Coalitional Bargaining in Stationary Markets

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Abstract

I study a model of coalitional bargaining in stationary markets featuring strategic choice of bargaining partners. The set of coalitions that form in the unique stationary equilibrium has a tier structure, with payoffs determined from the top tier down: Shocks propagate from higher to lower tiers, but not vice versa. In the limit as bargaining frictions vanish, the equilibrium payoff profile is the only one that gives each player her maximum Nash bargaining payoff—over all coalitions—subject to the other players' participation constraints. A player is made better off when she becomes less risk averse or the coalitions that she belongs to become more productive. But she may be made worse off when other players become *more* risk averse or some coalitions that she is not part of become *less* productive. Some players may be worse off after a uniform increase in the productivity of each coalition. In the special case of bilateral matching markets, independent tier structures—or *submarkets*—emerge in equilibrium, each with exactly one top-tier match: Shocks propagate within submarkets but not across them.

1 Introduction

Many economic activities involve coalitions of individuals who cooperate to achieve goals that they could not achieve separately. Different coalitions typically include similar individuals, which creates interconnections among them: For example, similar college graduates join investing, consulting and non-profit organizations, which links the wages paid by these companies. This article seeks to understand the nature of these interconnections, and their implications for how microeconomic shocks propagate through the economy.

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This article is organized as follows: section 2 describes the model, section 3 characterizes the essentially-unique stationary equilibrium, and section 4 describes the tier structure and comparative statics of this equilibrium. Finally, section 5 puts this article in the context of the related literature, and section 6 concludes.

2 Model

2.1 Primitives

The finite set of players is denoted by N. The output function $y : 2^N \to \mathbb{R}_{\geq 0}$ specifies the output that each coalition of players can produce. Output is perfectly divisible (e.g. money). Each player i has *vN-M utility function* u_i that only depends on the share of output that she receives, and is normalized so that $u_i(y(i)) = 0$.

2.2 The Game Γ

Consider the following infinite-horizon bargaining game Γ. At the beginning of each period $t = 0, 1, \ldots$, one player is selected uniformly at random to be the *proposer*. The proposer chooses one coalition S that she is a member of, and proposes a split of the output $y(S)$.

The members in S respond sequentially in (a pre-specified) order until one of them rejects or all of them accept. In the former case, no trade occurs this period and all players stay in the market for the next period. In the latter case, the coalition S forms, its members exit the market with the agreed shares, and are immediately replaced by replicas. Formally, there exists a sequence $i_0, i_1, \ldots, i_\tau, \ldots$ of players of type $i \in N$. The game starts with player set $\{i_0\}_{i \in N}$. For any $\tau \geq 0$ and any $i \in N$, if player i_τ exits the game, player $i_{\tau+1}$ immediately replaces her.

At the end of each period, the market breaks down with probability $0 < q < 1$, in which case the game ends, and each player i that has not yet formed a coalition obtains $y(i)$. All players have common knowledge of the game and perfect information about all the events preceding any of their decision nodes in the game.

2.3 Histories, Strategies and Equilibrium

Let T be the period at which breakdown occurs. For each period $t \leq T$, let h_t be a history of the game up to (but not including) period t, which is a sequence of t pairs of proposers and coalitions proposed—with corresponding proposals and responses.

There are two types of histories at which some player must take an action. First, (h_t, i) consists of h_t followed by player i being selected to be the proposer in period t. Second, $(h_t, i \to S, z, j)$ consists of (h_t, i) followed by player i proposing that coalition S shares its output according to the profile $z \in \mathbb{R}^S_{\geq 0}$, and all players preceding $j \in S$ in the response order having accepted.

A strategy σ_i for player i specifies, for all possible histories h_t, the offer $\sigma_i(h_t, i)$ that she makes following history (h_t, i) and her response $\sigma_i(h_t, j \to S, z, i)$ following history $(h_t, j \to S, z, i)$. I allow for mixed strategies, so $\sigma_i(h_t, i)$ and $\sigma_i(h_t, j \to S, z, i)$ are probability distributions over $2^N \times \mathbb{R}^N_{\geq 0}$ and {Yes, No}, respectively.

The strategy profile $(\sigma_i)_{i \in N}$ is a *stationary (Markovian-perfect) equilibrium* of the game Γ if it induces a Nash equilibrium in the subgame following every history, and if no player's strategy conditions behavior on the history of the game except—in the case of a response—on the going proposal and the identity of the proposer. I often refer to a stationary equilibrium simply as an *equilibrium*.

3 Essentially-Unique Equilibrium

3.1 Equilibrium Threshold Profile

Proposition 3.1 shows that different stationary equilibria of the game Γ differ only in non-essential ways.

Proposition 3.1. *Each player i has a threshold t_i such that, in every stationary equilibrium of Γ,*

1. *she always accepts offers that give her strictly more than t_i, and*
2. *she always rejects offers that give her strictly less than t_i.*

Note 3.1. The profile t determines the equilibrium strategy of each *non-dummy player*—that is, each player i with $t_i > y(i)$. Every period in which a non-dummy player i is selected to be the proposer, she proposes that one of her *preferred coalitions*—that is, one of the coalitions

$S \ni i$ with biggest net output $y(S) - \sum_{j \in S-i} t_j$—forms, and she offers each of its members $j \neq i$ the amount t_j, all of whom accept.[1]

Proof. Fix a stationary equilibrium of Γ. Let v_i denote player i's expected utility in any period conditional on not trading in that period, and let w_i denote player i's expected utility when she is the proposer. Let x_i and $p_i(x_i)$ be such that $v_i = u_i(x_i)$ and $w_i = u_i(p_i(x_i))$.

We have that[2]

$$u_i(x_i) = \chi u_i(p_i(x_i)) \text{ where } \chi := \frac{1-q}{1+(n-1)q} \tag{1}$$

The maximum amount of output that player i can obtain when she is the proposer is

$$\max_{C \subseteq N : i \in C} \left(y(C) - \sum_{j \in C-i} x_j \right). \tag{2}$$

since each player j rejects every offer that gives her strictly less than x_j. Player i can secure an amount of output arbitrarily close to (2) when she is the proposer, since each player j accepts every offer that gives her strictly more than a share x_j of output.

Together, these observations imply that

$$p_i(x_i) = \max_{C \subseteq N : i \in C} \left(y(C) - \sum_{j \in C-i} x_j \right) \text{ for all } i \in N, \tag{3}$$

The proof now follows from the fact that, by Proposition 3.2 and Proposition 3.3, there is a unique solution to the system (3); player i's equilibrium threshold t_i is x_i. □

3.2 Rubinstein Bargaining under Participation Constraints (RBPC)

Definition 3.2 defines an RBPC as a profile in $\mathbb{R}^n$ that is a fixed point of a natural map that is based on the traditional theory of bargaining. Proposition 3.2 shows that a profile satisfies Equation 3—and is hence an equilibrium threshold profile in Γ—if and only if it is an

[1] Even though each player i is indifferent between accepting and rejecting an offer that gives her an amount t_i, the notion of equilibrium requires that such offers are always accepted; otherwise, the proposer has no best response.

A dummy player i does not belong to any coalition S with strictly positive net output $y(S) - \sum_{j \in S} t_j$, so she can make unacceptable offers in equilibrium. She can also propose that one of the coalitions S with $y(S) = \sum_{j \in S} t_j$ forms (for example, $S = \{i\}$), offering t_j to each player $j \in S$, but these offers need not be accepted in equilibrium.

[2] To see this, note that $v_i = qu_i(y(i)) + (1-q)\left(\frac{1}{n}w_i + \frac{n-1}{n}v_i\right) = (1-q)\left(\frac{1}{n}w_i + \frac{n-1}{n}v_i\right)$ since equilibrium offers leave the responders indifferent between accepting and rejecting them, and $u_i(y(i))$ is normalized to 0. Hence, $v_i = \chi w_i$, with χ as defined in (1).

RBPC. This characterization is useful for two reasons. First, it gives us a way to think of the equilibrium threshold profile as a fixed point to an economically meaningful map. Second, as illustrated in subsection 3.3 below, the RBPC has a nice structure that can be easily computed, so this characterization allows us to understand the determinants of which coalitions form and how the resulting output is shared in equilibrium.

Definition 3.1. For each profile $x \in \mathbb{R}^N_{\geq 0}$, each coalition C and each player $i \in C$, let i's *x-(Rubinstein) share in C* be the ith element of the profile $s \in \mathbb{R}^C_{\geq 0}$ that satisfies

$$\begin{aligned} p_i(s_i) &= y(C) - \textstyle\sum_{j \in C-i} s_j \\ p_k(s_k) &= \max \left[p_k(x_k), y(C) - \textstyle\sum_{j \in C-k} s_j \right] \quad \forall k \in C \backslash \{i\} \end{aligned} \tag{4}$$

where the function p_i is defined in Equation 1. Player i's *θ-best share* is her maximum x-share over all coalitions that include her, and her *x-best coalitions* are those coalitions C for which her x-share in C is her x-best share.

Note 3.2. Lemma A.1 shows that there is indeed a unique profile $s \in \mathbb{R}^C_{\geq 0}$ that satisfies (4).

Note 3.3. Player i's x-share in C can be interpreted as her equilibrium threshold in a model of Rubinstein bargaining in coalition C subject to the *other player*'s participation constraints, given by x. Indeed, player i's x-share in C is her equilibrium threshold in the modification of the game Γ in which the output that each coalition $D \neq C$ is reduced to 0, and every $j \in C - i$ is required to reject all offers that give her strictly less than x_j.

Note 3.4. Player i's x-share s_i in C satisfies $p_i(s_i) \leq y(C) - \sum\limits_{j \in C-i} x_j$.

Note 3.5. The profile $s \in \mathbb{R}^C_{\geq 0}$ that solves (4) satisfies $p_i(s_i) - s_i = p_j(s_j) - s_j$ for all j with $s_j > x_j$.

Note 3.6. I say that player j's participation constraint in the computation of player i's x-share in C binds when the profile $s \in \mathbb{R}^C_{\geq 0}$ that satisfies (4) is such that $s_j = x_j$.

Definition 3.2. An RBPC is a profile $x \in \mathbb{R}^N$ such that, $\forall i \in N$, x_i is i's x-best share.

Proposition 3.2. *A profile $x \in \mathbb{R}^N$ is an RBPC if and only if it satisfies system (3).*

Proof. *Necessity*: Let $x \in \mathbb{R}^N$ be an RBPC, and let $i \in N$. By Note 3.4, we have that

$$p_i(x_i) \leq \max_{C \subseteq N : i \in C} \left(y(C) - \sum_{j \in C-i} x_j \right) \text{ for all } i \in N.$$

It only remains to show that there exists C such that $p_i(x_i) \geq y(C) - \sum_{j \in C-i} x_j$. Let C be i's x-best coalition, and suppose for contradiction that $p_i(x_i) < y(C) - \sum_{j \in C-i} x_j$. This implies

that the profile $s \in \mathbb{R}^C_{\geq 0}$ that solves system (4) has $s_i = x_i$, $s_j \geq x_j$ for all $j \in C - i$, and $s_l > x_l$ for some $l \in C - i$. Hence, the same profile s solves system (4) after interchanging the roles of l and i in this system, a contradiction of the assumption that x_l is l's x-best share.

Sufficiency: Suppose that $x \in \mathbb{R}^N$ is such that system (3) holds. Let $i \in N$ and $C \subseteq N$ be such that $p_i(x_i) = y(C) - \sum_{j \in C-i} x_j$. First, note that i's x-share y_i in any coalition $D \neq C$ satisfies $y_i \leq x_i$, since, using Note 3.4,

$$p_i(y_i) \leq y(D) - \sum_{j \in D-i} x_j \leq y(C) - \sum_{j \in C-i} x_j = p_i(x_i)$$

Hence, again using Note 3.4, it is enough to show that i's x-share in C is bounded below by x_i. Let $s \in \mathbb{R}^C_{\geq 0}$ be the profile that solves system (4). Suppose for contradiction that $s_i < x_i$. Then, $p_i(s_i) < y(C) - \sum_{j \in C-i} x_j$, which implies that $s_j > x_j$ for some $j \in C$. Using Note 3.5, the fact that $p_i(z_i) - z_i$ is increasing in z_i, and that system (3) holds, we get

$$p_j(s_j) - s_j = p_i(s_i) - s_i < p_i(x_i) - x_i = y(C) - \sum_{j \in C} x_j \leq p_j(x_j) - x_j$$

which implies that $s_j < x_j$, a contradiction. □

3.3 Computation of the RBPC

Proposition 3.3 shows that algorithm $\mathcal{A}$ defined in Definition 3.3 computes the unique RBPC.

Definition 3.3 (Algorithm $\mathcal{A}$)**.** Let $x^0 = 0 \in \mathbb{R}^N$ and $X^0 = \emptyset$. Proceed inductively as follows: In step $k \geq 1$, let X^k be the union of X^{k-1} and the set of all players in $N - X^{k-1}$ that are members of an x^{k-1}-*perfect coalition*—that is, an x^{k-1}-best coalition of all its members in $N - X^{k-1}$. For each such player i, let x^k_i be her x^{k-1}-best share; for each other player j, let $x^k_j = x^{k-1}_j$. End in the first step K for which $X^K = X^{K-1}$, and let $\chi := x^K$.

Note 3.7. $X^0 \subset X^1 \subset \cdots \subset X^K \subseteq N$, so algorithm $\mathcal{A}$ ends in at most $|N|$ steps. Its running time is bounded above by $|N|$ times m, where m denotes the number of productive coalitions. When every coalition is productive, the running time of the algorithm grows exponentially with the number of players—as does the number of different coalitions.

Proposition 3.3. *The profile χ defined by algorithm $\mathcal{A}$ is the unique RBPC.*

Proof. I prove by induction in k that X^k is a strict superset of X^{k-1} unless $X^{k-1} = N$ (so $X^K = N$ at the step K at which algorithm $\mathcal{A}$ ends) and that every RBPC gives x^k_i to each

player $i \in X^k$ (so χ is the only possible RBPC). The fact that χ is an RBPC then follows from the observation that, for every $k \leq K$ and for each player i in X^k, x_i^k is i's χ-best share.[3]

Let $k = 1, 2 \ldots$ be such that $X^{k-1} \neq N$, and suppose that every RBPC gives x_i^{k-1} to each player $i \in X^{k-1}$ (note that this induction hypothesis is vacuously true when $k = 1$, so there's no need to prove the base step separately). By Lemma 3.4 below, X^k is a strict superset of X^{k-1}.

It only remains to prove that every RBPC gives each of the members of an x^{k-1}-*perfect coalition* her x^{k-1}-best payoff. Let x be an RBPC (and hence, by the induction hypothesis, $x_i = x_i^{k-1}$ for all $i \in X^{k-1}$), let C be an x^{k-1}-perfect coalition, and suppose for contradiction that, for some i in $C - X^{k-1}$, x_i is strictly smaller than i's x^{k-1}-share in C.[4] This implies that i's x-share in C is strictly smaller than i's x^{k-1}-share in C, which in turn implies that, for some j in $C - X^{k-1}$, x_j is strictly bigger than j's x^{k-1}-share in C (that is, j's x^{k-1}-best share) which contradicts Lemma 3.5. □

Lemma 3.4. *For each k for which $X^{k-1} \neq N$, there is at least one x^k-perfect coalition.*

Proof. Let k be such that $X^{k-1} \neq N$. Denoting, for each coalition C, player i's x^{k-1}-share in C by x_i^C, by definition we have that $p_i(x_i^C) - x_i^C$ is the same for every player $i \in C \cap X^{k-1}$; denote by $\mu(C)$ this common value. A coalition C with maximum $\mu(C)$ is an x^{k-1}-perfect coalition, since, by the concavity of u_i, $p_i(x_i)) - x_i$ is increasing in x_i. □

Lemma 3.5. *For each $k = 0, 1, \ldots$, and for all $x \in \mathbb{R}^N_{\geq 0}$ such that $x_i = x_i^{k-1}$ when $i \in X^{k-1}$, each player's x^{k-1}-best share is an upper bound on her x-best share.*

Proof. Let $k = 0, 1, \ldots$, and let $x \in \mathbb{R}^N_{\geq 0}$ be such that $x_i = x_i^{k-1}$ when $i \in X^{k-1}$. Let $C \subseteq N$ and $i \in C$. Player i's x-share in C is equal to her x^{k-1}-share in C if none of the participation constraints in system (4) of players in $C - X^{k-1}$ bind, and smaller than that otherwise. □

[3]To see this, let i be in $X^k - X^{k-1}$, and let C be i's x^{k-1}-best coalition. On the one hand, x_i^k is a lower bound on i's χ-best share because her χ-share in C is x_i^k. On the other hand, x_i^k is an upper bound on i's χ-best share because i's x^l-share in any coalition C is not increasing in l (because the set of relevant constraints in the computation of x^l-shares only expands with l).

[4]The alternative case in which x_i is strictly larger than i's x^{k-1}-share in C immediately contradicts Lemma 3.5.

3.4 Nash Bargaining under Participation Constraints

Definition 3.4. For each $x \in \mathbb{R}^N_{\geq 0}$, each coalition C and each player $i \in C$, let i's *x-Nash share in C* be[5]

$$\text{(5)} \qquad \underset{s \in \mathbb{R}^C_{\geq 0}}{\operatorname{argmax}} \prod_{i \in C} u_i(s_i) \quad \text{s.t.} \quad \begin{array}{ll} \text{Feasibility:} & \sum_{i \in C} s_i \leq y(C), \\ \text{Participation:} & \forall j \in C - i,\ s_j \geq x_j. \end{array}$$

when (4) is well defined, and 0 otherwise. Player i's *x-Nash best share* is her maximum x-Nash share over all coalitions that include her, and her *x-Nash best coalitions* are those coalitions C for which her x-Nash share in C is her x-Nash best share.

Definition 3.5. An NBPC is a profile $\theta \in \mathbb{R}^N_{\geq 0}$ such that, $\forall i \in N$, θ_i is i's θ-Nash best share.

Corollary 3.6. *As the breakdown probability q goes to 0, the RBPC converges to the unique NBPC.*

Proof. The fact that there exists a unique NBPC follows from the argument analogous to that in Proposition 3.3 when we use algorithm $\mathcal{A}$ (Definition 3.3) after replacing—in its definition—the Rubinstein shares (Definition 3.1) with the Nash shares (Definition 3.4).[6]

The fact that the RBPC converges to the NBPC follows from the observation that, for each profile θ, each coalition C and each player $i \in C$, i's θ-Rubinstein share in C converges to i's θ-Nash share in C. This is only a slight generalization of the observation—first made by Binmore (1987)—that i's 0-Rubinstein share in C converges to i's 0-Nash share in C. □

4 Tier Structure and Comparative Statics

4.1 Tier Structure

An important property of the equilibrium threshold profile t of Γ that comes out directly from the characterization of the RBPC (Proposition 3.3) is that both the players and the coalitions that form in equilibrium can be organized into tiers, as follows:

[5]Without the participation constraints, (5) is the definition of the Nash Bargaining solution, where player i's *disagreement* (or *threat*) *point* is $u_i(y(i)) = 0$.

[6]The only part of the argument that is different in this limit is the proof of the statement analogous to Lemma 3.4. The argument in this case is essentially the same as that in Pycia (2012, pages 330-331): Denoting, for each coalition C, player i's x^{k-1}-Nash share by x_i^C, we have that $u_i(x_i^C)/u_i'(x_i^C)$ is the same for every player $i \in C \cap X^{k-1}$; denote by $\chi(C)$ this common value. A coalition C with maximum $\chi(C)$ is an x^{k-1}-perfect coalition, since each player's x^{k-1}-Nash share in C is increasing in $\chi(C)$.

The *tier*-1 *coalitions* are those that are a preferred coalition of all its members. The *tier*-1 *players* are those that are members of a tier-1 coalition. Proceeding inductively, after having defined tiers $1, 2, \ldots, k-1$, a coalition is in *tier-k* if and only if it contains at least one player in tier $k-1$ and is a preferred coalition of all its members who are not in any of the tiers above $(1, 2, \ldots, k-1)$. *Tier-k players* are those that are in a tier-k coalition and are not in any tier-k' coalition, for any $k' < k$.

4.2 Comparative Statics

4.2.1 Marginal Shocks

The tier structure of the equilibrium implies that—in the generic case in which each player has a unique preferred coalition[7]—a player's equilibrium threshold is not affected by marginal shocks that only hit coalitions and players in lower tiers.

A marginal increase in the output of a coalition in tier k can only decrease the equilibrium payoffs of players in tier $k+1$, but has ambiguous effects on players in tiers $k' > k+1$. Moreover, a uniform increase of $\Delta > 0$ units in the output of each coalition can decrease some players equilibrium payoffs, because the sum of the relevant participation constraints in a coalition can increase by more than Δ.

In the limit as the breakdown probability vanishes, Corollary 3.6 implies that the output in each coalition that forms in equilibrium is shared according to the Nash bargaining solution subject to the participation constraint of the players in higher tiers.

In the special case of bilateral matching markets, we can visualize the equilibrium by depicting the *preferred-neighbor* network, which has a link from player i to player j if player i makes offers to player j in equilibrium. Combining the observation that, generically, each player has exactly one preferred neighbor with the fact that each component of the preferred-neighbor network—or *submarket*—has at least two mutually-preferred neighbors, we conclude that, generically, only one pair of players in each submarket are in the top tier. In other words, independent tier structures—one for each submarket—emerge in equilibrium, each with exactly one top tier match. Marginal shocks propagate within submarkets, but not across them, and marginal shocks to the preferences of the top-tier players or the productivity of the top-tier match of a submarket affect all terms of trade in their submarket.

[7] Equivalently, letting x be the RBPC, the generic case in which each player has a unique x-best coalition.

4.2.2 Beyond Marginal Shocks

Definition 4.1. Player i is more risk averse than player j if there exists an increasing and concave function g such that $u_i = g \circ u_j$.

Proposition 4.1. *Player i's equilibrium threshold is nondecreasing in $y(C)$ for all $C \ni i$, and non-increasing in her own risk aversion.*

Proof. Let $C \subseteq N$ and $i \in C$. An increase in $y(C)$ cannot generate more stringent participation constraints at the step of algorithm $\mathcal{A}$ at which player i's RBPC share is defined. It follows that player i's RBPC share can only increase after such increase.

Even though an increase in i's risk aversion can generate less stringent participation constraints at the step of algorithm $\mathcal{A}$ at which player i's RBPC share is defined, player i's RBPC share cannot increase after such a change. To see this, let C be i's x^k-best coalition at the step k in which her post-shock RBPC share is defined. Note that the only relevant less stringent participation constraint can be that of a player $j \in C$ whose pre-shock RBPC share is defined at some step before i's pre-shock RBPC is defined, and whose post-shock RBPC share is defined at step k. But this implies that j's x^k-best share is not smaller than j's pre-shock RBPC share, which together with the fact that $p_i(x_i) - x_i$ is increasing in both x_i and i's risk aversion, implies that player i's RBPC share cannot increase after such shock. □

5 Related Literature

This paper contributes to the literature on bargaining in stationary markets started by Rubinstein and Wolinsky (1985) and that includes Gale (1987), De Fraja and Sakovics (2001), Manea (2011), Nguyen (2015) and Polanski and Vega-Redondo (forthcoming).

The three main differences with respect to Manea (2011) are that he uses a *random-match bargaining protocol* (where in each period, bargaining is restricted to occur among two players that are matched at random), that he assumes that each pair of players can generate the same surplus, and that he focuses on the limit as bargaining frictions vanish. In contrast, I use a *strategic-match bargaining protocol* (where in each period, one randomly selected player can choose whom to bargain with), I allow trade between different players to generate different surpluses, and I study both the case of arbitrary bargaining frictions and the limit as bargaining frictions vanish.

The model that is closest to the one I study here is that of Nguyen (2015), which is a generalization of the model in Manea (2011) to a setup in which coalitions of arbitrary size

can form: The two main differences between the model in Nguyen (2015) and the one I study here is that I don't restrict attention to the case of transferable utility, and that I allow players to strategically choose which coalitions to make offers to.

The structure of the equilibrium of the game Γ is similar to the no-delay equilibrium of a similar non-stationary model characterized by Chatterjee et al. (1993) in the case of transferable utility (see also Ray, 2007). This is natural, since in a stationary market no player can benefit by delaying trade, and players have a degree of bargaining power similar to that conferred by the rejector-proposes protocol of Chatterjee et al. (1993). From this perspective, this article presents a characterization of the no-delay equilibrium of the model analogous to the one in Chatterjee et al. (1993) in a setting in which utility is not necessarily perfectly transferable.

The equilibrium payoff profile of Γ in the limit as bargaining frictions vanish coincides with the *Nash Bargaining solution under Participation Constraints*: The unique payoff profile that gives each player her maximum Nash Bargaining payoff—over all coalitions—subject to the participation constraints of the other players.

The NBPC can be regarded as a cooperative solution concept for the stationary environment considered in this paper. A natural way to build such a solution concept is to assume that output within each coalition is shared according to a fixed sharing rule; see for example Farrell and Scotchmer (1988), Banerjee et al. (2001) and Pycia (2012), who study the conditions under which different such fixed rules lead to stable outcomes. However, this does not allow coalitions to make exceptions in their sharing rules in order to meet the participation constraints of their members. This leads to well-known holdup problems (see e.g. Pycia, 2012): Players with strong bargaining positions cannot commit to adequately reward others, making it difficult for them to find coalitional partners.

To overcome these problems, the sharing rule in each coalition must be allowed to depend on the sharing rule in other teams. In particular, the problem has to be solved for all possible coalitions *simultaneously*, in such a way that the surplus in each coalition is shared respecting the endogenous participation constraints of its members. But this has its own challenges. Most importantly, participation constraints can lose connection with the economic fundamentals. For example, a player can get a certain share of output in coalition C_1 because she gets it in coalition C_2, when the only reason that she gets it in coalition C_2 is because she gets it in coalition C_1. Hence, a satisfactory solution must impose additional requirements that rule out this type of circular reasoning in the determination of participation constraints.

The NBPC imposes a *credibility* requirement on participation constraints: Each player must be able to justify her payoff as being her Nash bargaining payoff in some coalition subject to the participation constraints of the other players. Similar credibility requirements play important roles throughout game theory: For example, *subgame perfection* in the theory of non-cooperative games rules out threats that cannot be justified by equilibrium play, and *farsighted stability* in the theory of cooperative games rules out blocks by coalitions that cannot be justified with stable allocations; see for example Dutta and Ray (1989), Chwe (1994), and Ray and Vohra (2014, 2015).

Other solutions based on Nash Bargaining under core-like constraints—albeit in a framework in which at most one coalition can form and utility is perfectly transferable—include Serrano and Shimomura (1998), Okada (2010), Compte and Jehiel (2010) and Burguet and Caminal (2016). The NBPC is closest in spirit to the SCOOP of Burguet and Caminal (2016): The main difference—on top of the two just mentioned—is that the SCOOP requires that *disagreement* (or *threat*) *points* in each coalition reflect the payoffs that players can obtain in other coalitions, whereas outside options in the NBPC act instead as *participation constraints*; that is, as lower bounds on players' payoffs. (see Binmore et al. (1986) on the importance of this distinction).

6 Conclusion

The classical theory of bargaining, as exemplified by the Nash Bargaining solution (Nash, 1950) and Rubinstein's alternating offers model (Rubinstein, 1982), explores the determinants of how surplus is shared in the absence of market forces. As expressed by Abreu and Gul (2000, page 86):

> In its *purest* form, [bargaining theory] is precisely about explaining a division of *residual* surplus that remains after one has accounted for market forces, outside options, and so on.

But, ultimately, market forces and outside options come from somewhere, so an important challenge for bargaining theory it to illuminate the source of these forces and their interaction with the pure bargaining problem.[8]

In this article, I characterize the equilibrium of a new model of coalitional bargaining in

[8] Chatterjee et al. (1993) is the pioneer in this regard. More recent contributions along these lines include Manea (2011), and Elliott and Nava (2016), among many others.

stationary markets. This characterization provides insight into the source of each player's bargaining power: The set of coalitions that form in equilibrium has a tier structure; output in each coalition is shared as is standard Rubinstein bargaining("pure bargaining") subject to the participation constraints of players that are members of coalitions in higher tiers ("market forces"). This tier structure illuminates each player's source of bargaining power, and how productivity and preference shocks propagate through the market.

The equilibrium payoff profile in the limit as bargaining frictions vanish coincide with the *Nash Bargaining solution under Participation Constraints*: The unique payoff profile that gives each player her maximum Nash bargaining payoff—over all coalitions—subject to the participation constraint of the other players. I leave the investigation of the axiomatic properties of the NBPC for future research.

A Supplement to section 3

Lemma A.1. *For each profile $x \in \mathbb{R}^N_{\geq 0}$, each coalition C and each player $i \in C$, there is a unique profile $s \in \mathbb{R}^C_{\geq 0}$ that satisfies system (4).*

Proof. Suppose for contradiction that that there are two solutions s and s' to Equation 4. Define K to be the set of all indices in which the solutions differ; that is,

$$K := \{i \in N \mid s_i \neq s'_i\}.$$

Pick the index $k \in K$ for which $p_k(s_k) - s_k$ is highest, and suppose without loss of generality that $p_k(s_k) - s_k$ is an upper bound on $\{p_k(s'_k) - s'_k\}_{i \in K}$.

Since, by the concavity of u_i, $p_i(s_i) - s_i$ is increasing in s_i, we also have that $s_k > s'_k$. We have that

$$p_k(s_k) = y(C) - \sum_{j \in C-k} s_j \tag{6}$$

and that[9]

$$p_j(s_j) - s_j \geq p_k(s_k) - s_k \text{ for all } j \in C \tag{7}$$

So, given our choice of $k \in K$, for all $j \in C$ we have that $p_j(s_j) - s_j \geq p_j(s'_j) - s'_j$ or, using again that $p_j(s_j) - s_j$ is increasing in s_j, that $s_j \geq s'_j$. But then, Equation 6 combined with the fact that, by definition,

$$p_k(s'_k) \geq y(C) - \sum_{j \in C-k} s'_j$$

implies that $s'_k \geq s_k$, a contradiction. □

[9]To see this, note that, for all $j \in C - i$,

$$p_j(s_j) \geq y(C) - \sum_{i \in C-j} s_i$$

Adding $-s_j$ to both sides of this inequality and using Equation 6 gives Equation 7.

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