frac{\rho_W H_W(\lambda_F + \lambda_W) Z_F}{\rho_W H_W(\lambda_F + \lambda_W) + r + \delta},$$

where f is the expected flow payoff to an occupied firm, V_F is the steady state value of a vacant firm, and Z_F is the steady state value of a firm currently employing a worker.

Rather than calculating the equilibrium values of w and f, we use the observation that $w + f = x$, the total flow surplus, to calculate the equilibrium values of V_W and V_F directly. Letting S denote the surplus a matched firm and worker divide, we have

$$S = Z_W + Z_F = \frac{x + \delta(V_W + V_F)}{r + 2\delta}.$$

Firms and workers bargain over the surplus S by making take-it-or-leave-it wage proposals with equal probability. In equilibrium, any such proposal makes the responding agent indifferent between accepting the proposal and rejecting, and so

$$Z_W = \frac{1}{2} V_W + \frac{1}{2}(S - V_F) \tag{3.4}$$

and

$$Z_F = \frac{1}{2} V_F + \frac{1}{2}(S - V_W). \tag{3.5}$$

Solving for V_W and V_F, we have the *value equations*

$$V_W = \frac{\rho_F(\lambda_F + \lambda_W)x}{(r + \delta)\left[(\rho_F + \rho_W H_W)(\lambda_F + \lambda_W) + 2(r + 2\delta)\right]}, \tag{3.6}$$

and

$$V_F = \frac{\rho_W H_W(\lambda_F + \lambda_W)x}{(r + \delta)\left[(\rho_F + \rho_W H_W)(\lambda_F + \lambda_W) + 2(r + 2\delta)\right]}. \tag{3.7}$$

Finally, suppose that no worker acquires skills, giving $H_W = 0$. Then substituting $H_W = 0$ and $\rho_F = 1$ in (3.6) gives the value to a worker who decides to become skilled when no other worker acquires skills,

$$V_W(0) \equiv \frac{(\lambda_F + \lambda_W)x}{(r + \delta)(\lambda_F + \lambda_W + 2(r + 2\delta))}.$$

We now have all the information required to define a symmetric steady state:

Definition 1. *A* ***symmetric steady-state*** *is a 5-tuple of values* $(H_W, \rho_F, \rho_W, V_W, V_F)$ *satisfying the skilled worker steady state condition (3.1), the vacancies steady state condition (3.2), the unemployment steady state condition (3.3), and the value equations (3.6) and (3.7).*

Definition 2. *A* ***symmetric equilibrium*** *is a symmetric steady-state in which every firm finds it optimal to search both colors of worker.*

If the opportunity cost of acquiring skills is too large, then the only symmetric steady state is trivial, in that no worker becomes skilled. The following proposition focuses attention on nontrivial equilibria—some workers acquire skills ($H_W > 0$)—by assuming that there are some workers with sufficiently low opportunity costs of skill acquisition ($C(V_W(0)) > 0$). Search frictions ensure that some (but not all) of these workers are unemployed ($\rho_W \in (0,1)$).

At the other extreme, low opportunity costs may lead all workers to become skilled. A sufficient condition for the equilibrium to be *interior*—not all workers acquire skills ($H_W < 1$)—is $C(x/(r+\delta)) \leq 1$. This ensures that there are some workers whose opportunity cost of acquiring skills exceeds the maximum possible payoff of $x/(r+\delta)$, the value of receiving the entire flow surplus x and being immediately rematched with a new firm upon firm death.[15] The proof of the following Proposition, as well as the other proofs not found in the text, is in Section 8.

Proposition 1. *Every symmetric steady state is a symmetric equilibrium. A symmetric equilibrium exists and is unique. If* $C(V_W(0)) > 0$*, then* $H_W > 0$ *and* $\rho_W \in (0,1)$*. If, in addition,* $C(x/(r+\delta)) \leq 1$*, then* $H_W < 1$*.*

It is immediate that every symmetric steady state is a symmetric equilibrium, since the two colors of worker act, and so can be treated, identically.

A basic asymmetry between firms and workers is built into the model, because only workers have the opportunity of opting out of skilled sector. This asymmetry is reflected in the equilibrium (when it is interior), since skilled workers are in short supply. Workers use this imbalance to extract a larger share of the surplus when bargaining with the firm, which in turn gives workers a larger value of participating in the market:

Proposition 2. *If the symmetric equilibrium is interior,*

$$\rho_F > \rho_W, \quad Z_F < Z_W, \quad \text{and} \quad V_F < V_W.$$

[15] While sufficient, this condition is not necessary for interior symmetric equilibria. In later sections, we will construct interior symmetric equilibria for parameter values that violate this condition.

If firms must also make an investment to participate in the skilled labor market, the asymmetry between firms and workers reflected in Proposition 2 would disappear.[16] However, the economic forces in which we are interested are most conveniently displayed in a model which ignores this complication.

Asymmetries between firms and workers in the ability to search have no effect on equilibrium. The search intensities λ_F and λ_W only affect the equilibrium through their sum $\lambda \equiv \lambda_F + \lambda_W$, while the distribution of search opportunities between firms and workers is irrelevant. As long as firms search both reds and greens, it is thus immaterial whether contacts are initiated by firms or by workers.

Increasing the sum λ of the search intensities has the expected effects of increasing the rate at which an unemployed worker encounters vacant firms ($\rho_F\lambda$) and the rate at which a vacant firm encounters unemployed workers ($\rho_W H_W \lambda$). This reduces the proportion of vacant firms and the unemployment rate of workers, increasing the value of acquiring skill and the proportion of workers acquiring skill:

Proposition 3. *The symmetric equilibrium is unchanged by variations in λ_F and λ_W that leave the sum $\lambda = \lambda_F + \lambda_W$ unchanged. Moreover,*

$$\frac{d(\rho_F\lambda)}{d\lambda} > 0, \quad \frac{d(\rho_W H_W \lambda)}{d\lambda} > 0$$

and hence

$$\frac{d\rho_F}{d\lambda} < 0, \quad \frac{d\rho_W}{d\lambda} < 0, \quad \frac{dV_W}{d\lambda} > 0, \quad \text{and} \quad \frac{dH_W}{d\lambda} > 0.$$

The value of vacant firms need not be increased by an increase in the aggregate search intensity λ. If the symmetric equilibrium is interior, so $H_W < 1$, there is an excess supply of firms. If an increase in search intensity produces only a modest increase in the supply of skilled workers, the attendant increase in the workers' value may suffice to move the balance of bargaining power sufficiently in favor of workers that firms are worse off. In particular, $V_F = 0$ when $\lambda = 0$. If $H_W < 1$, then we also have $V_F \to 0$ as $\lambda \to \infty$, since the workers capture all the surplus when matching frictions disappear. Together, these ensure that the value of a firm cannot be monotonic in the search intensity.

Since firms are in fixed supply, it is intuitive that an increase in the search intensity should reduce the vacancy rate of firms ($d\rho_F/d\lambda < 0$). In contrast, it initially appears as if the increased search intensity, and the corresponding increase in the value of a worker, could prompt a sufficiently large increase in the number

[16] Masters [8] studies such a model, examining the inefficiencies due to bargaining and search.

of skilled workers to raise the unemployment rate of skilled workers. However, an increase in the unemployment rate is inconsistent with the unemployment steady state condition (3.3).

The following Proposition describes how the symmetric equilibrium responds to supply-and-demand pressures. Parameterize the distribution of opportunity costs by fixing a distribution function $\mathcal{C}$ and setting $C(V_W) = \mathcal{C}(V_W - Y)$ for $Y \geq 0$. An increase in Y can be interpreted as an increase in the attractiveness of the unskilled labor sector and hence a decrease in the supply of skilled labor. Note that the symmetric equilibrium is interior for sufficiently large Y.

Proposition 4. *Let the distribution of opportunity costs be given by $\mathcal{C}(V_W - Y)$. If the symmetric equilibrium is interior ($H_W < 1$), then $d\tilde{\rho}/dY = d(\rho_W H_W)/dY < 0$, and hence,*

$$\frac{d\rho_F}{dY} > 0, \quad \frac{d\rho_W}{dY} < 0, \quad \frac{dV_F}{dY} < 0, \quad 0 < \frac{dV_W}{dY} < 1, \quad \textit{and} \quad \frac{dH_W}{dY} < 0.$$

An increase in Y, by increasing the opportunity cost of acquiring skills, reduces the quantity of skilled labor supplied ($dH_W/dY < 0$). As a result, the vacancy rate of firms increases, while the unemployment rate of workers decreases. This shift in unemployment rates is accompanied by a shift of bargaining power in favor of workers, and the wage rate increases. As a result, the equilibrium value of a firm falls, while the equilibrium value of a skilled worker must increase to offset the higher opportunity cost of acquiring skills.

In a frictionless world ($\lambda_W, \lambda_F = \infty$) in which $H_W < 1$, the value of a match between a worker and a firm is $\frac{x}{r+\delta}$ and workers capture all of the surplus. There is no unemployment, and hence no excess expenditure on acquiring skills, and all workers for whom $\alpha \leq \frac{x}{r+\delta}$ enter the skilled sector. In our model, time-consuming search and bargaining lead to two distortions. First, there is unemployment. The total expenditure on acquiring skills, including the opportunity cost of sacrificing participation in the unskilled market, is thus inefficiently higher than would be needed in a frictionless world to achieve the realized volume of employment. Second, too few workers acquire skill, both because unemployment reduces the value of acquiring skills and because costly bargaining prevents workers from capturing the entire surplus from a match.[17]

[17] Because an increase in the search intensity can decrease V_F, reducing search frictions need not lead to Pareto superior oucomes. However, increasing the search intensity must increase the total economic surplus, net of opportunity costs, generated by the market. In particular, an increased search intensity reduces the expected length of an unemployment spell for existing workers, increasing the surplus generated by each such worker (with no new opportunity costs

4. Asymmetric Steady States

We now analyze an asymmetric equilibrium in which firms search only green workers. There is clearly nothing special about greens here, and an analogous equilibrium exists in which firms search only for reds.

Consider first a market in which all workers automatically enter the skilled sector and in which every meeting between a firm and worker results in employment of the worker at an exogenously fixed wage rate. Suppose moreover that firms search only for green workers. Then there will be more unemployed reds than greens, because reds acquire employment only as a result of their own search, while greens acquire employment either as a result of their own search or firm search. As a result, firms earn a higher payoff from seeking reds than greens, and the firms' behavior cannot be sustained as an equilibrium.

In our model, two additional forces appear. First, the wage rate is determined endogenously. If firms search only greens, then red workers will be in a relatively weak bargaining position and will earn a lower wage rate than greens. This makes it even less likely that firms will find it optimal to search only for greens. However, fewer reds may acquire skills, making reds less attractive because it is harder to find a skilled red, and reinforcing the decision to search only for greens. The question is then whether the market wage rate for reds can fall enough to disrupt an asymmetric equilibrium by making it optimal for firms to search reds, or whether the lower wages will be overwhelmed by a sufficiently large adjustment in the number of workers acquiring skills as to make searching reds suboptimal.

We break our study of the asymmetric equilibrium into three parts. This section defines and characterizes the steady state induced by asymmetric firm behavior. Section 5 investigates the optimality of asymmetric search and hence the existence of an asymmetric equilibrium. Section 6 explores some implications of asymmetric equilibria. The notation follows that of the symmetric case, but with the single subscript "W" for workers now replaced by "R" and "G," for red and green workers.

The equilibrium conditions are analogous to those of the symmetric case, with two exceptions. First, in equilibrium, firms must find it optimal to only search greens. Second, if there are unemployed skilled red workers, then some vacant firms will meet them as a result worker search, and we must specify a firm's reaction to such a meeting. The steady state conditions are derived under the presumption that a vacant firm reaches an agreement with any unemployed skilled

incurred). New workers are attracted into the market only if the value of entry exceeds their opportunity cost, which in turn implies that the value of the additional surplus they create must exceed their opportunity cost, since they do not capture all of this surplus.

red worker it happens to meet. We then confirm that in the steady state, vacant firms strictly prefer to reach such an agreement rather than remaining vacant (Proposition 6).

The inflows of new skilled workers are given by $\delta C(V_G)/2$ for greens and $\delta C(V_R)/2$ for reds, since half of the workers are green and half are red. In a steady-state equilibrium, these inflows must match the outflows of skilled workers caused by death (given by δH_G and δH_R), so the *green* and *red skilled worker steady state* conditions are

$$H_G = C(V_G)/2 \tag{4.1}$$

and

$$H_R = C(V_R)/2. \tag{4.2}$$

The *vacancies steady state* condition is:

$$2\delta(1-\rho_F) = 2\rho_F\lambda_F H_G\rho_G + (\rho_G H_G + \rho_R H_R)\lambda_W\rho_F. \tag{4.3}$$

This again reflects a balancing of the rate at which deaths create firm vacancies, given by $(1-\rho_F)2\delta$, and the rate at which vacancies are filled. The first term on the right side captures the rate at which vacancies are filled as a result of firm search, since there are ρ_F firms searching and each firm is searching at rate $2\lambda_F\rho_G H_G$ (given a green unemployed skilled-worker population of size $\rho_G H_G$). The second term captures the effect of worker search. These terms are asymmetric because firms search only green workers while both types of workers search for firms.

The *green* and *red unemployed steady state* conditions are given by:

$$2\delta(1-\rho_G) = \rho_G\rho_F(\lambda_W + 2\lambda_F) \tag{4.4}$$

and

$$2\delta(1-\rho_R) = \rho_R\rho_F\lambda_W. \tag{4.5}$$

These are analogous to the unemployment rate for the symmetric equilibrium, given by (3.3), with the exceptions that red workers find employment only as a result of their own search and not firm search, and the entire unit measure of firms is now searching the half-unit measure of green workers, causing the effective firm search rate to be $2\lambda_F$.

Analogously to the symmetric case, the various value functions are (where, for example, w_R is the expected wage of a red skilled worker and $Z_{F,R}$ is the value of the firm when currently matched with a red worker):

$$Z_R = \frac{w_R + \delta V_R}{r+2\delta}, \qquad Z_G = \frac{w_G + \delta V_G}{r+2\delta},$$

$$Z_{F,R} = \frac{f_R + \delta V_F}{r + 2\delta}, \qquad Z_{F,G} = \frac{f_G + \delta V_F}{r + 2\delta},$$

$$V_R = \frac{\rho_F \lambda_W Z_R}{\rho_F \lambda_W + r + \delta}, \qquad V_G = \frac{\rho_F (2\lambda_F + \lambda_W) Z_G}{\rho_F (2\lambda_F + \lambda_W) + r + \delta},$$

and

$$V_F = \frac{(2\lambda_F + \lambda_W)\rho_G H_G Z_{F,G} + \lambda_W \rho_R H_R Z_{F,R}}{(2\lambda_F + \lambda_W)\rho_G H_G + \lambda_W \rho_R H_R + r + \delta}.$$

The surpluses of the different matches are given by

$$S_R = Z_R + Z_{F,R} = \frac{x + \delta(V_R + V_F)}{r + 2\delta}$$

and

$$S_G = Z_G + Z_{F,G} = \frac{x + \delta(V_G + V_F)}{r + 2\delta}.$$

Finally, bargaining gives

$$\begin{aligned}
Z_R &= \frac{V_R}{2} + \frac{(S_R - V_F)}{2} = \frac{x}{2(r + 2\delta)} + \frac{((r + 3\delta)\, V_R - (r + \delta)\, V_F)}{2(r + 2\delta)}, \\
Z_G &= \frac{V_G}{2} + \frac{(S_G - V_F)}{2} = \frac{x}{2(r + 2\delta)} + \frac{((r + 3\delta)\, V_G - (r + \delta)\, V_F)}{2(r + 2\delta)}, \\
Z_{F,R} &= \frac{V_F}{2} + \frac{(S_R - V_R)}{2} = \frac{x}{2(r + 2\delta)} + \frac{((r + 3\delta)\, V_F - (r + \delta)\, V_R)}{2(r + 2\delta)},
\end{aligned}$$

and

$$Z_{F,G} = \frac{V_F}{2} + \frac{(S_G - V_G)}{2} = \frac{x}{2(r + 2\delta)} + \frac{((r + 3\delta)\, V_F - (r + \delta)\, V_G)}{2(r + 2\delta)}.$$

Some tedious algebra (see Appendix B) allows us to solve for the value functions:

$$V_F = \frac{x}{(r + \delta)\,\Delta} \left[(2\lambda_F + \lambda_W)\rho_F \lambda_W \left(\rho_G H_G + \rho_R H_R\right) + 2\,(r + 2\delta)\left\{(2\lambda_F + \lambda_W)\rho_G H_G + \lambda_W \rho_R H_R\right\}\right], \tag{4.6}$$

$$V_R = \frac{\rho_F \lambda_W \left(\rho_F (2\lambda_F + \lambda_W) + 2\,(r + 2\delta)\right) x}{(r + \delta)\,\Delta}, \tag{4.7}$$

and

$$V_G = \frac{\rho_F (2\lambda_F + \lambda_W) \left(\rho_F \lambda_W + 2\,(r + 2\delta)\right) x}{(r + \delta)\,\Delta}, \tag{4.8}$$

where $\Delta \equiv 2\,(r + 2\delta)\left\{\rho_F(2\lambda_F + \lambda_W) + (2\lambda_F + \lambda_W)\rho_G H_G + \rho_F \lambda_W + \lambda_W \rho_R H_R + 2\,(r + 2\delta)\right\} + \rho_F \lambda_W (2\lambda_F + \lambda_W)\left(\rho_F + \rho_R H_R + \rho_G H_G\right)$.

Definition 3. *A* **green asymmetric steady state** *is an 8-tuple* $(H_G, \rho_G, V_G, H_R, \rho_R, V_R, \rho_F, V_F)$ *solving the balance equations (4.1)-(4.5) and the value functions (4.6)-(4.8).*

As before, we say that the green asymmetric steady state is *nontrivial* if some workers acquire skills, and nontriviality will require the existence of some workers with low opportunity costs of skill acquisition. The Appendix uses a fixed-point argument to establish:

Proposition 5. *There exists a green asymmetric steady state. If* $C(V_W(0)) > 0$*, every green asymmetric steady state is nontrivial.*

We have not asserted the uniqueness of a green asymmetric steady state. Exploiting the recursive nature of the steady-state conditions, we can use (4.3)–(4.5) to show that for any fixed worker entry rates H_G and H_R, there is a unique set of vacancy and unemployment rates. However, there may be multiple green asymmetric steady states, characterized by different entry rates H_G and H_R.

The following proposition characterizes the different treatment received by red and green workers in an asymmetric steady state.

Proposition 6. *In a nontrivial green asymmetric steady state, some green workers acquire skills* ($H_G > 0$)*, and*

$$\begin{array}{ccc} V_R < V_G, & H_R < H_G, & Z_{F,R} > Z_{F,G}, \\ Z_{F,R} > V_F, & w_R < w_G, & f_R > f_G, \quad \text{and} \\ & \rho_R > \rho_G. & \end{array}$$

In a green asymmetric equilibrium, red workers face a less attractive value of entering the market than do green workers ($V_R < V_G$), and so if any workers acquire skills, then some green workers do ($H_G > 0$), while fewer red than green workers acquire skills ($H_R < H_G$). Red workers are thus at a disadvantage when bargaining with firms. Firms exploit this weaker position to extract a larger share of the surplus from red workers. As a result, average red-worker wages fall short of green-worker wages ($w_R < w_G$), while the firm receives a larger portion of the flow surplus from a red worker ($f_R > f_G$). Given this wage difference, a vacant firm given a choice between a red worker and a green worker would strictly prefer a red worker ($Z_{F,R} > Z_{F,G}$), and a vacant firm prefers to enter an employment relationship with a red worker rather than continue searching ($Z_{F,R} > V_F$). Our assumption that the firm would hire any red workers it meets, used to calculate the steady state, is thus consistent with optimal behavior. At the same time, since reds are not being searched by firms, the unemployment rate is higher for reds than greens ($\rho_R > \rho_G$). In conventional terms, reds thus suffer lower wages, lower labor-force participation rates, and higher unemployment rates. Finally, notice that Proposition 6 does not rule out $H_R = 0$, and we will construct nontrivial asymmetric steady states in which no reds acquire skills.

5. Asymmetric Equilibria

In the symmetric case, red and green workers achieve identical outcomes, making the firm indifferent between the two colors of worker and ensuring that the firm's decision to treat red and green workers identically is automatically optimal. In contrast, (green) asymmetric equilibria involve an additional consideration beyond the steady state conditions: it must be optimal for the firm to search only green workers.

Instead of searching only green workers, the firm has the option of either searching only red workers or searching both types of workers. Searching only greens is optimal if

$$V_F \geq \max\{V_F(R|G), V_F(W|G)\}, \tag{5.1}$$

where $V_F(R|G)$ ($V_F(W|G)$) is the value of a firm searching red (all) workers in a steady state in which all other firms are searching green workers. Hence,

$$\begin{aligned} V_F(R|G) &= \frac{\lambda_W \rho_G H_G Z_{F,G} + (2\lambda_F + \lambda_W)\rho_R H_R Z_{F,R}}{\lambda_W \rho_G H_G + (2\lambda_F + \lambda_W)\rho_R H_R + r + \delta} \quad \text{and} \\ V_F(W|G) &= \frac{(\lambda_W + \lambda_F)(\rho_G H_G Z_{F,G} + \rho_R H_R Z_{F,R})}{(\lambda_W + \lambda_F)(\rho_G H_G + \rho_R H_R) + r + \delta}. \end{aligned}$$

Definition 4. *A **green asymmetric equilibrium** is a green asymmetric steady state that satisfies (5.1).*

Direct manipulation of the firm-search optimality condition $V_F \geq V_F(R|G)$ allows us to identify an essential feature of asymmetric equilibria:

Proposition 7. *In all nontrivial green asymmetric equilibria,*

$$\rho_R H_R < \rho_G H_G.$$

Since vacant firms in a green asymmetric equilibrium prefer to hire reds, conditional on meeting a skilled worker, a vacant firm would direct search at reds if red and green workers were equally easy to find. An equilibrium in which firms do not seek red workers can then only be supported if there are more unemployed skilled green workers than red workers, giving $\rho_R H_R < \rho_G H_G$.

We have already seen that skilled red workers in a green asymmetric steady state must have a higher unemployment rate than green workers. This is consistent with the paucity of skilled red workers required for the optimality of asymmetric search only if sufficiently few red workers acquire skills. The existence of a green asymmetric equilibrium then requires a sufficiently vigorous supply response on the part of workers. An asymmetric steady state for which $H_G \leq H_R$ thus cannot be a nontrivial green asymmetric equilibrium. In particular, a green asymmetric equilibrium is inconsistent with a degenerate opportunity cost distribution that fixes skilled acquisition rates so that $H_G = H_R$.

When does an asymmetric equilibrium exist? In addition to sufficiently responsive worker supply decisions, the relative sizes of the search intensities λ_W and λ_F are now important. This represents a departure from the symmetric equilibrium, where only the sum $\lambda_W + \lambda_F$ mattered. A nontrivial green asymmetric equilibrium requires that there be at least some firm search, or $\lambda_F > 0$. The conditions for the existence of an asymmetric equilibrium involve the interplay between the relative sizes of λ_F and λ_W and the worker supply responses induced by the opportunity cost distribution C, with larger values of λ_F allowing asymmetric equilibria to exist under weaker assumptions on C. We find it convenient to capture these forces by considering variations in λ_F and λ_W that preserve the sum $\lambda = \lambda_F + \lambda_W$.

We begin with the result that asymmetric equilibria exist when λ_F is large:

Proposition 8. *Let $C(V_W(0)) > 0$ and fix $\lambda = \lambda_F + \lambda_W$. There exists $\lambda_F^* < \lambda$ such that a nontrivial asymmetric equilibrium exists for all $\lambda_F \in (\lambda_F^*, \lambda]$.*

If virtually all contacts between firms and workers arise as a result of firm search, then a decision on the part of firms to search only green workers can impose an insurmountable obstacle to red workers. Red skilled workers face such limited employment prospects that red workers overwhelmingly opt into the unskilled sector of the economy. It is then much more likely that a firm will be successful in finding an unemployed green skilled worker than an unemployed red one, allowing firms to optimally search only greens.

It is not surprising that asymmetric equilibria exist if firm search is sufficiently more important than worker search. The more important observation is that if the distribution of the cost of acquiring skills tends to concentrate its mass near the value of being a skilled worker, worker supply responses will be large and asymmetric equilibria can then exist even for small values of λ_F. Fix total search intensity $\lambda = \lambda_W + \lambda_F$. Denote by $\widehat{V}_G(\lambda_F)$ and $\widehat{V}_R(\lambda_F)$ the values of acquiring skills, to a green and a red worker (respectively), in a hypothetical asymmetric steady state in which the firms' search intensity is λ_F, all green workers acquire

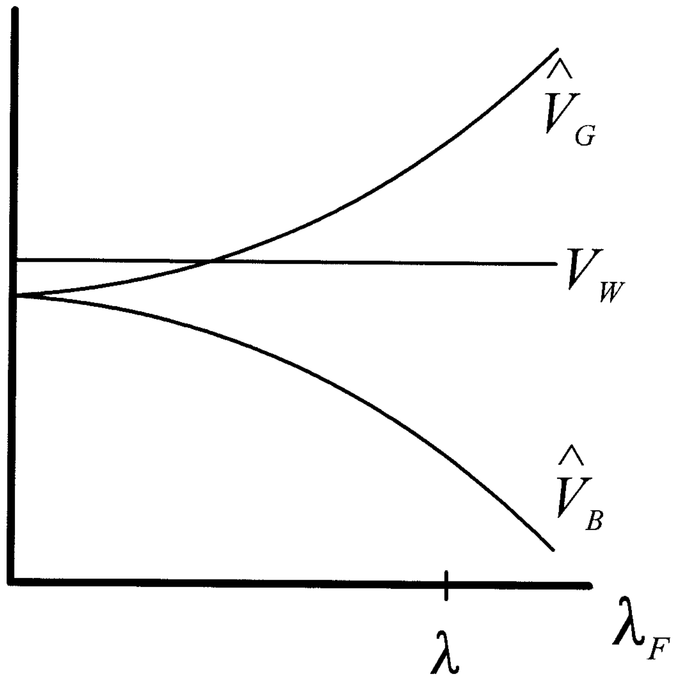

Figure 5.1: Extreme asymmetric value functions

skills ($H_G = 1/2$) , and no red workers do ($H_R = 0$). We refer to such a steady state as an *extreme steady state.* Since $\lambda_F \in [0, \lambda]$, $\widehat{V}_G, \widehat{V}_R : [0, \lambda] \to \Re_+$. A lengthy algebraic manipulation (see Appendix C) verifies:

$$\frac{d\widehat{V}_G(\lambda_F)}{d\lambda_F} > 0 \text{ and } \frac{d\widehat{V}_F(\lambda_F)}{d\lambda_F} < 0. \tag{5.2}$$

These functions are illustrated in Figure 5.1. Under the maintained assumption that all green workers enter and no red workers enter, the asymmetric steady state in which firms search only greens is equivalent to a symmetric equilibrium in which the total number of workers is $1/2$ ($= H_G$) and in which all of the workers necessarily enter. Increasing λ_F while holding the total of $\lambda_F + \lambda_W$ constant at λ has the effect of increasing the effective search rate, given by $2\lambda_F + \lambda_W$. But increasing the search rate in a symmetric equilibrium increases the value of workers, ensuring that $\widehat{V}_G(\lambda_F)$ is increasing in λ_F. To ascertain the behavior of $\widehat{V}_R(\lambda_F)$, notice that ρ_F decreases as λ_F increases, meaning that a red worker who did acquire skills would find it more difficult to find a vacant firm, which

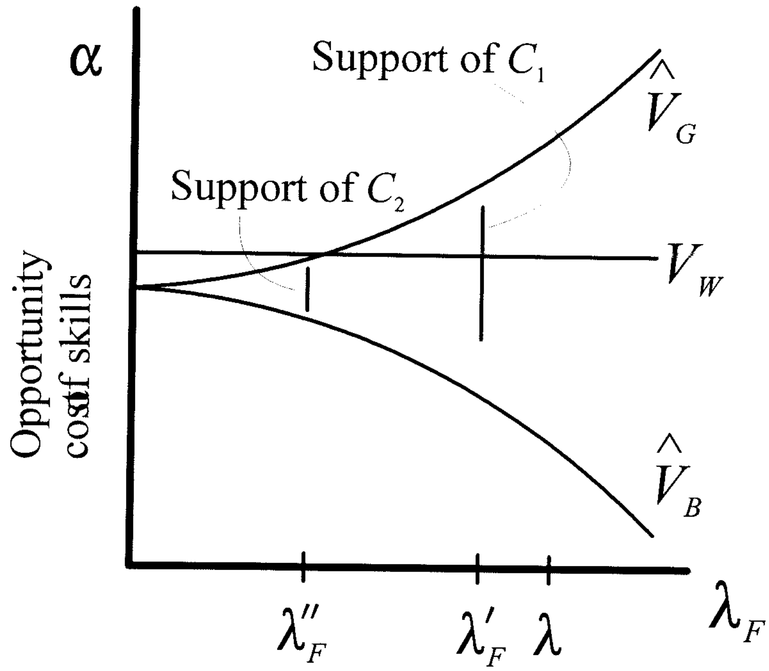

Figure 5.2: Existence of extreme green asymmetric equilibria. V_W is the symmetric value function for the distribution C_1 (but *not* for C_2). There is an extreme equilibrium when the cost distribution is C_1 and $\lambda_F = \lambda'_F$, and when the distribution is C_2 and $\lambda_F = \lambda''_F$.

tends to decrease the value $\widehat{V}_R(\lambda_F)$. An increase in λ_F may also decrease V_F, decreasing the firm's bargaining power and tending to increase $\widehat{V}_R(\lambda_F)$, but this force is overwhelmed by the decrease in ρ_F, ensuring that $\widehat{V}_R(\lambda_F)$ decreases.

Note that when $\lambda_F = 0$, all contacts between firms and workers come about as a result of worker search. It is then irrelevant whether firms search reds or greens, and red and green workers face identical environments and opportunities, giving $\widehat{V}_G(0) = \widehat{V}_R(0)$.

Figure 5.1 also displays the function $V_W(\lambda_F)$, showing the value of acquiring skills in a *symmetric* equilibrium. Since we have fixed $\lambda = \lambda_F + \lambda_W$, this function is constant in λ_F (Proposition 3). The relative positions of the curves V_W, $\widehat{V}_G$, and $\widehat{V}_R$ depend upon the number of workers who acquire skills in the symmetric equilibrium. If $H_W = 1/2$, then $V_W = \widehat{V}_G(0) = \widehat{V}_R(0)$ (note that when $\lambda_F = 0$, so that only workers search, an asymmetric steady state in which all greens but no reds acquire skills—$H_G = 1/2$, $H_R = 0$—gives precisely the same outcome as a symmetric steady state in which half of the workers acquire skills—$H_W = 1/2$). From Proposition 4, we then know that if more than half of the workers acquire skill in the symmetric equilibrium, then the value of worker skills will be lower, giving $V_W < \widehat{V}_G(0) = \widehat{V}_R(0)$. If less than half of the workers acquire skill in the symmetric equilibrium, then $V_W > \widehat{V}_G(0) = \widehat{V}_R(0)$, as shown in Figure 5.1.

We now examine the interplay between the importance of firm search, indicated by λ_F, and the distribution of skill costs. Suppose V_W is the value of skills in the unique symmetric equilibrium when the cost distribution is C. Then, for any cost distribution C' satisfying $C'(V_W) = C(V_W)$, V_W is also the value of skills in the unique symmetric equilibrium when the cost distribution is C'.

Proposition 9. *Fix a cost distribution C and $\lambda = \lambda_F + \lambda_W$. Suppose there exists $\bar{\lambda}_F$ such that $\widehat{V}_G(\bar{\lambda}_F) > V_W > \widehat{V}_R(\bar{\lambda}_F)$, where V_W is the value of skills in the unique symmetric equilibrium given C. Then, for all $\lambda_F \geq \bar{\lambda}_F$, there exists C_1 such that $C_1(V_W) = C(V_W)$ (so that V_W is the value of skills in the symmetric equilibrium for C_1) and such that an asymmetric equilibrium exists for the pair (λ_F, C_1).*

Figure 5.2 illustrates the construction used in the proof: letting $\lambda_F = \lambda'_F$, we can find a cost distribution, such as C_1, with a sufficiently small support so that, under the hypothesized behavior in the extreme steady state, the value to a green worker acquiring skills satisfies $C_1(\widehat{V}_G(\lambda_F)) = 1$, and the value to a red worker acquiring skills satisfies $C_1(\widehat{V}_R(\lambda_F)) = 0$. This ensures that the hypothesized behavior of workers is optimal in the extreme steady state. It is then optimal for the firm to search only greens, giving an asymmetric equilibrium. More generally, the key to existence of an asymmetric equilibrium is that workers' decisions of whether to acquire skills are sufficiently sensitive to changes in the value of skills. If they are, then the increase in the value of skills to greens and the decrease in the value of skills to reds, caused by firms searching only greens rather than all workers, will prompt enough greens to acquire skills and enough reds to forego skills as to make it optimal for the firm to search only greens. In the proof of Proposition 9, we have found it convenient to force such a dramatic worker response as to cause all greens and no reds to acquire skills, but this extreme case is unnecessary. Reds need not be excluded from the skilled labor market, nor must all greens be included, as long as the supply response to firms' decisions to search only greens is sufficiently large.

While Proposition 9 potentially yields existence for smaller values of λ_F than does Proposition 8, it does not apply to very small values of λ_F if $H_W \neq 1/2$, since it simultaneously asserts that in the symmetric equilibrium, V_W has a particular value. The same argument, however, immediately yields asymmetric equilibria for any value of λ_F:

Proposition 10. *For any* $\lambda_F \in (0, \lambda]$ *and* $V \in (\widehat{V}_R(\lambda_F), \widehat{V}_G(\lambda_F))$*, there exists a nontrivial asymmetric equilibrium for any cost distribution sufficiently concentrated around* V*.*

This is illustrated in Figure 5.2, where we take $\lambda_F = \lambda_F''$ and the cost distribution C_2.

The key to the existence of an asymmetric equilibrium is thus not that firm search is very important, but rather that by deciding to search only green workers, firms can prompt sufficiently large adjustments in worker skill decisions to justify this decision. This can occur even for very small $\lambda_F > 0$ if the cost of acquiring skills is likely to be near the benefits of being skilled.

6. Does Discrimination Lower Payoffs?

In the green asymmetric equilibria constructed in the previous section, green workers are better treated than in the symmetric equilibrium, while red workers are treated worse (i.e., $V_G > V_W > V_R$). This is the expected result that by discriminating in favor of greens when searching for workers, firms make green workers better off and red workers worse off. Is this a characteristic of all asymmetric equilibria? We first present two examples showing that these equalities may hold only weakly.

Example 1. Consider a symmetric equilibrium giving the value V_W illustrated in Figure 6.1. Notice that $H_W < 1/2$, since $V_W > \widehat{V}_G(0) = \widehat{V}_R(0)$. Now fix λ_F such that $\widehat{V}_G(\lambda_F) > V_W$. Then if $C_1(\widehat{V}_G(\lambda_F)) = 1$ and $C_1(\widehat{V}_R(\lambda_F)) = 0$, there exists an asymmetric equilibrium in which $H_G = 1/2$ and $H_R = 0$. But as λ_F declines toward the value λ_F^* at which $\widehat{V}_G(\lambda_F^*) = V_W$, V_G declines to V_W as V_R remains well below V_W. In the limit, we have an equilibrium corresponding to λ_F^*, supported by a cost distribution that places all of its mass on the value $\alpha = V_W$, for which $\widehat{V}_G(\lambda_F^*) = V_W$ and $\widehat{V}_R(\lambda_F^*) < V_W$. This demonstrates that being the targets of preferable search activity need not make greens better off (or at least, for nontrivial cost distribution, need not make greens very much better off). At the same time, reds do not suffer any payoff losses from discrimination in the limiting equilibrium corresponding to λ_F^*. The value of skills to a *skilled* red worker falls precipitously when firms search only greens, and reds accordingly opt out of the skilled labor market, but the opportunity cost of $\alpha = V_W$ ensures that they suffer no payoff loss in doing so.

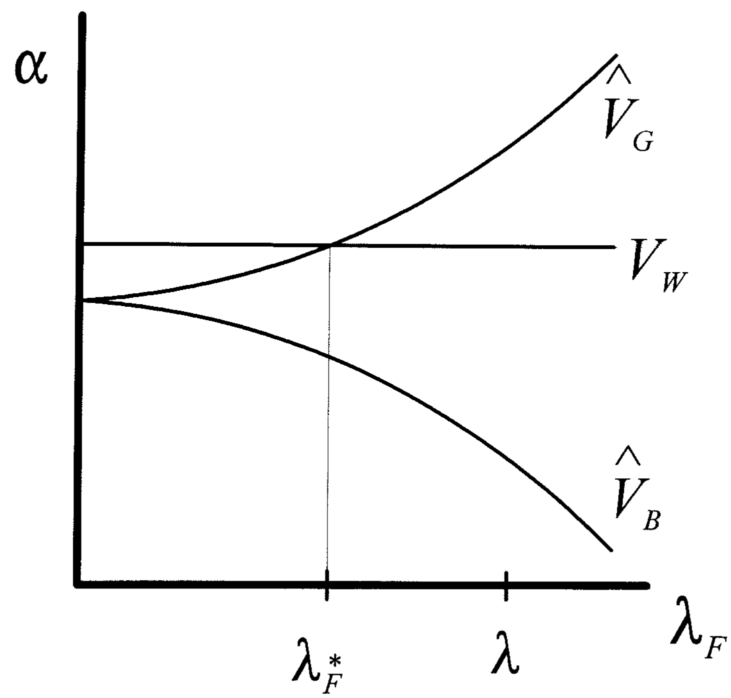

Figure 6.1: If the cost distribution is sufficiently concentrated around V_W and λ_F is sufficiently close to (but larger than) λ_F^*, then *all* workers are almost indifferent between the extreme green asymmetric equilibrium and the symmetric equilibrium.

Example 2. Consider a symmetric equilibrium giving the value V_W illustrated in Figure 6.2. Notice that $H_W > 1/2$, since $V_W < \widehat{V}_G(0) = \widehat{V}_R(0)$. Now fix λ_F such that $\widehat{V}_R(\lambda_F) < V_W$. Then if $C_1(\widehat{V}_G(\lambda_F)) = 1$ and $C_1(\widehat{V}_R(\lambda_F)) = 0$, there again exists an asymmetric equilibrium in which $H_G = 1/2$ and $H_R = 0$. But as λ_F declines toward the value λ_F^* at which $\widehat{V}_G(\lambda_F^*) = V_W$, V_R increases to V_W as V_G remains well above V_W. In the limit, we have an equilibrium for λ_F^* featuring a cost distribution that places all of its mass on the value $\alpha = V_W$ and featuring $\widehat{V}_R(\lambda_F^*) = V_W$ while $\widehat{V}_G(\lambda_F^*) > V_W$. This demonstrates that having firms ignore reds to search only greens need not make reds worse off (or at least, for nontrivial cost distribution, need not make reds very much worse off). Reds opt out of the skilled labor market when firms search only greens, but the value of skills to a red worker has not fallen and the opportunity cost of $\alpha = V_W$ ensures that they suffer no payoff loss from declining to become skilled. In this case, however, green workers gain significantly from being the targets of firm search, so that the discriminatory search produces a weak Pareto improvement in worker payoffs.

While both these examples have the feature that red workers are almost indifferent between the symmetric equilibrium and the green asymmetric equilibrium, this is of course not true in general. In the equilibrium illustrated in Figure 5.2, with the exception of the marginal red worker (for whom $\alpha = V_W$), all the red workers who choose to become skilled in the symmetric equilibrium strictly prefer the symmetric equilibrium to the green asymmetric equilibrium.

Is it necessarily the case that green workers prefer the green asymmetric equilibrium, while red workers prefer the symmetric equilibrium? In a model in which it is simply *assumed* that each party receives a fixed share of any flow surplus, the answer is easily shown to be yes. Our nontrivial bargaining procedure, however, complicates things dramatically. When firms search only for green workers, the unemployment rate of green workers falls while that of firms rises (as expected). Green workers thus find matches with vacant firms more quickly, increasing their values, and firms find it harder to find unemployed workers, decreasing their bargaining power and again increasing the value of a green worker. However, red workers find it much harder to find vacant firms, since no firms search for red workers. This decreases the value of a red worker, increasing the value of a firm and potentially making the firm a more aggressive bargainer when dealing with green workers. We must then show that this increase in firm bargaining power cannot decrease the value of a green worker. Alternatively, there is the possibility that the firms' bargaining position is sufficiently weak that even red workers do very well in bargaining, yielding high values.

We have not been able to provide a complete analytic answer. We proceed in two stages. First, let $V_W(H_G + H_R)$ be the value of an unemployed worker in a

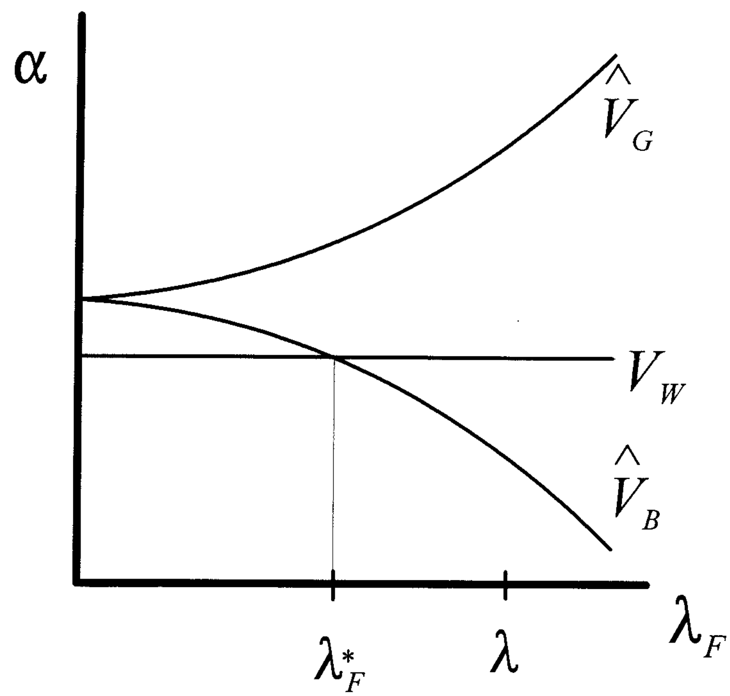

Figure 6.2: If the cost distribution is sufficiently concentrated around V_W and λ_F is sufficiently close to (but larger than) λ_F^*, then *red* workers are almost indifferent between the extreme green asymmetric equilibrium and the symmetric equilibrium, while green workers strictly prefer the asymmetric equilibrium

hypothetical symmetric steady state in which the proportion of skilled workers is arbitrarily fixed at $H_G + H_R$ (with equal numbers of reds and greens acquiring skills).

Proposition 11. *Suppose $(H_G^*, V_G^*, H_R^*, V_R^*)$ are the entry rates and values of green and red workers in a green asymmetric equilibrium. If*

$$V_G^* > V_W(H_G^* + H_R^*) > V_R^*, \tag{6.1}$$

and if a nontrivial symmetric equilibrium exists, then

$$V_G^* \geq V_W^* \geq V_R^*,$$

where V_W^* is the value of an unemployed worker in the symmetric equilibrium with no restrictions on entry.

Notice that Proposition 11 includes the case of $H_R^* > 0$, and hence is not limited to extreme equilibria.

Proof: Suppose $V_G^* < V_W^*$, and hence $V_W^* > V_W(H_G^* + H_R^*)$. An argument mimicking the proof of Proposition 4 establishes that a symmetric equilibrium can yield a higher worker value than $V_W(H_G^* + H_R^*)$ only if the number of workers entering in the symmetric equilibrium is less than $H_G^* + H_R^*$. But since $V_G^* < V_W^*$ and $V_R^* < V_W^*$, at least as many workers acquire skills in the symmetric equilibrium as in the asymmetric equilibrium, and hence at least $H_G^* + H_R^*$ workers acquire skills, a contradiction. A similar argument precludes the possibility that $V_W^* < V_R^*$. ■

It remains to verify (6.1). As before, we fix $\lambda = \lambda_W + \lambda_F$, and let λ_F vary. Denote the values to a green and red worker in an asymmetric steady state with entry rates of skilled workers arbitrarily fixed at H_G and H_R by $\widehat{V}_G(H_G, H_R, \lambda_F)$ and $\widehat{V}_R(H_G, H_R, \lambda_F)$ (respectively).[18] We again have $\widehat{V}_G(H_G, H_R, 0) = \widehat{V}_R(H_G, H_R, 0) = V_W(H_G + H_R)$, since firm search decisions and the composition of the skilled labor force are irrelevant when $\lambda_F = 0$. It is also immediate that

$$\widehat{V}_G(H_G, H_R, \lambda) > V_W(H_G + H_R) > \widehat{V}_R(H_G, H_R, \lambda) = 0. \tag{6.2}$$

Remark: A sufficient condition for (6.1) is that, for all H_G, H_R, and λ_F,

$$\widehat{V}_G(H_G, H_R, \lambda_F) > V_W(H_G + H_R) > \widehat{V}_R(H_G, H_R, \lambda_F). \tag{6.3}$$

[18] So that $\widehat{V}_G(\lambda_F) = \widehat{V}_G(1/2, 0, \lambda_F)$ and $\widehat{V}_R(\lambda_F) = \widehat{V}_R(1/2, 0, \lambda_F)$.

Verifying (6.3) is a much easier task than working with the full model, since we have exogenously fixed worker entry decisions, allowing us to ignore any considerations arising out of the specification of the cost distribution C. However, the inequality is still sufficiently complicated that we have not found a direct analytical approach. Extensive numerical investigation verifies the inequality for combinations of parameters satisfying $\lambda \in (1, 50]$, $\lambda_F \in (.05, \lambda]$, $r \in (1, 10]$, $\delta \in (1, 10]$, $H_G \in (.05, .4]$, and $H_R \in [0, H_G]$.

Proposition 11 establishes $V_G \geq V_W \geq V_R$ for the case in which a nontrivial symmetric equilibrium exists. It is easy to construct cases in which an asymmetric equilibrium exists, but the symmetric equilibrium is trivial (giving $V_G \geq V_W = V_R = 0$):

Proposition 12. *A nontrivial green asymmetric equilibrium may exist when the symmetric equilibrium is trivial.*

Proof: A special case of (6.2) is $\widehat{V}_G(\frac{1}{2}, 0, \lambda) > V_W(\lambda) > \widehat{V}_R(\frac{1}{2}, 0, \lambda) = 0$. If the opportunity cost distribution concentrates all of its mass in $(V_W(\lambda), \widehat{V}_G(\frac{1}{2}, 0, \lambda))$, then the symmetric equilibrium will be trivial, but an asymmetric equilibrium will exist in which all greens and no reds acquire skills. ■

An asymmetric equilibrium increases the value of acquiring skills for the advantaged group. If the opportunity cost of becoming skilled is relatively high, then an asymmetric equilibrium may be necessary in order to induce any workers to acquire skills. Discrimination may thus be necessary to the opening of a skilled labor market.

7. Discussion

The workers in our model impose an important externality on one another. It is only the existence of the green workers that allows firms to direct their search to the detriment of red workers. This externality leads to markedly different policy considerations than coordination-failure models. The coordination-failure aspect of statistical discrimination is "separable," so that any policy intervention in the red market has no effect on green workers. One can subsidize red skill acquisition, impose restrictions on red wages, or impose red hiring quotas without the slightest impact on green workers. In contrast, our red and green workers participate in a single labor market. Any policy designed to affect the outcome of red workers inevitably has consequences for greens, and greens may have a great deal to lose or gain from discrimination policy.

We have argued that groups of people can be trapped in outcomes featuring low skill levels, low wages, and high unemployment, not because there are barriers to their seeking employers or securing a job once an employer is found, but because employers are optimally choosing not to seek them. Given that workers always have the option of seeking firms, can firm search decisions really be that important? We suspect that firm search is vitally important in real labor markets, especially markets for skilled labor. Jobs are frequently filled not through formal procedures by which potential employees apply to firms, but through formal and informal efforts on the part of firms to identify candidates for the job.[19] Academic labor markets are a superb example, where the hiring process for new Ph.D.s typically begins with a department search phase. More generally, "headhunting" firms exist because firm search is important.

We also suspect that the importance of employer search is growing in our economy. Employers face increasingly stringent legal restrictions on the information they can seek from job applicants. In many settings, it is illegal to ask about an applicant's religion, marital and family status, nationality, health, criminal record, and a variety of personal habits, even though many of these may be important in ascertaining the value of the employee to the firm. As a substitute for seeking this information, a firm can offer incentives for existing employees or other contacts to recommend new employees, with the recommendation of the existing employee signaling information that the firm cannot legally seek. As a result, we can expect firm search to become an increasingly important force in shaping economic outcomes.

Red and green workers are identical, but receive different wage rates in our equilibrium. This is discrimination in its starkest form. What can be done about such discrimination?

The first step in any program to eliminate discrimination is typically to prohibit the practice of paying different wages to workers who are identical except for their color, i.e., to impose the constraint

$$w_R = w_G.$$

For example, we might implement this constraint by forcing firms to participate in "color-blind" bargaining in which they make wage offers without knowing the color of their bargaining partners. Because red and green workers are equally productive, this will imply $f_R = f_G$ and $Z_{F,R} = Z_{F,G}$, meaning that firms receive identical shares of the flow surplus and equal identical present values from red and green workers.

[19]Scoones [11] discusses firm search.

While an equal-wage requirement has great normative appeal, it can enhance the firm's incentive to discriminate between workers. Without mandated wage equality, skilled red workers have the attraction that they are cheaper than green workers, though firms do not find this advantage sufficient to overwhelm the paucity of skilled red workers. With mandated wage equality, red workers are no longer less expensive, causing their scarcity to pose a more powerful deterrent to firm search. The result can be an even more concentrated focus on searching for green workers and higher unemployment rates for red workers. Asymmetric equilibria will continue to exist in which firms search only greens. Employed red workers earn the same payoffs as employed green workers in such an equilibrium, but red workers still suffer from higher unemployment rates and lower expected values from acquiring skills, and so fewer red workers acquire skills. This suggests not that equal-pay provisions are misguided, but that they are most likely to be effective if coupled with measures to address firm search and hiring behavior.

Affirmative action programs designed to address search behavior fall into two categories, those based on procedures and those based on outcomes. Outcome-based schemes involve requirements that firms hire more reds, in turn prompting more reds to acquire skills, while leading employers to revise their assessments of red skill levels and voluntarily hire reds.[20] Procedure-based approaches involve requirements that firms devote sufficient energy to making vacancies known to minority candidates and seeking such candidates.

In our model, outcome-based affirmative action schemes are unnecessary for convincing firms of the merits of red workers. The value of skilled reds is evident once a match is made. However, search procedures play an important role in shaping the equilibrium and procedure-based programs have great potential to be effective.

One possibility is to eliminate the asymmetric equilibrium by prohibiting firms from seeking workers. However, it seems very unlikely that firms will ever be told that they cannot seek candidates for jobs, or that the efficiency costs of such a move would be worth the benefits. There are two potentially effective alternatives. First, steps can be taken to increase the rate and effectiveness of worker search (i.e., to increase λ_W). The provision of labor market information, training in job search techniques, and logistical support in the job search process may all enhance the search rate for disadvantaged workers. This in turn may increase the value of acquiring skills, and attract sufficiently many workers into the skilled sector, as to make it optimal for firms to search such workers, breaking the asymmetric equilibrium.

Second, firms can be required to search both reds and greens. This will

[20] See Coate and Loury [4] for a discussion.

again eliminate asymmetric equilibria. Color-blind search is the goal behind a host of procedural affirmative action requirements, including requirements that firms advertise positions widely, include members of disadvantaged groups in their target search pools, and include them in the interviewing process. It remains an unfortunate characteristic of such programs that a token effort is difficult to distinguish from a sincere one, though it is important to note that once a symmetric equilibrium is established, then searching both colors is optimal.

Motivated by our interest in cases in which identical agents are faced with different outcomes, we have examined only the case in which red and green workers are identical. There may well be differences between red and green workers. For example, they may have different search opportunities, leading to different search intensities for red and green workers. They may also have different cost distributions for acquiring skills. Unfortunately, is easy to imagine circumstances in which asymmetric equilibria, in which firms search only greens, lead to reductions in the search opportunities and increases in the costs of acquiring skills facing reds, reinforcing the asymmetry in the equilibrium and creating a market-induced "poverty trap."

The mere fact that one color of worker faces higher costs of acquiring skills than the other does not doom the former to being disadvantaged in an asymmetric equilibrium. As long as the distributions of schooling costs are not too dissimilar, it is possible that the high-cost color of worker is the advantaged worker in an asymmetric equilibrium. For example, it could be that greens face higher costs of acquiring skills than reds, but concentration of firms on searching greens confers a sufficient advantage that more greens acquire skills (especially if the greens are ex ante a larger group). If we interpret higher costs of acquiring skills as arising out of lower natural abilities, we then have a case in which the less able group fares better in equilibrium.[21] Considerable attention has recently been devoted to the question of whether the inferior economic performance of various demographic and racial groups should be attributed to deficiencies in individuals' abilities or in some aspect of their environment. Our asymmetric equilibrium provides yet another reason for caution in linking seemingly inferior outcomes to differences in ability.

8. Proofs

Proof of Proposition 1: Because $V_W \leq V_W(0)$, it is immediate that there is a unique, trivial symmetric steady state when $C(V_W(0)) = 0$. Suppose then

[21] Shimer [12] examines a model in which information frictions may induce firms to prefer less qualified workers to more qualified workers.

$C(V_W(0)) > 0$ and set $\lambda \equiv \lambda_F + \lambda_W$. The equations determining steady state equilibria are recursive, with the unemployment rate entering most equations only in the form $\rho_W H_W \equiv \tilde{\rho}$. From (3.2), we have

$$\rho_F = \frac{2\delta}{(2\delta + \tilde{\rho}\lambda)}, \tag{8.1}$$

from (3.1), (3.2), and (3.3),

$$\tilde{\rho} = C(V_W) - 1 + \rho_F,$$

and finally, from (3.6),

$$V_W = \frac{\rho_F \lambda}{(r+\delta)\left[(\rho_F + \tilde{\rho})\lambda + 2(r+2\delta)\right]} x.$$

Combining these three equations yields

$$\tilde{\rho} = C\left(\frac{2\delta\lambda}{(r+\delta)\left[(2\delta(1+\tilde{\rho}) + \tilde{\rho}^2\lambda)\lambda + 2(r+2\delta)(2\delta + \tilde{\rho}\lambda)\right]} x\right) - 1 + \frac{2\delta}{2\delta + \tilde{\rho}\lambda}. \tag{8.2}$$

There is a unique value of $\tilde{\rho} \in (0,1)$ solving (8.2), since the right side is continuous (because C is a continuous distribution function) and decreasing in $\tilde{\rho}$, is strictly positive at $\tilde{\rho} = 0$ (the argument of C at $\tilde{\rho} = 0$ is just $V_W(0)$) and is less than one at $\tilde{\rho} = 1$.

Given $\tilde{\rho}$, the values of ρ_F and V_W are uniquely determined by (8.1) and (3.6), with H_W then determined by (3.1), and V_F by (3.7).

Because the unique equilibrium satisfies $\tilde{\rho} = \rho_W H_W \in (0,1)$, we have $\rho_W \in (0,1)$ and $H_W > 0$. If $C(x/(r+\delta)) < 1$, then $H_W < 1$, since $V_W < x/(r+\delta)$. ∎

Proof of Proposition 2: The number of occupied firms and employed skilled workers must be equal. However, some workers face opportunity costs arbitrarily close to $\frac{x}{r+\delta}$, which is strictly larger than the payoff to acquiring skills (because it ignores the possibilities of having to wait to find a vacant firm and of firm deaths), and hence opt into the unskilled sector. As a result, $H_W < 1$, and so, from (3.2) and (3.3),

$$1 - \rho_F = H_W(1 - \rho_W) < 1 - \rho_W, \tag{8.3}$$

so that $\rho_W < \rho_F$. The inequality $V_F < V_W$ then follows from (3.6)–(3.7) while $Z_F < Z_W$ follows from (3.4)–(3.5). ∎

Proof of Proposition 3: Letting $m = \tilde{\rho}\lambda$, (8.2) can be rewritten as

$$m = \lambda C - \lambda + \frac{2\delta\lambda}{2\delta + m}.$$

Differentiating this expression with respect to λ and then substituting from (8.2) shows that $dm/d\lambda > 0$, that is,

$$\frac{d(\rho_W H_W \lambda)}{d\lambda} > 0.$$

Equation (8.1) then shows that $d\rho_F/d\lambda < 0$.

We now construct an argument by contradiction. Suppose $d(\rho_F\lambda)/d\lambda \leq 0$. Then, $d\left(\rho_W H_W\right)/d\lambda > 0$ (from (3.2)), and hence $dV_W/d\lambda < 0$ (3.6), which in turn implies $dH_W/d\lambda < 0$ (3.1), and so $d\tilde{\rho}/d\lambda < 0$ (8.3). But, from (8.1),

$$\frac{d(\rho_F\lambda)}{d\lambda} = \frac{2\delta}{(2\delta + \tilde{\rho}\lambda)^2}\left[2\delta + \tilde{\rho}\lambda - \lambda^2\frac{d\tilde{\rho}}{d\lambda}\right],$$

so that $d(\rho_F\lambda)/d\lambda > 0$, a contradiction. Hence, $d\rho_F\lambda/d\lambda > 0$ and (3.3) then gives $d\rho_W/d\lambda < 0$.

Suppose now that $dV_W/d\lambda \leq 0$. This is equivalent to

$$\frac{d(\rho_F\lambda)}{d\lambda}\left(\tilde{\rho}\lambda + 2(r + 2\delta)\right) \leq \rho_F\lambda\frac{d(\tilde{\rho}\lambda)}{d\lambda}. \tag{8.4}$$

But differentiating the equation $\tilde{\rho}\lambda = \lambda H_W - \lambda + \lambda\rho_F$ yields

$$\frac{d(\tilde{\rho}\lambda)}{d\lambda} = H_W + \lambda\frac{dH_W}{d\lambda} - 1 + \frac{d(\rho_F\lambda)}{d\lambda} = \tilde{\rho} - \rho_F + \lambda\frac{dH_W}{d\lambda} + \frac{d(\rho_F\lambda)}{d\lambda},$$

and substituting into (8.4) gives, after rearrangement,

$$\frac{d(\rho_F\lambda)}{d\lambda}\left((\tilde{\rho} - \rho_F)\lambda + 2(r + 2\delta)\right) \leq \rho_F\lambda\left(\tilde{\rho} - \rho_F + \lambda\frac{dH_W}{d\lambda}\right).$$

Since $d(\rho_F\lambda)/d\lambda > 0$, we have (recall that $dV_W/d\lambda \leq 0$ implies $dH_W/d\lambda \leq 0$),

$$\frac{d(\rho_F\lambda)}{d\lambda}\left((\tilde{\rho} - \rho_F)\lambda\right) < \rho_F\lambda\left(\tilde{\rho} - \rho_F\right).$$

From Proposition 2, $\tilde{\rho} < \rho_F$, and so $d(\rho_F\lambda)/d\lambda > \rho_F$. That is, $d\rho_F/d\lambda > 0$, a contradiction. Hence, we have $dV_W/d\lambda > 0$, and, from (3.1), $dH_W/d\lambda > 0$. ■

Proof of Proposition 5: The green and red unemployed and vacancies steady state conditions (4.3)–(4.5) have the form

$$2\delta(1-\rho_X) = \rho_X K_X,$$

where $K_X \geq 0$ is a function of the search intensities and the other unemployment and/or vacancy rates but not ρ_X. Solving this equation for ρ_X gives

$$\rho_X = 2\delta/(2\delta + K_X) \in [0,1].$$

Since (4.1) and (4.2) imply H_G, $H_R \in [0,1]$, and (4.6)-(4.8) imply V_F, V_R, $V_G \in [0, \frac{x}{r+\delta}]$, the 8 equation system described by (4.1)-(4.8) maps the compact set $[0,1]^5 \times [0, \frac{x}{r+\delta}]^3$ into itself in a continuous manner. Thus, by Brouwer's fixed point theorem, the system has a fixed point, and this is a green asymmetric steady state.

If the steady state is trivial, then $H_G = H_R = 0$ and $\rho_F = 1$. Evaluating (4.8) at these values gives

$$V_G = \frac{(2\lambda_F + \lambda_W)x}{(r+\delta)(2\lambda_F + \lambda_W + 2(r+2\delta))} > V_W(0),$$

and $C(V_W(0)) > 0$ thus implies $H_G > 0$, so the steady state cannot be trivial. ■

Proof of Proposition 6: The inequality $V_R < V_G$ follows immediately from (4.7)–(4.8), and in turn implies $H_G > H_R$, and so $H_G > 0$ if $\max\{H_G, H_R\} > 0$. The inequality $V_R < V_G$ also implies $Z_{F,R} > Z_{F,G}$, and so $Z_{F,R} > V_F$. The inequality $Z_{F,R} > Z_{F,G}$ can hold only if the firm extracts a larger portion of the flow surplus from red workers, giving $w_R < w_G$ and $f_R > f_G$. The inequality on unemployment rates, $\rho_R > \rho_G$, follows directly from (4.4)–(4.5). ■

Proof of Proposition 8: If there is no worker search ($\lambda_W = 0$ and $\lambda_F = \lambda$) and firms search only greens, then $V_R = 0$ and hence $H_R = 0$. From Proposition 5, the asymmetric steady state is nontrivial, and so $V_G > 0$ and $H_G > 0$. Moreover, (4.4) implies $\rho_G > 0$. It is then optimal for firms to search only greens, and we have a green asymmetric equilibrium.

Since steady states are upper-hemicontinuous in λ_F, both the number of green workers who acquire skills (H_G) in a green asymmetric steady state and the number of unemployed skilled green workers ($\rho_G H_G$) are bounded away from zero, while H_R and hence $\rho_R H_R$ approach zero, as λ_F approaches λ. For sufficiently large λ_F, it is then again optimal for firms to search only greens, yielding an asymmetric equilibrium. ■

Proof of Proposition 9: For sufficiently concentrated C, we have $C(\widehat{V}_G(\lambda_F)) = 1$ and $C(\widehat{V}_R(\lambda_F)) = 0$, so that all workers' opportunity costs of acquiring skills α satisfy $\widehat{V}_R(\lambda_F) < \alpha < \widehat{V}_G(\lambda_F)$, yielding an asymmetric steady state in which $H_G = 1/2$ and $H_R = 0$. But then there are no skilled red workers, making it pointless for the firm to search red workers and ensuring $V(R|G) < V_F$ and $V(W|G) < V_F$. ■

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ENDOGENOUS INEQUALITY IN INTEGRATED LABOR MARKETS WITH TWO-SIDED SEARCH

by George J. Mailath, Larry Samuelson, and Avner Shaked

Omitted calculations

A. Symmetric Equilibrium

This section provides alternative calculations for the value functions of a symmetric equilibrium and calculates the equilibrium flow payoffs. It initially reproduces some calculations of Section 3 in order to be self-contained. Numbered equations duplicate those of the paper and retain their original numbers.

Let V_W denote the value of skills to a worker. The fraction of new workers who become skilled is

$$h_W = C(V_W).$$

Since there is a death rate of δ, the change in the stock of skilled workers is given by the difference between the measure of newly entering workers acquiring skills, or δh_W, and the proportion of skilled workers who die, or δH_W, giving

$$\dot{H}_W = \delta(h_W - H_W).$$

In a steady state equilibrium, $\dot{H}_W = 0$ and so

$$h_W = H_W = C(V_W). \tag{A.1}$$

We turn now to the determination of the vacancy rate among firms. There are two ways that additional jobs appear in the vacancy pool: an occupied firm dies, and is replaced by a new firm without a worker, or an employed worker dies. Thus, new jobs join the vacancy pool at rate $(1-\rho_F)\delta + (1-\rho_F)\delta = 2(1-\rho_F)\delta$. Vacancies are filled by successful searches on the part of both firms and workers. There are ρ_F firms searching at rate $\lambda_F \rho_W H_W$, so that the rate at which vacancies are filled as a result of firm search is $\rho_F \lambda_F \rho_W H_W$. There are also $\rho_W H_W$ workers searching at rate $\lambda_W \rho_F$, so that the rate at which vacancies are filled as a result of worker search is $\rho_W H_W \lambda_W \rho_F$. Thus, the change in the number of vacancies is given by

$$\dot{\rho}_F = 2(1-\rho_F)\delta - \rho_F \rho_W H_W(\lambda_F + \lambda_W),$$

giving, for a steady state equilibrium,

$$2\delta(1-\rho_F) = \rho_F \rho_W H_W(\lambda_F + \lambda_W). \tag{A.2}$$

Next, consider the unemployment rate among workers. This differs from the case of firms because a newborn worker is unemployed only if she chooses to acquire skills, with unskilled workers opting into the unskilled sector of the economy. If an occupied firm dies, the skilled worker is now unemployed, which occurs at rate $(1-\rho_F)\delta$. There is an inflow of δh_W into the unemployed skilled worker pool from newborns and an outflow of $\delta\rho_W H_W$ from death. Unemployed workers are hired at the same rate as vacancies are filled (given by $\rho_F\rho_W H_W(\lambda_W+\lambda_R)$). This yields

$$\frac{d}{dt}(\rho_W H_W) = \dot{\rho}_W H_W + \rho_W \dot{H}_W = (1-\rho_F)\delta + (h_W - \rho_W H_W)\delta - \rho_F\rho_W H_W(\lambda_F+\lambda_W).$$

Since the number of employed workers and filled jobs is the same, we have

$$H_W(1-\rho_W) = 1-\rho_F,$$

so that

$$\dot{\rho}_W H_W + \rho_W \dot{H}_W = (H_W(1-\rho_W) + h_W - \rho_W H_W)\delta - \rho_F\rho_W H_W(\lambda_F+\lambda_W).$$

Since in a steady state, $H_W = h_W$, $\dot{\rho}_W = 0$, and $\dot{H}_W = 0$, we have

$$2\delta(1-\rho_W) = \rho_F\rho_W(\lambda_F+\lambda_W). \tag{A.3}$$

We next calculate V_W, the value to a worker of entering the skilled market. Since we are constructing a symmetric equilibrium in which both red and green workers are searched, their value functions, and hence skill decisions, will be identical, and so the firms will be indifferent over all their search possibilities.

Let $Z_W(s)$ denote the expected value of an employed worker at time s. Then we have:

$$\begin{aligned} Z_W(s) &= \int_s^\infty \left\{ \int_s^\omega \left\{ \int_s^\upsilon w(\tau)e^{-r(\tau-s)}d\tau + e^{-r(\upsilon-s)}V_W(\upsilon|\omega) \right\} \delta e^{-\delta(\upsilon-s)}\,d\upsilon \right. \\ &\qquad \left. + \; e^{-\delta(\omega-s)} \int_s^\omega w(\tau)e^{-r(\tau-s)}\,d\tau \right\} \delta e^{-\delta(\omega-s)}\,d\omega, \end{aligned}$$

where ω is the date of the worker's death, υ is the date of the firm's death, $w(\tau)$ is the expected flow payoff of an employed worker at date τ, and $V_W(\upsilon|\omega)$ is the value of being an unemployed worker at date υ, conditional on death at date ω. The first line of this expression captures the payoff in the event that the firm dies before the worker, and consists of the sum of the wage payments received from the firm and the value of being pushed back into unemployment. The second line captures the payoff in the event the worker dies first. Since the value of being unemployed at time v is given by

$$V_W(v) = E_{\omega \geq v}\left[V_W(v|\omega)\right] = \int_v^\infty V_W(v|\omega)\delta e^{-\delta(\omega - v)}\, d\omega,$$

we can simplify to obtain

$$\begin{aligned} Z_W(s) = & \int_s^\infty \int_s^\omega \int_s^v w(\tau)e^{-r(\tau-s)}\delta e^{-\delta(v-s)}\delta e^{-\delta(\omega-s)}\, d\tau\, dv\, d\omega \\ & + \int_s^\infty e^{-r(v-s)}V_W(v)\delta e^{-2\delta(v-s)}\, dv \\ & + \int_s^\infty \int_s^\omega w(\tau)e^{-r(\tau-s)}\delta e^{-2\delta(\omega-s)}\, d\tau\, d\omega. \end{aligned}$$

In a steady state, $w(\tau) = w$, $V_W(v) = V_W$, and $Z_W(s) = Z_W$, and so we can perform the integration to find

$$Z_W = \frac{w + \delta V_W}{r + 2\delta}.$$

If s is the time that a match arrives, then

$$V_W = E_s\left[e^{-\delta s}e^{-rs}Z_W\right],$$

where s is the time that a match with a vacant firm occurs, $e^{-\delta s}$ is the probability that the worker is still alive at time s, and e^{-rs} provides the appropriate discounting of the value $Z_W(s)$ to the present. The time s at which the worker meets a vacant firm has density $\rho_F(\lambda_F + \lambda_W)e^{-\rho_F(\lambda_F+\lambda_W)s}$.[22] We have an expected value of:

$$\begin{aligned} V_W &= \frac{(w + \delta V_W)}{(r + 2\delta)} \int_0^\infty \rho_F(\lambda_F + \lambda_W)e^{-(\rho_F(\lambda_F+\lambda_W)+r+\delta)s}\, ds \\ &= \frac{(w + \delta V_W)}{(r + 2\delta)} \frac{\rho_F(\lambda_F + \lambda_W)}{(\rho_F(\lambda_F + \lambda_W) + r + \delta)}. \end{aligned}$$

Solving this equation for V_W yields

$$V_W = \frac{\rho_F(\lambda_F + \lambda_W)w}{(r + \delta)(\rho_F(\lambda_F + \lambda_W) + r + 2\delta)}.$$

[22] The arrival rate of matches is the sum of matches from worker search ($\lambda_W \rho_F$) and firm search ($\lambda_F \rho_F$).

A similar calculation gives the value to a firm or participating in the market:

$$V_F = \frac{\rho_W H_W(\lambda_F + \lambda_W) f}{(r+\delta)\left(\rho_W H_W\left(\lambda_F + \lambda_W\right) + r + 2\delta\right)},$$

where f is the expected steady-state flow payoff of an occupied firm and V_F is the steady-state value of a vacant firm. We now determine the expected flow payoffs w and f. Firms and workers bargain over the surplus created in a match by making wage proposals with equal probability, and any such proposal will make the responding agent indifferent between accepting the proposal and rejecting. Suppose the firm is chosen to make a proposal, and offers a wage of $\underline{w}$ to the worker. Accepting this offer gives an expected payoff of $(\underline{w} + \delta V_W)/(r + 2\delta)$. which is the value of Z_W calculated at the wage $\underline{w}$. If the worker rejects this offer, her continuation value is V_W. The firm will choose $\underline{w}$ so as to make the worker indifferent between accepting and rejecting, giving:

$$\underline{w} = (r+\delta)V_W.$$

Since the firm must similarly be made indifferent by an offer received from the worker, should the worker be called upon to make the offer, the worker will offer $(r+\delta)V_f$ to the firm. We then have, in the steady state

$$\begin{aligned} w &= \frac{1}{2}\underline{w} + \frac{1}{2}\left\{x - (r+\delta)V_F\right\} \\ &= \frac{x}{2} + \frac{1}{2}(r+\delta)(V_W - V_F), \end{aligned}$$

and symmetrically for the firm,

$$f = \frac{x}{2} + \frac{1}{2}(r+\delta)(V_F - V_W).$$

Inserting the expressions for the value of a firm V_F and worker V_W, we then solve for:

$$\begin{aligned} w &= \frac{\rho_F(\lambda_F + \lambda_W) + r + 2\delta}{(\rho_F + \rho_W H_W)(\lambda_F + \lambda_W) + 2(r + 2\delta)} x, \\ f &= \frac{\rho_W H_W(\lambda_F + \lambda_W) + r + 2\delta}{(\rho_F + \rho_W H_W)(\lambda_F + \lambda_W) + 2(r + 2\delta)} x, \end{aligned}$$

giving

$$V_W = \frac{\rho_F(\lambda_F + \lambda_W)}{(r+\delta)\left[(\rho_F + \rho_W H_W)(\lambda_F + \lambda_W) + 2(r + 2\delta)\right]} x, \tag{A.6}$$

and

$$V_F = \frac{\rho_W H_W(\lambda_F + \lambda_W)}{(r+\delta)\left[(\rho_F + \rho_W H_W)(\lambda_F + \lambda_W) + 2(r + 2\delta)\right]} x. \tag{A.7}$$

B. Asymmetric Equilibria

This section presents the calculations leading to (4.6)–(4.8). Define $\lambda_G = 2\lambda_F + \lambda_W, \tilde{\rho}_G = \rho_G H_G, \tilde{\rho}_R = \rho_R H_R$, and

$$Z_F = \frac{\lambda_G \tilde{\rho}_G Z_{F,G} + \lambda_W \tilde{\rho}_R Z_{F,R}}{(\lambda_G \tilde{\rho}_G + \lambda_W \tilde{\rho}_R)}.$$

The value Z_F is the expected value of an occupied firm, where the probabilities reflect the relative likelihood of meeting a red and green worker. This allows us to simplify the system of value functions to:

$$\begin{aligned}
Z_R &= \frac{x}{2(r+2\delta)} + \frac{1}{2(r+2\delta)} \left((r+3\delta) V_R - (r+\delta) V_F\right), \\
V_R &= \frac{\rho_F \lambda_W Z_R}{\rho_F \lambda_W + r + \delta}, \\
Z_G &= \frac{x}{2(r+2\delta)} + \frac{1}{2(r+2\delta)} \left((r+3\delta) V_G - (r+\delta) V_F\right), \\
V_G &= \frac{\rho_F \lambda_G Z_G}{\rho_F \lambda_G + r + \delta}, \\
Z_F &= \frac{x}{2(r+2\delta)} + \frac{1}{2(r+2\delta)} \left((r+3\delta) V_F - \frac{(r+\delta)}{(\lambda_G \tilde{\rho}_G + \lambda_W \tilde{\rho}_R)} (\lambda_G \tilde{\rho}_G V_G + \lambda_W \tilde{\rho}_R V_R)\right),
\end{aligned}$$

and

$$V_F = \frac{(\lambda_G \tilde{\rho}_G + \lambda_W \tilde{\rho}_R) Z_F}{(\lambda_G \tilde{\rho}_G + \lambda_W \tilde{\rho}_R + r + \delta)}.$$

Eliminating the unmatched value functions gives

$$\begin{aligned}
2(r+2\delta) Z_R &= x + \left((r+3\delta) \frac{\rho_F \lambda_W Z_R}{\rho_F \lambda_W + r + \delta} - (r+\delta) \frac{(\lambda_G \tilde{\rho}_G + \lambda_W \tilde{\rho}_R) Z_F}{(\lambda_G \tilde{\rho}_G + \lambda_W \tilde{\rho}_R + r + \delta)}\right) \\
2(r+2\delta) Z_G &= x + \left((r+3\delta) \frac{\rho_F \lambda_G Z_G}{\rho_F \lambda_G + r + \delta} - (r+\delta) \frac{(\lambda_G \tilde{\rho}_G + \lambda_W \tilde{\rho}_R) Z_F}{(\lambda_G \tilde{\rho}_G + \lambda_W \tilde{\rho}_R + r + \delta)}\right) \\
2(r+2\delta) Z_F &= x + \left\{(r+3\delta) \frac{(\lambda_G \tilde{\rho}_G + \lambda_W \tilde{\rho}_R) Z_F}{(\lambda_G \tilde{\rho}_G + \lambda_W \tilde{\rho}_R + r + \delta)}\right. \\
&\quad \left. - \frac{(r+\delta)}{(\lambda_G \tilde{\rho}_G + \lambda_W \tilde{\rho}_R)} \left(\lambda_G \tilde{\rho}_G \frac{\rho_F \lambda_G Z_G}{\rho_F \lambda_G + r + \delta} + \lambda_W \tilde{\rho}_R \frac{\rho_F \lambda_W Z_R}{\rho_F \lambda_W + r + \delta}\right)\right\}.
\end{aligned}$$

Simplifying,

$$Z_R = \frac{(\rho_F \lambda_W + r + \delta)}{(\rho_F \lambda_W + 2(r+2\delta))} \left\{\frac{x}{(r+\delta)} - \frac{(\lambda_G \tilde{\rho}_G + \lambda_W \tilde{\rho}_R) Z_F}{(\lambda_G \tilde{\rho}_G + \lambda_W \tilde{\rho}_R + r + \delta)}\right\}$$

$$
\begin{aligned}
Z_G &= \frac{(\rho_F\lambda_G+r+\delta)}{(\rho_F\lambda_G+2\,(r+2\delta))}\left\{\frac{x}{(r+\delta)}-\frac{(\lambda_G\tilde{\rho}_G+\lambda_W\tilde{\rho}_R)\,Z_F}{(\lambda_G\tilde{\rho}_G+\lambda_W\tilde{\rho}_R+r+\delta)}\right\}\\
Z_F &= \frac{(\lambda_G\tilde{\rho}_G+\lambda_W\tilde{\rho}_R+r+\delta)}{(\lambda_G\tilde{\rho}_G+\lambda_W\tilde{\rho}_R+2\,(r+2\delta))}\left\{\frac{x}{(r+\delta)}-\frac{1}{(\lambda_G\tilde{\rho}_G+\lambda_W\tilde{\rho}_R)}\right.\\
&\quad\left.\times\left(\lambda_G\tilde{\rho}_G\frac{\rho_F\lambda_G Z_G}{(\rho_F\lambda_G+r+\delta)}+\lambda_W\tilde{\rho}_R\frac{\rho_F\lambda_W Z_R}{(\rho_F\lambda_W+r+\delta)}\right)\right\}.
\end{aligned}
$$

Substituting for Z_R and Z_G into the expression for Z_F gives

$$
\begin{aligned}
Z_F &= \frac{(\lambda_G\tilde{\rho}_G+\lambda_W\tilde{\rho}_R+r+\delta)}{(\lambda_G\tilde{\rho}_G+\lambda_W\tilde{\rho}_R+2\,(r+2\delta))}\left\{\frac{x}{(r+\delta)}-\frac{1}{(\lambda_G\tilde{\rho}_G+\lambda_W\tilde{\rho}_R)}\right.\\
&\quad\times\left[\lambda_G\tilde{\rho}_G\frac{\rho_F\lambda_G}{(\rho_F\lambda_G+2\,(r+2\delta))}\left(\frac{x}{(r+\delta)}-\frac{(\lambda_G\tilde{\rho}_G+\lambda_W\tilde{\rho}_R)\,Z_F}{(\lambda_G\tilde{\rho}_G+\lambda_W\tilde{\rho}_R+r+\delta)}\right)\right.\\
&\quad\left.\left.+\lambda_W\tilde{\rho}_R\frac{\rho_F\lambda_W}{(\rho_F\lambda_W+2(r+2\delta))}\left(\frac{x}{(r+\delta)}-\frac{(\lambda_G\tilde{\rho}_G+\lambda_W\tilde{\rho}_R)\,Z_F}{(\lambda_G\tilde{\rho}_G+\lambda_W\tilde{\rho}_R+r+\delta)}\right)\right]\right\}.
\end{aligned}
$$

Collecting terms, the term involving x is

$$
\begin{aligned}
&\frac{x}{(r+\delta)}\frac{(\lambda_G\tilde{\rho}_G+\lambda_W\tilde{\rho}_R+r+\delta)}{(\lambda_G\tilde{\rho}_G+\lambda_W\tilde{\rho}_R+2\,(r+2\delta))}\left\{1-\frac{1}{(\lambda_G\tilde{\rho}_G+\lambda_W\tilde{\rho}_R)}\right.\\
&\quad\left.\times\left(\frac{\lambda_G\tilde{\rho}_G\rho_F\lambda_G}{(\rho_F\lambda_G+2\,(r+2\delta))}+\frac{\lambda_W\tilde{\rho}_R\rho_F\lambda_W}{(\rho_F\lambda_W+2(r+2\delta))}\right)\right\}\\
&= \frac{x}{(r+\delta)}\frac{(\tilde{\rho}_G\lambda_G+\lambda_W\tilde{\rho}_R+r+\delta)\,2\,(r+2\delta)}{(\lambda_G\tilde{\rho}_G+\lambda_W\tilde{\rho}_R+2\,(r+2\delta))}\\
&\quad\times\frac{(\lambda_G\rho_F\lambda_W\,(\tilde{\rho}_G+\tilde{\rho}_R)+2\,(r+2\delta)\,(\lambda_G\tilde{\rho}_G+\lambda_W\tilde{\rho}_R))}{(\tilde{\rho}_G\lambda_G+\lambda_W\tilde{\rho}_R)\,(\rho_F\lambda_G+2r+4\delta)\,(\rho_F\lambda_W+2r+4\delta)}.
\end{aligned}
$$

The coefficient of Z_F on the right hand side is

$$
\begin{aligned}
&\frac{(\lambda_G\tilde{\rho}_G+\lambda_W\tilde{\rho}_R+r+\delta)}{(\lambda_G\tilde{\rho}_G+\lambda_W\tilde{\rho}_R+2\,(r+2\delta))\,(\lambda_G\tilde{\rho}_G+\lambda_W\tilde{\rho}_R)}\\
&\times\left(\frac{\lambda_G\tilde{\rho}_G\rho_F\lambda_G\,(\rho_F\lambda_G+r+\delta)\,(\lambda_G\tilde{\rho}_G+\lambda_W\tilde{\rho}_R)}{(\rho_F\lambda_G+r+\delta)\,(\rho_F\lambda_G+2\,(r+2\delta))\,(\lambda_G\tilde{\rho}_G+\lambda_W\tilde{\rho}_R+r+\delta)}\right.\\
&\left.+\frac{\lambda_W\tilde{\rho}_R\rho_F\lambda_W\,(\rho_F\lambda_W+r+\delta)\,(\lambda_G\tilde{\rho}_G+\lambda_W\tilde{\rho}_R)}{(\rho_F\lambda_W+r+\delta)\,(\rho_F\lambda_W+2(r+2\delta))\,(\lambda_G\tilde{\rho}_G+\lambda_W\tilde{\rho}_R+r+\delta)}\right)
\end{aligned}
$$

$$
=\frac{\rho_F\left(2\lambda_G^2\tilde{\rho}_G r+4\lambda_G^2\tilde{\rho}_G\delta+\lambda_G^2\tilde{\rho}_G\rho_F\lambda_W+2\lambda_W^2\tilde{\rho}_R r+4\lambda_W^2\tilde{\rho}_R\delta+\lambda_W^2\tilde{\rho}_R\rho_F\lambda_G\right)}{(\rho_F\lambda_W+2r+4\delta)\,(\rho_F\lambda_G+2r+4\delta)\,(\lambda_G\tilde{\rho}_G+\lambda_W\tilde{\rho}_R+2r+4\delta)}.
$$

So, the equation determining Z_F can be written as

$$\left(1-\frac{\rho_F\left(2\lambda_G^2\tilde{\rho}_G r+4\lambda_G^2\tilde{\rho}_G\delta+\lambda_G^2\tilde{\rho}_G\rho_F\lambda_W+2\lambda_W^2\tilde{\rho}_R r+4\lambda_W^2\tilde{\rho}_R\delta+\lambda_W^2\tilde{\rho}_R\rho_F\lambda_G\right)}{\left(\rho_F\lambda_W+2r+4\delta\right)\left(\rho_F\lambda_G+2r+4\delta\right)\left(\lambda_G\tilde{\rho}_G+\lambda_W\tilde{\rho}_R+2r+4\delta\right)}\right)Z_F$$

$$\begin{aligned} = {} & \frac{x\left(\tilde{\rho}_G\lambda_G+\lambda_W\tilde{\rho}_R+r+\delta\right)2\left(r+2\delta\right)}{\left(r+\delta\right)\left(\tilde{\rho}_G\lambda_G+\lambda_W\tilde{\rho}_R+2r+4\delta\right)} \\ & \times\frac{\left(\lambda_G\rho_F\lambda_W\left(\tilde{\rho}_G+\tilde{\rho}_R\right)+2\left(r+2\delta\right)\left(\lambda_G\tilde{\rho}_G+\lambda_W\tilde{\rho}_R\right)\right)}{\left(\tilde{\rho}_G\lambda_G+\lambda_W\tilde{\rho}_R\right)\left(\rho_F\lambda_G+2r+4\delta\right)\left(\rho_F\lambda_W+2r+4\delta\right)},\end{aligned}$$

or

$$\begin{aligned} & \{\left(\rho_F\lambda_W+2r+4\delta\right)\left(\rho_F\lambda_G+2r+4\delta\right)\left(\lambda_G\tilde{\rho}_G+\lambda_W\tilde{\rho}_R+2r+4\delta\right) \\ & -\rho_F\left(2\lambda_G^2\tilde{\rho}_G r+4\lambda_G^2\tilde{\rho}_G\delta+\lambda_G^2\tilde{\rho}_G\rho_F\lambda_W+2\lambda_W^2\tilde{\rho}_R r+4\lambda_W^2\tilde{\rho}_R\delta+\lambda_W^2\tilde{\rho}_R\rho_F\lambda_G\right)\}\,Z_F \\ = {} & \frac{x\left(\tilde{\rho}_G\lambda_G+\lambda_W\tilde{\rho}_R+r+\delta\right)2\left(r+2\delta\right)}{\left(r+\delta\right)} \\ & \times\frac{\left(\lambda_G\rho_F\lambda_W\left(\tilde{\rho}_G+\tilde{\rho}_R\right)+2\left(r+2\delta\right)\left(\lambda_G\tilde{\rho}_G+\lambda_W\tilde{\rho}_R\right)\right)}{\left(\tilde{\rho}_G\lambda_G+\lambda_W\tilde{\rho}_R\right)},\end{aligned}$$

and hence,

$$\begin{aligned} Z_F = {} & \frac{x\left(\tilde{\rho}_G\lambda_G+\lambda_W\tilde{\rho}_R+r+\delta\right)}{\left(r+\delta\right)\left(\tilde{\rho}_G\lambda_G+\lambda_W\tilde{\rho}_R\right)} \\ & \times\frac{\left(\lambda_G\rho_F\lambda_W\left(\tilde{\rho}_G+\tilde{\rho}_R\right)+2\left(r+2\delta\right)\left(\lambda_G\tilde{\rho}_G+\lambda_W\tilde{\rho}_R\right)\right)}{\Delta},\end{aligned}$$

where $\Delta\equiv 2\left(r+2\delta\right)\left(\rho_F\lambda_G+\lambda_G\tilde{\rho}_G+\rho_F\lambda_W+\lambda_W\tilde{\rho}_R+2\left(r+2\delta\right)\right)+\rho_F\lambda_W\lambda_G\left(\rho_F+\tilde{\rho}_R+\tilde{\rho}_G\right)$.

We can use this result to calculate:

$$V_F=\frac{x}{\left(r+\delta\right)}\frac{\left(\lambda_G\rho_F\lambda_W\left(\tilde{\rho}_G+\tilde{\rho}_R\right)+2\left(r+2\delta\right)\left(\lambda_G\tilde{\rho}_G+\lambda_W\tilde{\rho}_R\right)\right)}{\Delta}. \tag{B.6}$$

Similarly,

$$\begin{aligned} Z_R = {} & \frac{x\left(\rho_F\lambda_W+r+\delta\right)}{\left(r+\delta\right)\left(\rho_F\lambda_W+2(r+2\delta)\right)} \\ & \times\left\{1-\Delta^{-1}\left(\lambda_G\rho_F\lambda_W\left(\tilde{\rho}_G+\tilde{\rho}_R\right)+2\left(r+2\delta\right)\left(\lambda_G\tilde{\rho}_G+\lambda_W\tilde{\rho}_R\right)\right)\right\} \\ = {} & \frac{x}{\left(r+\delta\right)\Delta}\left(\rho_F\lambda_W+r+\delta\right)\left(\rho_F\lambda_G+2\left(r+2\delta\right)\right),\end{aligned}$$

and

$$
\begin{aligned}
Z_G &= \frac{x\left(\rho_F\lambda_G + r + \delta\right)}{\left(r+\delta\right)\left(\rho_F\lambda_G + 2\left(r+2\delta\right)\right)} \\
&\quad \times \left\{1 - \Delta^{-1}\left(\lambda_G\rho_F\lambda_W\left(\tilde{\rho}_G + \tilde{\rho}_R\right) + 2\left(r+2\delta\right)\left(\lambda_G\tilde{\rho}_G + \lambda_W\tilde{\rho}_R\right)\right)\right\} \\
&= \frac{x}{\left(r+\delta\right)\Delta} x\left(\rho_F\lambda_G + r + \delta\right)\left(\rho_F\lambda_W + 2\left(r+2\delta\right)\right),
\end{aligned}
$$

giving

$$
V_R = \frac{\rho_F\lambda_W\left(\rho_F\lambda_G + 2\left(r+2\delta\right)\right)}{\left(r+\delta\right)\Delta} x \tag{B.7}
$$

and

$$
V_G = \frac{\rho_F\lambda_G\left(\rho_F\lambda_W + 2\left(r+2\delta\right)\right)}{\left(r+\delta\right)\Delta} x. \tag{B.8}
$$

Thus,

$$
\begin{aligned}
V_G &= \frac{\rho_F\lambda_G\rho_F\lambda_W + \rho_F\lambda_G 2\left(r+2\delta\right)}{\left(r+\delta\right)\Delta} x = V_R + \frac{\rho_F 2\left(r+2\delta\right)\left(\lambda_G - \lambda_W\right)}{\left(r+\delta\right)\Delta} x \\
&= V_R + \frac{\rho_F 2\left(r+2\delta\right) 2\lambda_F}{\left(r+\delta\right)\Delta} x > V_R.
\end{aligned}
$$

C. The Extreme Asymmetric Steady State

This appendix verifies (5.2). In the extreme asymmetric steady state, $H_G = 1/2$ and $H_R = 0$, so the vacancies steady state condition is

$$
4\delta(1-\rho_F) = \rho_F\rho_G(\lambda + \lambda_F). \tag{C.1}
$$

Similarly, the green unemployment steady state condition is

$$
2\delta(1-\rho_G) = \rho_G\rho_F(\lambda + \lambda_F). \tag{C.2}
$$

These imply

$$
\rho_G = 2\rho_F - 1. \tag{C.3}
$$

From (3.3) and (3.1),

$$
\frac{d\rho_F}{d\lambda_F} = \rho_F' = \frac{-\rho_F(2\rho_F - 1)}{\{4\delta + (4\rho_F - 1)(\lambda + \lambda_F)\}}. \tag{C.4}
$$

We first show that $\hat{V}_G$ is increasing in λ_F. The argument parallels our demonstration that $dV_W/d\lambda > 0$ in the symmetric steady state. First, we can write

$$\widehat{V}_G = \frac{\rho_F(\lambda+\lambda_F)}{(r+d)[(\rho_F+\frac{1}{2}\rho_G)(\lambda+\lambda_F)+2(r+2\delta)]}.$$

Notice that this is (3.6), with ρ_W replaced by ρ_G and H_W by $\frac{1}{2}$ (because only green workers enter, and all green workers enter), and with the total search intensity given by $2\lambda_F + \lambda_W = \lambda + \lambda_F$ (because the decision of firms to search only greens doubles the firm's effective search intensity). Alternatively, it is straightforward to verify that this expression equals (4.6). Using (3.3), we have

$$\widehat{V}_G = \frac{\rho_F(\lambda+\lambda_F)}{(r+d)[(2\rho_F-1)(\lambda+\lambda_F)+2(r+2\delta)]}.$$

Suppose $d\widehat{V}_G/d\lambda_F < 0$. Then, taking the derivative, it must be the case that

$$\frac{d(\rho_F(\lambda+\lambda_F))}{d\lambda_F}[(2\rho_F-\frac{1}{2})(\lambda+\lambda_F)+2(r+2\delta)]-\rho_F(\lambda+\lambda_F)\frac{d((2\rho_F-\frac{1}{2})(\lambda+\lambda_F))}{d\lambda_F}<0.$$

Simplifying, we must have

$$\frac{d(\rho_F(\lambda+\lambda_F))}{d\lambda_F}[(-\frac{1}{2})(\lambda+\lambda_F)+2(r+2\delta)]-\rho_F(\lambda+\lambda_F)\frac{d((-\frac{1}{2})(\lambda+\lambda_F))}{d\lambda_F}<0.$$

It is immediate from (3.1)–(3.2) that $d(\rho_F(\lambda+\lambda_F))/d\lambda_F > 0$. We can then delete the (positive) term involving $2(r+2\delta)$ and divide by $\lambda+\lambda_F$ to find that a necessary condition is

$$\frac{d(\rho_F(\lambda+\lambda_F))}{d\lambda_F} > \rho_F.$$

But this implies that $d\rho_F/d\lambda_F > 0$, which contradicts (3.1)–(3.2). Hence, $\hat{V}_G$ is increasing in λ_F.

We next show that $\hat{V}_R$ is decreasing in λ_F. From (4.7),

$$\hat{V}_R = \frac{\rho_F^2(\lambda^2-\lambda_F^2)+\rho_F(\lambda-\lambda_F)2\,(r+2\delta)\,x}{(r+\delta)\,\hat{\Delta}},$$

where $\hat{\Delta} \equiv 2\,(r+2\delta)\,\{\rho_F(\lambda+\lambda_F)+(\lambda+\lambda_F)\rho_G/2+\rho_F(\lambda-\lambda_F)+2\,(r+2\delta)\}+(\lambda^2-\lambda_F^2)\,(\rho_F^2+\rho_F\rho_G/2) = \rho_F^2(\lambda^2-\lambda_F^2)+\rho_F(\lambda-\lambda_F)2\,(r+2\delta)+4\,(r+2\delta)^2+2\,(r+2\delta)\,(\lambda+\lambda_F)\{\rho_F+\rho_G/2\}+(\lambda^2-\lambda_F^2)\rho_F\rho_G/2$. Now, $\hat{V}_R$ is decreasing in λ_F if and only if $((r+\delta)\hat{V}_R/x)^{-1}$ is increasing in λ_F. Substituting,

$$\frac{x}{(r+\delta)\hat{V}_R} = 1+\frac{4\,(r+2\delta)^2+2\,(r+2\delta)\,(\lambda+\lambda_F)\{\rho_F+\rho_G/2\}+(\lambda^2-\lambda_F^2)\rho_F\rho_G/2}{\rho_F^2(\lambda^2-\lambda_F^2)+\rho_F(\lambda-\lambda_F)2\,(r+2\delta)}.$$

Differentiating the denominator with respect to λ_F yields

$$2\rho_F\rho_F'(\lambda^2-\lambda_F^2)-2\rho_F^2\lambda_F+\rho_F'(\lambda-\lambda_F)2\,(r+2\delta)-\rho_F 2\,(r+2\delta)<0,$$

since $\lambda_F \leq \lambda$.

Turning to the numerator, its derivative is

$$2\,(r+2\delta)\,\{\rho_F+\rho_G/2\}+2\,(r+2\delta)\,(\lambda+\lambda_F)\{\rho_F'+\rho_G'/2\}-\lambda_F\rho_F\rho_G+(\lambda^2-\lambda_F^2)(\rho_F'\rho_G/2+\rho_F\rho_G'/2).$$

Using $\rho_G' = 2\rho_F'$ and (3.3), we can rewrite this as

$$\begin{aligned} & (r+2\delta)\,(4\rho_F-1)+4\,(r+2\delta)\,(\lambda+\lambda_F)\rho_F'-\lambda_F\rho_F(2\rho_F-1)+(\lambda^2-\lambda_F^2)(\rho_F'(\rho_F-1/2)+\rho_F\rho_F') \\ = \; & (r+2\delta)\,(4\rho_F-1)+4\,(r+2\delta)\,(\lambda+\lambda_F)\rho_F'-\lambda_F\rho_F(2\rho_F-1)+(\lambda^2-\lambda_F^2)\rho_F'(2\rho_F-1/2), \end{aligned}$$

which has the same sign as, substituting (3.4) and ignoring the positive denominator $\{4\delta+(4\rho_F-1)(\lambda+\lambda_F)\}$,

$$\begin{gathered} [(r+2\delta)\,(4\rho_F-1)-\lambda_F\rho_F(2\rho_F-1)]\,\{4\delta+(4\rho_F-1)(\lambda+\lambda_F)\} \\ -\rho_F(2\rho_F-1)4\,(r+2\delta)\,(\lambda+\lambda_F)-\rho_F(2\rho_F-1)(\lambda^2-\lambda_F^2)(2\rho_F-1/2) \end{gathered}$$

$$\begin{gathered} =(r+2\delta)\,(4\rho_F-1)\,\{4\delta+(4\rho_F-1)(\lambda+\lambda_F)\}-\lambda_F\rho_F(2\rho_F-1)4\delta \\ -\rho_F(2\rho_F-1)(\lambda+\lambda_F)\lambda_F(4\rho_F-1)-\rho_F(2\rho_F-1)4\,(r+2\delta)\,(\lambda+\lambda_F) \\ -\rho_F(2\rho_F-1)(\lambda^2-\lambda_F^2)(2\rho_F-1/2) \end{gathered}$$

$$\begin{gathered} =(r+2\delta)\,(4\rho_F-1)\,\{4\delta+(4\rho_F-1)(\lambda+\lambda_F)\}-\lambda_F\rho_F(2\rho_F-1)4\delta \\ -\rho_F(2\rho_F-1)(\lambda+\lambda_F)\,\{\lambda_F(4\rho_F-1)+4\,(r+2\delta)+(\lambda-\lambda_F)(2\rho_F-1/2)\}\equiv\Theta. \end{gathered}$$

It suffices to show that $\Theta > 0$. We first observe that variations in the interest rate r affect none of the other variables appearing in Θ. We can accordingly take the derivative

$$\frac{d\Theta}{dr}=(4\rho_F-1)[4\delta+(4\rho_F-1)(\lambda+\lambda_F)]-4\rho_F(2\rho_F-1)(\lambda+\lambda_F),$$

which will be positive if

$$(4\rho_F-1)^2-4\rho_F(2\rho_F-1)=16\rho_F^2-8\rho_F+1-8\rho_F^2-4\rho_F=8\rho_F^2-4\rho_F+1>0,$$

which holds for all $\rho_F \in (\frac{1}{2}, 1]$. As a result, it suffices to examine the value of r that minimizes Θ, namely $r = 0$, giving

$$\begin{gathered} 2\delta(4\rho_F-1)\,\{4\delta+(4\rho_F-1)(\lambda+\lambda_F)\}-\lambda_F\rho_F(2\rho_F-1)4\delta \\ -\rho_F(2\rho_F-1)(\lambda+\lambda_F)\,\{\lambda_F(4\rho_F-1)+8\delta+(\lambda-\lambda_F)(2\rho_F-1/2)\}\equiv\Lambda. \end{gathered}$$

Next, we note that, from (3.1)–(3.2), the vacancy rate ρ_F depends only upon the sum $\lambda+\lambda_F$. We can accordingly take a derivative of Λ with respect to λ_F, letting $d\lambda/d\lambda_F = -1$ to as to preserve the sum $\lambda+\lambda_F$, to obtain

$$\begin{aligned}\frac{d\Lambda}{d\lambda_F} &= -\rho_F(2\rho_F-1)4\delta - \rho_F(2\rho_F-1)(\lambda+\lambda_F)[4\rho_F - 1 - 2(2\rho_F - \frac{1}{2})] \\ &= -\rho_F(2\rho_F-1)4\delta < 0.\end{aligned}$$

We can then again confine attention to the worst case, namely the value of λ_F that minimizes Λ, or $\lambda_F = \lambda$. Our task is then to show

$$2\delta(4\rho_F-1)\left\{4\delta+(4\rho_F-1)(2\lambda)\right\}-\lambda\rho_F(2\rho_F-1)4\delta-\rho_F(2\rho_F-1)(2\lambda)\left\{\lambda(4\rho_F-1)+8\delta\right\}>0.$$

Dividing by λ^2, this is

$$2\frac{\delta}{\lambda}(4\rho_F-1)\left\{4\frac{\delta}{\lambda}+2(4\rho_F-1)\right\}-4\rho_F(2\rho_F-1)\frac{\delta}{\lambda}-2\rho_F(2\rho_F-1)\left\{(4\rho_F-1)+8\frac{\delta}{\lambda}\right\}>0.$$

From (3.1) and (3.3), we have

$$\frac{\delta}{\lambda}=\frac{\rho_F(2\rho_F-1)}{2(1-\rho_F)}.$$

Substituting, we have

$$\begin{aligned}&2\tfrac{\rho_F(2\rho_F-1)}{2(1-\rho_F)}(4\rho_F-1)\left\{4\tfrac{\rho_F(2\rho_F-1)}{2(1-\rho_F)}+2(4\rho_F-1)\right\}-4\rho_F(2\rho_F-1)\tfrac{\rho_F(2\rho_F-1)}{2(1-\rho_F)}\\ &\quad -2\rho_F(2\rho_F-1)\left\{(4\rho_F-1)+8\tfrac{\rho_F(2\rho_F-1)}{2(1-\rho_F)}\right\}>0.\end{aligned}$$

Extracting and deleting the positive factor $\rho_F(2\rho_F-1)/(1-\rho_F)$, we have

$$\begin{aligned}&(4\rho_F-1)\left\{\tfrac{2\rho_F(2\rho_F-1)+2(4\rho_F-1)(1-\rho_F)}{(1-\rho_F)}\right\}-2\rho_F(2\rho_F-1)\\ &\quad -2(1-\rho_F)\left\{(4\rho_F-1)+\tfrac{4\rho_F(2\rho_F-1)}{(1-\rho_F)}\right\}>0.\end{aligned}$$

Extracting and deleting the positive factor $(1-\rho_F)$, we have

$$\begin{aligned}&(4\rho_F-1)\left\{4\rho_F^2-2\rho_F+2(4\rho_F-1-4\rho_F^2+\rho_F\right\}-2\rho_F(1-\rho_F)(2\rho_F-1)\\ &\quad -2(1-\rho_F)\left\{(4\rho_F-1)(1-\rho_F)+4\rho_F(2\rho_F-1)\right\}>0.\end{aligned}$$

A series of simplifications now gives:

$$\begin{aligned}&(4\rho_F-1)\left\{4\rho_F^2-2\rho_F+8\rho_F-2-8\rho_F^2+2\rho_F\right\}-2\rho_F(2\rho_F-1-2\rho_F^2+\rho_F)\\ &\quad -2(1-\rho_F)\left\{(4\rho_F-1)(1-\rho_F)+4\rho_F(2\rho_F-1)\right\}>0.\end{aligned}$$

$$(4\rho_F - 1)\left\{-4\rho_F^2 + 8\rho_F - 2\right\} - 2\rho_F(-2\rho_F^2 + 3\rho_F - 1)$$
$$-2(1-\rho_F)\left\{4\rho_F - 1 - 4\rho_F^2 + \rho_F + 8\rho_F^2 - 4\rho_F)\right\} > 0.$$

$$-16\rho_F^3+32\rho_F^2-8\rho_F+4\rho_F^2-8\rho_F+2+4\rho_F^3-6\rho_F^2+2\rho_F-2(1-\rho_F)\left\{4\rho_F^2+\rho_F-1)\right\} > 0.$$

$$12\rho_F^3 + 30\rho_F^2 - 14\rho_F + 2 - 2[4\rho_F^2 + \rho_F - 1 - 4\rho_F^3 - \rho_F^2 + \rho_F] > 0.$$

$$12\rho_F^3 + 30\rho_F^2 - 14\rho_F + 2 - 8\rho_F^2 - 2\rho_F + 2 + 8\rho_F^3 + 2\rho_F^2 - 2\rho_F] > 0.$$

$$-4\rho_F^3 + 24\rho_F^2 - 18\rho_F + 4 > 0.$$

$$-2\rho_F^3 + 12\rho_F^2 - 9\rho_F + 2 \equiv \Phi > 0.$$

We now examine the cubic equation Φ. It is straightforward to calculate:

$$\begin{aligned}\lim_{\rho_F\to-\infty}\Phi(\rho_F) &> 0, \quad \Phi(0)=2, \quad \Phi(\frac{1}{2})=\frac{1}{4}, \\ \Phi(1) &= 3, \quad \text{and} \quad \lim_{\rho_F\to\infty}\Phi(\rho_F)<0.\end{aligned}$$

In addition,

$$\frac{d\Phi(\rho_F)}{d\rho_F} = -6\rho_F^2 + 24\rho_F - 9.$$

This derivative is positive for all $\rho_F \in [\frac{1}{2}, 1]$, and hence $\Phi(\rho_F) > 0$ for all $\rho_F \in [\frac{1}{2}, 1]$. ■